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A COMPARATIVE ANALYSIS OF GARCH AND ARIMA MODELS IN FORECASTING ZWL/USD EXCHANGE RATE VOLATILITY, A CASE OF ZIMBABWE.

BY

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A DISSERTATION SUBMITTED IN THE PARTIAL FULFILMENT OF THE REQUIREMENTS OF THE BACHELOR OF SCIENCE HONOURS DEGREE IN STATISTICS AND FINANCIAL MATHEMATICS

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Approval From

This is to certify, that this research project is the result of my own research work and has not been copied from past sources without acknowledgement. I hereby declare that no part of it has been presented for another degree in this University or elsewhere.

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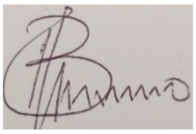

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Dedication

I duly dedicate this research project to my loving family for their unquenchable support, caring, encouragement and motivation. They apprised me of their proud feelings and I had every reason to remain focused.

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Initially, I would like to express my sincere gratitude to the almighty God for his unmerited favour throughout my research project. Without his grace, this journey would not be a supernova and I will forever be indebted to him. Extended gratitude goes to all those who played critical and pivotal role towards making this research a triumph. Special acknowledgement goes to my immediate supervisor, Mr. B.Kusotera, my co-supervisor Ms.P.Hlupo and Ms. JC Pagan'a for consistently supporting and encouraging my research project through their immense knowledge and attention. Furthermore, I am so grateful to my institution of study, Bindura University of Science Education, particularly, the department of statistics and mathematics for facilitating my researches. Last but not least, a heartfelt recognition and appreciation goes to my family and friends for their unconditional love, enormous encouragement and vibrant support during tough times.

Abstract

This research is a comparative analysis of GARCH and ARIMA model in forecasting exchange rate volatility in Zimbabwe the study applied GARCH (1, 1), Generalized Error Distribution and ARIMA (16,20) models in forecasting ZWL/USD exchange rates volatility using performance evaluation techniques such RMSE, Symmetric MAPE, AIC, BIC. The major objective was to compare the performance of these models in predicting future exchange rates and to assist Zimbabwean policy makers, the Central Bank, the Ministry of Finance and Economic Development and interested entities with recommendations. The research used data of weekly exchange rates extracted from the Reserve Bank of Zimbabwe's website which ranges from January 2020 to November 2022. The forecasting was based on In-Sample and Out-Of-Sample predictive horizon. Based on the findings, the study recommends the use of GARCH (1,1) Gaussian error distribution for forecasting exchange rates since it had the lowest Symmetric MAPE, RMSE, AIC, BIC and RMS values. According to the findings, the research study further recommends the use of other currencies other than the USD. A Rand Monetary Union was recommended to facilitate international trade with high hopes of strengthening the value of ZWL.

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Acronyms

1. GARCH - Generalized Auto Regressive Conditional Heteroscedasticity
2. ARIMA - Auto Regressive integrated Moving Average
3. RBZ - Reserve Bank Of Zimbabwe
4. BIC - Bayesian Information Criteria
5. AIC - Akaike Information Criteria
6. RMSE - Root Mean Squared Error
7. MSE - Mean Squared Error
8. ACF - Auto correlation Function
9. PACF - Partial Auto correlation Function
10. ADF - Augmented Dicky Fuller Test

CHAPTER 1
INTRODUCTION

1.0 Introduction

This Chapter introduces the research topic: ***Comparison of GARCH and ARMA models in forecasting exchange rate volatility, a case of Zimbabwe.***

At the heart of economics, international trade facilitates a network of foreign transactions in the so-called foreign exchange market. After the Great Depression of 1976, the government of the United States of America proposed to replace gold with the US dollar as the standard currency of exchange. The proposal marked the introduction of exchange rates at various international institutions such as the IMF, EU and the World Bank. The foreign exchange market is the basic and fundamental platform for trade (imports and exports). Now we look closely at the exchange rate and the foreign exchange market. In the foreign exchange market, economic units participate in the exchange of currencies at the prevailing price (exchange rate).

Then why should we predict the price of one currency to another (exchange rate)? The exchange rate changes randomly over time posing great risk to the world's treasures, financial and real assets, for example the financial crisis of 1976, the Great Depression and the recession of 2007. As the currency value or exchange rate becomes volatile, the future expectations become increasingly uncertain. Understanding exchange rate varying patterns helps in risk management, hedging and insurance.

This study evaluates the predictive performance of the GARCH and ARIMA models. ZWL/USD exchange rate data is available on the website of the Central Bank of Zimbabwe. Data ranges from January 2020 to November 2022. In particular, performance evaluation is related to the results of measurement approaches such as MSE, RMSE, Symmetric MAPE, AIC, and BIC. The study is divided into 5 chapters. Chapter 1 presents the context of the study, the statement of the problem, the research question, the hypothesis, the limits and the delimitation. Chapter 2 is the literature review. Chapter 3 includes the research methodology. Chapter 4 consists of data presentation, analysis and interpretation. Finally, Chapter 5 consists of summary, conclusions and recommendations. ***Key words: Reserve Bank of Zimbabwe, exchange rate volatility, forecasting, ARIMA model, GARCH model***

1.1 Background

Comparative analysis of financial time series models has got many economists at work, a history of past decades. Time series data (exchange rate data) used by economists in building models showed how information was distributed and processed in the foreign exchange market. Time after the 1976 financial crisis, investors, businesses and Industry became risk averse. The collapse of the Bretton Woods system did not only struck fear among investors, but it negatively effects currency markets and global economies. The US foreign exchange market faced high volatility as investors became less confident and withdrew their money from investments. On the other hand, Zimbabwe made history of recessions, hyperinflation, the worst recorded in 2008.

The popularity of the comparative analysis of financial time series models was driven by anxiety triggered by episodic depression crises in the United States, Europe, and parts of Africa (1976). At that time, economic entities (market participants) derived aspiration from the concept that the economy can be compared to a cyclical system. It was thought that predictions could be made and accidents avoided. Economist’s strong aspirations created an environment of hope with time series becoming more and more prominent. The idea of the time series was to understand the business cycle and, by whatever means possible, manage the risk posed. With risk is well managed, investors would return to business. The development of time series models along with computer software assisted in market analysis and risk management. From the ancient application of the automatic regression model (Udng Yule, 1920s) to Professor Rob Hindman's 1970 prediction competition, the topic of predicting random fluctuations of variables has been a challenge all the way.

In 1982, Robert Angle identified other ways of understanding risk in markets (the ARCH model). In 1986 Bollerslave supported Engle's work and generalized the model to the Generalized Automatic Regression Conditional Variance Model. After Bollerslave, various models of the GARCH family emerged, NGARCH, PGARCH, IGARCH, etc. In 1970, Box and Jenkins developed the auto regressive integrated moving average ARIMA(p,d,q). In the rapid development of time series models, George Box quoted "All models are wrong, but some are useful" in 1978. This makes research papers useful as they attempt to articulate useful models through performance metrics. Today's popular GARCH and ARIMA models are compared in estimating volatility in the foreign exchange markets.

Zimbabwe Foreign Exchange Market US/ZWL records exchange rates of Rhodesia since 1976 to Zimbabwe since 2022. Between this period, the data reflects market activities, for example, the 1980 sanctions relief, the 1998 Black Friday crash, the 2009 multi-currency regime, the 2020 Foreign Exchange Auction System, and the 2022 gold coins. This research paper is on exchange rate interest which results from instability of economic fundamentals rather than the fundamentals themselves.

Due to the soaring rates of inflation in Zimbabwe, economists and investors forecasts future exchange rates in order to rely on better estimates in decision making. Volatility poses a significant risk to the macro and micro activities of the economy. In this risky environment, accurately forecasting exchange rates is crucial in every aspect of trading, e.g asset pricing, complex allocation and hedging policies. The exchange rate is required for spot speculation, portfolio investments, exposure to hedge transactions, calculating economic openness and hedging, short- and long-term financing, decision making and investment, strategic planning, foreign payment balance determination and foreign direct investment. The forces of the economy are causing exchange rates to rise and fall every day, putting at risk the market participants. Therefore, accurate estimates will enable firms, investors, and policy makers to make effective decisions when conducting foreign policy making in the business and economic spheres (Kevich, 2001). Fluctuations in the exchange rate negatively affect the business cycle and capital flows of any economy.

1.2 Statement Of Problem

Based on Zimbabwe's national policy framework, the central bank and government are working to alleviate the exchange rate crisis. The growing fear of investment among some economic units adds to the momentum of economic collapse and devalues currencies. Risk has become a measurement tool for evaluating investments. Depreciation on assets due to increased inflationary pressure and gaps now has the element of exchange rate volatility. Answering questions such as "what will the exchange rate volatility look like after a certain period of time due to the financial crisis?" and which model performs best in predicting exchange rate volatility? Predictive analysis includes both model specification and parameter estimation, hence the research study.

1.3 Research Objectives

- To identify a model that performs better in forecasting exchange rate volatility, comparison of GARCH and ARIMA.
- To forecast in-sample and out-of-sample exchange rates.
- To recommend on better model based on predictions.

1.4 Research Questions

- Which model GARCH and ARIMA performs better in forecasting exchange rate volatility ZWL/USD, data from January 2020 to November 2022?
- What will be the position of exchange rates in the future?
- Which model can be recommended for forecasting exchange rates?

1.5 Assumptions

Now due to random market complexities, time series GARCH and ARIMA possesses some implicit assumptions to work well.

1. The models assume volatility can be predicted from limited information such as Standard deviation, Variance, bid ask spread, arrival rate of information, regression constants, probability distribution and trading intensity.
2. GARCH models are conditionally heteroscedastic but yet have unconditional variance.
3. Return series follows a specific distribution.
4. Rate of information arrival and rate of trading per unit time remains constant over the forecast horizon.
5. The error term in GARCH and ARIMA models contains relevant explanatory information and are sufficient for predicting deviation in model results.
6. Assume model selection criterion will be based on the measures of ;

1. AIC
2. BIC
3. MSE, RMSE and Symmetric MAPE.

1.6 Significance Of The Study

Prediction of exchange rate volatility is usually settled in the short run. Exchange rate volatility determinants are the main guideline for evaluating the benefits and risks resulting from the development and use of forecasting techniques and models. In risk management, accuracy of forecasts is of utmost importance, hence the need to examine and evaluate models by comparing results accordingly. In this research paper, the main objective will be to benchmark the model's ability to predict reliable results.

Why is a better estimate of the exchange rate more important? At the macro level of economic activity, the state, along with other large firms, eventually reaches equilibrium value point. A trade equilibrium point, where aggregate demand for domestic goods and imports equal aggregate supply of domestic and foreign goods, provides a true indication of the balance of payments, government accounts, national income, economic growth, and determining the market price. At the micro level of economic activity, traders make decisions about investing in alternative assets, locally or abroad. In this regard, assessing the ability of ARIMA and GARCH models to predict better estimates will provide reassurance in the face of the financial crisis. Using various evaluation techniques and model selection criteria, a better model for predicting and predicting exchange rate volatility will be proposed.

1.7 Limitation Of The Study

The absence of literature on the Time Series modeling of exchange rates in Zimbabwe was a challenge. For this reason, the researcher used international articles and journals for the literature review. Due to differences in economies some ideas do not fit well in the Zimbabwean economy. As stated in the research, the researcher used secondary data on the central bank's (RBZ) website of which the researcher is not familiar with the data collection processes and the methodology used and how well it was done. Apparently, data collection processes, error writing and lack of observation cannot be traced. This may hinder the results and conclusions.

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1.8 Delimitation

this study encompasses some delimitation such as

- Research objective

To determine a model between ARIMA and GARCH that performs better in forecasting exchange rate volatility ZWL/USD.

- Variables

The major research variables are, currency prices, time, rate of exchange ZWL/USD and its corresponding volatility.

- Target sample

The study targets a sample of ZWL/USD exchange rate from Jan 2020 to November 2022 from RBZ statistics.

- Statistical analysis technique

Predictive analysis of time series data with ARIMA and GARCH models. Also
Comparative analysis with model selection criterion will be based on
AIC, BIC, MSE, RMSE and Symmetric MAPE tests.

- Technique (tools)

Eviews 12 Student lite version was used for prediction computation and analysis

1.9 Definition Of Terms

- Exchange rate

Exchange rate is the rate at which the price of one currency change value against the other currency at a given time period.

- Exchange rate volatility

It is the corresponding movement of exchange rate. In this case the volatility measures the rate by which exchange rates fluctuate over time.

- Forecasting

It is the process of deriving futuristic estimates value from the given or available information by means of predictive analysis.

- ARIMA

Auto-regressive Integrated Moving Average is a time series analysis model that is used to forecast data estimate in the future by making use of the past records.

- GARCH

Generalized auto-regressive conditional heteroscedasticity is the generalized model of ARCH model and it is a statistical model used for time series data analysis. In predictive analysis, the model is used for estimating futuristic values. It is also used to predict market volatility.
volatility.

- AIC

The Akaike information criterion is an estimator of prediction error and is used in evaluating model quality for a given set of data.

- BIC

Bayesian information criterion (BIC) is a criterion for model selection based in part, on the likelihood function.

- MSE

It is used to measure the forecast accuracy of a model and it stands for mean squared error.

- RMSE

It is a useful metric for calculating forecast accuracy and it shows how spread residuals are. It stands for rooted squared error.

- MAPE

Mean absolute percentage error is a measure that uses relative errors to enable you to compare forecast accuracy between time-series models.

1.10 Chapter Summary

Research topic: ***Comparative analysis of GARCH and ARMA models in forecasting exchange rate volatility, a case of Zimbabwe.***

Chapter 1 introduced the basic concepts, ideas and information related to the background of the study, statement of the problem, research question, assumptions, limitations and delimitation.

The study seeks to answer the question, which model performs better in forecasting exchange rate volatility ZWL/USD data, from January 2020 to November 2022, regarding weekly forecasting results in comparison of GARCH and ARIMA?In achieving the major objective, the research faces the following limitations and delimitation. The process of forecasting is only to estimate future values that can be affected with uncertainty. The forecasters, analysts and strategists trying to estimate such values will depend on specific fraction of the data set in this case empirical records. These are boundaries and conditions the research use as guidelines in the execution process. the boundaries are elements of research objectives, questions, variables, sample, statistical analysis tools and technical tools.

CHAPTER 2
LITERATURE REVIEW

2.0 Introduction

Modeling and forecasting have been one of the important developments in empirical finance. It has been applied to solve major business and financial problems such as quantitative trading, financial derivatives and risk modeling. The main pitfall in financial data modeling comes with an underlying variable, volatility. It is the nature of volatility that makes predictive statistical models essential. Sometimes highly volatile data can be very challenging to understand. The presence of anomalies, clusters, and structural breaks in financial data has made time series analysis popular. Comparing time series models with their processes is of utmost importance to capture volatility and leverage effects. Historically, there have been several studies comparing models such as GARCH and ARIMA in forecasting market volatility with thousands of reports each year. The contributions and applications of these well-known models will be reviewed in this chapter.

2.1 Conceptual Framework

Financial forecasting is how analysts envision and develop strategies for the future by using past and present financial data. Forecasting is a dynamic and puzzling task in the finance Industry for a number of reasons. First of all, it helps financial market analysts to avoid trading losses and earn huge profits by formulating the right business policies. In international trade, the state is interested in exchange rate fluctuations and inflation. The need for a best estimate is the argument of the whole concept. This scrutiny has an effect on any financial institution because it leads to the lack of trust on any failed financial model presented by financial institutions. Given the impact of estimates on private and state-owned companies, it is important to understand how estimates are generated from volatile financial data with conceptual interpretations.

