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LEARNERS' ERRORS AND MISCONCEPTIONS IN GEOMETRICAL TRANSFORMATIONS.

BY

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ABSTRACT

Geometrical transformations are a fundamental concept in mathematics, but learners often struggle to understand and apply them accurately. This research seeks to Curriculum change emerged as key focus in restructuring the educational system and strong emphasis was placed on its implementation. Curriculum changes are intended to improve the quality of education for the benefit of learners and teachers. However, the curriculum changes that have happened in Zimbabwe education system do not seem to achieve its intended goals. As a result, it might be argued that some of the changes are contributing to the challenges faced by the school management teams, teachers and learners in schools. This qualitative case study identified concerns and barriers teachers report when implementing a new curriculum. Data were collected from 16 participants (4 teachers and 12 learners) through a questionnaire, interviews, and observations. Participants were interviewed to identify any barriers experienced with the revised science curriculum. Teachers were observed to determine which components of the curriculum were present in or omitted from their lessons. Participants completed a questionnaire to determine their levels of concern when asked to implement a revised curriculum. Results indicated that teachers required additional information before the expected implementations occur and an understanding of demands on their personal time. Common themes showed a desire for professional development, peer-collaboration, and access to curriculum resources, which served as the basis for the project. The resulting project integrated to address concerns connected to reoccurring themes. Implications for social change include change at a systematic level by providing administrators with data to support teachers during curriculum changes and substantiation for the benefits of understanding concerns prior to a change for improving the implementation of revised curriculum.

Dedication

Above all, I give my infinity heartfelt thanks and admiration to my wife: Sharon Mazodze, Uncle Br Jonathan J Chazura, brothers, sisters, my friends, lastly but not least my boys Thamsanqa and Itumeleng who always want the best from me. This work would have not been completed without the love, care, support and prayers from them.

The dedication would not be complete if I did not mention my late mother, Ethel Zulu, who was my first mathematics teacher, were the journey all began. Even when you are not around physically but in spirit you continue to pester me to soldier on.

ABREVIATIONS/ ACCRONYMS

ZIMSEC	Zimbabwe Schools Examination Council
BUSE	Bindura University of Science Education.
MoPSE	Ministry of Primary and Secondary education.

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My heart overflows with deep sense of gratitude to the Almighty God, for in him I live, move and have my being and he is the prime source behind the completion of my study. This dissertation would not have been possible without the support and guidance of several individuals and organisations.

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CHAPTER 1

1.0 Motivation for the study.

Learners' mathematical performance in the transformations geometrical has been a major concern to the researcher as it has been persistently poorly done over the years as stated by According to Van der Sandt (2007). Though, the topic is very interesting and the researcher sees it as being easy to grasp but unfortunately the topic usually poses as being difficult and complex to learners. The researcher wishes to identify common errors and misconceptions relating geometrical transformations with the aim of finding insights into the areas were learners struggle. This will enhance the professional development of the researcher and others teachers alike, as they will become better mathematics teachers.

In geometry, such as transformation geometry, information must be communicated at levels of reasoning of the learner, to avoid misconceptions arising (Luneta 215).

According to Panaoura and Gagatsis (2009), errors were seen in a constructive way through the work of Piaget, as they allowed for the tracing of reasoning mechanism used by the learners. Therefore being able to identify areas were common errors and misconceptions occur, will help in addressing them. This will help in identifying areas were additional instructions and support is needed. Thereby helping educators to use more effective teaching strategies and materials that will help to improve learners' understanding and learning outcomes and also, they (educators) will be able to provide additional instruction and support in challenging areas. The researcher observed that learners may lack acquisition of geometric skills such as the ability to imagine, rotate and twist an object which is essential in facilitating the learning of transformation geometry and other geometrical concepts.

The difficulty in comprehending geometrical transformation can be best explained by looking at work done by van Hiele (1986), which had its roots in Piaget's work on five levels of geometric conceptualization. The levels are visualization, descriptive, abstraction, formal deduction and rigor.

The study will be interested in trying to have a better understanding on areas were learners have difficulties when solving geometrical transformations. By so doing. It will be an opportunity to analyse learners' work, conduct interviews and review existing literature to identify recurring errors and misconceptions. Presenting valuable insights into the specific areas where learners tend to struggle and where misconceptions tend to arise. Taking into consideration that errors and misconceptions are an integral part of the learning process (Hartnett and Gelman, (1998), Leu and Wu (2005), Santagata (2005)). This will present educators with an opportunity to develop effective instructional strategies, such as developing lesson plans that focus on the weaknesses of learners and also develop activities that provide learners with the chance to rectify their understanding. As these activities will aim to take into account of learners' prior knowledge, cognitive abilities and learning styles.

It is hoped that the study on errors and misconceptions for implications for educational policies and curriculum reforms. As they will be a better understanding on how errors and misconceptions are related and how they differ in many aspects (Luneta and Makonye, 2010). This will be done by identifying areas for improvement in mathematics education, the research will provide evidence and recommendations for teacher training programmes. The study will aim to contribute to the improvement of the curriculum in mathematics education.

By identifying and addressing errors and misconceptions related to geometrical transformations will help inform curriculum developers about areas that need more attention and clarification in educational standards and materials. According to Mathaba and Bayanga (2021), if learners do not grasp a concept, it could lead to errors. Thus errors and misconceptions affect the learning of mathematics (Makonye and Stepwell (2016)).

The study will involve 70 learners doing, Form 4 at St Francis High School, a school in Mashonaland West Province. These will be sampled through random sampling. 3 Mathematics teachers and the Head were purposely will also purposely selected. This was done to seek their views with regards to the causes of errors and misconceptions in geometrical transformation. Data was collection was done using questionnaire, interviews, focus group discussions, observations and literature reviews.

It is my hope that the study will contribute to the research community especially on errors and misconceptions in geometrical transformations. Maybe in the end, it will be published in academic journals, presented at conferences or even collaborate with other with other scholars in the field of mathematical education. Hence fostering a collective understanding of the challenges related to geometrical transformations and promoting the implementation of effective interventions across various educational contexts.

1.1 Statement of the problem.

Over the past years (spanning a decade) of teaching mathematics at secondary school level, the researcher has discovered that learners do not understand principles involved in learning transformation of geometric shapes such as translation, reflection, shear, stretch and enlargement. This might be because learners inaccurately apply transformation rules to coordinate geometry.

These errors and misconceptions may be caused by various factors such as the difficulty of the topic (complexity), lack of enough assumed knowledge or inadequate instruction or cognitive difficulties. Most learners tend to confuse different transformations and cannot differentiate between rotations and reflections.

Some learners have difficulties in understanding the concept of invariant properties in geometric transformations. These properties are characteristics of a shape that remains the same after transforming it. Other learners have difficulties in applying scale factors from transformations such as shear, enlargement and stretch. For instance when the centre of enlargement is between the image and object.

When it comes to transforming a shape using rotation, learners are unable to understand the effects of the centre of rotation and angle of rotation. They struggle to understand how the two affect the transformed shape; and that it does not produce a different shape altogether but only the orientation of the shape is changed.

1.2 Research question/ hypotheses

1) What are the common errors and misconceptions learners have when learning about geometrical transformations?
2) What are the possible causes of errors and misconceptions learners have when doing geometrical transformations?
3) What are the possible ways of mitigating learners’ errors and misconceptions?
4) What instructional strategies or teaching methods are more effective in addressing and correcting errors and misconceptions in geometrical transformations?

1.3 Objectives

The objective of conducting the research on errors and misconceptions in geometrical transformations vary depending on the goals of the study. The study will look at the following objectives:

1. Identifying common errors and misconceptions that learners have when learning and applying geometrical transformations. This will help teachers understand the specific areas where learners struggle most.
2. The research aims to investigate the underlying causes of these errors and misconceptions. By being aware why learners make certain mistakes, researchers will be able to develop more effective instructional approaches that target these root causes.
3. The research should be able to provide valuable information on effective instructional strategies for teaching geometrical transformations. By understanding the misconceptions and errors that learners are prone to, teachers can develop interventions and teaching methods that address and correct these misconceptions leading to more effective learning outcomes.
4. Contribute to the broader field of mathematics education by helping develop theoretical framework and models that explain how learners learn and understand geometrical concepts.
5. Research on errors and misconceptions could help inform the design of instructional and teaching strategies. It will help on how to develop lessons, provide appropriate examples and exercises and scaffolding learning in a way that addresses common misconceptions.

1.4 Significance of the study

The findings of this study are significant in a number of ways. The study and recommendations could assist teachers of mathematics at St Francis High School to choose and implement strategies that best suit learner needs. Hence the learners, mathematics teachers, and the school will greatly benefit.

Identification of common errors and misconceptions by educators will help them develop teaching strategies aimed at addressing these challenges and also improve learner performance. As errors are an important part of the learning process. Since geometric transformations are interconnected with other mathematical concepts and problem solving skills. It gives educators an insight of the knowledge gained from other topics such as gradient, matrices or ratios/ scale factor.

The study can also be used as a diagnostic indicator of learners' understanding. The teacher will be presented with a better understanding of the learners' thought processes and identify areas where they need remedial. This will also help to guide how the teacher will give instructions, such as adjusting the pace of teaching, providing additional examples or explanations or re-teaching some concepts.

The study will help learners be self-motivated and self-efficacy, by helping them recognise that errors are a natural part of the learning process. Therefore learners can develop a growth mind-set and appreciate that mistakes are opportunities for improvement. This will make learners believe in their own ability to learn and overcome difficulties.

The improved performance will be as a result of many learners excelling in mathematics and taking up STEM subjects at tertiary level. Since geometrical transformations are fundamental concepts in geometry and are used in various fields such as computer graphics, architecture, and physics. Hence the Nation will see an improvement in its science and technology sectors in the economy.

From the findings it is hoped that it provided useful information to the Curriculum Development Unit, which prepares the mathematics syllabus on good teaching practices

1.5 Scope of the study

The scope of the study will be to:

- Investigate learners' difficulties with visualising shapes and objects in different positions, confusing translations with reflections, misunderstanding congruence, failing to identity corresponding angles and sides, and struggling with identifying the centre of rotation or applying rules for rotation.
- Investigate common errors and misconceptions so as to identify the most prevalent errors and misconception encountered by learners in transformation geometry.
- Examine factors contributing to the persistence of errors and misconceptions in transformation geometry such as learners' prior knowledge, instructional practices and cognitive processes.
- Develop and implement evidence based intervention strategies to address errors and misconceptions in transformation geometry.

- Evaluate the effectiveness of various instructional approaches, including approaches such as hands-on activities and technology enhanced resources in promoting conceptual understanding and problem solving proficiency.
- Investigate the role of curriculum materials, textbooks and technology in shaping learners' understanding of transformation geometry.
- Use the constructivism approach that advocates for learners to actively develop their own understanding of mathematical concepts. Errors and misconceptions usually occur when learners try to grasp new information based on their existing mental models. Educators need to design learning experiences that challenge and refine learners' mental models.

1.6 Limitations of the study

In undertaking this research, the researcher might encounter several problems however, to a lesser extent compromise the reliability and validity of research findings. The learners at the school may not cooperate with the researcher while he questions them about the study. Hence gaining access to a sufficient number of will prove to be difficult. The researcher will therefore, create an environment which is conductively motivating to the learners.

Answers to questions often depend on the knowledge of the respondents therefore, the researcher would not have control over extraneous variable such as teaching methods used by teachers, and cognitive abilities, which can influence learners' errors and misconceptions. The researcher may face difficulties in trying to identify and categorize errors and misconceptions. As learners' mistakes may not always fit neatly into predefined categories. The researcher findings, may not be applicable to all learners or educational contexts due to individual differences and varying teaching approaches.

The researcher's own knowledge and understanding of geometrical transformations and mathematics education may limit the depth and accuracy of the analysis. Even if errors and misconceptions are identified, creating effective interventions to address them can be a significant hurdle. Furthermore, learners' unique experiences, abilities, and learning styles can affect their errors and misconceptions, making it difficult to develop targeted interventions. Additionally, the researcher suffered from lack of resources due to lack of finance and Inadequacy of time. This was one of the huddles which consequently narrow the area of study. The researcher was only restricted to one secondary school, meaning to say the results substantiate the situation in other schools.

1.7 Delimitations of the study

This study was only confined to small sample that contained form 4 learners at a selected school. Data was collected from Mathematics teachers and learners. The school is located in Mashonaland West Province and the study focuses on Mathematics learners' at “O” level. The study will rely on questionnaires, tests and interviews for data collections and the researchers has limited expertise in the field of research. The research will be carried out within a short time frame, this would not allow for longitudinal exploration of learners’ development and change over time.

1.8 Definition of key terms

1. **Geometric transformations:** According to Akarsu, M., & Öçal, M. F. (2022), a geometric transformation is the process by which a two dimensional figure is moved on a plane by mapping the pre-image set of points to a second points called the image. It involves a physical or mental manipulation of a figure to a new position or orientation on a plane. Geometrical transformations can also be defined as functions whose domain and ranges are sets of points and are often used to study geometry”.
2. **Misconceptions**: According to Vosniadou, S. (2020), Misconceptions can be defined as misunderstanding of concepts or ideas often due to a lack of deeper understanding or incorrect prior knowledge. Misconceptions can usually lead to errors. While errors can be best defined as inaccuracies or mistakes, often due to lapses in concentration, misunderstandings or misconceptions.

   According to Luneta and Makonye (2010), “an error is inaccuracies but a misconception is the result of misunderstanding. A misconception happens when a learner believes a false concept is a true concept and errors are caused by misconceptions.”
3. Casanova, Cantoria, & Lapinid (2021), defines misconceptions as “conceptual or reasoning difficulties that hinder students’ mastery of any discipline.”
4. **Learning:** According to John Dewey, “learning is an active process and interactive process where learners form their own knowledge and understanding through experience and social interaction”.

   According to Jerome Bruner’s theory of learning, which emphasises that children build knowledge and meaning through active experience with the world around them.