In this study, exchange rates are used in forecasting future projections. In general, these rates tend to be more volatile and, therefore, have the character of heteroscedasticity. Economic time series models are better used to address such issues. Heteroscedasticity does not cause ordinary least squares coefficient estimates to be biased, although it can cause ordinary least squares estimates of coefficient variances (and thus, standard errors) to be biased, potentially above or below the true population variance. Thus, regression analysis using heteroscedastic data will still provide an unbiased estimate of the relationship between the predictor variable and the outcome, but the standard errors and therefore the inferences obtained from the data analysis are suspect. Biased standard errors lead to biased inferences, so the results of hypothesis tests are likely to be incorrect.

Essentially, as long as there is heteroscedasticity, the observations follow a non-linear pattern. Instead, they tend to cluster together. Therefore, if statistical models are used assuming a constant variance of these data, the results and the predictive value that can be extracted from the model will not be reliable. In review, many models are available and the most popular are GARCH and ARIMA. It's amazing how these models understand volatile data and process it according to their inbuilt rules. The modeling process can be very interesting, especially when dealing with frequent data. Evaluation of model accuracy and performance is based on respective estimates and errors.

GARCH models are used when the variance of the error term is not constant. The variance of the error term in GARCH models is assumed to vary systematically, depending on the average size of the error terms in previous periods. In other words, it has conditional heteroscedasticity, and for this reason the error term follows an auto regressive moving average pattern. This means that it is a function of the average of its own past values. Heteroscedasticity is an error term, or description of an irregular pattern of variation of the variable. The general process of GARCH consists of three steps. The first is to estimate a best-fitting auto-regressive model. The second is to compute the auto-correlations of the error term. The third step is relevance testing.

Now it has come to light, the study is a comparative analysis of ARIMA and GARCH model in predicting exchange rate volatility in Zimbabwe. The future estimates will help in business and economic decision as mentioned in Chapter 1. To achieve the three basic objective of this study, that is, to identify a model that performs better in forecasting exchange rate volatility (GARCH and ARIMA), to forecast in and out-of-sample exchange rates and to recommend on better model based on predictions, a selection criterion is used (Symmetric MAPE, MSE, AIC, BIC and RMSE). This is the general conceptual framework of the study.

2.2 Theoretical Frame Work

Several theories lead the topic of research, a comparative analysis of GARCH and ARIMA in forecasting exchange rate Volatility in Zimbabwe. The field of study includes disciplines in statistics and economics with related theories. In statistics, theoretical analysis of time series GARCH and ARIMA. In economics, the market theory of supply and demand exchange rates, and exchange rate determination. It is the hypothetical descriptions of these theories that give meaning to the study.

Time Series Analysis

series data refers to the information of events or observations indexed over a regular period of time. Collectively, the information forms a data set. Analysis of these data sets by data mining, pattern recognition, and predictive analysis is called time series analysis. Since the birth of this field, rapid growth has been experienced and has become one of the most progressive disciplines in Statistics. Time series analysis is easier nowadays after the development of models, for example GARCH and ARIMA. In 1987, Makridakis and Hibon held an M-competition Blhadjali et al (1987), using data from 1001 time series from the journal Economics, Industry and Demography. After the competition, the participants presented their results, and the findings regarding the accuracy of the revised prediction estimates were that the complexity of the model was independent of the accuracy of the predictions, that the performance ranking of a model was relative as it was dependent on the accuracy of the measurement instrument used, and that in many models, the length of the prediction of the time horizon affected the accuracy of the results. Now thanks to this empirical evolution of time series we can now look at models of interest.

The GARCH Model

It is one of the most popular and powerful models in time series analysis. Generalized auto regressive conditional heteroscedasticity (GARCH) was developed by Bollerslave and Tylor (1986) as an extension of the 2003 Nobel laureate R. Engle (1982, ARCH model) and is the fundamental denominator for most volatility models. The GARCH model belongs to the conditional volatility models and they make predictions based on an optimal exponential weighting of historical returns. The GARCH model adds a lagged conditional variance term as a new term to the GARCH model.

The Box-Jenkins ARIMA(p,d,q,) model

Adapting discrete-time filtering methods of Norbert Wierner et al.(1930's - 1940's), Geoge Box an Jenkins developed systematic methods (ARIMA) applicable in business and economics in 1970. ARIMA model become the most famous models since tier development with over thousand citations in many researches and studies. It is the combination of Auto-regressive differenced with Moving averages that makes Auto-regressive integrated Moving average a power modeling capturing volatility. The parameters of the model are p which is the number of Auto-regressive (AR) terms, d is the number difference taken and q is the number of moving averages (MA) terms. Importantly this model assumes variance to be constant.

ARIMA Process

As a generalized random walk model, it removes all residual auto correlations. As a generalized exponential smoothing model, it incorporates long-term trends and seasonality Use the lags of the dependent variable and/or the lags of prediction errors as regressors.

Returns

For average investors, return of an asset is a complete and scale free summary of investment opportunity. Return series are easier to handle than price series because the returns have attractive statistical properties.
The purpose of time series modeling for returns is to discover the internal auto-correlations of the data and to make a judgement about the future return assuming the characteristics can be repeated in the future. If we want to generate trading signals based on this profit forecast, for example, we buy if the expected return is

positive and if the expected return is negative, we sell. This works vice versa in the case of exchange rates. When the exchange rate returns are positive, the risk in forex trading high and when the return is negative, it means there is less risk.

Model Comparative Theory

In time series modelling, the performance of models is evaluated with the error measurement methods such as MSE, MAPE, RMSE, AIC and BIC. These theories provide the results the model in forecasting by measuring the errors of the forecasted from the actual values. After estimating volatility using ARIMA and GARCH (p, q) models, the researcher aims to predict performance using mean square error (MSE) and mean absolute percentage error (MAPE). Bowerman et al., (2005), assert that, MSE and MAPE are able to assist the researcher in monitoring a prediction system to detect when something has gone "wrong" with the model. (MSE) and (MAPE) are discussed below.

MSE

The MSE is obtained by dividing the error sum of squares by its degrees of freedom, and the result is the error variance or mean square error Yaffee and McGee (1999). According to the SAS Institute Inc. (2012), MSE is useful for drawing on the concepts of bias, accuracy, and precision in statistical estimation. In this study, MSE is used to check the accuracy and precision of the statistical estimation. When making decisions, the smaller the error, the better forecasting ability of the model.

MAPE

MAPE is the average of the sum of the absolute values of the percentage errors Yaffee and McGee (1999). MAPE also helps build on the concepts of bias, precision and accuracy in statistical estimates (SAS Institute Inc., 2012). For the purposes of this study, MAPE is also used to verify the accuracy and precision of the statistical estimate. When making decisions, the smaller the error the better the forecasting ability of the model.

Akaike (AIC)and Schwartz Criteria (BIC)

When modeling a time series, we estimate and select a model that best fits the data series. The model can be determined by increasing its probability of occurring though it results in over fitting or using complexities of the structural expressions of the model. The criteria of Akaike and Schwartz, impose a penalty on the number of parameters. They differ in terms of the penalty associated with increasing the order of the model.

$$\mathbf{AIC = 2K - 2\ln(L)}$$

Where L is the maximum likelihood function and k is the number of parameters

BIC = ln(n)k - 2ln(L)

Where L is the maximum likelihood function and k is the number of parameters

2.06 Foreign exchange Market

It is a platform that facilitates trading through currency exchange. Members of this platform meet to trade currencies at a predetermined rate. This market does not establish the absolute value of the currency but determines the relative value. The price of one currency against another is called the exchange rate. In this market, the exchange rate is determined by supply and demand forces, government intervention, or changes in other economic fundamentals such as trade restrictions and interest rates. The modern foreign exchange market was born in the 1970s after the Bretton Woods system. It is the largest market in the world followed by the credit market due to its trading volumes. According to the Bank for International Settlements, global findings from the central bank's triennial survey of foreign exchange and over-the-counter derivatives markets for 2019 showed that FX traded averaged $6.6 trillion per day in April 2019, up from $5.1 trillion in April. April 2016. In April 2019, FX swaps traded at $3.2 trillion per day, followed by spot trades at $2 trillion.

2.3 Empirical Findings Of Related Literature.

A lot of studies have been conducted analyzing and comparing Time series models in forecasting future estimates of time series data. From Great depressions and the collapse of Bretton Woods Systems several studies have been conducted in bid to understand best performing model among thousands of them. In this section, studies relating to GARCH and ARIMA are being reviewed and identified.

Bhardwa et al. (2014) conducted research comparing ARIMA and GARCH model in forecasting Agricultural prices. Using performance evaluation criteria, RMSE, MAE and MAPE, The GARCH (1,1) was found to be a better model in forecasting spot price of Gram. The value of RMSE, MAE and MAPE obtained were smaller than those in ARIMA (0,1,1) model. The AIC and SIC value from GARCH model were smaller than that from ARIMA model. Therefore, it shows that GARCH is better than ARIMA for estimating daily price of Gram.

Ramzan et al. (2012) conducted a study "Modeling and Forecasting Exchange Rate Dynamics in Pakistan Using the ACH Family of Models". The study focused on building a time series model of exchange rates in Pakistan. Because the exchange rate fluctuates. Modeling using GARCH and ARCH models can provide better forecast estimates. For this purpose, the average monthly exchange rate returns from July 1981 to May 2010 were used. Forecasting performance was evaluated by measures such as MAE, RMSE and MAPE.

This result indicated that the GARCH model was the best at capturing exchange rate volatility and leverage effects in sample exchange rate returns.

Ramasami and Munisami (2012) conducted a study in which they tested the GARCH, GJR (Glosten, Jagannathan and Runkle GARCH) models and the predictive adjusted EGARCH model in the daily exchange rate for four categories. currencies: Australian dollars, Thai baht, Singaporean dollars and Philippine pesos. Errors were estimated using the predicted values and compared to the actual 2011 values. They found that the GARCH model was effective in predicting Exchange Rate Fluctuations and Leverage Improvements in EGARCH and GJR did not significantly improve the forecast.

Laurent, Rombouts and Violante (2012) investigated which multivariate GARCH models are the best performers for 10 stocks. They compared 125 different GARCH models with accuracy over a 10-year period and 1-, 5- and 20-day forecasts ahead. They found that multivariate GARCH models perform poorly in volatile markets.

Marcucci (2005) found that the GARCH model performs much better than complex Models over long-term horizons such as MRS-GARCH (Markov Regime-Switching GARCH). But the MRS-GARCH model outperforms the GARCH model in the short run. Maccucci meant that because the properties of the GARCH model "represent smooth forecasts and excessive volatility".

Canbold et al (2017) conducted a study to investigate the time series models ARIMA, SARIMA and SVAR in forecasting exchange rate volatility in Turkey. Using datasets from 2005-2017. The application of the GARCH and ARCH family of models showed interesting results in examining exchange rate volatility. In the GARCH family, EGARCH is the best model for forecasting exchange rate movements after dummy inclusion and has been successful in controlling for leverage effect. By evaluating three models, SARIMA, ARIMA and SVAR using RMSE and MAE, SARIMA proved to be more accurate than the others.

Dritsaki (2019) conducted a study to examine ARCH, GARCH and EGARCH models in predicting the monthly return of the EUR/USD exchange rate between August 1953 and January 2017 with 763 observations. The idea was to capture the volatility of exchange rate returns by leveraging strong trading patterns. Tested models work well in making future estimates. However, the results still showed differences in prediction performance. The results indicated that ARIMA (0,0,1)-EGARCH (1,1) satisfactorily described exchange rate returns and leverage. It also provides better estimates than dynamics.

Pahlavani and Roshan (2015) conducted a research on the comparison between ARIMA and hybrid-ARIMA models in forecasting the exchange rate of Iran. In the study model performance was the concern using daily date of exchange rate returns of Iran against US dollar (IRR/USD) data ranging from 20 March 2014 to 20

June 2015. Using RMSE, MAE and TIC parameters, the results showed that ARIMA (7,2) (12)-EGARCH (2,1) is the best model. The model was successful in capturing volatility and leverage in exchange rate returns and it was also better at predicting performance.

In conclusion, these studies provide us with important empirical results on the research topic. Analysis of empirical evidence clearly shows that model performance varies in strength when forecasting future estimates of a volatile data set. The time horizon of the forecast has implications on the performance of the model as some are better predictors in the short term while poor in the long term and vice versa. Measuring tools also allows examinations as to their definitions. There is no doubt that the comparative analysis of models in forecasting exchange rate volatility has been a topic of interest for many researchers.

2.4 Chapter Summary

This chapter has focused on understanding the comparative analysis of GARCH and ARIMA time series models in forecasting exchange rate volatility by reviewing the literature through conceptual, theoretical and empirical frameworks. Since the 1970s there have been many competing methods available for forecasting volatility. The major revelation in forecasting exchange rate volatility has come through the basic and most fundamental models GARCH and ARIMA. Their predictive performance can be examined by measures such as RMSE, MSE, MAPE, etc.

CHAPTER 3
RESEARCH METHODOLOGY

3.0 Introduction

This chapter focuses on how the research will be done. It is divided into research methods, data collection, statistical methods and techniques implemented to achieve the main objectives described in Chapter 1. All methodological sections have a framework, from initial data preparation to introducing results. It covers study design, research methods, population and sampling methods, data collection, presentation and analysis.

3.1 Research Design

A research design is a mapping or plan that defines the overall framework for conducting research. It answers questions such as how do you do research? Saunders, et al.(2012) defined research design as a plan that addresses research questions. Research includes data management, strategies, component collection, data collection methods and analysis. However, Bliemer (1970) considered the research design to be irrelevant to flexibility and duration. MC Combe (2019) defines a research design as a plan or strategy for answering a set of questions. As mentioned, the aim of the research design is to answer the research question in successive phases. Then the design appears to be primitive as it is a necessary condition Vogt et al. (2012). Well-planned study designs have minimal margins of error. The research design determines the value of your conclusion from your findings Bordens and Abbott (2018). Good research design must be neutral, valid, reliable, and generalization.

Descriptive research

It is the type of study design used in this research. The purpose of this research design is to describe the case, the phenomenon and the situation by answering the research questions. The idea is to understand the search problem and get some idea of the problem before understanding why it happened. This research design is used because it is appropriate for the research problem and its ability to provide a more informed comparative understanding.

3.2 Data Collection

This is a very important aspect in statistics and data analysis. Almost all statistical methods and data analysis techniques need data to process and provide results. Data collection is the process of collecting, measuring and recording information for a specific entity. Data can be collected from primary sources using techniques such as interviews, surveys, questionnaires, experiments, observations and case studies to extract information. Through literature research of existing information, data from secondary sources can also be collected as second hand information. This research study will use a secondary source where the exchange rate is available and updated weekly on the Reserve Bank of Zimbabwe website.

3.3 Research Target

Research targets are designed to answer research questions such as, what should we measure? How should we measure (quantitative or qualitative), What data or information is available? Is the data of sufficient quality and scope to support our intended measurement? Now this research study attempts to measure the performance of the time series of forecasting or forecasting USD/ZWL exchange rate volatility from 2017 to 2022. The study used a quantitative approach to forecast exchange rates and it requires quantitative measuring analysis. The data is a list of exchange rates available on the Reserve Bank of Zimbabwe website. Within the model processes specification (ARIMA and GARCH), the data provides sufficient quality and coverage support for the intended procedure. Due to the objectives of this study, it is more appropriate and convenient to choose this research objective as the above answers are justified.

3.4 Research Instrument

Now we open the survey toolkit. Research tools are basic tools used for data acquisition, measurement, presentation and analysis of data related to the overall study. Several tools have made it possible to access, measure and analyze data. The use of software and programming languages has taken predictive analysis to a further level where computation is a matter of seconds and the accuracy of results is managed. Eviews 12 lite student version and Microsoft Excel are used to prepare and implement the data analysis process. Eviews is an open-source software for computing and analyzing statistics. It creates a computing environment and was developed to cater for time series analysis. Microsoft Excel will be used for data preparation and management. Finally, the Internet will help with data availability and package downloads.

3.5 Data Presentation and Analysis.

Data analysis involves organizing, classifying, and demonstrating data themes and patterns (Marshall and Rossman (1995). According to Roseman (1995), data analysis is a process of bringing order, structure and meaning to the collected research features. The conclusions were presented in tables and graphs. Data representation means presenting information through illustrations through visualizations. In this study, the research uses tables, graphs, descriptive and inferential statistics to draw reasonable conclusions of comparative analysis. To ensure reliability and better results, the following preliminary tests were carried out: stationarity test, auto correlation test, and error test. This is the bulk of the entire research study where the collaborative analysis will be conducted. This is where the results of the findings are presented and analyzed. It includes graphs, numbers, response rates, demographics, and bench-marking. The results can be presented orally or as a document (Robins Research Consulting, 2020). Presenting the data in this section using text, tables and graphs will provide a clear visual understanding.

Sampling

The data consist of a sample with 150 observations of weekly exchange rates ZWL/USD ranging from January 2020 to November 2022. The data was collected from a secondary source, the Reserve bank of Zimbabwe website. In this analysis of models, the data will be enough to explain how the models perform in predicting exchange rate volatility. The sample analysis will be used for inference of the total population although it is cautiously understood that the uni-variate of time series using the time series model identifies model that suites the data. This model identification is due to sample size and characteristics of inclusive events in the series. So, the sample cannot be the perfect explain-er of the population since financial time series are time varying events.

Stationarity test

A series is said to be stationary if it exhibits mean reversion, has a finite and time-invariant variable, has a theoretical correlogram that diminishes as lag length increases. As Brooks (2008) stated, all time series variables need to be tested for stationarity using a unit root test. Determine if the USD/ZWL exchange rate is stable. Augmented Dickey Fuller (ADF) test was used. In time series analysis, checking for stationarity is paramount because stationarity or non-stationarity can affect the behavior or parts of a trend, and therefore the results.

Augmented Dickey Fuller Test

This is the hypothetical testing that a unit root is present in a time series sample. The statistic value of Augmented Dickey Fuller test is a negative number. The more negative it is, the stronger the rejection of the hypothesis that there is a unit root at some level of confidence. The intuition behind the test is that if there is a unit root in the series, then the lagged level of the series provides no useful prediction information than the lagged changes. In this research, the exchange rates data shall be first differenced to ensure stationarity in data.

Auto correlation Test

Auto-correlation is another important feature of time series. Ideally, the covariance and correlation between error terms must be zero. In other words, all disturbances must be independently and identically distributed. Usually, Auto correlation is caused by the omitted variables miss specification of the model. To evaluate auto correlation or exchange rates return and squared returns, the research used Ljung-Box test.

Ljung -Box test

It tests auto-correlation of a time series to confirm if there are different from zero instead of test randomness of each distinct lag, it tests overall randomness based on a number of lags

$H_0 \; : \; Y_k = 0$ Where k = 1…………..K

$H_0 \; : \; Y_k \neq 0$

The test statistic is given by

Q(k) = T(T+2)$\sum_{k=1}^{K} \frac{Y_k^2}{T-1} \sim H_0 \chi^2(k)$

The researcher will use lags to calculate Q- statistic in R. the test statistic will be compared with the critical value with degrees of freedom.

Sampling, Training and Testing

In statistical modeling, a data set is usually split into two disjoint sets for training and validation. The idea of splitting a data set into two separate sets allows to evaluate and compare forecasting performance of various models without worrying about over fitting on the training set. A common ratio for splitting data is the 80:20 where 80% is used for training and 20% for testing. The split ratio is a derivation from a well-known Pareto principle, Dunford (2014).

The sample contains 150 observations, ranging from January 2020 to November 2021 for weekly exchange rates. The sample is divided into 120 observations from week 1 to week 120 for model training, and the remaining 30 observations from week 121 to week 150 are used for validation. The forecast will be in a sample of 120 observations and out of a sample of 30 observations. Performance criteria will be based on RMSE, Symmetric MAPE, AIC, BIC and MAE.

Model Selection

Most commonly used methods for model selection are Akaike Information Criteria (AIC) and Bayesian Information criterion (BIC) values for two/more models, the lowest AIC or BIC value should be chosen. In this research the BIC shall be used. The maximum likelihood estimation procedures in the formulation of the test statistic from a log like hood functions for BIC is

BICt = -2 ln(L) + 2 ∗ (N) Where k=model degrees of freedom N=number of observations.

Model Specification

The study is going to compare the models GARCH (1, 1) and the ARIMA () based on

Performance capability. To evaluate the effectiveness of the forecasting performance several error measurements are used and AIC and BIC are also considered.

GARCH (1,1)

For simplicity and reliability, a GARCH (1,1) model shall be used

$\sigma_t^2 = \propto_0 + \sum_{i=1}^{1} \propto_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_t^2$

Where σ_t^2 will be replaced by h_t ;

$$h_t = \propto_0 + \sum_{i=1}^{1} \propto_i \epsilon_{t-i}^{2} + \sum_{j=1}^{1} \beta_j h_t$$

Where;

h_t = Conditional Variance

h_{t-j}^{2} = Past Conditional Variance

ϵ_{t-1}^{2} = Past Squared Residual Return

$\propto_0 > 0, \quad \propto_j >= 0, \quad \beta_j >= 0$

ARIMA(p,d,q)

The model allows Yt to be explained by the past or lagged value of y itself and stochastic error term (innovation and shocks). The series is simply explaining itself using its historical data thus it is called atheoretic model since they are not derived from any economic theory. The model is composed of two distinct models that explains the behavior of a series from two different perspectives. The first is the Auto regressive model (AR) and the last is the Moving Average model. These models move in opposite directions of one another. ARIMA(p,d,q) tells us the number of lags of the dependent variable (p) , how many times the variable is differences to become stationary (d) and the number of lags of the error term (q)

Modelling Conceptual overview

The modeling of GARCH and ARIMA follows procedure and assumptions. To properly model a time series data, the data must some presumptions and follow some distributions as explained above. If the fails to meet the assumptions, the model is either discarded or the data is either transformed to meet the necessities. The modeling of GARCH and ARIMA will both follow the following Methodologies**;**

ARIMA modeling approach to exchange rates

ARIMA modeling procedure will go through the following methodological steps;

4. Model identification
5. Estimation
6. Diagnostic Checking
7. Forecasting a series

3.6 Chapter Summary

This chapter provided the methodology that will be used during the research. The area of study, research design, data collection techniques and data presentation and analysis procedure were highlighted. On data analysis the test necessary to be carried out before the data is used for model fitting were discussed. Moreover, the model selection and the GARCH and ARIMA models to be fitted were explained in terms of their general fitted model equations. Finally, the performance evaluation techniques to recommend on the best model in forecasting exchange rate volatility was explained. The chapter was laid in relation to recommendations made by the previous literature reviewed in the second chapter. The subsequent chapter focuses on data representation, analysis and discussion.

CHAPTER 4
DATA ANALYSIS AND PRESENTATION

4.0 Introduction

This chapter is an important part of the study which provides a detailed analysis of the GARCH and ARIMA models. By examining the performance of these popular time series models, the chapter will provide an evidence-based benchmark against the main research questions and objectives. The Chapter will express focus on the analytical presentation of Financial Modeling, forecasting, performance evaluation and interpretation of results. Evaluation and decision-making are based on the theory of measurement error.

4.1 Model identification

In our comparative analysis, we seek to identify the best performing model between ARIMA and GARCH in forecasting exchange rate volatility. Now the ARIMA model can be specified differently given the choice of auto regressive component (AR) and moving averages component (MA). Selecting an ARIMA model is more of an art than a science, thus the model is called an atheoretical model. After identifying various tentative ARIMA (p, d, q) models, p (number of lags for the dependent variable from the AR model), q (number of lags for the error term from MA) and d (number of times the series differs from its stability correction), we then estimate the best ARIMA model. To identify this specific ARIMA(p,d,q) model, a correlogram can be drawn

Preliminary Analysis

The comparative analysis used Zimbabwe's weekly exchange rates of ZWL/US Dollar from January 2020 to November 2022 comprising of 150 observations.

Data Properties:

As illustrated in figure (4.1), the plot shows a an upward increasing trend with non-constant mean, showing results of non-stationary series. The plot portrays exchange rate variation generally increasing as time passes. To confirm non stationarity of ZWL/USD exchange rates, a correlogram was plotted (figure 4.2) and results from Dicky Fuller Test observed. As clearly illustrated, the correlogram shows some significant auto correlations that are outside the standard error bound (broken lines) or the 5% confidence interval and the auto correlation exponentially decay from lag 1 up-to lag 22 on the ACF. the lags are very significant and the decline is very gradual. While he PACF shows significance on the first lag while others cut off. With the help of a correlogram we primarily doubt the stationarity of the series. ADF test also confirms non stationarity of the series (Table 4.1). The ADF statistic tests the presence of unit root against the null hypothesis of no unit root. The null hypothesis is rejected if the test statistic is less than the critical value.

From the table the statistic is greater than the critical value at 1%, 5% and 10% and hence we cannot reject the presence of unit root. According to ADF hypothesis Testing, the series is non stationary.

In correction of non-stationary exchange rates, an exchange rate return series was computed using the formula

$$r_t \ = log(\frac{p_{t-1}}{p_{t-1} - p_t})$$

Where r_t is the return series

p_t is the rate at time t

In eveiws, the formula is given as

R_Rates = d(log(rates))

Now we plot the correlogram of the return series. Generally, the significance of the plot outside the standard error-bound is acceptable and the plot shows signs of stationarity. This shows an ARIMA pattern because ACF and PACF have the same pattern (figure 4.4) and specifying the fitting model will be covered on step 2 (estimation procedure).

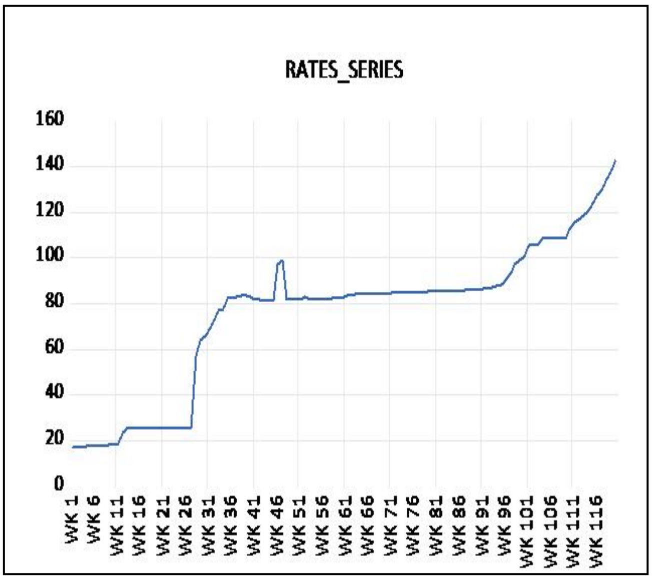
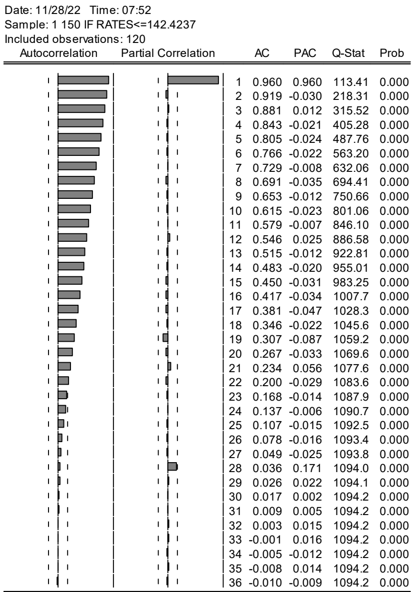


Figure 4.1 The plot of exchange rates



Date: 11/28/22 Time: 07:52
Sample: 1 150 IF RATES<=142.4237
Included observations: 120

Lag	AC	PAC	Q-Stat	Prob
1	0.960	0.960	113.41	0.000
2	0.919	-0.030	218.31	0.000
3	0.881	0.012	315.52	0.000
4	0.843	-0.021	405.28	0.000
5	0.805	-0.024	487.76	0.000
6	0.766	-0.022	563.20	0.000
7	0.729	-0.008	632.06	0.000
8	0.691	-0.035	694.41	0.000
9	0.653	-0.012	750.66	0.000
10	0.615	-0.023	801.06	0.000
11	0.579	-0.007	846.10	0.000
12	0.546	0.025	886.58	0.000
13	0.515	-0.012	922.81	0.000
14	0.483	-0.020	955.01	0.000
15	0.450	-0.031	983.25	0.000
16	0.417	-0.034	1007.7	0.000
17	0.381	-0.047	1028.3	0.000
18	0.346	-0.022	1045.6	0.000
19	0.307	-0.087	1059.2	0.000
20	0.267	-0.033	1069.6	0.000
21	0.234	0.056	1077.6	0.000
22	0.200	-0.029	1083.6	0.000
23	0.168	-0.014	1087.9	0.000
24	0.137	-0.006	1090.7	0.000
25	0.107	-0.015	1092.5	0.000
26	0.078	-0.016	1093.4	0.000
27	0.049	-0.025	1093.8	0.000
28	0.036	0.171	1094.0	0.000
29	0.026	0.022	1094.1	0.000
30	0.017	0.002	1094.2	0.000
31	0.009	0.005	1094.2	0.000
32	0.003	0.015	1094.2	0.000
33	-0.001	0.016	1094.2	0.000
34	-0.005	-0.012	1094.2	0.000
35	-0.008	0.014	1094.2	0.000
36	-0.010	-0.009	1094.2	0.000

Figure 4.2 The correlogram of exchange rates

Table 4.1 ADF Stationarity Test of ZWL/USD exchange rates at level form

	Test Statistic	**1% Critical Value**	**5% Critical Value**	**10% Critical Value**	**Conclusion**
Constant	-0.451827	-3.486551	-2.886074	-2.579931	Not Stationary
Constant/Linear Trend	-1.586324	-4.037668	-3.448348	-3.149326	Not Stationary

None	2.063247	-2.584707	-1.943563	-1.614927	Not Stationary

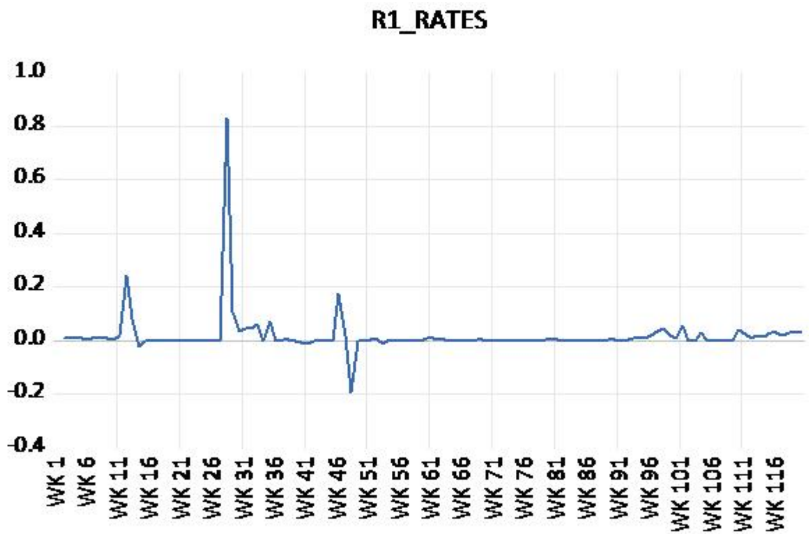
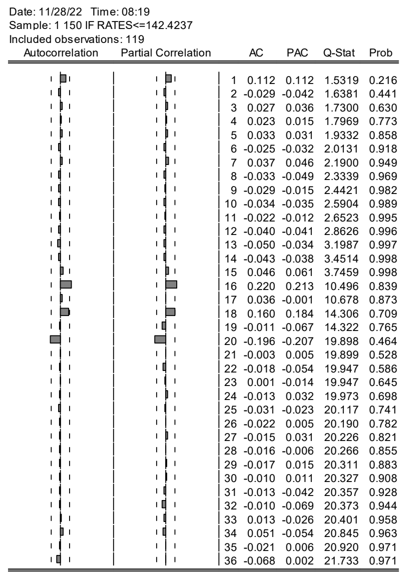