1.9 Summary

In this chapter the researcher presented on the background to the problem, statement to the problem, significance of the study, research objectives, research questions, assumptions limitations of the study and delimitations of the study. The definition of terms was also presented as well as the summary for chapter one. In the next chapter literature review shall be presented.

CHAPTER TWO

LITERATURE REVIEW

2.0 INTRODUCTION

This chapter will review the literature of the study that looked at the errors and misconceptions in geometrical transformation. The researcher relied on literature as a source of information which were available on the internet. The literature on what errors and misconceptions will be discussed in this chapter and also classified these errors into categories. The application of spatial reasoning and how it affects the learning of transformation geometry is also discussed in this chapter. That is, its role and visualisation in the learning of transformation geometry. The chapter also looks at issues and common mistaken encountered by during the period they will be learning transformation geometry.

Mathematics is a subject that emphasizes more on many concepts which are interrelated for instance transformation Geometry material can include concepts from matrices, gradient and geometrical construction. Learners will have the concepts or material from other topics which they would have studied in earlier. Therefore, learners are expected to be able to connect concepts that have been thought earlier during the “O” level course with new concepts learned in this topic. But many students have difficulty understanding the topic on transformation geometry. According to Jannah, Sujadi and Siswanto, (2022), that there are many learners who are not able to understand even in the simplest parts and there are many mistakes or errors in understanding concepts during learning the topic on transformation geometry. Hence this will lead to many learners making conceptual errors in understanding concepts of transformation geometry.

2.1 Misconceptions and errors

Kennedy (2020), defined a, “misconception as students’ perception which systematically produces mistakes. They are not the mistakes themselves, but the reason of them.” Bingölbali & Özmantar, (2009), noticed that in almost every topic of mathematics, learners make mistakes due to misconceptions they have.

The need for teachers to identify errors and misconceptions learners make and also addressing them, plays a vital role in trying to improve learner performance in mathematics, particularly transformation geometry. According to Panaoura & Gagatsis (2009), the work of Piaget saw errors were seen for the first time in a positive way, whereby they allowed the tracing of the reasoning mechanisms adopted by learners. This means that making errors in addition or subtraction, for example, is not completely bad because they become an important part of the learning process. They need to be dealt with diagnostically. Errors and misconceptions are related, even though they are different in many aspects. Chitiyo (2022) define "An error is a mistake, slip, blunder or inaccuracy." A basic premise in differentiating between an error and a misconception is that errors are easily detected in learners' work such as written text or speech, while misconceptions are often hidden from mere observation without scrutiny. Jory (2021) state that, "If errors and misconceptions were to be put on a continuum, one would have non-systematic errors on one end, and the more serious systematic errors, which are deeply rooted in misconceptions, on the opposite end."

2.2 Classification of Errors

Errors have been classified in different ways by various authors. Most of these classifications fall under two categories, namely, non-systematic and systematic errors.

1. Non-systematic errors are normally referred to as slips and are caused by students' carelessness, misreading information or forgetting some piece of information, unintentionally. Usually, learners will easily correct such errors by themselves. According Wiens, (2007: 5), "errors in the learning of mathematics are not simply the absence of correct answers or the result of unfortunate accidents. They are the consequence of definite processes whose nature must be discovered." Even if these errors were simple slip ups, they have a consequence of demoralising learners, impeding their performance when committed, thus becoming demotivated. Schnepper & McCoy, (2013: 1) states that "a serious inhibitor to learning as mathematics builds on itself."

2. Systematic errors according to VanLehn (1990) indicated that systematic errors mean frequently appling faulty methods, or rules. According to Xiaobao (2006), the cause of systematic errors maybe connected to students' procedural knowledge, conceptual knowledge, or links between these two types of knowledge. Egodawatte, (2011) suggested examples of systematic errors, such as errors are due to deficiencies of mastery prerequisite skills, facts and concepts and errors due to the application of irrelevant rules or strategies. According to Schnepper & McCoy (2013), other examples that were suggested include making logically invalid inferences, using an

improper version of a definitions or theorem and having the right solution to the wrong question, or making a mistake when applying a basic skill.

The International Journal of Research in Education, 1(1), January 2021, 6-15 7 Errors in learning mathematics can be divided into several types. According to Utami (2016), errors are divided into five, namely errors in decoding, comprehension, transformation, process skill, and encoding. Nolting (2011) divides errors into three, namely conceptual errors, misunderstanding problems, and careless errors. Riccomini (2016) divides errors into four, namely conceptual errors, procedural errors, factual errors, and carelessness.

Skemp (1976), classified the types defined operational and conceptual understanding as the classification of mathematical understanding as the types of mathematical information and which are reflection of relational and instrumental understanding produced in mathematics education. According to Van De Walle et al, (2012), Conceptual understanding refers to the perception about ideas and relations as regards a topic. Operational understanding involves knowing the symbols of mathematical terms and the information about the rules and expressions used while making mathematical operations. Based on the various classifications, the researcher is going emphasise more on conceptual errors, for which improvements in the understanding of concepts are one of the focuses in learning mathematics or transformation geometry.

Errors are a very important and integral part of learning mathematics. Brodie, (2014) states that, "Errors arise because of interactions between the mathematics features, the learning process, and social practices." Furthermore, Sulistyorini (2017) states that errors are an inseparable part of learning mathematics. In line with this, Ingram, Baldry, & Pitt (2013) also states that errors are important in learning mathematics, both for students and teachers. From the perspective of students, making errors is the basis for constructing a concept. From the teacher's perspective, errors can be used as a basis for knowing how students understand mathematical concepts.

Another way that can be used to repair errors is to conduct error analysis in learning transformation geometry. According to Rushton (2018), Error analysis is an effective step in overcoming misunderstandings and allowing students to reflect on their learning. Fitriani, Turmudi, & Prabawanto, (2018) state that error analysis also helps students in reflecting on their problem-solving skills. Error analysis is not only beneficial for students but also has a positive impact on teachers. Sapire, et al (2016) state that, "error analysis is an inseparable part of the knowledge possessed by teachers". It means that any remedial action the teacher takes related to students' mistakes and depends on how deep knowledge and concepts of teachers that related to the errors. According Fitriani, Turmudi, & Prabawanto, (2018) error analysis can be a reference for teachers in choosing suitable strategies, models or media learning material that are used to reduce students' errors.

Shalem, Sapire, & Sorto (2014) explain that six criteria need to be taken into consideration by teachers in conducting error analysis are:

1) The procedural understanding of the correct answer that emphasizes the quality of the teacher's procedural explanation.
2) The conceptual understanding of the correct answer that emphasizes the quality of the teacher's conceptual explanation.
3) Awareness of errors that emphasizes the teacher's explanation of actual mathematical errors not on the reasoning of students.
4) Diagnosis of students' reasoning related to students' errors.
5) The use of relationships in everyday life in the explanation of errors.
6) Errors that emphasize the offering of various alternative explanations in correcting students' errors.

According to Brodie (2014) states, in error analysis, three important points are emphasized, which are: identifying, interpreting and engaging with students' errors. International Journal of Research in Education, 1(1), January 2021, 6-15 8 Efi (2016) stated, "that mind mapping can used for connect the student's idea in mind with new concept they got." According to Dahar (1998) concept maps can be used for specific purposes, learners actively think of the relationship with their concepts with new concepts that are received through the construction of concept maps. The concept map can encourage students' curiosity in learning materials.

Moreso, mathematical concepts are almost all related. In other words, concept maps are very useful for detecting learners' conceptual errors. The acquisition of new knowledge is a function of the cognitive structure that must link their concepts with new concepts that already learned. Previous studies such as Sulistyorini, (2017a), "emphasized the analysis of errors in solving Euclid's Geometry problems by considering the assimilation and accommodation of the concepts being learned." While this research also looks at the analysis of errors, namely conceptual errors, by involving the construction of concept maps.

2.3 Spatial reasoning

Yusnia (2018) states that in curriculum of mathematics, problem solving is an integral part. As an effort to support achieving problem solving skills, the facilitator should put problem solving as one of the mathematics learning standard process. According to NCTM (2000), by applying skills on how to solve mathematics problems, learners will understand how to think, to develop their high curiosity, and to have a confidence in facing the unfamiliar situation.

Teachers need to design problems which are routine problems and non-routine problems. According to Pehkonen, et al (2013), a routine problem is a problem which students can find or recognize the solving strategy easily because they often find it in textbook or other sources. Özreçberoğlu & Çağanağa, (2018) state that a non-routine problem is a problem that needs a particular technique and deep thinking to find the solving strategy because it is unfamiliar for students. According to Kolovou, et al, (2009) and Yazgan (2016) states that non routine problem is a problem that there is no explicit solving approach such that it is difficult to predict. Giving non routine problem is important because problems faced by students in daily life have complex and complicated structure. Daguplo (2013) posits that non-routine problem solving also develops mathematical reasoning

Sholl, et al. (2000) says that, "spatial orientation is the ability associated to navigation or orientation in unfamiliar environment." Considering Ramful, et al. (2016), in spatial orientation task, learners are asked to determine the position of object by self-positioning mentally or physically. Ontario (2014) states that a learner with a high spatial reasoning will perform the better in mathematics. In mathematics, transformation geometry is considered as a very difficult topic. According to Retnawati, et al. (2017) explains that learners had some misconceptions and less knowledge towards the concepts about geometrical problems, difficulties to understand the information given in the test item, and in applying the geometrical concepts. Hence requires the use of spatial reasoning or orientation.

However there is limited available research on learners' understanding and learning of transformation geometry. In one such study, according to Edwards (1997), who states that transformation geometry provides enough opportunity for learners to develop their spatial visualisation skills and geometrical reasoning ability.

From Hanfi, (2017) and Aini, d. k. (2020), state that in learning transformation geometry, visualization assistance is needed which can use mathematical applications. This aspect requires the ability to reason. The ability to reason by visualising information is linked to the cognitive abilities of the learners. The way a learner receives information is called a cognitive style. The information that learners receive in the form of visual symbols and verbal symbols can differ from one learner to another depending on their cognitive style. These are the differences in receiving information visually and verbally are visualizer and verbaliser cognitive styles.

Learners with a cognitive verbalizer style tends to be easier to receive, process, store, and use information in the form of text or writing. Of course this will affect the strategies used in solving mathematical problems, including those related to geometric problems. Subjects with a cognitive verbalizer style tend to be easier to receive and convey information in words. As for subjects with visualizer abilities, it is easier to receive and convey information with pictures. (McEwan, 2007). Visualizer individuals like things in the form of pictures, diagrams, tables and graphs, because they are easier to understand than explanations using words for the same subject. Meanwhile, verbaliser individuals prefer to write, according to their habits which make it easier to understand information in the form of spoken or written (related to text).

The important thing related to the process of entering information into the sensory register is attention. In the learning process in the classroom, what the teacher explains is easy for certain students to understand, but it may be difficult for other students to understand. So the teacher needs to know the difficulties faced by students in solving problems so that this is related to their cognitive style. Based on the explanation above, various authors are interested in trying to explore the difficulties experienced by prospective mathematics teacher students in transformation geometry courses in terms of visualizer verbalizer cognitive style.

2.4 Issues students experience with transformations concepts

2.4.1 Translation.

Chultz & Austin (1983) and Shultz (1978) found that the direction of the movement of the translation had a definite impact on the difficulty of the problem; they found that translations to the right, then to the left were easier than diagonal translations, either up and to the right or up and to the left. Learners often forget that translation involves moving an object and does not change its orientation or shape. They also found that as the distance between the initial and final figure increased in the translation, the students experienced increasing difficulty in performing the translation tasks. Some learners may incorrectly identify the image of a point as they forget to consider the direction and magnitude of the translation leading to errors in locating the new position of the point. According to a study conducted by Gulfem and Melihan,

(2017) that focused on analysing the learners' common mistakes in Transformation Geometry. The research was carried out and the results showed that pupils understood that translation transformation was a movement of replacement, but they had difficulty in the topics such as the direction of the transformation, and the position of the figure within the transformation.

Flanagan (2001) indicated that learners have difficulties in recognising that the movement of the figure in a translation is the magnitude of movement and is related to the length of the vector shaft represented on the coordinate graph. Hollebrands (2003) asserts that learners should recognise that a figure and its image are parallel and that the distances between the preimage and image points are equal and the same length as the translating vector. Flanagan (2001) and Wesslen and Fernandez (2005) found that learners did not realise that translating a figure moves every point on the figure the same distance and in a parallel orientation. The findings above illustrate that it is important to look at the direction of the translation of the figure since certain directional movements are easier for students to perform than others, especially the movement of a figure in a translation that is in a diagonal direction to the horizontal.

Flanagan (2001), postulates that, learners forget the commutative property of translations, meaning that the order in a translation is performed does not affect the result. Hence translations can be done in any order without changing the final position of the object. Most learners fail to connect the topic on vector with translation. Whereby translation involves vector addition or subtraction, which often leads to difficulties in inverse translations. Since every translation has an inverse that undoes its effect.