Figure 4.3 The plot of exchange rate return series rate


Date: 11/28/22 Time: 08:19
Sample: 1 150 IF RATES<=142.4237
Included observations: 119

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.112	0.112	1.5319	0.216
		2	-0.029	-0.042	1.6381	0.441
		3	0.027	0.036	1.7300	0.630
		4	0.023	0.015	1.7969	0.773
		5	0.033	0.031	1.9332	0.858
		6	-0.025	-0.032	2.0131	0.918
		7	0.037	0.046	2.1900	0.949
		8	-0.033	-0.049	2.3339	0.969
		9	-0.029	-0.015	2.4421	0.982
		10	-0.034	-0.035	2.5904	0.989
		11	-0.022	-0.012	2.6523	0.995
		12	-0.040	-0.041	2.8626	0.996
		13	-0.050	-0.034	3.1987	0.997
		14	-0.043	-0.038	3.4514	0.998
		15	0.046	0.061	3.7459	0.998
		16	0.220	0.213	10.496	0.839
		17	0.036	-0.001	10.678	0.873
		18	0.160	0.184	14.306	0.709
		19	-0.011	-0.067	14.322	0.765
		20	-0.196	-0.207	19.898	0.464
		21	-0.003	0.005	19.899	0.528
		22	-0.018	-0.054	19.947	0.586
		23	0.001	-0.014	19.947	0.645
		24	-0.013	0.032	19.973	0.698
		25	-0.031	-0.023	20.117	0.741
		26	-0.022	0.005	20.190	0.782
		27	-0.015	0.031	20.226	0.821
		28	-0.016	-0.006	20.266	0.855
		29	-0.017	0.015	20.311	0.883
		30	-0.010	0.011	20.327	0.908
		31	-0.013	-0.042	20.357	0.928
		32	-0.010	-0.069	20.373	0.944
		33	0.013	-0.026	20.401	0.958
		34	0.051	-0.054	20.845	0.963
		35	-0.021	0.006	20.920	0.971
		36	-0.068	0.002	21.733	0.971

Figure 4.4 The correlogram of exchange

Table 4.2 Stationarity Test of Return Series

	Test Statistic	**1% Critical Value**	**5% Critical Value**	**10% Critical Value**	**Conclusion**
Constant	-9.623844	-3.486551	-2.886074	-2.579931	Stationary
Constant/Linear Trend	-9.675661	-4.037668	-3.448348	-3.149326	Stationary
None	-9.284956	-2.584707	-1.943563	-1.614927	Stationary

4.2 Descriptive Statistics

Table 4.3 Descriptive Statistics of Return Series

	R1 RATES
Mean	0.017945
Median	0.001066
Maximum	0.830440
Minimum	-0.192203
Std. Dev.	0.083846
Skewness	7.848449
Kurtosis	76.28341
Jarque-Bera	27850.22
Probability	0.000000
Sum	2.135471
Sum Sq. Dev.	0.829549
Observations	119

Table 4.3. Summarizes the descriptive statistics of the Zimbabwe Dollar against the US Dollar series of returns. Generally, the return series reached an average value of 0.017945 with a minimum value of -0.192203 and maximum of 0.830440. A standard deviation of 0.083846 of returns represent a significant

risk premium and shows that traders in this forex market had to hedge against risk of price changes. The skewness is positively skewed. This is an indication of non-smooth series. Kurtosis is much larger than 3 of the normal distribution. Skewness indicates non-normality, while a large kurtosis suggests that the distribution of the exchange rate return series is leptokurtic, that is, it exhibits fat tails. The Jarque-Bera normal test statistic also indicates that there is no normal distribution for the return series.

Estimation of ARIMA model

A very important note on the estimation of ARIMA model is parsimony. As specified by Box-Jenkins methodology, it is advised to select the model with few number of lags. Parsimonious models give better forecasts than over parameterized models. From figure 4.4, ACF for MA model and PACF for AR model are significant at 16, 18, 20 and 16, 18, 20 lags respectively and resulting in tentative models,
ARMA(16,16), ARMA(16,20), ARMA(20,16), ARMA(20,20), ARMA(16,18), ARMA(18,16), ARMA(18,20), ARMA(20,18), ARMA(18,18).
The estimation of these models are as follows;

Table 4.4 Results of estimated Tentative ARIMA models

Exchange rate returns	**Significant coefficient**	**Sigma volatility**	**Adj R^2**	**AIC**	**SBIC**
ARMA(16,16)	0	0.006611	0.026926	-2.108065	-2.014649
ARMA(16,20)	2	0.006288	0.074443	-2.150041	-2.056625
ARMA(20,16)	2	0.006321	0.069528	-2.146603	-2.053187
ARMA(20,20)	0	0.006470	0.047669	-2.110939	-2.017523
ARMA(16,18)	2	0.006402	0.057605	-2.135704	-2.042289
ARMA(18,16)	2	0.006424	0.054494	-2.133166	-2.039750
ARMA(18,20)	2	0.006433	0.053103	-2.128567	-2.035151
ARMA(20,18)	2	0.006412	0.056176	-2.131078	-2.037662
ARMA(18,18)	0	0.006610	0.027087	-2.100458	-2.007042

In our decision criteria, the appropriate model should have the most significant coefficients, lowest volatility, highest adjusted R^2 and lowest AIC and BIC. Looking at the table, ARMA (16,20) meets the required conditions so it selected as the best model. Note that the selection criteria is more of an art than a science. Having identified the ARIMA model the next step is to perform some diagnostic checking to be certain that there is no uncaptured information by plotting the correlogram of the residuals.

4.3 Diagnostic Checking

Under diagnostics checking the ideal model (ARIMA (16,20)) is tested to be certain that there is no uncaptured information by plotting the correlogram of the residuals. An ideal correlogram for the residuals should be flat, that is the lag structures should be within the standard error bound. If a lag is significant, that is outside the standard error bound, we re-estimate the model. We will cautiously try to avoid over fitting the model.

Date: 11/28/22 Time: 09:35
Sample: 1 150 IF RATES<=142.4237
Q-statistic probabilities adjusted for 2 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.107	0.107	1.4011	
		2	-0.030	-0.041	1.5084	
		3	0.048	0.057	1.7979	0.180
		4	0.089	0.077	2.7858	0.248
		5	0.045	0.032	3.0458	0.385
		6	-0.029	-0.034	3.1509	0.533
		7	0.037	0.040	3.3279	0.650
		8	-0.041	-0.064	3.5473	0.738
		9	-0.044	-0.034	3.8060	0.802
		10	-0.037	-0.033	3.9831	0.859
		11	-0.040	-0.036	4.1962	0.898
		12	-0.057	-0.046	4.6379	0.914
		13	-0.049	-0.025	4.9638	0.933
		14	-0.041	-0.032	5.1956	0.951
		15	0.033	0.055	5.3477	0.967
		16	0.021	0.023	5.4093	0.979
		17	0.017	0.028	5.4512	0.988
		18	0.172	0.177	9.6671	0.883
		19	0.005	-0.039	9.6710	0.917
		20	-0.012	-0.011	9.6925	0.941
		21	0.012	-0.011	9.7129	0.960
		22	-0.021	-0.071	9.7767	0.972
		23	0.001	-0.011	9.7769	0.982
		24	0.012	0.019	9.7986	0.988
		25	-0.017	-0.040	9.8453	0.992
		26	-0.021	0.013	9.9146	0.995
		27	-0.004	0.016	9.9177	0.997
		28	-0.019	-0.011	9.9726	0.998
		29	-0.015	0.020	10.010	0.999
		30	-0.008	0.010	10.021	0.999
		31	-0.032	-0.030	10.185	1.000
		32	-0.071	-0.053	11.014	0.999
		33	-0.004	-0.017	11.018	1.000
		34	0.010	-0.012	11.033	1.000
		35	-0.013	-0.011	11.064	1.000
		36	-0.028	-0.052	11.199	1.000

Figure 4.5 The correlogram of residual

As can be seen from the correlogram Figure 4.5, Lag 18 is Significant, which means there is information in this model that is not captured with the omission of lag 18. Clearly lag 18 needs to be included in the model. In this case we are going to re-estimate the model ARMA (16,20) by adding AR(18) and MA(18)

Table 4.5 Results of Adjusted ARIMA models

Exchange rate returns	**Significant coefficient**	**Sigma volatility**	**Adj** R^2	**AIC**	**SBIC**
ARMA (16,20)	2	0.006288	0.074443	-2.150041	-2.056625
ARMA (16,20)+AR(18)	3	0.006090	0.095775	-2.160896	-2.044126

ARMA(16,20)+MA(18)	3	0.006004	0.108549	-2.169362	-2.052592

Between two new model formed by adding AR (18) and MA (18), We need to decide the most appropriate. The Table 4.5 above shows the results of the adjusted ARIMA models. Using our model selection criteria of selecting the model with the most significant coefficients, low volatility, highest Adjusted R^2, lowest AIC and SBIC, we can see that the model with the inclusion of MA (18) is the most ideal. It has the lowest volatility, it has the highest Adjusted R^2, it has the lowest AIC and SBIC. So, this is the final model we are going to base our forecasting on. Now we check again the correlogram of the residual to make sure the model have captured all the information.

The correlogram of the residual for the re-estimated ARMA model.

Date: 11/28/22 Time: 10:13
Sample: 1 150 IF RATES<=142.4237
Q-statistic probabilities adjusted for 3 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.111	0.111	1.5027	
		2	-0.024	-0.037	1.5742	
		3	0.044	0.052	1.8156	
		4	0.107	0.097	3.2569	0.071
		5	0.058	0.039	3.6878	0.158
		6	-0.023	-0.029	3.7526	0.289
		7	0.051	0.053	4.0929	0.394
		8	-0.036	-0.065	4.2596	0.513
		9	-0.039	-0.033	4.4572	0.615
		10	-0.027	-0.025	4.5568	0.714
		11	-0.051	-0.054	4.9080	0.767
		12	-0.054	-0.040	5.3002	0.807
		13	-0.054	-0.031	5.6891	0.841
		14	-0.054	-0.043	6.0841	0.868
		15	0.027	0.056	6.1832	0.907
		16	0.031	0.039	6.3162	0.934
		17	-0.000	0.009	6.3162	0.958
		18	-0.021	-0.008	6.3786	0.973
		19	-0.010	-0.015	6.3921	0.983
		20	0.015	-0.001	6.4263	0.990
		21	0.009	0.002	6.4386	0.994
		22	-0.041	-0.056	6.6843	0.996
		23	-0.009	-0.007	6.6975	0.998
		24	0.025	0.020	6.7941	0.999
		25	-0.023	-0.032	6.8759	0.999
		26	-0.012	0.006	6.8985	1.000
		27	0.010	0.017	6.9127	1.000
		28	-0.012	-0.016	6.9343	1.000
		29	-0.003	0.017	6.9359	1.000
		30	0.005	0.005	6.9393	1.000
		31	-0.023	-0.039	7.0286	1.000
		32	-0.056	-0.050	7.5398	1.000
		33	-0.010	-0.006	7.5569	1.000
		34	-0.001	-0.010	7.5570	1.000
		35	-0.011	-0.001	7.5793	1.000
		36	-0.015	-0.008	7.6177	1.000

Figure 4.6 The correlogram of residuals

As shown in Fig 4.6 above correlogram, the re-estimated model now have managed to capture all the information, thus a flat correlogram with all lags falling within the standard error bound or the 95% confidence interval. Now we can conclude by saying that the adjusted ARMA model is the most ideal. This is the model we are going to use for forecasting.

Now we perform the Ljung-Box test, which is the test for autocorrelation.

4.4 Ljung-Box test

A test for Auto correlation

Date: 11/28/22 Time: 10:27
Sample: 1 150 IF RATES<=142.4237
Included observations: 119

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.003	-0.003	0.0009	0.976
		2	-0.012	-0.012	0.0172	0.991
		3	-0.012	-0.012	0.0337	0.998
		4	0.001	0.001	0.0339	1.000
		5	-0.006	-0.006	0.0377	1.000
		6	-0.012	-0.012	0.0554	1.000
		7	-0.007	-0.007	0.0619	1.000
		8	-0.013	-0.013	0.0832	1.000
		9	-0.013	-0.013	0.1038	1.000
		10	-0.013	-0.014	0.1260	1.000
		11	-0.012	-0.013	0.1445	1.000
		12	-0.011	-0.012	0.1617	1.000
		13	-0.011	-0.012	0.1792	1.000
		14	-0.010	-0.012	0.1939	1.000
		15	-0.006	-0.007	0.1990	1.000
		16	0.126	0.124	2.4026	1.000
		17	-0.011	-0.011	2.4182	1.000
		18	-0.013	-0.012	2.4427	1.000
		19	-0.012	-0.011	2.4648	1.000
		20	-0.011	-0.013	2.4824	1.000
		21	-0.013	-0.014	2.5069	1.000
		22	-0.011	-0.010	2.5257	1.000
		23	-0.011	-0.012	2.5443	1.000
		24	-0.013	-0.012	2.5707	1.000
		25	-0.013	-0.012	2.5976	1.000
		26	-0.014	-0.013	2.6264	1.000
		27	-0.004	-0.004	2.6290	1.000
		28	-0.004	-0.003	2.6313	1.000
		29	-0.004	-0.004	2.6338	1.000
		30	-0.004	-0.004	2.6365	1.000
		31	-0.004	-0.005	2.6387	1.000
		32	0.001	-0.018	2.6388	1.000
		33	-0.004	-0.004	2.6417	1.000
		34	-0.004	-0.004	2.6447	1.000
		35	-0.004	-0.004	2.6478	1.000
		36	-0.004	-0.004	2.6506	1.000

Figure 4.7 Test for Auto correlation

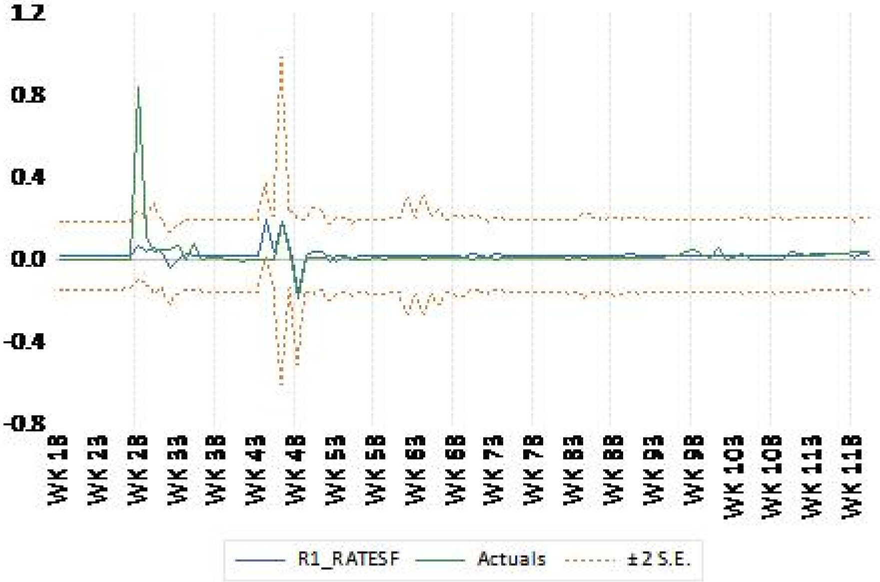
As can be seen from **Figure 4.7**, lag 1 to lag 36, the probability values are higher than 5% which indicate that there is no auto correlation in the adjusted model. So, this model is good.

In conclusion, we going to use the Adjusted ARIMA (16,20) with additional MA (18) to forecast our exchange rate returns.

4.5 Forecasting

Remember the essence of fitting an ARIMA model is to forecast the future value of the series based on the final selected Adjusted (ARMA (16,20) with additional MA (18). In other words, we are using the past values of the series to get some insights of the future values. After forecasting the future exchange rates, we will plot the forecast graph against the actual graph to see if the forecast is good. Our forecasting will be based on two approaches, in-sample forecasting and out-of- sample forecasting. Since our data set consist of 150 ZWL/USD weekly exchange rate observations from January 2020 - November 2022. The data is split into 120 observations for training and testing and 30 observations for out-of-sample forecasting using the 80:20 % ratio criteria. In-Sample forecasting will range from week 1 to week 120 and the out-of-Sample forecasting will range from week 121 to week 150. Figure 4.7 and Figure 4.8 below shows the Actual and forecast graphs of in-sample and out-of-sample forecasting.

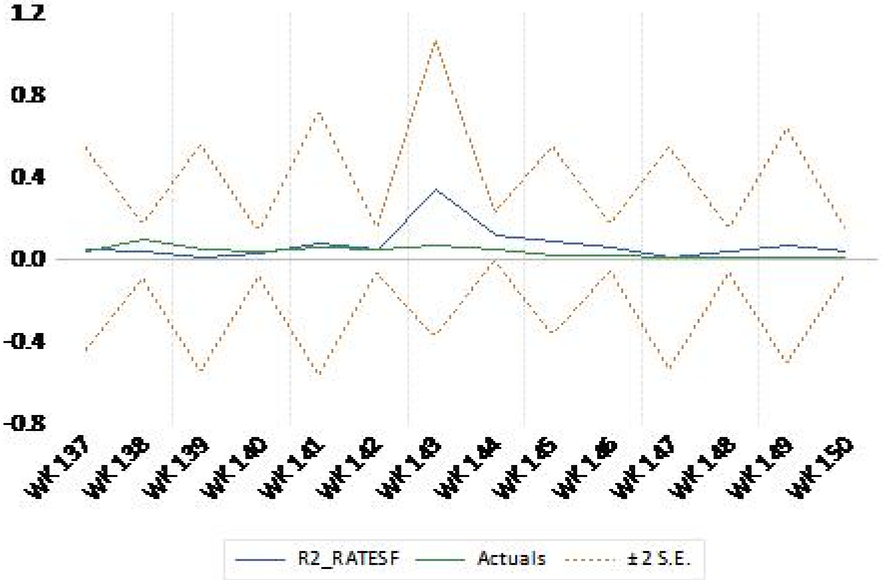
In -Sample ZWL/USD Forecasting results (week 1-120)



Forecast: R1_RATESF	
Actual: R1_RATES	
Forecast sample: 1 150 IF RATES<=1...	
Adjusted sample: 1 150 IF RATES<=1...	
Included observations: 103	
Root Mean Squared Error	0.080795
Mean Absolute Error	0.026116
Mean Abs. Percent Error	NA
Theil Inequality Coef.	0.649141
Bias Proportion	0.000278
Variance Proportion	0.464109
Covariance Proportion	0.535613
Theil U2 Coefficient	NA
Symmetric MAPE	145.8690

Figure 4.8 In-Sample forecast graph

Out-of-Sample ZWL/USD Forecast results (week 121-150)



Forecast: R2_RATESF	
Actual: R2_RATES	
Forecast sample: 1 150 IF RATES>14...	
Adjusted sample: 1 150 IF RATES>14...	
Included observations: 14	
Root Mean Squared Error	0.085358
Mean Absolute Error	0.052496
Mean Abs. Percent Error	449.5693
Theil Inequality Coef.	0.581767
Bias Proportion	0.179085
Variance Proportion	0.406035
Covariance Proportion	0.414880
Theil U2 Coefficient	9.304481
Symmetric MAPE	100.2585

Figure 4.9 Out-of-sample forecast graph

As can be seen from the graphs, the forecast line lays between the +/- standard error or the 95% confidence interval. Amazingly, the forecast graph appears to provide good results in the short run.

4.6 Graphical Observations

In-Sample Forecast

In -Sample results are shown in Figure 4.8. As illustrated the forecast graph was almost exact from week 1 to week 23. From week 23 up to week 43, the forecast graph deviated from the actual graph with high deviation on week 25 and week 24. Between week 33 and week 43, the graph had almost exact forecast until it deviated again on week 53. From week 53 up to week 120, the forecast graph was almost the exact.

Generally, the forecast graph is good. The error metrics computation is also shown, RMSE = 0.085358, MAE = 0.052496, Symmetric MAPE =100.2585.

Out-Of-Sample Forecast

As illustrated in Figure 4.9, The forecast graph shows almost exact forecast values week 137 up to week 142. The forecast graph deviated from week 142 to week 143 and began to converge from week 143 to week 148. We can observe a very small deviation from week 148 up to week 150. The error metric results are also shown, RMSE =0.085358, MAE = 0.052496 and MAPE = 100.2585

4.2.1 GARCH Modeling Of ZWL/USD Exchange Rates.

Basics Of GARCH Modelling

Exchange rates fluctuates with time. In Zimbabwe ZWL/USD shows peak points during recession periods of soaring inflation. The GARCH model was designed to handle heteroscedasticity issues with mean reversion patterns. The returns of exchange rates possess attractive statistical properties for GARCH modeling.In practice researchers have uncovered several stylized facts about the GARCH model. We consider three of them;

1. Volatility clustering
2. Fat tails
3. Volatility mean reversion

4.2.2 Volatility clustering

For many daily and weekly financial time series large value of h_{t-1} will be followed by large values of h_t and also small values of h_{t-1} will be followed by small value of h_t.

ZWL/USD Exchange Rates Return Series

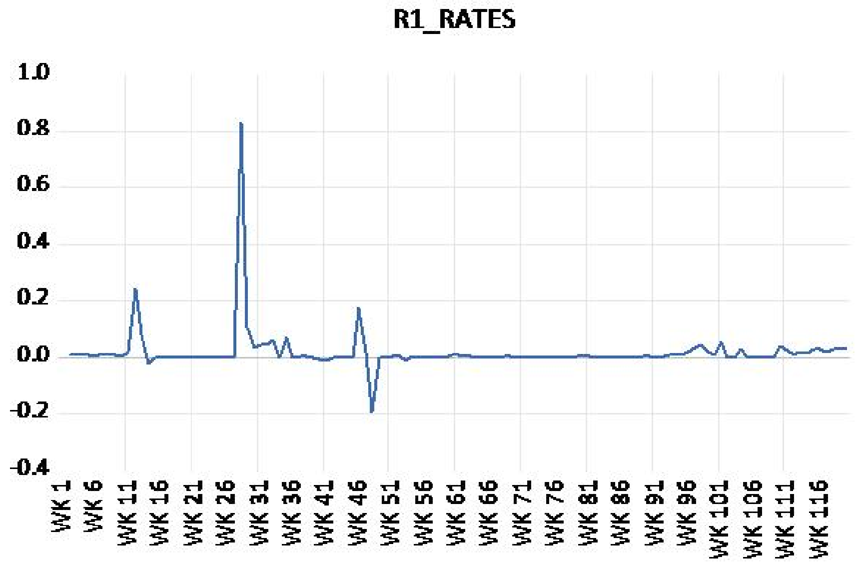
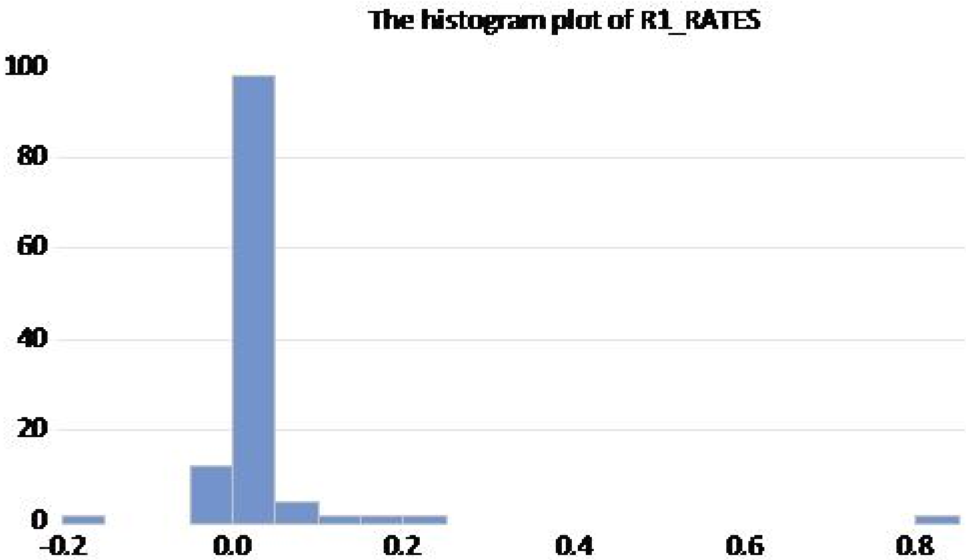


Figure 4.10 Return Series graph

As the graph Figure 4.10 illustrates past high values followed by current high values and past small values followed by current small values,
The effect of volatility clustering holds and is evident. We can also see mean reversion is evident at 0.0.

4.2.3 Fat Tails

It is factually known that the distributions of high frequency financial data have fatter tails, that is they are leptokurtic, than the normal distribution. A fat-tailed distribution is a probability distribution that exhibits a large skewness or kurtosis, indicated with a kurtosis greater than 3. The GARCH model will easily replicate fat tails in financial time series.



Series: R1_RATES	
Sample 1 150 IF RATES< =142.4237	
Observations 119	
Mean	0.017945
Median	0.001066
Maximum	0.830440
Minimum	-0.192203
Std. Dev.	0.083846
Skewness	7.848449
Kurtosis	76.28341
Jarque-Bera	27850.22
Probability	0.000000

Figure 4.11 Return Series Histogram graph

The histogram Figure 4.10 clearly shows the present of fat tails which is a clear feature of financial time series, that is leptokurtic, also evidence of ARCH effects. The value of kurtosis is 76.28341 which is far greater than the kurtosis value of 3 exceptional for normal distribution.

4.2.4 Volatility mean reversion

Though volatility maybe experienced in financial markets, it is expected that in the long run the economy will revert to ‘calmness’. The mean reversion is evident from the line plot as the volatility of returns escalate around the mean zero (0.0).
From the estimation of GARCH (1,1) the long run variance (Roman Kozhan ,2010), is computed as

$$\frac{\varphi}{1-\theta_1-b_1}$$

4.2.5 Test For Arch Effects

We are going to test for the possible presence of ARCH effect to know whether the model requires ARCH estimation method or OLS. If there is no ARCH effect then there is no need to evaluate the ARCH model. But if the test shows the presence of ARCH effects, then we continue with GARCH modeling.

ARCH LM Test

It is a test used to test for ARCH effects by regressing the squared errors on its intervals. The null hypothesis is that the lag regression coefficient $b_1 = 0$ (homoscedasticity), there is no ARCH effect, and the opposite alternative hypothesis $b_1 \neq 0$ (heteroscedasticity) holds. The results of the ARCH effect test Table 4.6 show that there is an ARCH effect in the parameter since the coefficient $b_1 \neq 0$. This is evident from the results of the ARCH LM test as well as the results of the auto correlation (AC) and Q statistics as presented by the results. The ARCH LM test showed rejection of the null hypothesis, and the coefficients were not zero indicating rejection of the 'no ARCH effect' hypothesis. Now, after that, we proceed to apply the GARCH model.

Table 4.6 ARCH LM Test

Heteroskedasticity Test: ARCH

F-statistic	0.018632	Prob. F(1,135)	0.8916
Obs*R-squared	0.018906	Prob. Chi-Square(1)	0.8906

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 11/14/22 Time: 09:58
Sample: 1 150 IF RATES<=494.9883
Included observations: 137

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.007173	0.004955	1.447623	0.1500
RESID^2(-1)	-0.011747	0.086060	-0.136501	0.8916

R-squared	0.000138	Mean dependent var	0.007090
Adjusted R-squared	-0.007268	S.D. dependent var	0.057349
S.E. of regression	0.057557	Akaike info criterion	-2.857600
Sum squared resid	0.447225	Schwarz criterion	-2.814972
Log likelihood	197.7456	Hannan-Quinn criter.	-2.840277
F-statistic	0.018632	Durbin-Watson stat	2.000271
Prob(F-statistic)	0.891629		

4.2.6 Estimation of the GARCH (1,1) model

The GARCH modeling approach is an original way of modeling assumed heteroskedasticity over time, which is some form of process heterogeneity (making the process non-stationary) as an observable characteristic that arises from the presence of a memory for the process, essentially involving stationarity at the unconditional level. In other words, we took the two main antagonists in stochastic process analysis (contrast and memory) and used one to reverse the other, and in effect it is a directed strategy. When modeling GARCH models, parsimony is a key concept. In our case, we retain the assumption of GARCH (1,1) GARCH (2,1) GARCH (2,2) GARCH (1,2) in the return exchange rate series

Table 4.7 Estimation of Parsimonious GARCH(p,q) models

MODELS	GARCH(1,1)	Prob	GARCH(2,1)	Prob	GARCH(2,2)	Prob	GARCH(1,2)	Prob
C	0.016431	0.6247	0.015973	0.6133	0.016337	0.6229	0.020603	0.4837
D(log(RATES(-1)))	0.107192	0.9060	0.128885	0.8872	0.126051	0.6682	0.096128	0.9016
Variance Equation								
C	0.004381	0.3502	0.004732	0.4205	0.004688	0.6045	0.004333	0.5001
RESID(-1)^2	-0.016021	0.0000	0.025938	0.7678	0.020737	0.7688	-0.382332	0.7022
RESID(-2)^2			-0.028830	0.5984	-0.024389	0.7822		
GARCH(-1)	0.587529	0.1859	0.528567	0.3712	0.476478	0.8228	0.523018	0.7452
GARCH(-2)					0.036468	0.9806	0.034447	0.9784

Table 4.7 the results of GARCH (1,1) GARCH (2,1) GARCH (2,2) GARCH (1,2). Remember, our concept of parsimony, over-parameterize affects model ability to forecast better results. As the results shows negative values on the coefficients and their statistical insignificance, the model is expected to behave poorly. Considering the results, we have only a choice to choose the GARCH (1,1) which is better though poor.

Given the GARCH (1,1) model

The sum of the coefficients must be less than 1 for stationarity to hold and vice versa

Table 4.8 Model Selection Criterion

	GARCH (1,1)	GARCH (2,1)	GARCH (1,2)	GARCH (2,2)
AIC	-2.106731	-2.031889	-2.045181	-2.025939
BIC	-1.989329	-1.891007	-1.904299	-1.861576

The results provided in **Table 4.08**. indicate that the GARCH (1,1) model proved to be the best model supported by data. Also due to the concept of parsimonious we opt for GARCH (1,1) than the competing

In our decision criteria, we choose a model with the lowest AIC and BIC. According to the results above, GARCH (1,1) is the best model.

4.2.7 GARCH Models and Diagnostics

I Gusti Ngurah Agung (2009) In statistical theory, all the simple ARCH/GARCH presented are theoretically acceptable or good models. However, their statistical results depend heavily on the data available to the researchers. Based on any of these models, it is possible to achieve false or unrealistic convergence near a single matrix. This means that your data is very accurate in predicting your results.