2.4.2 Reflections.

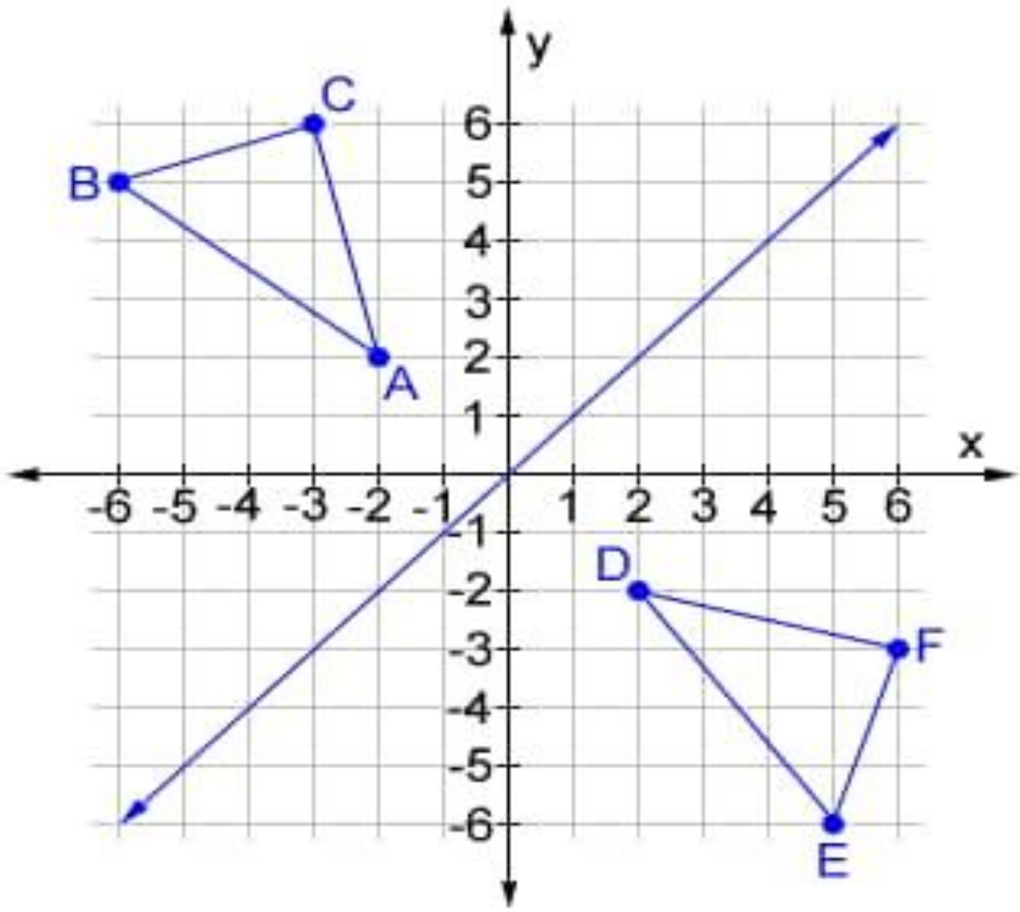

Figure 2. 1

According to a research by Rollick (2007, 2009), found that leaners had various problems with reflections. The specific reflection that the participants found the easiest was the movement of a figure from the left to right position over the y-axis or a vertical line. The participants had problems performing the right to left reflection and had a tendency to interpret the movement as being top to bottom instead. Further, Gulfem & Melihan, (2017) found that the learners had also problems in identifying and writing the equation of the axis of symmetry of shapes under reflection, as well as finding and using the angle and centre of rotation

Many of the participants identified a reflection as a translation when symmetric shapes were used. Hence confusing the two transformations. This shows that learners would not have understood the distinction between these transformations. Additionally, sometimes they misunderstood reflections and confused them with rotating the figure. Rollick (2009) explains that, “developing the concept of invariant relationships between the figure and its image is needed to help dismiss these misconceptions.” Yanik and Flores (2009) and Edwards and

Zazkis (1993) both indicated that learners interpreted the line of the mirror as cutting the figure in half, or alternatively interpreting the edge of the figure as the predetermined line of reflection. Kuchemann (1980, 1981) found that students had the most difficulties with reflection over a diagonal line, the students were found to often ignore the angle or slope of the reflection line and perform a horizontal or vertical reflection instead. This often results in images that are flipped in the wrong direction.

According to Burger and Shaugnessy (1986), Perham, Perham, and Perham, (1976), and Schultz (1978), the most difficult type of reflection for students is reflecting a figure over a line of reflection that intersects the preimage, this type of transformation reflects the image to overlap itself. Edwards & Zazkis, (1993); and Yanik & Flores, (2009), the learners usually forget to maintain distances, since it is very important to maintain the distances between the points and the line of reflection. Hence most learners make the mistake of preserving the distance hence resulting in distorted reflected images. According to Soon, (1989), learners fail to recognise symmetrical properties and forget that reflection is an isometric transformation.

Through a research carried by, Rollick (2007), and (2009) found learners had various problems with reflections. Most of the (learners) participants found reflection transformations of a figure from the left to right position over the y-axis or a vertical line or from below to above over the horizontal axis or the x-axis were found to be easiest. The learners had difficulties performing the right to left reflection and had a tendency to interpret the movement as being top to bottom instead. Many of the learners identified a reflection as a translation when symmetric shapes were used. They tended to forget that reflection involves crossing/ flipping an object across a line. Additionally, sometimes they misunderstood reflections and confused them with rotating the figure. As they think, that reflection involves the rotation of an object.

Rollick (2009) explains there is a need to develop the concept of invariant relationships between the figure and its image as this poses a big challenge to the learners. Zarina, Skilters et al (2024) and Edwards and Roy (2022) both indicated that learners interpreted the mirror line as cutting the figure in half and they usually forget that the distance of each point from the line of reflection (mirror line) remains the same. Most learners that struggle with reflection overlook some or most of the key properties of congruence, that is, the reflected shape is congruent to the original shape. Understanding language and vocabulary such as “reflection,” “line of symmetry” and “mirror image” can be challenging. Many learners struggle to identify the centre of rotation, fail to maintain certain properties such as distance, symmetry and angles of a particular shape. This also an indication that they may be lacking knowledge on concepts such as symmetry. The study by Gulfem and Melihan, (2017) also observed that the pupils had misconceptions concerning reflection and discovered that learner confused similarity with congruent property of shapes.

2.4.3 Rotation.

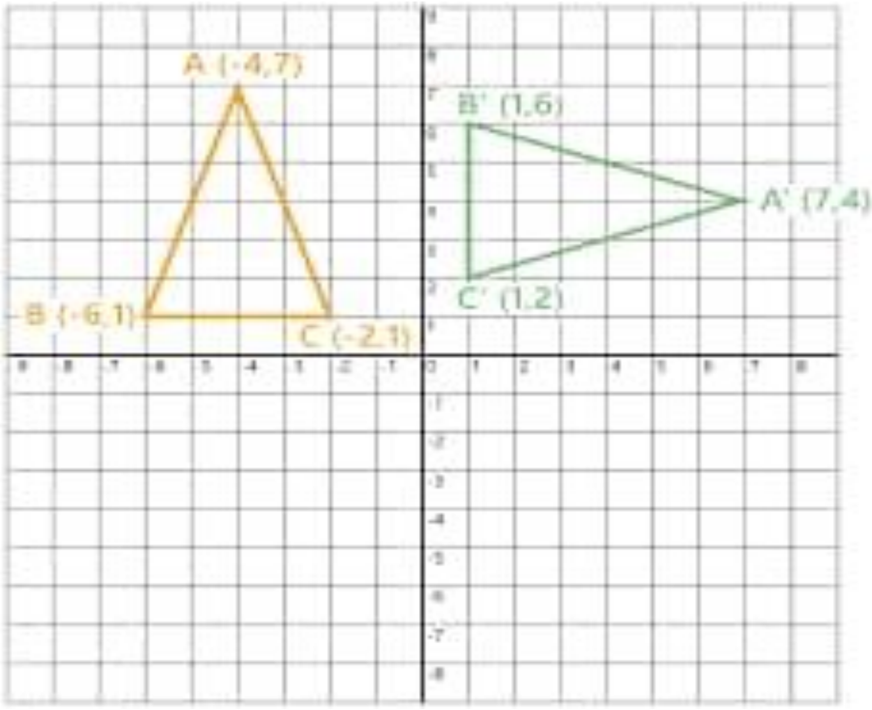

Figure 2. 2

Of all of the isometric transformations, Moyer (1975, 1978) and Shah (1969) noticed that most learners at secondary school had the most difficulty on working with rotations. Kidder (1976) found, in his learners of average mathematical ability, that learners were often unable to comprehend the concept of the angle and the drawing of radii or lines necessary for a rotation to occur. Learners are unable to hold some factors constant (same) while varying other factors to perform a rotation. Kidder (1976) also found that learners had difficulty holding the distance from the centre of rotation to the vertices of the figure constant while performing a rotation. Failure to follow the correct procedure when rotating results in distorted images. Since learners mistakenly change distances of the radii from the centre of rotation. This showed that the concept about the centre of rotation and how it affected each point is rather complicated and confusing for the learners.

Edwards (2003) identified a particular misconception about rotations. She found that instead of seeing rotation as mapping all the points of the plane around a centre point, the students in her study expected the shape to slide to the given centre point and then turn around it- showing that they had a torrid time seeing rotation as taking place ‘at a distance’ from the object. Another study Sproule, (2005) was carried out with Grade 7 learners, and sought to identify the strategies that were best able to support learners in correctly completing reflection tasks. Sproule (2005) found that although measuring distances in the diagrams was the most common strategy used by the learners, those participants who folded along the axis of symmetry were the most successful.

According to Olson, Zenigami and Okazaki (2008) learners were unaware that angle measures of the figure remain unchanged under the turn. This also indicates that learners do not have enough knowledge concerning rotation symmetry, as object that have rotational symmetry can be rotated at a certain angle and still look the same. They also found that students had a weak understanding that the radii from the centre of rotation of different lengths rotated the same number of degrees the same angle measure resulted. Clements, & Battista, (1990); Krainer, (1991) found that learners demonstrated common misconceptions about the angle of rotation which determines the amount of turn and object experiences.

Additionally, Clements, Battista and Sarama (1998) found that learners had difficulty in transforming a shape through a certain angle of degrees to the angle of rotation, but they had little difficulty when using the measures of 90 and 180 degrees. As these two angles are equivalent to a quarter turn or a half turn which is non – intuitive. Edwards and Zazkis (1993), Yanik and Flores (2009), and Wesslen and Fernandez (2005) all agree that learners’ image of rotation was usually at the centre of the object being rotated, and most of them had little challenges with this type of rotation. Wesslen and Fernandez (2005) discovered that learners were not confident with rotating shapes where the centre of rotation was defined (such as using given coordinates) instead of using the centre of the shape or a vertex of the figure. Soon (1989) and Soon and Flake (1989) noticed that learners encountered the most challenge in the rotation of a shape with the centre of rotation given as a point external to the figure. Even when using the shapes’ vertices as a centre of rotation, learners still faced difficulties with the direction of rotation, that is, clockwise or anti – clockwise direction. This show that learners have very little knowledge about the clock analogy. Whereby the hour hand and minute hand turn in the same direction, which is the clockwise direction, about the centre of the clock. Hence failing to accurately identify the centre of rotation, resulting in incorrectly drawn shapes.

According Soon and Flake, (1989) learners were usually ignorant about a prescribed centre of rotation and instead rotated a given shape about the centre of the shape or a vertex of the figure; and they also disregarded the direction of rotation indicated in the transformation. Soon (1989) and Wesslen and Fernandez (2005) found that students did not exhibit enough knowledge about the angle of rotation or centre of rotation or both. Ünal and Ürün, (2021) and Baiduri, Ismail and Sulfiyah (2020) discovered that average learners have many misconceptions and have difficulty learning the concepts of angle and rotation; these concepts are key to the understanding of rotation.

Ünal and Ürün, (2021) suggest that the static definition of angle (An angle is the part of the plane between two rays meeting at a vertex) may be part of the cause for the misconception. Ünal and Ürün, (2021) found that students did not give notice to the directionality of right or left of a turn in performing a rotation. The studies presented above describe numerous problems that students experience with performing rotational tasks. Among the problems that appear most frequently are the concept of angle measure, measure of angle of rotation, and centre of rotation as they appear to be overwhelming for learners. Hence they is need for learners to learn and apply these terms correctly. Additionally, learners cannot make a mental note or visualise the resulting image after rotating and guess its final position.

2.4.4 Enlargement

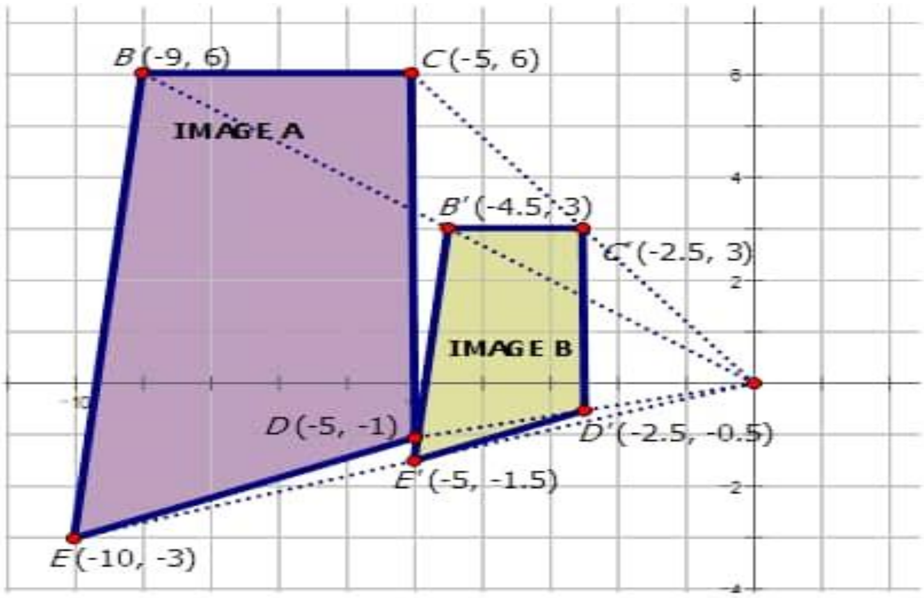

Figure 2. 3

In Enlargement Transformation Geometry, these tasks require pupils to identify a transformation, find or use the centre and scale factor of enlargement to perform an enlargement, as well as describing an enlargement using correct terminologies and reasonable arguments in arriving at their answers (Arican, and Özçakir, 2021). Enlargement transformation maps an object onto its image through a particular centre known as centre of enlargement which could be on the point on the object, or outside the object in the plane. It could also be represented by matrix or coordinates of the image of the original shape. The image of the enlargement is proportional to the original shape (object).

According to Liu (2020), enlargement transformation was a challenging concept for learners as evidenced by their work (assignments). Some of the learners showed confusion about the property of the scale factor, showing that the learners struggle with how the scale factor affects the shape. They assumed that a positive scale factor meant an increase in the shape and a negative scale factor resulted in the reduction in size, hence misunderstanding the scale factor. Learners mistakenly assume that the scale factor determines the size of the image, rather than the ratio between the lengths of corresponding sides.