The preferred model must have;

1. Least number of parameters
2. Significant ARCH and GARCH coefficients
3. High adjusted R2
4. High log-likelihood ratio
5. Lowest SBIC
6. No heteroscedasticity
7. No auto correlations

Its not likely that a model can pass all the conditions and so a trade off must be made

Table4.9 GARCH Model Error Constructs

	GARCH (1,1) Gaussian Normal Distribution		**GARCH (1,1) Generalized Error Distribution**		**GARCH (1,1) Student t's Distribution**	
	Value	P-value	Value	P-value	Value	P-value
Mean equation						
C	0.016431	0.6247	0.000199	0.0000	0.007754	0.7406
	0.107192	0.9060	0.018938	0.0000	0.123203	0.7124
Variance equation						
C	0.004381	0.3502	0.000477	0.0046	0.004467	0.5382
RESID (-1) ^2	-0.016021	0.0000	12.26906	0.2293	-0.010962	0.7098
GARCH (-1)	0.587529	0.1859	0.002646	0.0000	0.400234	0.6850
AIC	-2.106731		-6.299215		-2.659348	
BIC	-1.989329		-6.158332		-2.518466	
H&Q	-2.059062		-6.242012		-2.602146	

ARCH-LM Test (lag 1)	No heteroscedasticity		No heteroscedasticity	No heteroscedasticity
ARCH-LM Test (lag 36)	No heteroscedasticity		No heteroscedasticity	No heteroscedasticity
Correlation Q-test	No serial correlation		No serial correlation	No serial correlation

Results of GARCH (1,1) Gaussian Normal Distribution

As shown in Table 4.9. , the results of the GARCH(1,1) Gaussian Normal Distribution indicates coefficient is very significant but the ARCH coefficient is not very significant. The overall addition of the coefficient is positive and significant. Now we want to know if the model passes the residual test by testing for heteroscedasticity. Residual diagnostics was performed at lag 1 and at lag 36. As the F-statistic Obs*R-squared Clearly shows there is no heteroscedasticity at different lag values. After residual diagnostics, serial correlation test was conducted to check if there is serial correlation on the residuals and squared residuals. Since the probabilities were not significant at 5% confidence level, there was no serial correlation on both residuals and squared residuals, hence the model has passed heteroscedasticity test.

Results of GARCH (1,1) Student t's Distribution.

GARCH (1,1) Results under student t distribution show insignificant coefficients which mean the model can be expected to behave poorly. Also, the model passed the tests for heteroscedasticity and auto correlation but it has AIC, BIC and H&Q bigger than that of GED error construct, so the model cannot be the best.

Results of GARCH (1,1) Generalized Error Distribution

As the F-statistic Obs*R-squared Clearly shows there is no heteroscedasticity at different lag values.
Test for serial correlation. Again, since the probabilities are not significant at 5% confidence level, there is no serial correlation of the residuals squared residuals, hence the model has passed heteroscedasticity and serial correlation test. AIC, BIC and H&Q has the lowest values, so the model can be desirable

Table 4.10 Selection Criteria Table

	Normal Distribution	**GED**	**Student t's**
Significant coefficients	Not All	**Not All**	Not Sig
ARCH Significant?	yes	**No**	No
GARCH Significant?	No	**yes**	No
Likelihood	129.2971	**377.6537**	162.9015
Adj R2	0.0040009	**-0.032005**	-0.005468

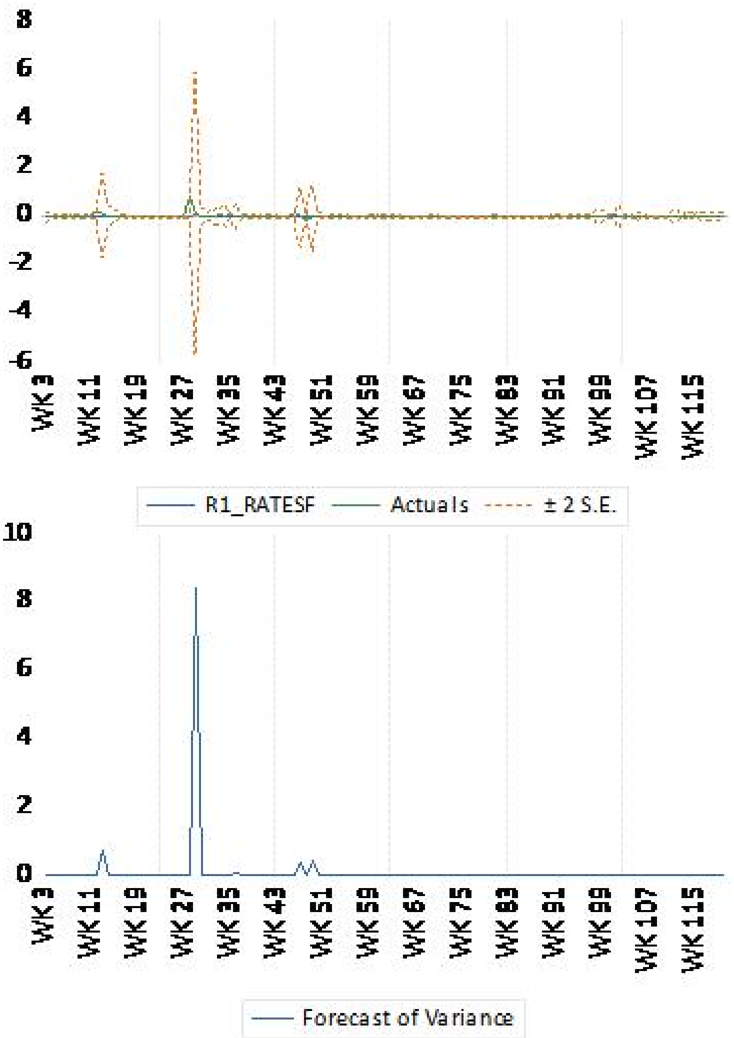
Schwarts IC	-1.989329	**-6.158332**	-2.518466
Heteroscedasticity	NO	**NO**	NO
Auto correlation	NO	**NO**	NO

The student t distribution could not be computed due to insignificance of GARCH (1,1) coefficients. Now, using our selection criteria as stated above the ideal model is GED.

4.2.8Forecasting with GARCH (1,1) (GED)

Forecasting is important because predicting future events is used for decision making in many organizations (Bowerman, et al. 2005). One of the objectives of this study is to calculate forecasts of exchange rate fluctuations. The figure is shown in Fig 4.11. The sample estimate of exchange rate volatility from GARCH (1,1) (GED) is 150 weeks.

In-sample Forecasting (wk 1 - wk 120) ZWL/USD

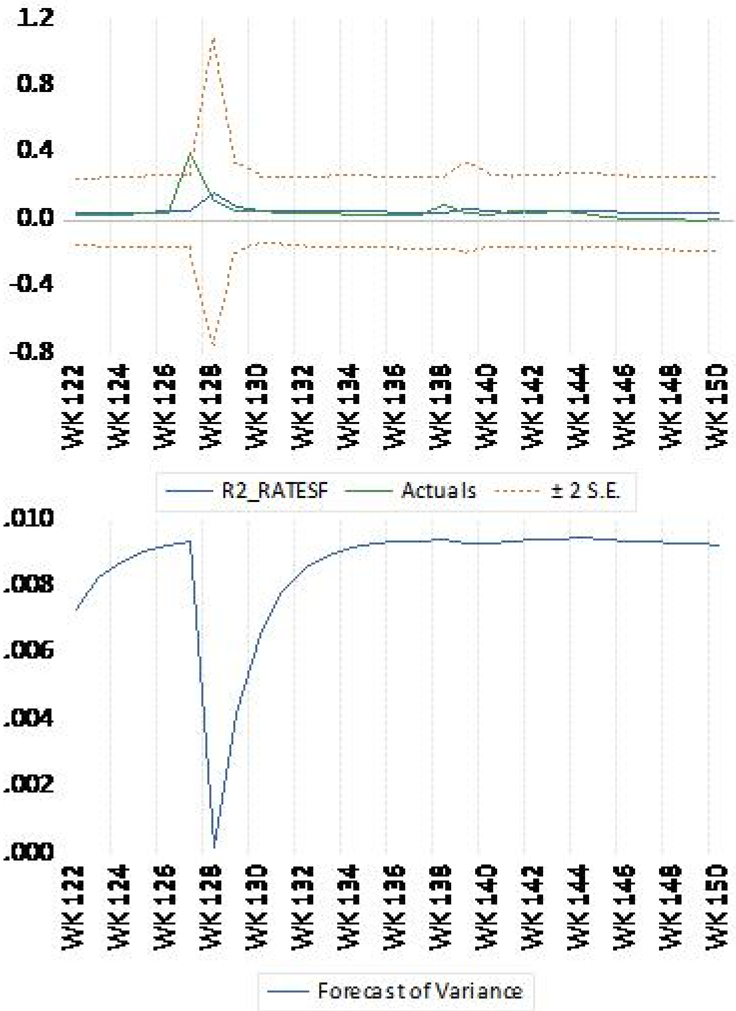


Forecast: R1_RATESF	
Actual: R1_RATES	
Forecast sample: 1 150 IF RATES<=1...	
Adjusted sample: 1 150 IF RATES<=1...	
Included observations: 118	
Root Mean Squared Error	0.084805
Mean Absolute Error	0.020889
Mean Abs. Percent Error	NA
Theil Inequality Coef.	0.889261
Bias Proportion	0.034932
Variance Proportion	0.771398
Covariance Proportion	0.193669
Theil U2 Coefficient	NA
Symmetric MAPE	158.0177

Figure 4.12 In- Sample forecast graph

Figure 4.12 shows how the forecast values of exchange rates behave; the graph shows that the forecasts for the volatility of exchange rates in most cases are lower than the actual values. In the last weeks of 35 to 120 including the week of November, it can be seen that the rate of return of the exchange rate is very low, which can be good news for Zimbabwe as the weak exchange rate makes the currency more attractive. type of investment Attractive and increased demand for money (Mirchandandi, 2013). It can be concluded that these forecasts are valid because, if we compare these forecasts with previous exchange rate movements in Zimbabwe, the actual values of the exchange rate differ slightly from the predicted values due to the shocks or news entering the market. A low exchange rate return in November 2022, recommend the government to continue monitoring the financial system and monetary policies.

Out-of-sample Forecasting Results



Forecast: R2_RATESF	
Actual: R2_RATES	
Forecast sample: 1 150 IF RATES>14...	
Adjusted sample: 1 150 IF RATES>14...	
Included observations: 29	
Root Mean Squared Error	0.068974
Mean Absolute Error	0.032733
Mean Abs. Percent Error	201.7422
Theil Inequality Coef.	0.470348
Bias Proportion	0.005502
Variance Proportion	0.491304
Covariance Proportion	0.503194
Theil U2 Coefficient	2.655292
Symmetric MAPE	60.93675

Figure 4.13 Return Series graph

As illustrated in Figure 4.13, the forecast graph showed good performance with almost exact forecast values as compared to the actual. Only between week 126 and 130, the forecast graph deviated from the actual by predicting low values than the actual. As we discussed above, low returns mean that the value of money is converging, which is good news for the government. Along with the IMF Group, the IMF mission notes the authorities' efforts to stabilize the local foreign exchange market and reduce inflation. And the parallel market has played an important role in narrowing the exchange gap. These predictions are great because we can actually use them to relate what's happening on the ground.

4.2.9 Comparative Schedule of Adjusted ARIMA (16,0,20) Vs GARCH (1,1) with GED error construct

Since both models were very good in predicting exchange rate returns, now the question is how good and which model managed to do it the best way. From our comparative schedule below Table 4.10, We are going to evaluate the performance of the two models using AIC, BIC, MAE, RMSE and Symmetric MAPE. A best model is the one the minimize the errors or simply the difference between the actual value and the forecasts.

Table 4.11 In-Sample Forecast Comparative Schedule

	AIC	BIC	MAE	RMSE	Symmetric MAPE
ARIMA(16,20)	-2.169362	-2.052592	0.026116	0.080795	145.8690
GARCH (1,1)	-6.299215	-6.158332	0.020889	0.084805	158.0177

Table 4.12 Out-sample forecast Comparative Schedule

	AIC	BIC	MAE	RMSE	Symmetric MAPE
ARIMA(16,20)	-2.169362	-2.052592	0.032496	0.085358	100.2585
GARCH (1,1)	-6.299215	-6.158332	0.032733	0.068974	60.93675

From the results above, it can be seen that GARCH (1,1) had the minimum value of AIC, BIC, MAE, RMSE and Symmetric MAPE on both forecasts. This makes GARCH model the best in predicting exchange rate volatility.

4.3 Chapter Summary

Chapter 4 presents the results obtained using E-views 12 Student Lite Version. The purpose of this chapter is to provide results on exchange rate movements in the Zimbabwean context. Data are investigated using formal and informal tests for stability. The data is transformed into a returned series, which is recommended mainly because of its attractive statistical properties. Stationarity is then checked using the formal ADF test and the data were stationary at the 5% significant level. Using the ARIMA method, Adjusted ARMA (16,20) was used after examining the Box-Jenkins model procedure for model identification, model estimation, model evaluation, and model prediction. Similarly, for a GARCH approach, GARCH (1,1) GED was used after observing a similar approach and some tests for rainbow effects and some error constructions. In our comparative analysis, the GARCH (1,1) GED model proved to be the best model as it produced the smallest AIC and BIC values compared to ARIMA (16,20). Exchange rate fluctuations and their results show convergence. This means that policies and other measures taken by the government, such as the issuance of gold coins, can help reduce inflation and exchange rate volatility. Finally, the MSE and Symmetric MAPE prediction accuracy test was applied and the accuracy test preferred GARCH (1,1) GED because the model has the smallest error compared to ARIMA (16,20). The next chapter presents a summary of the studies and recommendations. which is one of the aims of education.

CHAPTER 5
CONCLUSION

5.0 Introduction

This chapter is a concluding summary of the research paper, the comparative analysis of GARCH and ARIMA models in forecasting the exchange rate volatility of ZWL/USD. The research findings have provided the author with an in-depth understanding and interesting facts of modeling financial time series data. In this chapter conclusions are drawn from the comparative schedule created in previous chapter where the performance of the models was being evaluated and analyzed. Recommendations regarding Time series modeling and ZWL/USD exchange forecasting are suggested at the end of the chapter. This research paper provides useful insights into the world of business and finance, insights of risks imposed with high volatile financial data. Despite the great impressive findings, the study faced some constrains that will also be discussed in this chapter.

5.2 Summary Of Findings

The research study was a comparative analysis of GARCH and ARIMA models in forecasting exchange rate volatility in Zimbabwe. As explained in chapter 1, the worst exchange rates in Zimbabwe were recorded periods of hyperinflation and economic instability. The research covers the weekly exchange rates of the current period from January 2019 to November 2022. A comparative analysis was conducted so as to identify between these two famous models of all time, which model performs better and which can be recommended for future use by Policy makers. After analyzing the exchange rates and making some predictions, it was observed that the Zimbabwean foreign exchange market is influenced a lot by shocks and news. The news coming from short term priorities and the data characterized with leverage effect, surprisingly, could not be captured by Symmetric GARCH models and requires the GARCH (1,1). The model seemed poor but had the best results as compared with ARIMA (16,20). The ARIMA model performed well enough but could not stand the ground with the GARCH model both In-Sample and Out-Of-Sample forecasts.