According to Liu, (2020), found that learners inconsistently applied the scale factor across the shape, hence producing an uneven enlarged shape. It is important to apply the scale factor to corresponding sides to produce proportional enlargement.

When plotting coordinates, some learners are not careful on how to accurately plot the corresponding points of the original shape and the image.

2.5 Theoretical considerations.

Constructivism is one of the research philosophies that tries to explain learners' errors and misconceptions in geometrical transformations (Mosia, Matabane and Moloi 2023). As it posits that learners build their own knowledge and understanding through being active participants (with their environment and through social interactions). In the case of geometrical transformations, constructivists state that learners' errors and misconceptions are brought about from their individual constructions of meaning, which may not align with the conventional mathematical understanding. One of the principles put forward by constructivists, is that learners' understanding is dependent on the specific context and representation used (for example visual, algebraic or spatial).

Accordiong to Mahlaba, S. C. (2021), constructivism states that knowledge is constructed and not discovered, meaning that learners formulate their understanding of geometrical transformations through active engagement, rather than simply discovering pre-existing truths. It also suggests that the learners' understanding of geometrical transformations is shaped by their individual experiences, intuitions, and cognitive biases, leading to diverse and sometimes incorrect interpretations. The constructivist theorists view errors and misconceptions as an important part of the learning process, since learner will refine and reorganise their understanding through experience and social interactions. These errors and misconceptions are seen as natural stepping stones towards deeper understanding rather than as failures or mistakes.

2.6 Conclusion

The chapter examined at literature that was writer by other scholars/ researchers that focused on errors and misconceptions faced by learners when dealing with transformation geometry. These errors and misconceptions usually have a negative impact on the overall performance of a learner. Within the chapter, several types of errors were identified. The chapter also looked at how spatial reasoning affects the way learners perform in transformation geometry. It lastly, looked at the mistakes made by learners when tackling the different transformations. It also looked constructivist theory as a way to see how learners' errors and misconception are beneficial to their learning process.

CHAPTER 3:

RESEARCH METHODOLOGY

3.0 introduction

This chapter presents and explains the research methodology that will be used in this study. This research aims to investigate the nature and causes of errors and misconceptions in transformation geometry, with a goal of identifying the key concepts that learners deem to be challenging and the specific misconceptions that they have. The chapter will be by giving an overview of the researcher design. The study will be guided by a mixed method approach, combining both quantitative and qualitative data collection methods. This chapter will encompass the research design, instruments to collect data, population, sample and sampling techniques, procedures for collecting data, data analysis as well as ethical considerations, will be discussed in this chapter.

3.1 Research design.

Research design is also known as a research strategy, which a plan to answer a set of questions (Hunziker and Blankenagel, (2021). It provides a framework that includes the methods and procedures to collect, analyse and interpret data. It will outline an approach that will be used to approach the problem. McCombes (2019), referred to a research design as a strategy, is a plan to answer a set of questions. It is a framework that encompasses the methods and procedures to collect, analyse and interpret data. In short, the research design describes how the researcher will investigate the main problem of the research.

Umar, Purwanto, and Al Firdaus (2023), defines research design as a plan for a study, providing the overall framework for collecting data. Taherdoost (2021) defines it as a plan for selecting subjects, research sites, and data collection procedures to answer the research question(s). They further indicate that the goal of a sound research design is to provide results that are judged to be credible. As it involves giving the overall approach and methods that will be used to collect and analyse data.

3.2 Research paradigm/philosophy

The study was guided by a mixed method approach and a constructivist philosophical paradigm. "…the constructivist paradigm entails that knowledge is constructed through social interactions, emphasising subjective experiences and meanings" (Pilarska, 2021). Learners will need to build their understanding by being hands on rather being passive participants as errors are not mistakes but are an inherent part of learning. This perspective encourages a more empathetic approach and is supportive approach to teaching and learning.

3.3 Research methodology

The research design for this research/ study is a mixed- methods approach that combines qualitative and quantitative research methods. According to Hall (2020), this approach allows for a comprehensive understanding of errors and misconceptions in geometrical transformations by exploring participants' experiences and perspectives while also providing statistical data to support the findings. It also appropriate in answering research question that are neither quantitative nor qualitative are able to answer alone. It gives participates a chance to have a strong voice and share their experiences across the research process and can facilitate different area of exploration which will enrich the evidence and enable questions to be answered in depth.

The researcher is going to use the descriptive research design as it is empirical. The data collected will seek to establish the cause and effect relationship among variables using different statistical and computational methods (Ghanad 2023). The researcher will also make use of the qualitative research method in data collection. As the research seeks to understand and explore the meanings, experience and perspective of the learners, (Akyildiz, and Ahmed, 2021)), as they tackle geometric transformations.

3.4 Data Collection Instruments

Data collection methods and tools are the techniques, instruments and tools used in research to collect various information, measurements of observations for the purpose of research (Taherdoost, 2021)). It forms a very important part in the research process. The data which are processed quantitatively or qualitatively, then arranged systematically. Data collection is defined as research activities that aim to collect field data needed to answer research questions in qualitative research or to test hypotheses in quantitative research.

The researcher will need to design clear and consistent data collection tools which will capture the data in an accurate and ethical way. This will be done by, use of simple and precise language that is appropriate for your target audience, avoiding jargon, ambiguity, and leading questions. A research instrument can include interviews, tests, surveys, observations, or focus groups.

3.4.1 Interviews

One of the mostly common used tool for data collection is the interview. It is best defined as a purposeful interaction in which one person obtains information from one or more individuals (Seidman, 2006). Interviews are best to be used when describing the process by which a phenomenon occurs. It is basically the oral, in-person administration of a questionnaire to each member of the sample. Whereby two or more people exchange information through a series of questions and answers. There are various types of interviews such as structured interviews, unstructured interviews, semi – structured interviews, personal interviews and group interviews/ focus group.

The researcher will prefer to use structured interviews as they will ensure that the interviewer asks a predetermined set of questions, by using the same wording and following the same order of questions (Roberts, 2020). This will offer the interviewer the opportunity to explain the research personally. The main ain of an interview is to solicit for information form the respondents and to understand phenomena within its social context, the social context could be an organisation or a certain unit within an institution such as the school (Islam, and Aldaihani, 2022).

This offered the possibility of explaining the intent of the research personally. Moreover, it was possible to add explanatory comments and ask additional questions for increased clarification. Interviews are helpful especially were a lengthy response/ explanation is required (Langley, & Meziani, 2020). The question will be developed in such a way to information from individuals who have expertise in geometry education such as mathematics teachers. Interviews will present the researcher with an opportunity to gather information that cannot be collected through observations, mainly teachers' emotions.

In this study, the main purpose is to understand and get insights into the challenges that are experienced by the learners, their misconceptions in geometrical transformations. The researcher choose interviews as it allows the researcher to have control over the interview, by ensuring that the interviewee remains focused and on track to complete the research. It is easy to carry out obtain reliable results quickly (Islam, & Aldaihani, 2022). In personal face to face interviews, the researcher will also ask follow-up questions that will probe further, so as to clarify misunderstandings immediately. However, there are certain disadvantages of interviews such as provide less anonymity, which is a big concern for many respondents and the respondent’s answers can be affected by his reaction to the interviewer’s class, age or physical appearance. However, the researcher is uses multiple instruments to check the reliability.

The researcher will use semi-structured interviews to collect data from selected participants. Semi-structured interviews will allow for a specialized form of communication between people for the specific purpose associated with (errors and misconceptions in geometrical transformations) some agreed subject matter (Mannan, & Afni, 2020). Thus, the interview was a highly purposeful task which went beyond mere conversation. Face-to-face interviews were conducted with subject teachers. Semi-structured and unstructured interview questions were used in both interviews. (Naz, Gulab, & Aslam, 2022), stated that the strength of the semi-structured interview is that it can clarify questions and allow for probing of answers from the participants, providing more complete information that would be available in written form.

3.4.2 Questionnaires

A questionnaire is a list of carefully planned questions or items used to gather useful data from respondents (Monday, 2020). Jangam (2021), defines a questionnaire as a set of written questions on a document given to respondents in a research. In this study a questionnaire will be used as a form of inquiry tool containing systematic and organised questions that will be sent to a sampled group of respondents. The questionnaire were very crucial in evaluating participants’ understanding of fundamental concepts, identifying specific errors and misconceptions and exploring individual difference, related to geometrical transformations.

The researcher will personally design and deliver the questionnaires to sampled learners and teachers who are part of the respondents of this study. In this research study, questionnaires will provide anonymity, which is most likely to encourage participants to be more honest and open in their responses especially when investigating sensitive topics such as error and misconceptions (Yoshizawa, Singelée, et al 2023). This indicates that questionnaire are confidential tools in research. Questionnaires are cosy effective, scalable, allow for rapid collection of data and efficient in collecting research information. Stolk, (2024), argue that questionnaire are of benefit to a researcher as they bear advantage of standardisation and reduce biasness as the researcher’s presence does not influence responses.

It is also more economical and can generate a large amount of data over relatively low cost and time. A self-administered questionnaire requires no special skills and use of pre-defined answers makes it easy for respondents and for analysis. They also exert less pressure on the respondents.

Despite, advantages mentioned above, questionnaires poses some shortfalls in researches. Questionnaires are fixed in their nature, meaning that respondents can only provide information that fits within the predefined answers. Hence they cannot probe respondents if they provide an inadequate answer and if they are structured properly they become ambiguous in nature (Einola, & Alvesson, 2021). Questionnaires has disadvantage that it cannot allow one to express his/her subjective or imaginary feeling towards the problem understudy. However, in support of the advantages mentioned above questionnaires are going to be used as disadvantages will be complemented by advantages of interviews in probing errors and misconceptions in geometrical transformations.

3.4.3 Pre-tests

According to Mbusi (2019), Pre-tests are an important instrument that can be used to investigate learners' existing understanding of geometrical transformation with the aim of trying to identify/ or unveil areas of strength and weakness. They also provide a baseline to measure what learners know and also to determine the effectiveness of instructional interventions after administering the post test. Pre – tests can provide valuable insights into learners 'errors and misconceptions in geometrical transformations, informing instructional design. Post tests will be given at the end so as to measure learners' retention, understanding and application of geometrical transformation concepts (Setiana, and Purwoko, 2021).

3.5 Population

Mweshi and Sakyi (2020), define population as the entire group or set of individuals, objects that possess specific characteristics are of interest to the researcher or wishes to make inferences to. It represents the larger population from which individuals are drawn. Casteel and Bridier (2021), define population as a defined group of individuals or cases that are the focus of a study. The population is very important in any research as it determines how sampling is going to be done and determines how the manoeuvres the whole research methodology (Nayak, & Singh, 2021). In this study a population would all form 4 learners that are learning about geometrical transformations or a general group of individuals that have varying levels of mathematical knowledge and experience with geometric transformations. The number of learner drawn from twelve (12) form 4 classes will be one hundred and fifty (150) and thirty -

four (34) mathematics teachers. The study will be carried out with any discrimination on gender, age and ethnicity of learners and teachers. In this study subject teachers and learners were the target population because they were thought to be reliable informants with accurate information which could answer best questions of the researcher.

3.5.1 Sample and sampling techniques to be used in the study

A sampling procedure is the process by which the researcher, will use to choose a sub-group from a population that will form the sample. The sampling procedure will follow a set of rules that will specify how to select the sample (Takwi, 2022). Mweshi and Sakyi (2020) define a sample as a subset of the population which is meticulously selected to represent its characteristics. This refers to the actual group that the research will collect data from. This group is smaller and more manageable so as to draw inferences that can be used to generalise to the larger population. The study population involved a subset of forty (40) form four learners, and four (4) mathematics teachers.

The way that the researcher will use to select sample of individual to be sample participants is very important. So that sample is representative of the target population, as much as possible, with the least possible error and without incompleteness (Juddoo, & George, 2020). Hence allowing to inferences and generalisations to be made from the data. There are several sampling technique available, the research is going to use the simple random method. It has an advantage that each member of the sample will have an equal chance of being picked. The learners will each be assigned a unique number from one (1) to one hundred and fifty (150). Then the lottery system will be used, whereby the researcher randomly picks numbers, with each number corresponding to a subject or item, in order to create a sample (Allibang, 2020).

For the teachers, the researcher will use the computer-generated random tags (four names). The teachers, just like the learners will be assigned a tag. The random tag appearing should correspond with a particular teacher, s/he will be included into the sample. A sample was important in this research study as reduce the cost and time consumed during the research study as supported by, (Eichelberger, 2023).

3.6 Procedures of collecting data

In this study a sample of forty-four individuals will is going to be used. That will be forty and four teachers. The researcher is going to design data collection instruments to collect primary data, whereby pupils and teachers are going to be interviewed and they will respond to questionnaires. Learners will write pre- and post-tests.

The researcher is going to obtain an introductory letter from Bindura University of Science Education (BUSE) and also seek permission and approval from the relevant offices of authority as especially from the Ministry of Primary and Secondary Education (MoPSE) and also from the school head (Taherdoost, 2021).

The research will design informed consent forms. This will make sure that all learners will provide informed consent to participate in the research. This form will clearly explain the purpose of the research and also will state/ preserve their right to withdraw at any time. The research will ensure that the data collected is handled with uttermost care to protect the privacy confidentiality of the participants (Tiersma, Reichman, and Popok, et al (2022). The researcher will try by all means to try and minimise harm such avoiding very difficult and frustrating tasks that could destroy a learner’s confidence

3.7 Data presentation and analysis procedures

Data analysis in this research will involve the process of inspecting, analysis, cleansing, transforming, and data modelling with the aim of deriving meaningful insights, so as to draw inferences from it (Chen, Van Hillegersberg, and Topan, et al, M. 2021). Data analysis for this study will involve a combination of quantitative and qualitative approaches.