In common practice, GARCH (1,1) is most commonly used to predict volatility because, due to its economical concept, it is the most ideal model for representing an infinite ARCH process. On the other hand, using ARIMA (16,20), the GARCH (1,1) model was shown to perform best in the short and long term, as indicated by RMSE, symmetric MAPE, and MAE error value comparisons. Although ARIMA (16,20) is competitive, GARCH is strongest when it comes to forecasting financial time series data. These results are consistent with Bhardwaj et al. (2014) research comparing ARIMA and GARCH model in forecasting Agricultural prices. Using performance evaluation criteria, RMSE, MAE and MAPE, The GARCH (1,1) was found to be a better model in forecasting spot price of Gram. The value of RMSE, MAE and MAPE obtained were smaller than those in ARIMA (0,1,1) model. The AIC and BIC value from GARCH model were smaller than that from ARIMA model. Therefore, it shows that GARCH is better than ARIMA for estimating daily price of Gram.

5.3 Conclusions

In Econometrics and Financial Mathematics, modeling of time series data do not only provides insights into the future, but also helps to avoid large trading losses, optimize profits, and manage risks. So much information can be extracted from the data using several statistical methods under the philosophy of "let the data speak for itself". However, the problem arises when these data (time series) are highly volatile and have characteristics of heteroscedasticity. It is very difficult to analyze and predict from this data. In finance, volatility is a measure of risk and its estimation is fundamental to forecasting future dynamics. As evident from the literature, heteroscedasticity is the main focus, understanding heteroscedasticity means understanding variability. Various models have been proposed and compared since the great depression of the 1970.

Now we conclude by proposing GARCH model as a better predictor of exchange rate volatility as compared to ARIMA model. Clearly the results shows that GARCH Model can perform better under both in-sample and out-of-sample time horizons. The ability of a GARCH model in dealing with heteroscedasticity or randomness in financial data was very convincing.

5.3 Recommendations

Foreign exchange market participants, investors and Reserve banks may consider the GED GARCH (1,1) model to forecast exchange rate movements as it has proven to be highly predictive. strong guess. Thanks to its parsimonious concept, the model is simple and easy to interpret. The model has also received several recommendations from several researchers, for example Mattei (2009) and Brooks (2008) support that the model provides quality results compared to other models.

- Based on the dynamics of the Forex market in Zimbabwe, the study recommends the reintroduction of the fixed exchange rate system. Under the fixed exchange rate system, investors can be very sure of their future capital preservation, and the return on capital is not affected by exchange rate fluctuations or inflation. As exchange rate volatility imposes risk on investment and volatility in other economic indicators, the risk involves foreign investment and destroys the food security economy of the Development Committee of Southern Africa, Zimbabwe. Nischith (2013) points out that apart from a fixed exchange rate, there will be no volatility thereby minimizing risk.
- A fixed exchange rate allows effective policy implementation to control exchange rate and promote international trade. The government should promote research by funding researchers and academics to

investigate other GARCH family models and do more comparative analysis. This will ensure continuous monitoring of high inflation rates and interests.

- Abandonment of United State dollar as a trading currency in Zimbabwe will promote international trade since local production in USD implies USD marked prices and this is big problem when trading with countries like South Africa as Zimbabwean products will tend to be more expensive than South African. In this case, more South African products are imported and less Zimbabwean products are exported making balance of payment deficit
- Vieira et al 2012, posit that one of the downsides of dollarization is the increased volatility and susceptibility of the country to external shocks. Del Cristo & Gomez-Puig (2016) concur with this view when they reviewed the Ecuadorian economy. They explained who the economy could not managed the shocks by implementing its own monetary policies. A recommended suggestion is the Rand Monetary Union as it provides a soft peg and allows regional trade since some southern countries are using the rand.

5.4 Further Research

The study is not a conclusive one, rather it created a platform for further future research. The study stretched focus on the weekly exchange rates other researchers can analyze the daily or monthly or even increasing the sample to check if they can get the same results. The study further suggest estimation of other GARCH family to identify the other best performing extensions of GARCH model.

5.5 Chapter Summary

This chapter was result oriented thus provided us with research conclusions and suggested recommendations. On the analytical comparison of GARCH and ARIMA models it was shown that GARCH was a better model in forecasting exchange rate volatility in Zimbabwe. The study had supportive reviews from several authors and confidentially we can conclude our research paper by proposing GARCH model for forecasting financial data like exchange rates. On recommending research findings, adopting a Rand Monetary policing can help to stabilize the Zimbabwean dollar, hence exchange rates. Not limited to this study are other possible research areas such as a comparative analysis of the GARCH model with other GARCH Family models.

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APPENDICES

Appendix A

Correlogram exchange rate stationarity Test

Date: 11/28/22 Time: 07:52
Sample: 1 150 IF RATES<=142.4237
Included observations: 120

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.960	0.960	113.41	0.000
		2	0.919	-0.030	218.31	0.000
		3	0.881	0.012	315.52	0.000
		4	0.843	-0.021	405.28	0.000
		5	0.805	-0.024	487.76	0.000
		6	0.766	-0.022	563.20	0.000
		7	0.729	-0.008	632.06	0.000
		8	0.691	-0.035	694.41	0.000
		9	0.653	-0.012	750.66	0.000
		10	0.615	-0.023	801.06	0.000
		11	0.579	-0.007	846.10	0.000
		12	0.546	0.025	886.58	0.000
		13	0.515	-0.012	922.81	0.000
		14	0.483	-0.020	955.01	0.000
		15	0.450	-0.031	983.25	0.000
		16	0.417	-0.034	1007.7	0.000
		17	0.381	-0.047	1028.3	0.000
		18	0.346	-0.022	1045.6	0.000
		19	0.307	-0.087	1059.2	0.000
		20	0.267	-0.033	1069.6	0.000
		21	0.234	0.056	1077.6	0.000
		22	0.200	-0.029	1083.6	0.000
		23	0.168	-0.014	1087.9	0.000
		24	0.137	-0.006	1090.7	0.000
		25	0.107	-0.015	1092.5	0.000
		26	0.078	-0.016	1093.4	0.000
		27	0.049	-0.025	1093.8	0.000
		28	0.036	0.171	1094.0	0.000
		29	0.026	0.022	1094.1	0.000
		30	0.017	0.002	1094.2	0.000
		31	0.009	0.005	1094.2	0.000
		32	0.003	0.015	1094.2	0.000
		33	-0.001	0.016	1094.2	0.000
		34	-0.005	-0.012	1094.2	0.000
		35	-0.008	0.014	1094.2	0.000
		36	-0.010	-0.009	1094.2	0.000

Correlogram (level) Stationarity Test of exchange rates return series

Date: 11/12/22 Time: 12:44
Sample: 1 150 IF RATES<=494.9883
Included observations: 139

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.149	0.149	3.1415	0.076
		2	0.007	-0.015	3.1487	0.207
		3	0.045	0.048	3.4454	0.328
		4	0.038	0.025	3.6533	0.455
		5	0.045	0.037	3.9487	0.557
		6	-0.006	-0.020	3.9542	0.683
		7	0.045	0.049	4.2608	0.749
		8	-0.014	-0.033	4.2888	0.830
		9	-0.011	-0.004	4.3058	0.890
		10	-0.013	-0.017	4.3330	0.931
		11	0.019	0.025	4.3860	0.957
		12	-0.015	-0.026	4.4229	0.975
		13	-0.034	-0.023	4.6072	0.983
		14	-0.033	-0.029	4.7772	0.989
		15	0.038	0.053	5.0108	0.992
		16	0.186	0.179	10.528	0.838
		17	0.038	-0.007	10.757	0.869
		18	0.128	0.133	13.431	0.765
		19	-0.017	-0.074	13.476	0.813
		20	-0.172	-0.183	18.358	0.564
		21	-0.012	0.012	18.383	0.625
		22	-0.024	-0.040	18.484	0.677
		23	0.001	0.003	18.484	0.731
		24	-0.019	0.013	18.545	0.776
		25	-0.036	-0.027	18.762	0.808
		26	-0.009	0.010	18.775	0.846
		27	-0.010	0.005	18.792	0.877
		28	-0.008	-0.002	18.802	0.904
		29	-0.001	0.015	18.803	0.926
		30	0.001	0.016	18.803	0.944
		31	-0.008	-0.012	18.814	0.958
		32	-0.008	-0.045	18.826	0.969
		33	0.009	-0.013	18.840	0.977
		34	0.042	-0.023	19.164	0.981
		35	-0.023	-0.006	19.265	0.986
		36	-0.064	-0.007	20.046	0.985

ADF non stationary level (constant)

Null Hypothesis: RATES SERIES has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.451827	0.8953
Test critical values:	1% level	-3.486551	
	5% level	-2.886074	
	10% level	-2.579931	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RATES SERIES)
Method: Least Squares
Date: 11/28/22 Time: 08:03
Sample: 1 150 IF RATES<=142.4237
Included observations: 118

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RATES SERIES(-1)	-0.005231	0.011577	-0.451827	0.6522
D(RATES SERIES(-1))	0.146325	0.092791	1.576928	0.1176
C	1.306881	0.941289	1.388395	0.1677

R-squared	0.022012	Mean dependent var	1.063368
Adjusted R-squared	0.005003	S.D. dependent var	3.935923
S.E. of regression	3.926064	Akaike info criterion	5.598247
Sum squared resid	1772.608	Schwarz criterion	5.668688
Log likelihood	-327.2966	Hannan-Quinn criter.	5.626848
F-statistic	1.294173	Durbin-Watson stat	1.964975
Prob(F-statistic)	0.278086		

ADF non stationary level (constant/trend)

Null Hypothesis: RATES SERIES has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.586324	0.7927
Test critical values:	1% level	-4.037668	
	5% level	-3.448348	
	10% level	-3.149326	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RATES SERIES)
Method: Least Squares
Date: 11/28/22 Time: 08:14
Sample: 1 150 IF RATES<=142.4237
Included observations: 118

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RATES SERIES(-1)	-0.040994	0.025842	-1.586324	0.1154
D(RATES SERIES(-1))	0.164236	0.092961	1.766725	0.0800
C	1.767114	0.981894	1.799699	0.0746
@TREND("1")	0.036616	0.023690	1.545614	0.1250

R-squared	0.042085	Mean dependent var	1.063368
Adjusted R-squared	0.016877	S.D. dependent var	3.935923
S.E. of regression	3.902568	Akaike info criterion	5.594457
Sum squared resid	1736.224	Schwarz criterion	5.688379
Log likelihood	-326.0730	Hannan-Quinn criter.	5.632592
F-statistic	1.669509	Durbin-Watson stat	1.966993
Prob(F-statistic)	0.177526		

Appendix B

ADF non stationary level (None)

Null Hypothesis: RATES SERIES has a unit root
Exogenous: None
Lag Length: 1 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		2.063247	0.9906
Test critical values:	1% level	-2.584707	
	5% level	-1.943563	
	10% level	-1.614927	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RATES SERIES)
Method: Least Squares
Date: 11/28/22 Time: 08:16
Sample: 1 150 IF RATES<=142.4237
Included observations: 118

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RATES SERIES(-1)	0.009521	0.004615	2.063247	0.0413
D(RATES SERIES(-1))	0.149232	0.093137	1.602276	0.1118

R-squared	0.005619	Mean dependent var	1.063368
Adjusted R-squared	-0.002953	S.D. dependent var	3.935923
S.E. of regression	3.941731	Akaike info criterion	5.597921
Sum squared resid	1802.320	Schwarz criterion	5.644882
Log likelihood	-328.2773	Hannan-Quinn criter.	5.616988
Durbin-Watson stat	1.966829		

ADF Stationary returns series (constant)

Null Hypothesis: R1 RATES has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.623844	0.0000
Test critical values:	1% level	-3.486551	
	5% level	-2.886074	
	10% level	-2.579931	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(R1 RATES)
Method: Least Squares
Date: 11/28/22 Time: 08:28
Sample: 1 150 IF RATES<=142.4237
Included observations: 118

Variable	Coefficient	Std. Error	t-Statistic	Prob.
R1 RATES(-1)	-0.887935	0.092264	-9.623844	0.0000
C	0.016041	0.007908	2.028316	0.0448

R-squared	0.443961	Mean dependent var	0.000198
Adjusted R-squared	0.439167	S.D. dependent var	0.112201
S.E. of regression	0.084026	Akaike info criterion	-2.098573
Sum squared resid	0.819006	Schwarz criterion	-2.051613
Log likelihood	125.8158	Hannan-Quinn criter.	-2.079506
F-statistic	92.61838	Durbin-Watson stat	1.990469
Prob(F-statistic)	0.000000		

ADF Stationary returns series (constant/trend)

Null Hypothesis: R1 RATES has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.675661	0.0000
Test critical values:	1% level	-4.037668	
	5% level	-3.448348	
	10% level	-3.149326	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(R1 RATES)
Method: Least Squares
Date: 11/28/22 Time: 08:29
Sample: 1 150 IF RATES<=142.4237
Included observations: 118

Variable	Coefficient	Std. Error	t-Statistic	Prob.
R1 RATES(-1)	-0.897540	0.092763	-9.675661	0.0000
C	0.030026	0.016066	1.868909	0.0642
@TREND("1")	-0.000228	0.000228	-1.000013	0.3194

R-squared	0.448754	Mean dependent var	0.000198
Adjusted R-squared	0.439167	S.D. dependent var	0.112201
S.E. of regression	0.084026	Akaike info criterion	-2.090283
Sum squared resid	0.811945	Schwarz criterion	-2.019841
Log likelihood	126.3267	Hannan-Quinn criter.	-2.061681
F-statistic	46.80921	Durbin-Watson stat	1.989407
Prob(F-statistic)	0.000000		

ADF Stationary returns series (None)

Null Hypothesis: R1 RATES has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on AIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.284956	0.0000
Test critical values:	1% level	-2.584707	
	5% level	-1.943563	
	10% level	-1.614927	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(R1 RATES)
Method: Least Squares
Date: 11/28/22 Time: 08:25
Sample: 1 150 IF RATES<=142.4237
Included observations: 118

Variable	Coefficient	Std. Error	t-Statistic	Prob.
R1 RATES(-1)	-0.848981	0.091436	-9.284956	0.0000

R-squared	0.424240	Mean dependent var	0.000198
Adjusted R-squared	0.424240	S.D. dependent var	0.112201
S.E. of regression	0.085137	Akaike info criterion	-2.080671
Sum squared resid	0.848053	Schwarz criterion	-2.057190
Log likelihood	123.7596	Hannan-Quinn criter.	-2.071137
Durbin-Watson stat	1.996980		

Appendix c

Return series correlogram

Date: 11/28/22 Time: 08:19
Sample: 1 150 IF RATES<=142.4237
Included observations: 119

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.112	0.112	1.5319	0.216
		2	-0.029	-0.042	1.6381	0.441
		3	0.027	0.036	1.7300	0.630
		4	0.023	0.015	1.7969	0.773
		5	0.033	0.031	1.9332	0.858
		6	-0.025	-0.032	2.0131	0.918
		7	0.037	0.046	2.1900	0.949
		8	-0.033	-0.049	2.3339	0.969
		9	-0.029	-0.015	2.4421	0.982
		10	-0.034	-0.035	2.5904	0.989
		11	-0.022	-0.012	2.6523	0.995
		12	-0.040	-0.041	2.8626	0.996
		13	-0.050	-0.034	3.1987	0.997
		14	-0.043	-0.038	3.4514	0.998
		15	0.046	0.061	3.7459	0.998
		16	0.220	0.213	10.496	0.839
		17	0.036	-0.001	10.678	0.873
		18	0.160	0.184	14.306	0.709
		19	-0.011	-0.067	14.322	0.765
		20	-0.196	-0.207	19.898	0.464
		21	-0.003	0.005	19.899	0.528
		22	-0.018	-0.054	19.947	0.586
		23	0.001	-0.014	19.947	0.645
		24	-0.013	0.032	19.973	0.698
		25	-0.031	-0.023	20.117	0.741
		26	-0.022	0.005	20.190	0.782
		27	-0.015	0.031	20.226	0.821
		28	-0.016	-0.006	20.266	0.855
		29	-0.017	0.015	20.311	0.883
		30	-0.010	0.011	20.327	0.908
		31	-0.013	-0.042	20.357	0.928
		32	-0.010	-0.069	20.373	0.944
		33	0.013	-0.026	20.401	0.958
		34	0.051	-0.054	20.845	0.963
		35	-0.021	0.006	20.920	0.971
		36	-0.068	0.002	21.733	0.971