Data presentation will involve presenting the data in a clear and concise manner so as to communicate the research findings (Nayak, & Singh, 2021). The research findings are going to be represented using various or a variety of data visualisation techniques such as frequency tables or suing a diagrammatic display (graphs, and charts)

3.8 Summary

This chapter outlined the research design, research paradigm and methodology. It also discussed data collection instruments, population, sample, data presentation and analysis procedures, and ethical measures. The next chapter will present and discuss the findings of the research project.

Chapter 4

DATA PRESENTATION, ANALYSIS AND INTERPRETATION

4.0 Introduction

The previous chapter presented the description of the methodology used in this study. This chapter presents the findings of the study on errors and misconceptions in geometrical transformations in the selected high school. The findings are presented according to the aim and the research questions which guided the study. The information derived from the questionnaires and interview guides.

4.1 Learner information.

4.1.1 What is your understanding of geometrical transformations?

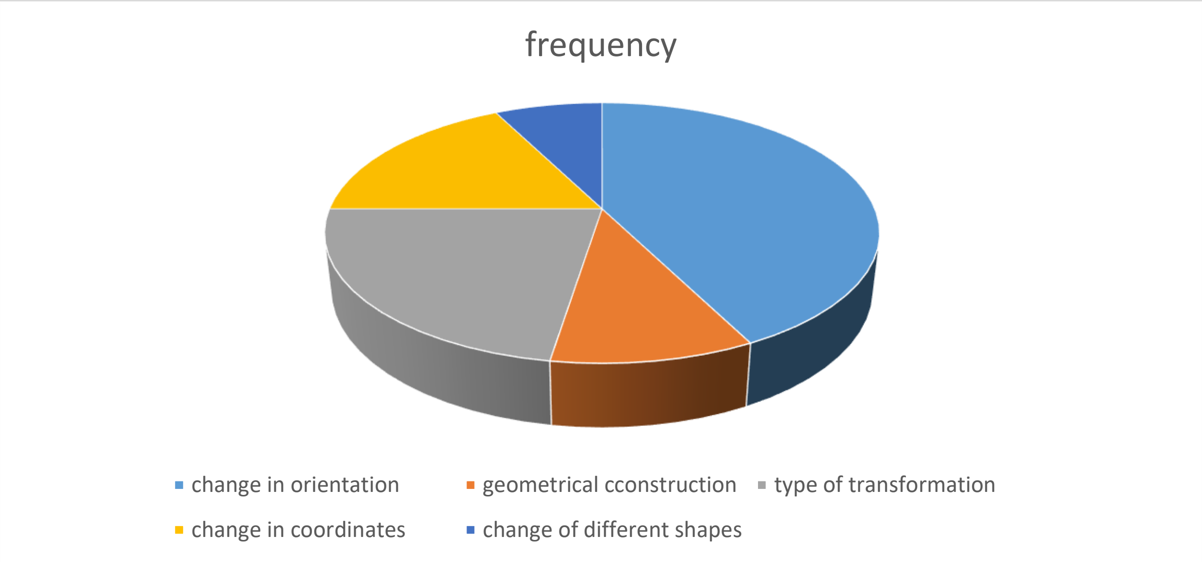

Figure 4. 1

42.5% of the learners were able to demonstrate their understanding of geometrical transformations by being able to define what a transformation is. They were able to identify the various types of transformations such as isometric and non-isometric transformations, on top of the definition. 22.5% of the learners gave examples of the different types of transformations, 10% as the use of geometrical construction instruments to answer a given question and 17.5 % of the learners, defined geometrical transformation as the change of coordinates and 7.5% considered geometrical transformation to the change and study of different shapes. The question was opened ended, but it was meant to test if learners can define what a transformation is. The percentage of learners below, illustrates that learners have lack of knowledge or understanding because it is an indicator that there is a gap in leaners knowledge of fundamental geometric concepts (Khormi, 2023). This may be attributed to lack of exposure.

4.1.2 What geometrical transformation are you familiar with?

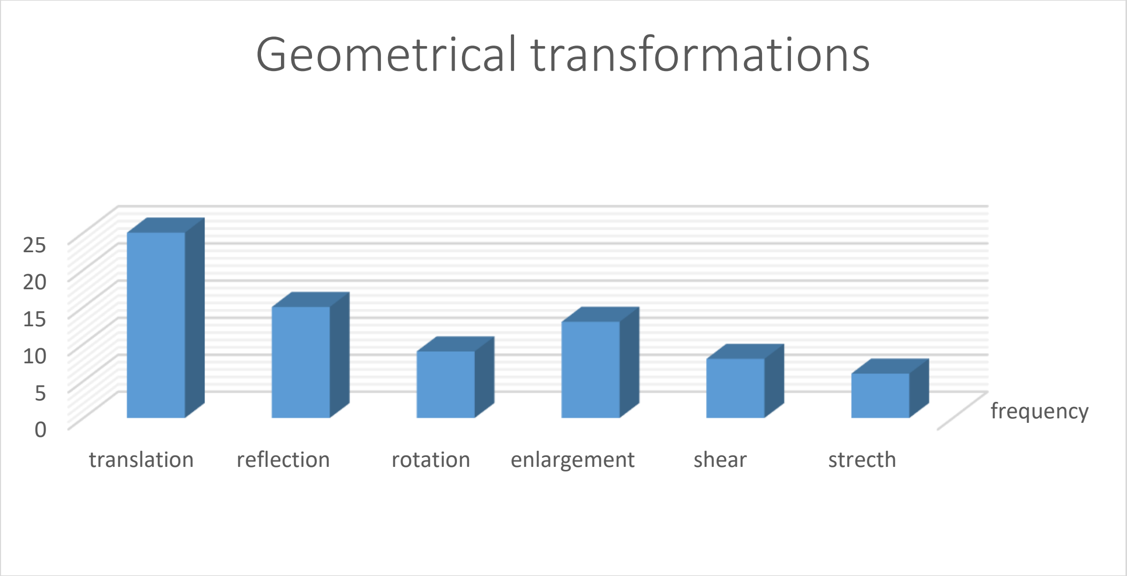

Figure 4. 2

Translation is the most popular geometric transformation, as learners are able to slide an object without flipping it or rotating (Uygun, 2020). From the information above, it can be seen that isometric transformations are more popular amongst the learners. It may be due to the fact that these transformations preserve the properties of an object such as the shape and size, which makes them easier to visualize and conceptualise. In addition, the popularity of isometric transformation, may be aided by the emphasis placed on them by the curriculum and textbooks (Nadarajan, Abdullah, et al 2022). As they are introduced and practiced extensively. Non-Isometric such as shear and stretch seem to be more challenging for learners to visualize and understand, as they involve changes in the size, shape or orientation of an object. Hence learners struggle to image the appearance after undergoing non-isometric transformation, leading to more errors and misconceptions. As learners may assume incorrectly that all transformations preserve these properties.

4.1.3 Have you encountered difficulties in understanding or applying geometrical transformation?

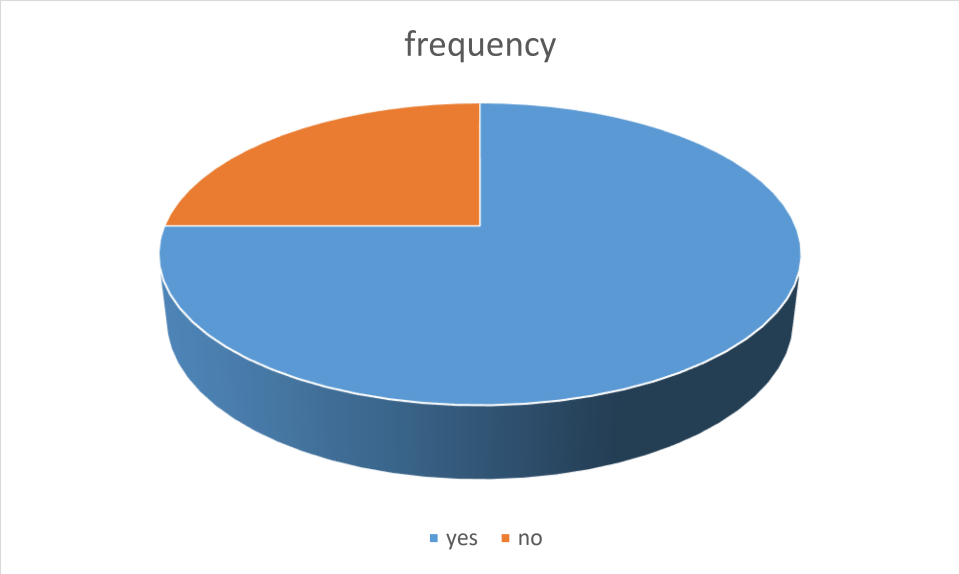

Figure 4. 3

The majority of learners acknowledged to having challenges in understanding or applying geometrical transformations. Acknowledgement of difficulties is a normal learning process and shows that learners are aware of the areas they need improvement. This shows that they are struggling with certain concepts or transformations and are willing to learn and improve (Dahal, Pant, Shrestha, et al, 2022). Rather than being stubborn and uncooperative during the learning process. This will also help teachers know the specific areas were learners are struggle. It will also help learners focus and practice more time in challenging areas. The researcher sees the acknowledgement of difficulties with geometrical transformations as a sign of intellectual maturity and a crucial part of the learning process.

4.1.4 Which geometrical transformations do you find most difficult?

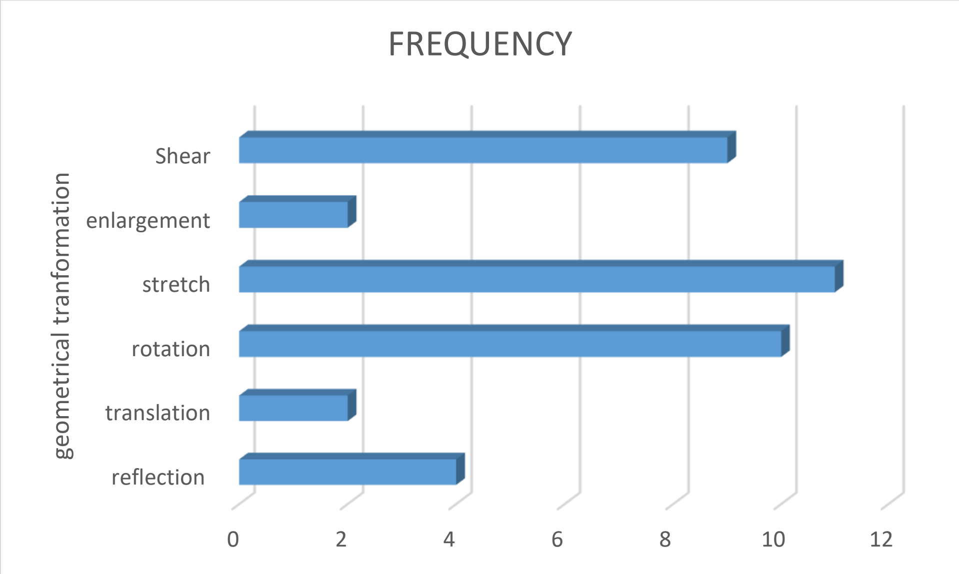

Figure 4. 4

From the above diagram, it can be seen that the geometrical transformation that is regarded to be, the second most difficult is shear. This may result from the fact that shear requires more careful mental model of how it works. Shear transformations, requires shifting one part of an object while keeping the other fixed, resulting in a distorted shape (Distante, Distante, Distante and Distante, 2020). The distortion results in changes in the angles and lengths within a shape. It also involves the use of matrix multiplication. Stretch transformation are viewed as being challenging because they involve matrix multiplication, the altering of angles within a shape. The learners had less challenges when it came to solving transformation that dealt with translations, reflections and enlargement (especially when the centre was/ is the origin)

4.1.5 Do you find it challenging to identify corresponding angles and sides or the centre of rotation in geometric transformations?

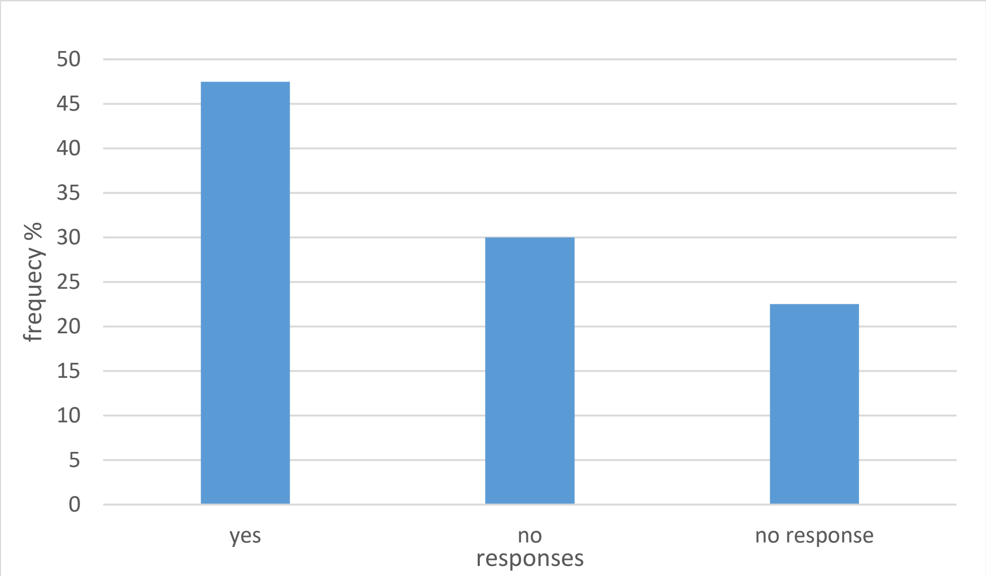

Figure 4. 5

From the responses provided by the learners, it can be seen that 50% of the learner acknowledge that they had challenges in identifying corresponding angles and sides or centre of rotation in geometric transformations. This may be due to the fact that these learners have challenges in matching angle concepts across different contexts (Ng , Shi and Ting, 2020). They may also have trouble visualizing and imagining angles or the centre of rotation with no visible lines or rays. 27.5% of the learners stated that they did not have any with concepts involving corresponding angle or identifying the centre of rotation. 22.5% of the respondent did not answer the given question, suggesting that they may not have solidly grasped the related concepts hence they felt unable to provide meaningful responses. Some of the learners could have felt intimidated by the technical nature question as they might have perceived the question to be beyond their current capabilities (Benning, Linsell, and Ingram 2023).

4.1.6 Do you struggle with the concept of symmetry and identifying symmetrical figures or line of symmetry?

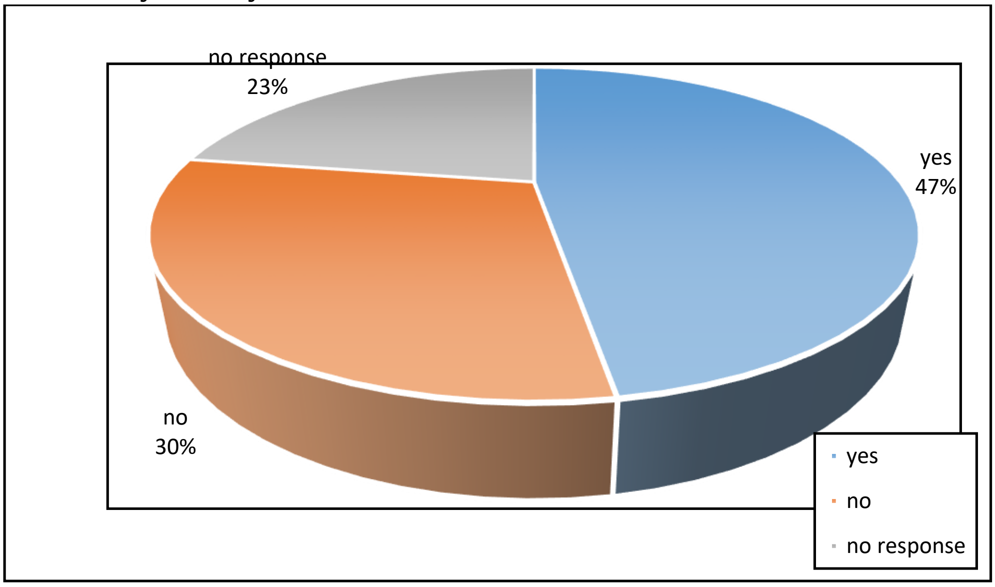

Figure 4. 6

23% of the respondents, did not give a response to the question. Such learners exhibit a gap in foundational knowledge of basic geometric concepts such as lines and shapes. According to Moleko and Mosimege (2020), the learners may have limited vocabulary to communicate their understanding of the concepts effectively. Thus making it difficult to comprehend the more advanced idea of symmetry. 30% of the respondents, stated that they did not struggle with the concepts of symmetry and identifying symmetrical figures or lines of symmetry. 47% of the respondents stated that they have challenges with the concept on symmetry. This might suggest that learners haven't had enough practice working with different types of symmetrical shapes, hence they may not recognize the patterns and properties that define symmetry (Dhasmana, 2021).

4.2.0 Errors and misconceptions

4.2.1 When learning about geometrical transformations, what errors or misconceptions have you experienced or observed?

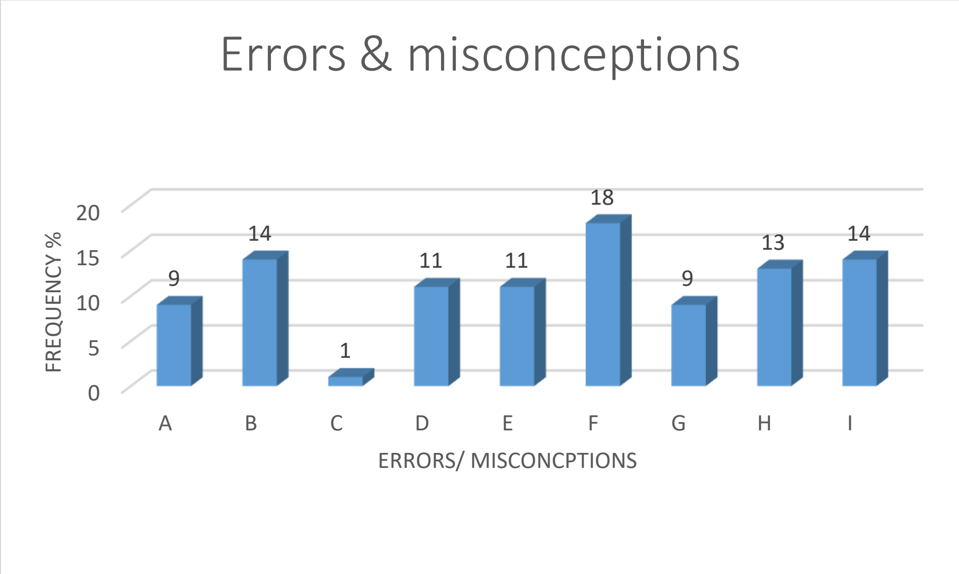

Figure 4. 7

Key:

A – Difficulty understanding rotation. B – Confusing clockwise and anti – clockwise rotations.

C – Forgetting to maintain shape size and orientation. D – Difficulty understanding reflection.

E – Confusing horizontal and vertical reflections. F – Forgetting to flip shape over axis

G - Difficulty understanding translation. H – Confusing left and right translations.

I – forgetting to move shape correct distance

From the responses collected from the learners, it shows that most of them produce the most errors and misconceptions, when dealing with reflection transformation. Forgetting to flip a shape over the correct axis, has percentage of 18%. It was followed the error or misconception of confusing clockwise and anti-clockwise rotations and forgetting to move a shape's correct distance that had 14%. Confusing left and right translations had 13%. Difficulty understanding reflection and confusing horizontal and vertical reflections had 11%. Difficulty in understanding rotation or translation had 9% and the least was forgetting to maintain shape size and orientations. The above information illustrates that learners perform some reflective learning as they take time to review their work. This also shows that learners are actively monitoring their own understanding and checking for flaws in their understanding (Rivers, 2021).

4.2.3 Do you believe that your understanding of basic geometric concepts such as angles, lines and shapes affects your comprehension of geometric transformations?

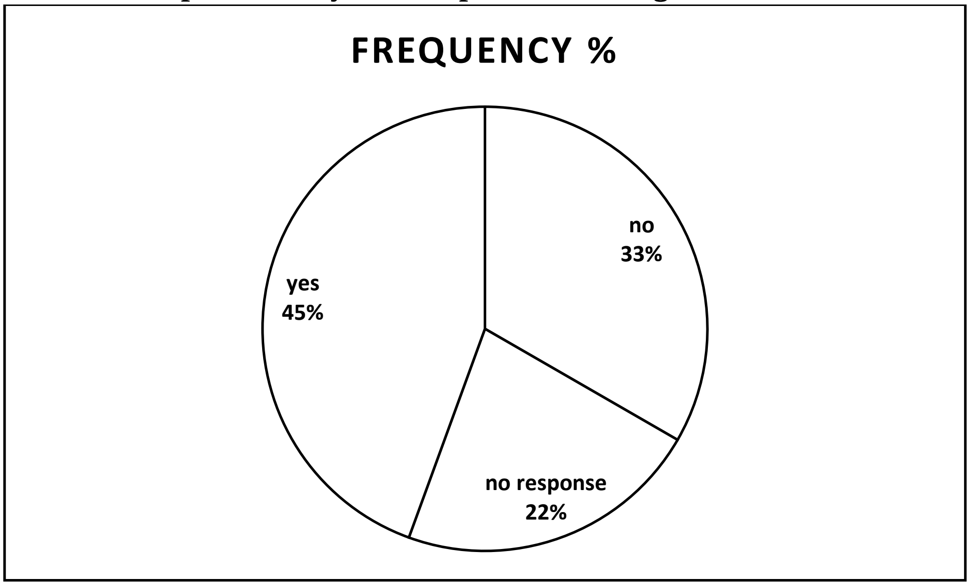

Figure 4. 8

33% of the learners did not believe that their understanding of basic geometry affects their comprehension of geometric transformations and 22% of the learners did not respond to the question. This may mean that learners may not fully understand the connection between geometric concepts and transformation as they may feel they cannot comprehend how angles, lines and shapes are related. 45% of the learners believed that their understanding is affected by basic geometric concepts. According to Prahmana, and D'Ambrosio (2020), learners think these concepts form the foundation to understanding geometric transformations, as all transformations operate on the basic elements of geometry. It will be difficult for them to visualize and mentally manipulate the object undergoing the various geometric transformations.

4.2.4 Have you ever relied solely on memorization rather than understanding when performing geometric transformations.

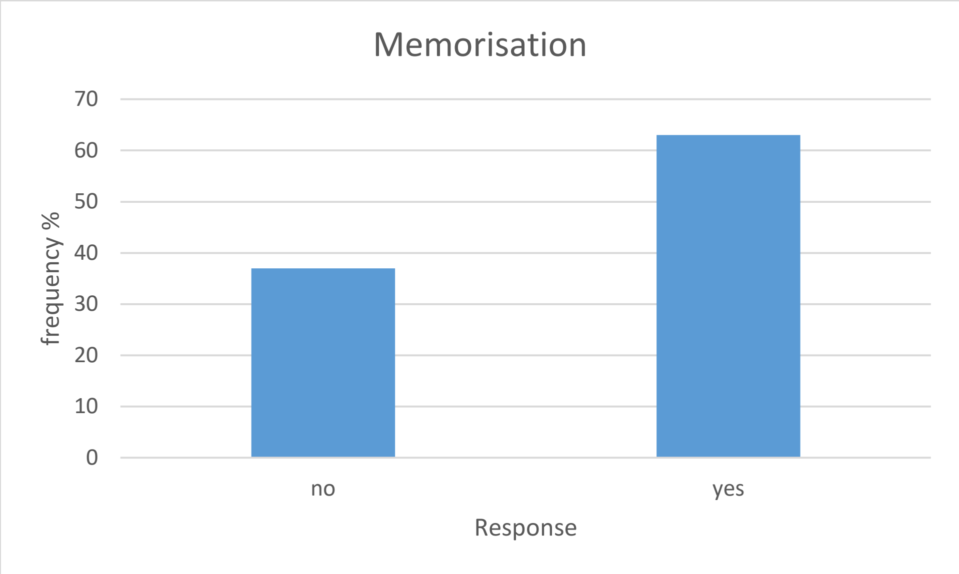

Figure 4. 9

37% of the learners said that they did not rely solely on memorization when performing geometric transformation but they tried by all means to understand the concepts involved. 63% of the learners stated that they dependent on memorization rather than understanding when performing geometric transformations. Hence the quickly forgot key concepts. Without a deep understanding of underlying geometric principles, learners easily make mistakes. Due to memorization learners usually confuse similar transformations and can lead to applying the wrong transformation in an unfamiliar situation or they might forget key details and also leading to difficulties in applying procedures (Panjaitan, 2023).

4.2.5 Do you find it challenging to visualize the changes in objects undergoing geometrical transformations?

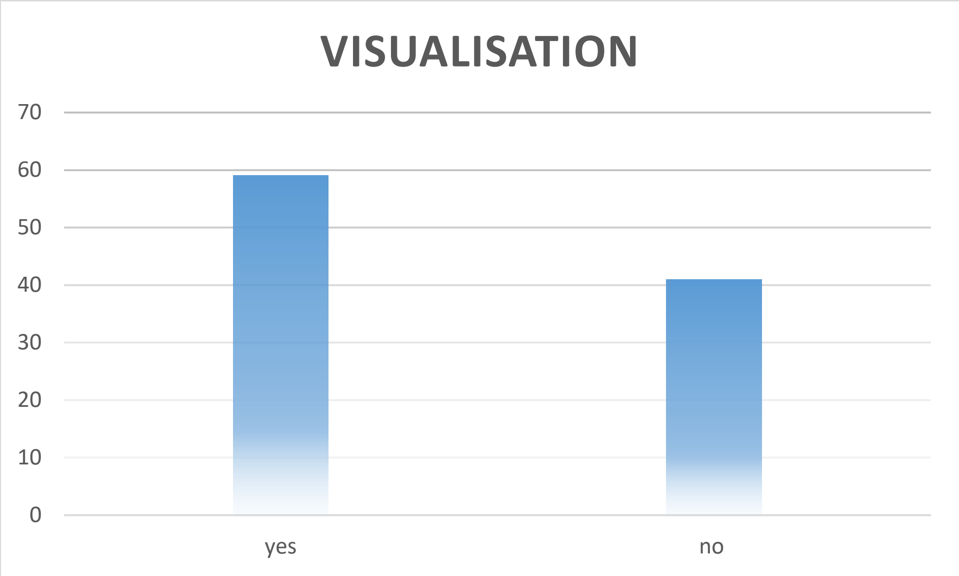

Figure 4. 10

41% 8of the learners stated that they did not have any difficulties in visualizing the changes in objects undergoing geometrical transformations. 59% of the learners acknowledged that they struggled to visualize the position of an object after it had undergone geometric transformations. From the information gathered, learners find it difficult to understand and predict the effects of various transformations on shapes. Learners end up not seeing the shape changes under a given transformation, making it impossible for them to understand the relationship between the original and resulting shape. According to Dorel (2023), visualization is very important when it comes to identifying and describing the relationships between shapes, such as congruence and similarity.

4.2.6 Do you agree that understanding geometrical transformations is crucial in developing spatial ability and problem solving skills?

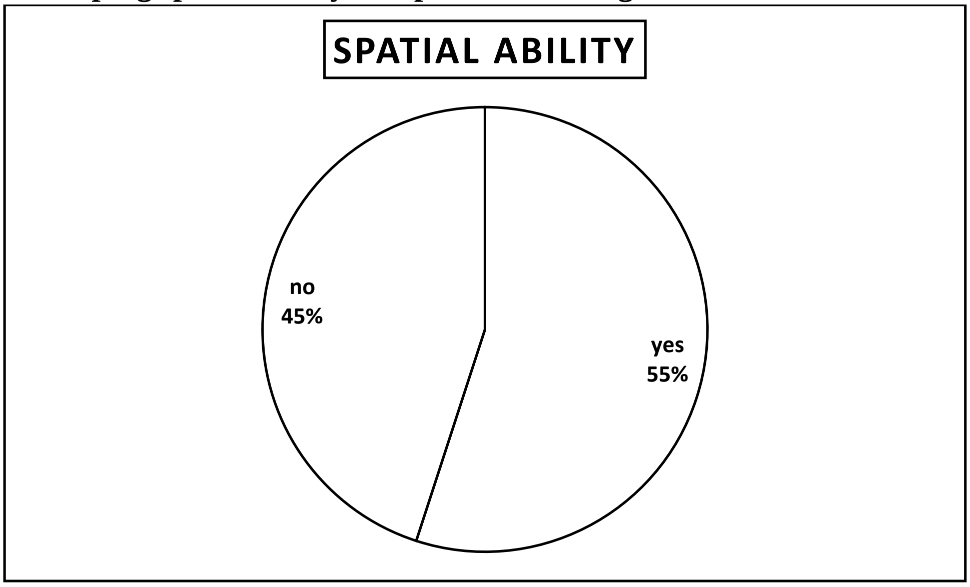

Figure 4. 11

55 % of the learners agreed to the fact that geometrical transformations is crucial in developing spatial ability and problem solving skills. 45% of the learners disagreed that understanding geometrical transformations is important in the development of spatial and problem solving skills. According Anggraini, Setyaningrum, W and Retnawati (2020, understanding geometric transformations is important as it allows learners to visualize and manipulate shapes in their minds thereby enhancing their spatial reasoning. Being able to solve geometric transformations such as translations, rotations and reflections, helps learners visualize how objects and shapes can be manipulated in space. This allows individuals to manipulate and understand spatial patterns, shapes and relationships, which is vital for solving complex problems in various domains.

4.3.0 Instructional preferences

4.3.1 How do you think technology such as dynamic geometry software, can support the learning of geometrical transformations?

None of the learners were able to respond this question. It was noted that they are not familiar or have not been to the use of dynamic geometry software, this has made it difficult for them to understand that technology can help the learning of geometric transformations. This lack of exposure to dynamic software or other technological tools has hindered their ability to recognize the potential benefits of technology in enhancing their learning (Hennessy, D'Angelo, McIntyre, et al 2022). Learners are not aware of the specific features and capabilities of dynamic geometric figures that can greatly enhance the learning of geometric transformations. Some of the learners could have not seen the connection between technology and learning hence they cannot put a link between dynamic software and improved understanding of geometric transformations, hence they have no benefits they can write home about (Viberg, Grönlund and Andersson 2023).

4.3.2 What teaching strategies do you find the most effective in understanding geometric transformations?

This questions was not answered by most learners as it seems, they have limited exposure to diverse teaching methods. Hence they have challenges in coming up with alternatives that could be more effective for the understanding geometric transformations. They are only exposed to the traditional teaching methods which often rely on formal definitions and examples which can be difficult for learners to understand and apply (Cronje, 2020).

Do you have any suggestions for improving the teaching and learning of geometrical transformations?

Learners found this question rather difficult to answer, as they have limited exposure to different teaching and learning approaches for geometric transformations. Another limiting issue, is that, learners are unfamiliar with pedagogical principles (Yang and Kuo, 2020).

4.4Pre-test

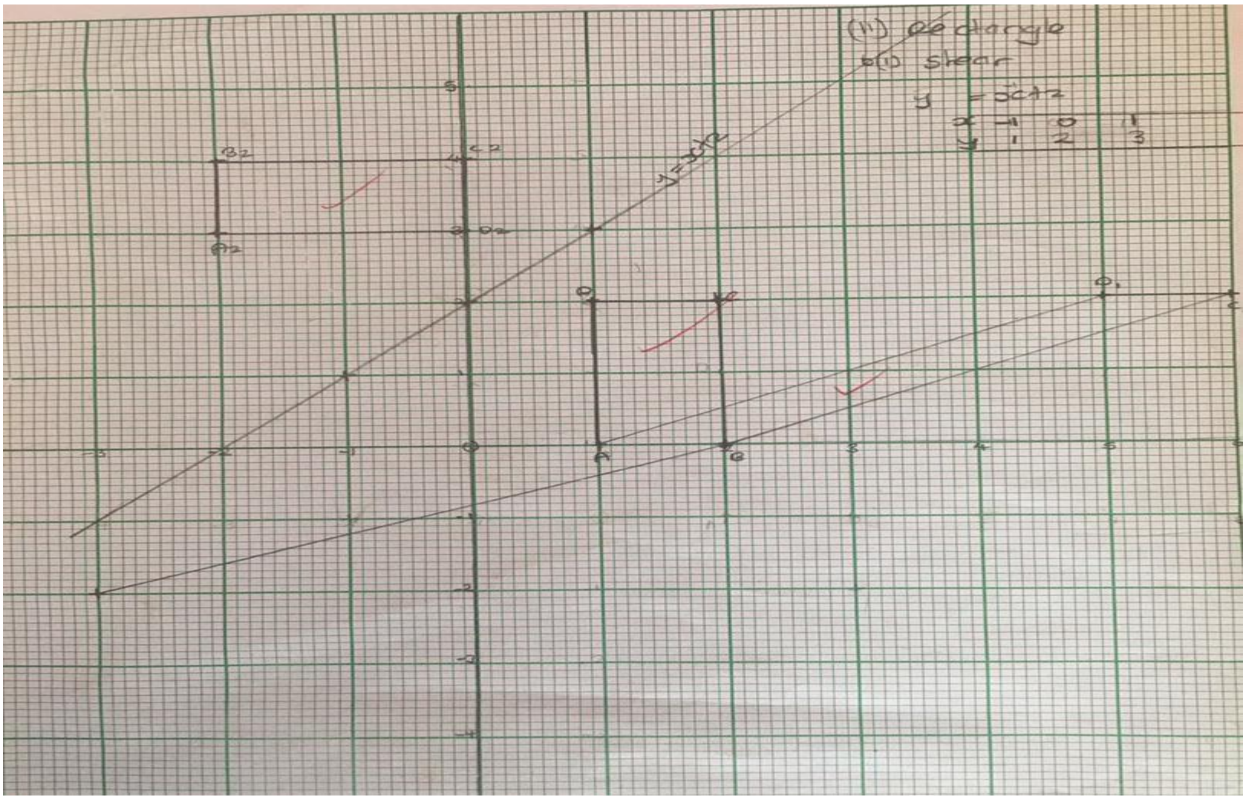

Figure 4. 12

In the above diagram, it can be seen that the learner failed to carry out an enlargement geometrical transformation with centre (-1;-1) and scale factor -1. It shows that the learner has not developed proportional reasoning skills, and it may indicate that the learner has difficulties with spatial visualization and mental imagery hence s/he found it difficult to mentally picture the transformed shape. The learner confused shear transformation with enlargement.

Reflection

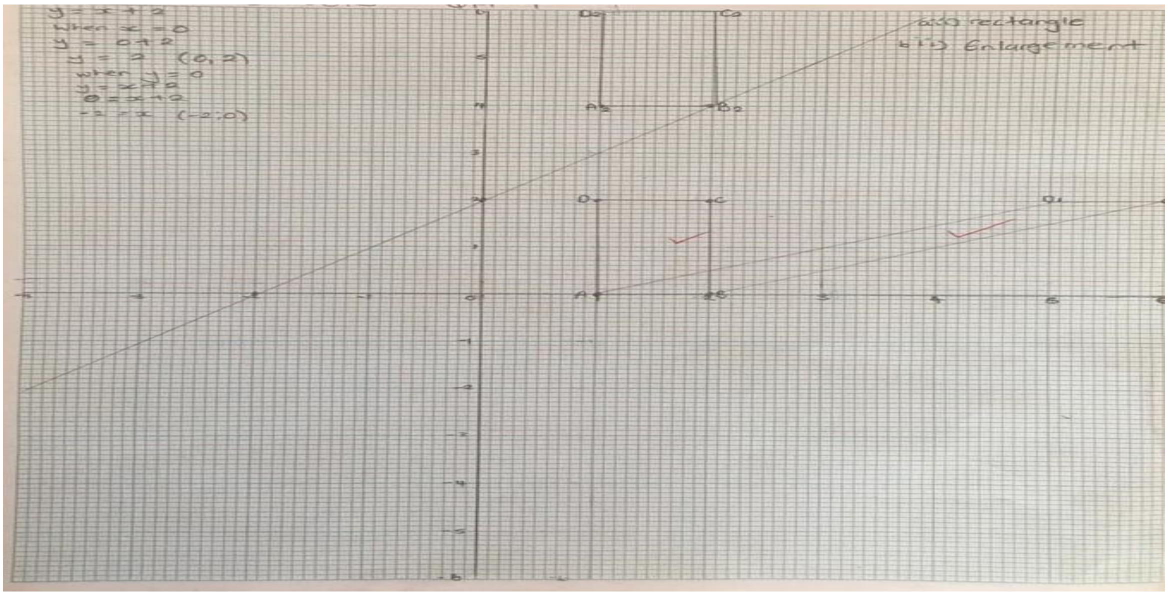

Figure 4. 13

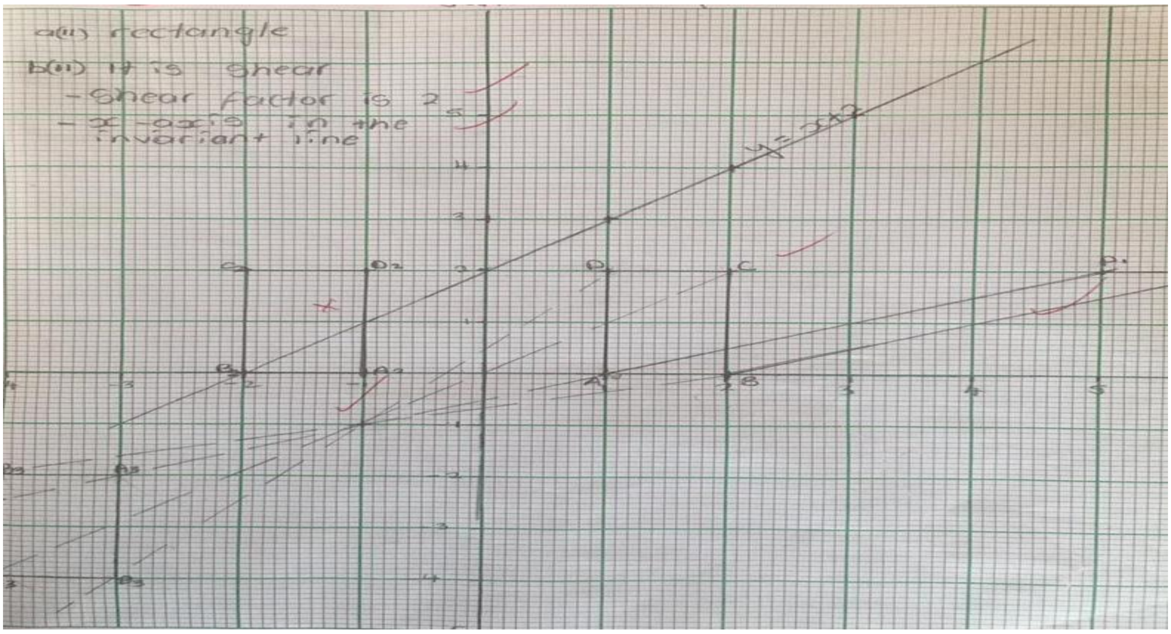

Figure 4.13(b)

In figure 4.13 (a), it can be seen that the learner grasped how to reflect an object across a horizontal line and in figure 4.13 (b), the learner is accustomed to reflecting the object across a vertical line. This shows that the learners have little or knowledge about the fact that, the reflection axis must be perpendicular to the desired flip.

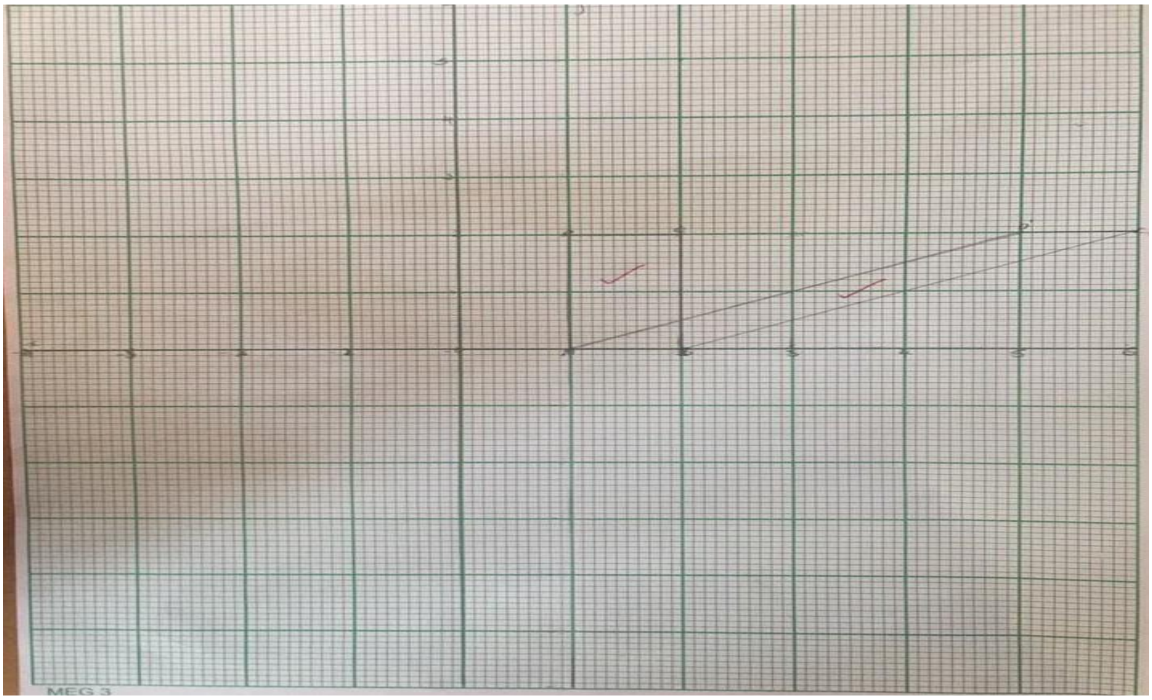

Figure 4. 14

In figure 4.14, it can be seen that the learner has mastered coordinate geometry as s/he was able to point given coordinates. This shows that the learner has limited problem-solving skills, such as analyzing the problem hence s/he was unable to execute the set of problems correctly. This also shows that the learner may not have grasped the fundamental concepts and properties of geometric transformations. Due to the way geometrical transformations are taught or the instructions are not tailor made to suit the learner's needs.

4.5 Chapter Summary

This chapter looked at the presentation, interpretation and analysis of the data that was collected by the researcher through questionnaires and observations from pupils at the selected school on errors and misconceptions in geometrical transformations. The next chapter will focus on the conclusions and recommendations.

Chapter Five

Summary, Conclusion and Recommendations.

5.1 Introduction

This chapter presents the summary, recommendations and conclusions based on the research findings in this study. The purpose of the study was to investigate learners' errors and misconceptions in geometrical transformations at a selected school in Chegutu District, Mashonaland West Province. The conclusions presented in this chapter are based on information obtained from the participants in a selected high school. The study explored learners' errors and misconceptions in geometrical transformations as a way of presenting them.

The aim of the study was to try to answer the following research questions:

1) What are the common errors and misconceptions learners have when learning about geometrical transformations?
2) What are the possible causes of errors and misconceptions learners have when doing geometrical transformations?
3) What are the possible ways of mitigating learners' errors and misconceptions?
4) What instructional strategies or teaching methods are more effective in addressing and correcting errors and misconceptions in geometrical transformations?

This allowed the researcher to have a better appreciation and to gain a deeper understanding of the various difficulties and challenges learners encounter or are likely to encounter when learning about geometrical transformations. Understanding the above questions will likely be important for improving for improving both teaching methods and learner comprehension of geometrical transformations.

5.2 Summary of the study

Mathematics is considered as a difficult and high profile subject at secondary school level in Zimbabwe. At the ordinary level, and in preparation for the Zimbabwe Examination Council examinations (ZIMSEC), most learners find the topic on geometrical transformation to be very difficult and taxing. Though it is a very important topic in the mathematics curricula, as learners often struggle with developing a deep conceptual understanding of the various concepts on the topic. The aim of the study was to examine and identify the type of errors and misconceptions exhibited when learning about geometric transformations.

Most learners misunderstand instructions and perform the wrong transformation, as they would not to be able to fully comprehend the instructions due to language deficiencies as they (learners) struggle to effectively communicate their ideas, leading to incorrect descriptions of geometric transformations. Since language plays an important part in the understanding of mathematical concepts. Therefore language barrier can make it difficult to follow the logical steps required to perform the required transformation. Hence making it difficult for learners to establish the relationships between different types of geometrical transformations and their underlying principles.

The researcher also discovered that matrices multiplication of coordinates also led to learners' errors and misconceptions. Some of the learners could not multiply or made errors when multiplying matrices or learners confused the order/ mixed up x and y coordinates leading to incorrect transformations.

The research tried to explore the common mistakes learners show when being taught about concepts from geometric transformation such as translations, rotations, reflections and dilations that is shear, stretch and enlargement.

For reflection, most learners seem to be have more practice that focus on reflections across standard lines such as the x-axis, y-axis, or diagonal lines like $y = x$ or $y = -x$. they then, have challenges when faced with other slant lines and struggle to determine the correct line. Some seems not have any knowledge with prerequisite knowledge such as finding the gradient and equation of a line. The researcher also discovered that some learners had difficulties in trying to visualising how an object has to appear after reflecting, thereby, struggling to mentally "flip" the object over the correct line. Hence they cannot reverse the orientation of the object.

A good command of geometrical construction usually helps learners develop a sound understanding of coordinate geometry and its role in describing the position and orientation of given objects. Lacking the skill to apply geometric construction makes it difficult for learners, to correctly apply and interpret some of the transformations. Hence most learners fail to quickly recognise and apply transformation.

5.3 Conclusion

The teachers should look for strategies that help learners develop conceptual understanding of the properties and effects of each type of geometrical transformation rather than just memorising procedures, step by step guidance and a lot of practice will help the learners master the concepts on geometric transformations. This can be done through the use of visual models, dynamic geometry software and hands on activities to cater for learners that understand concepts through experience and explore the various transformations as they manipulate and observe the effects of transformations. Lessons should also include activities that help build develop the spatial visualisation skills such as mental folding, paper cutting and figure sketching (these interactive tools will enhance spatial visualisation skills).

Learners need to be encouraged to discuss their thinking, share strategies, and learn from one another by working in pairs or small groups. It was observed that learners struggle to communicate is evident that the learners lack conceptual understanding about a transformation process. The use of visual strategy can be very useful, if used together with the analytic strategies. In these groups learners should also be encouraged to think about transformations in both forward and backward directions. This will help foster transformation-based reasoning.

Some learners lack basic geometry concepts, such as points, lines and angles usually struggle with geometrical transformations, other have insufficient skill in measuring lengths and angles led to errors in geometrical transformation. These skills are required for a learner to be able to analyse and describe geometric transformations. As a results learners who showed that they lacked an understanding of underlying concepts, made procedural mistakes that resulted in some incorrectly rotated shapes or reflected shapes.

Some teachers lack confidence in teaching transformation geometry, this may be due to inadequate training and support. Teacher training programmes need to include the content and methods that should be covered. Such teachers need on the job training through seminar and staff development programmes, so as to improve their knowledge of how to teach geometrical transformations. They also need to improve their perceptions of learners' abilities and their attitude towards teaching and learning mathematics.

5.4 Implications of the findings

The findings of this study suggests that teacher should place greater emphasis on developing the conceptual understanding of learners on geometrical transformations, instead of concentrating solely on procedural skills (teaching step by step technique). Learners can be encouraged to explain their reasoning and properties associated with each transformation, by so doing learners will be developing a more robust and flexible understanding of the underlying principles and properties of transformations. This help learners have a lasting understanding of geometric transformations. The use of traditional approaches to teaching transformations that look at or concentrating on memorising steps and practicing procedures, may not sufficiently help learners develop a deep conceptual understanding.

The recurrence of errors and misconceptions observed is an indicator that many learners lack foundational knowledge and mental representations needed to properly apply and reason about transformations. This can lead to lack of conceptual understanding. Thus hindering learners' ability to transfer knowledge to new situations or make connections between different transformation concepts.

Educators can make use of dynamic geometry software (simulations) such GeoGebra, Geometer's Sketchpad, Desmos or online platforms that offer interactive geometry tool. These tools give learners a chance to interactively explore transformations by applying them. The software helps learners visualise the effects of various transformations and hence gain a deeper understanding of the underlying principles. This hands on approach can help clarify learners' misconceptions by providing immediate feedback.

Teachers need to be empowered with better knowledge and skills to teach mathematics effectively. This can be done by providing targeted interventions and professional development for both pre-service and in-service teachers so as to improve their content knowledge. This includes addressing their lack of general knowledge of mathematics for teaching, particularly geometric transformation. As many teachers lack confidence and adequate training to effectively teach this topic. When the teacher are well versed with knowledge and know the concepts concerning geometric transformation, they can provide clearer explanations and more effective demonstrations. The teacher can also be able to anticipate common errors and misconceptions, and address them proactively. For example teachers with strong knowledge can make use of visual aids to demonstrate how shapes transform, this will help learners to grasp abstract concepts more accurately. Better equipped teachers tend to focus instruction on developing learners' conceptual understanding of transformation rather than just teaching procedures.

Developing teachers' pedagogical content knowledge will assist them understand how their learners learn and think about geometric transformations hence they can be able to anticipate common learner errors, plan targeted instructional strategies and provide appropriate feedback and scaffolding.

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APPENDICES

APPENDIX 1:

Learners' Questionnaire: learners Errors and misconception in Geometrical transformations.

The researcher Zulu Jonathan Munyaradzi, Reg No. 225569B is a student at Bindura University of Science Education and is conducting a research on Learners' errors and misconceptions in geometrical transformations. Please note that the information will be confidential and will remain anonymous.

Instruction for completing the questionnaire.

- **Do not write your name on the questionnaire.**

Section 1: Learner Information

Geometrical Transformations Understanding

a. What is your understanding of geometrical transformations?

   ………………………………………………………………………………….

b. What geometrical transformations are you familiar with?

   ……………………………………………………………………………….

c. Have you encountered difficulties in understanding or applying geometrical transformation?

   …………………………………………………………………………………

d. If yes please describe: ……………………………………………………………

e. Which specific geometrical transformations do you find most difficult? Why?

   …………………………………………………………………………………..

   ……………………………………………………………………………………….

f. Do you find it challenging to identify corresponding angles and sides or the centre of rotation in geometric transformations? Please explain.

   ……………………………………………………………………………………….

g. Do you struggle with the concept of symmetry and identifying symmetrical figures or lines of symmetry? Please explain.

h. …………………………………………………………………………………

Section 3: Errors and misconceptions

a. When learning about geometrical transformations, what errors or misconceptions have you experience or observed? (Tick all that apply)

- Difficulty understanding rotation.
- Confusing clockwise and anti-clockwise rotations.
- Forgetting to maintain shape size and orientation
- Difficulty understanding reflection.
- Confusing horizontal and vertical reflections
- Forgetting to flip shape over correct axis
- Difficulty understanding translation
- Confusing left and right translations
- Forgetting to move shape correct distance
- Other (please specify)

b. Do you believe that your understanding of basic geometric concepts such angles, lines and shapes affects your comprehension of geometric transformations? Please explain.

………………………………………………………………………………..

c. Have you ever relied solely on memorisation rather than understanding when performing geometric transformations? Please explain.

………………………………………………………………………………..

d. Do you find it challenging to visualise the changes in objects undergoing geometrical transformations? If yes why?

……………………………………………………………………………….

e. Do you agree that understanding geometrical transformations is crucial in developing spatial ability and problem-solving skills? Why or why not?

………………………………………………………………………………

Section 4: Instructional Preferences.

a. How do you think technology, such as dynamic geometry software, can support the learning of geometrical transformations?

……………………………………………………………………………..

b. What teaching strategies do you find most effective in understating geometric transformations? Please explain.

……………………………………………………………………………….

c. Do you have any suggestions for improving the teaching and learning of geometrical transformations?

APPENDIX 2: REQUEST TO CARRY OUT A RESEARCH

SAMED

P Bag 1020
BINDURA
ZIMBABWE

Tel: 0271 - 7531 ext 1038
Fax: 263 - 71 - 7616

BINDURA UNIVERSITY OF SCIENCE EDUCATION

Date: 10-05-2024

TO WHOM IT MAY CONCERN

NAME: ZULU J.M REGISTRATION NUMBER: B225569B

PROGRAMME: HBScEd MATHS PART: 2.2

This memo serves to confirm that the above is a bona fide student at Bindura University of Science Education in the Faculty of Science Education.

The student has to undertake research and thereafter present a Research Project in partial fulfillment of the Bachelor of Science Education honours degree in Mathematics programme. The research topic is:

Learners' errors and Misconceptions in geometrical transformations.

In this regard, the department kindly requests your permission to allow the student to carry out his/her research in your institutions.

Your co-operation and assistance is greatly appreciated.

Thank you

BINDURA UNIVERSITY OF SCIENCE EDUCATION
DEPARTMENT OF EDUCATIONAL FOUNDATIONS
9 APR 2024
P. BAG 1020
BINDURA

Z Ndemo (Dr.)
CHAIRPERSON - SAMED

ST. FRANCIS HIGH SCHOOL
THE HEAD
P.O. BOX 197, CHEGUTU

MINISTRY OF PRY. & SEC. EDUCATION
MASHONALAND WEST PROVINCE
DISTRICT SCHOOLS INSPECTOR
10 MAY 2024
CHEGUTU DISTRICT

permission granted.

APPENDIX 3: PERMISSION FROM THE SCHOOL HEAD TO CARRY OUT A RESEARCH

ST FRANCIS HIGH SCHOOL

P.O BOX 197
CHEGUTU
PHONE: 068-215-2224/3786
CELL: +263 786 98 1486

13 May 2024

The Head of Department

Bindura University of Science Education

P. Bag 1020

Bindura

Zimbabwe

Dear Sir/ Madam

RE: Permission to carry out a research

Zulu Jonathan Munyaradzi, EC number 0951785Y, has been granted permission to carry out a research at St Francis High School on " Learners errors and Misconceptions transformations."

Yours faithfully

Mutandwa Shepherd

(School Head)

+263 77 325 7498

ST FRANCIS HIGH SCHOOL
THE DEPUTY HEAD
27 JUN 2024
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Answer the whole of this question on a sheet of graph paper.

Quadrilateral ABCD has vertices at A(1; 0), B(2; 0), C(2; 2) and D(1; 2).

Using a scale of 2 cm to represent 1 unit on each axis, draw the x and y axes for $-4 \leq x \leq 6$ and $-5 \leq y \leq 5$.

(a) (i) Draw and label ABCD.

(ii) State the special name given to quadrilateral ABCD. [2]

(b) Quadrilateral ABC_1D_1 has coordinates at A(1; 0), B(2; 0), $C_1(6;\ 2)$ and $D_1(5;\ 2)$.

(i) Draw and label quadrilateral ABC_1D_1.

(ii) Describe **fully the single** transformation that maps ABCD onto ABC_1D_1. [4]

(c) Quadrilateral ABCD is mapped onto quadrilateral $A_2B_2C_2D_2$ by a reflection in the line $y = x + 2$.

(i) Draw and label line $y = x + 2$.

(ii) Draw and label quadrilateral $A_2B_2C_2D_2$. [3]

(d) $A_3B_3C_3D_3$ is the image of ABCD under an enlargement of scale factor -1 with $(-1;\ -1)$ as centre.

Draw and label quadrilateral $A_3B_3C_3D_3$. [3]

4028/2 J2015 Q9