Tentative ARMA(16,16)

Dependent Variable: R1 RATES
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/28/22 Time: 08:50
Sample: 2 120
Included observations: 119
Convergence achieved after 110 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.017919	0.021384	0.837952	0.4038
AR(16)	0.082619	2.026354	0.040772	0.9675
MA(16)	0.126317	2.156605	0.058572	0.9534
SIGMASQ	0.006611	0.000337	19.64062	0.0000

R-squared	0.051665	Mean dependent var	0.017945
Adjusted R-squared	0.026926	S.D. dependent var	0.083846
S.E. of regression	0.082709	Akaike info criterion	-2.108065
Sum squared resid	0.786690	Schwarz criterion	-2.014649
Log likelihood	129.4299	Hannan-Quinn criter.	-2.070132
F-statistic	2.088409	Durbin-Watson stat	1.790882
Prob(F-statistic)	0.105600		

Inverted AR Roots	.86	.79+.33i	.79-.33i	.61+.61i
	.61+.61i	.33-.79i	.33+.79i	-.00+.86i
	-.00-.86i	-.33-.79i	-.33+.79i	-.61+.61i
	-.61+.61i	-.79-.33i	-.79+.33i	-.86
Inverted MA Roots	.86+.17i	.86-.17i	.73-.49i	.73+.49i
	.49+.73i	.49-.73i	.17+.86i	.17-.86i
	-.17+.86i	-.17-.86i	-.49-.73i	-.49+.73i
	-.73+.49i	-.73-.49i	-.86-.17i	-.86+.17i

Tentative ARMA(16,20)

Dependent Variable: R1 RATES
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/28/22 Time: 08:59
Sample: 2 120
Included observations: 119
Convergence achieved after 141 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.018137	0.020802	0.871885	0.3851
AR(16)	0.215957	0.046736	4.620806	0.0000
MA(20)	-0.209487	0.133193	-1.572809	0.1185
SIGMASQ	0.006288	0.000312	20.12509	0.0000

R-squared	0.097974	Mean dependent var	0.017945
Adjusted R-squared	0.074443	S.D. dependent var	0.083846
S.E. of regression	0.080664	Akaike info criterion	-2.150041
Sum squared resid	0.748275	Schwarz criterion	-2.056625
Log likelihood	131.9274	Hannan-Quinn criter.	-2.112107
F-statistic	4.163591	Durbin-Watson stat	1.785436
Prob(F-statistic)	0.007702		

Inverted AR Roots	.91	.84-.35i	.84+.35i	.64+.64i
	.64+.64i	.35-.84i	.35+.84i	.00+.91i
	-.00-.91i	-.35-.84i	-.35+.84i	-.64+.64i
	-.64+.64i	-.84+.35i	-.84-.35i	-.91
Inverted MA Roots	.92	.88+.29i	.88-.29i	.75+.54i
	.75-.54i	.54-.75i	.54+.75i	.29-.88i
	.29+.88i	.00-.92i	-.00+.92i	-.29-.88i
	-.29+.88i	-.54+.75i	-.54-.75i	-.75-.54i
	-.75+.54i	-.88-.29i	-.88+.29i	-.92

Tentative ARMA(20,16)

Dependent Variable: R1 RATES
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/28/22 Time: 09:02
Sample: 2 120
Included observations: 119
Convergence achieved after 103 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.018002	0.020705	0.869476	0.3864
AR(20)	-0.182012	0.099816	-1.823473	0.0708
MA(16)	0.216316	0.054179	3.992599	0.0001
SIGMASQ	0.006321	0.000308	20.52271	0.0000

R-squared	0.093184	Mean dependent var	0.017945
Adjusted R-squared	0.069528	S.D. dependent var	0.083846
S.E. of regression	0.080878	Akaike info criterion	-2.146603
Sum squared resid	0.752248	Schwarz criterion	-2.053187
Log likelihood	131.7229	Hannan-Quinn criter.	-2.108670
F-statistic	3.939138	Durbin-Watson stat	1.784953
Prob(F-statistic)	0.010223		

Inverted AR Roots	.91-.14i	.91+.14i	.82+.42i	.82-.42i
	.65+.65i	.65+.65i	.42-.82i	.42+.82i
	.14-.91i	.14+.91i	-.14-.91i	-.14+.91i
	-.42+.82i	-.42-.82i	-.65-.65i	-.65-.65i
	-.82-.42i	-.82+.42i	-.91+.14i	-.91-.14i
Inverted MA Roots	.89+.18i	.89-.18i	.76-.50i	.76+.50i
	.50-.76i	.50+.76i	.18-.89i	.18+.89i
	-.18+.89i	-.18-.89i	-.50-.76i	-.50+.76i
	-.76-.50i	-.76+.50i	-.89-.18i	-.89+.18i

Appendix D

Tentative ARMA(20,20)

Dependent Variable: R1 RATES
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/28/22 Time: 09:03
Sample: 2 120
Included observations: 119
Convergence achieved after 50 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.017507	0.017751	0.986248	0.3261
AR(20)	0.492985	0.735464	0.670305	0.5040
MA(20)	-0.731261	0.726250	-1.006899	0.3161
SIGMASQ	0.006470	0.000918	7.047033	0.0000

R-squared	0.071881	Mean dependent var	0.017945
Adjusted R-squared	0.047669	S.D. dependent var	0.083846
S.E. of regression	0.081823	Akaike info criterion	-2.110939
Sum squared resid	0.769920	Schwarz criterion	-2.017523
Log likelihood	129.6009	Hannan-Quinn criter.	-2.073005
F-statistic	2.968852	Durbin-Watson stat	1.785884
Prob(F-statistic)	0.034866		

Inverted AR Roots	.97	.92+.30i	.92-.30i	.78-.57i
	.78+.57i	.57+.78i	.57-.78i	.30+.92i
	.30-.92i	.00-.97i	-.00+.97i	-.30+.92i
	-.30-.92i	-.57+.78i	-.57-.78i	-.78-.57i
	-.78+.57i	-.92-.30i	-.92+.30i	-.97
Inverted MA Roots	.98	.94-.30i	.94+.30i	.80+.58i
	.80-.58i	.58+.80i	.58-.80i	.30+.94i
	.30-.94i	.00-.98i	-.00+.98i	-.30+.94i
	-.30-.94i	-.58-.80i	-.58+.80i	-.80-.58i
	-.80+.58i	-.94+.30i	-.94-.30i	-.98

Tentative ARMA(16,18)

Dependent Variable: R1 RATES
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/28/22 Time: 09:06
Sample: 2 120
Included observations: 119
Convergence achieved after 127 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.017680	0.023288	0.759178	0.4493
AR(16)	0.207842	0.046936	4.428183	0.0000
MA(18)	0.168072	0.242530	0.692994	0.4897
SIGMASQ	0.006402	0.000257	24.87355	0.0000

R-squared	0.081565	Mean dependent var	0.017945
Adjusted R-squared	0.057605	S.D. dependent var	0.083846
S.E. of regression	0.081395	Akaike info criterion	-2.135704
Sum squared resid	0.761887	Schwarz criterion	-2.042289
Log likelihood	131.0744	Hannan-Quinn criter.	-2.097771
F-statistic	3.404311	Durbin-Watson stat	1.779904
Prob(F-statistic)	0.020098		

Inverted AR Roots	.91	.84-.35i	.84+.35i	.64-.64i
	.64+.64i	.35+.84i	.35-.84i	.00-.91i
	.00+.91i	-.35-.84i	-.35+.84i	-.64+.64i
	-.64-.64i	-.84-.35i	-.84+.35i	-.91
Inverted MA Roots	.89-.16i	.89+.16i	.78-.45i	.78+.45i
	.58+.69i	.58-.69i	.31+.85i	.31-.85i
	.00+.91i	-.00-.91i	-.31+.85i	-.31-.85i
	-.58+.69i	-.58-.69i	-.78-.45i	-.78+.45i
	-.89-.16i	-.89+.16i		

Tentative ARMA(18,16)

Dependent Variable: R1 RATES
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/28/22 Time: 09:08
Sample: 2 120
Included observations: 119
Convergence achieved after 95 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.017570	0.023599	0.744522	0.4581
AR(18)	0.148341	0.199242	0.744524	0.4581
MA(16)	0.211427	0.055659	3.798579	0.0002
SIGMASQ	0.006424	0.000279	23.01164	0.0000

R-squared	0.078533	Mean dependent var	0.017945
Adjusted R-squared	0.054494	S.D. dependent var	0.083846
S.E. of regression	0.081529	Akaike info criterion	-2.133166
Sum squared resid	0.764402	Schwarz criterion	-2.039750
Log likelihood	130.9234	Hannan-Quinn criter.	-2.095233
F-statistic	3.266979	Durbin-Watson stat	1.779332
Prob(F-statistic)	0.023912		

Inverted AR Roots	.90	.85+.31i	.85-.31i	.69-.58i
	.69+.58i	.45-.78i	.45+.78i	.16-.89i
	.16+.89i	-.16-.89i	-.16+.89i	-.45-.78i
	-.45+.78i	-.69+.58i	-.69-.58i	-.85+.31i
	-.85-.31i	-.90		
Inverted MA Roots	.89+.18i	.89-.18i	.75-.50i	.75+.50i
	.50+.75i	.50-.75i	.18+.89i	.18-.89i
	-.18+.89i	-.18-.89i	-.50-.75i	-.50+.75i
	-.75+.50i	-.75-.50i	-.89-.18i	-.89+.18i

Tentative ARMA(18,20)

Dependent Variable: R1 RATES
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/28/22 Time: 09:09
Sample: 2 120
Included observations: 119
Convergence achieved after 42 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.017911	0.023934	0.748382	0.4558
AR(18)	0.162094	0.291943	0.555223	0.5798
MA(20)	-0.223534	0.207682	-1.076326	0.2840
SIGMASQ	0.006433	0.000467	13.77526	0.0000

R-squared	0.077176	Mean dependent var	0.017945
Adjusted R-squared	0.053103	S.D. dependent var	0.083846
S.E. of regression	0.081589	Akaike info criterion	-2.128567
Sum squared resid	0.765528	Schwarz criterion	-2.035151
Log likelihood	130.6498	Hannan-Quinn criter.	-2.090634
F-statistic	3.205840	Durbin-Watson stat	1.768762
Prob(F-statistic)	0.025835		

Inverted AR Roots	.90	.85+.31i	.85-.31i	.69+.58i
	.69-.58i	.45+.78i	.45-.78i	.16-.89i
	.16+.89i	-.16+.89i	-.16-.89i	-.45+.78i
	-.45-.78i	-.69+.58i	-.69-.58i	-.85-.31i
	-.85+.31i	-.90		
Inverted MA Roots	.93	.88+.29i	.88-.29i	.75+.55i
	.75-.55i	.55-.75i	.55+.75i	.29-.88i
	.29+.88i	.00-.93i	.00+.93i	-.29-.88i
	-.29+.88i	-.55-.75i	-.55+.75i	-.75+.55i
	-.75-.55i	-.88-.29i	-.88+.29i	-.93

Appendix E

Tentative ARMA(20,18)

Dependent Variable: R1 RATES
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/28/22 Time: 09:11
Sample: 2 120
Included observations: 119
Convergence achieved after 65 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.017937	0.024387	0.735523	0.4635
AR(20)	-0.196105	0.157494	-1.245158	0.2156
MA(18)	0.209554	0.416532	0.503091	0.6159
SIGMASQ	0.006412	0.000412	15.55129	0.0000

R-squared	0.080172	Mean dependent var	0.017945
Adjusted R-squared	0.056176	S.D. dependent var	0.083846
S.E. of regression	0.081456	Akaike info criterion	-2.131078
Sum squared resid	0.763043	Schwarz criterion	-2.037662
Log likelihood	130.7991	Hannan-Quinn criter.	-2.093144
F-statistic	3.341120	Durbin-Watson stat	1.769355
Prob(F-statistic)	0.021771		

Inverted AR Roots	.91-.14i	.91+.14i	.82+.42i	.82-.42i
	.65-.65i	.65+.65i	.42-.82i	.42+.82i
	.14-.91i	.14+.91i	-.14-.91i	-.14+.91i
	-.42+.82i	-.42-.82i	-.65-.65i	-.65-.65i
	-.82+.42i	-.82-.42i	-.91+.14i	-.91-.14i
Inverted MA Roots	.90+.16i	.90-.16i	.79-.46i	.79+.46i
	.59+.70i	.59-.70i	.31+.86i	.31-.86i
	.00+.92i	-.00-.92i	-.31+.86i	-.31-.86i
	-.59+.70i	-.59-.70i	-.79-.46i	-.79+.46i
	-.90-.16i	-.90+.16i		

Tentative ARMA(18,18)

Dependent Variable: R1 RATES
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/28/22 Time: 09:13
Sample: 2 120
Included observations: 119
Convergence achieved after 61 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.017133	0.021245	0.806454	0.4216
AR(18)	-0.465015	1.289562	-0.360599	0.7191
MA(18)	0.667990	0.943253	0.708176	0.4803
SIGMASQ	0.006610	0.000520	12.71357	0.0000

R-squared	0.051822	Mean dependent var	0.017945
Adjusted R-squared	0.027087	S.D. dependent var	0.083846
S.E. of regression	0.082702	Akaike info criterion	-2.100458
Sum squared resid	0.786560	Schwarz criterion	-2.007042
Log likelihood	128.9773	Hannan-Quinn criter.	-2.062525
F-statistic	2.095098	Durbin-Watson stat	1.768363
Prob(F-statistic)	0.104722		

Inverted AR Roots	.94+.17i	.94-.17i	.83-.48i	.83+.48i
	.62+.73i	.62-.73i	.33+.90i	.33-.90i
	.00+.96i	-.00-.96i	-.33+.90i	-.33-.90i
	-.62+.73i	-.62-.73i	-.83-.48i	-.83+.48i
	-.94-.17i	-.94+.17i		
Inverted MA Roots	.96+.17i	.96-.17i	.85-.49i	.85+.49i
	.63+.75i	.63-.75i	.33+.92i	.33-.92i
	.00+.98i	-.00-.98i	-.33+.92i	-.33-.92i
	-.63+.75i	-.63-.75i	-.85-.49i	-.85+.49i
	-.96-.17i	-.96+.17i		

Correlogram of the residual ARMA(16,20)

Date: 11/28/22 Time: 09:35
Sample: 1 150 IF RATES<=142.4237
Q-statistic probabilities adjusted for 2 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.107	0.107	1.4011	
		2	-0.030	-0.041	1.5084	
		3	0.048	0.057	1.7979	0.180
		4	0.089	0.077	2.7858	0.248
		5	0.045	0.032	3.0458	0.385
		6	-0.029	-0.034	3.1509	0.533
		7	0.037	0.040	3.3279	0.650
		8	-0.041	-0.064	3.5473	0.738
		9	-0.044	-0.034	3.8060	0.802
		10	-0.037	-0.033	3.9831	0.859
		11	-0.040	-0.036	4.1962	0.898
		12	-0.057	-0.046	4.6379	0.914
		13	-0.049	-0.025	4.9638	0.933
		14	-0.041	-0.032	5.1956	0.951
		15	0.033	0.055	5.3477	0.967
		16	0.021	0.023	5.4093	0.979
		17	0.017	0.028	5.4512	0.988
		18	0.172	0.177	9.6671	0.883
		19	0.005	-0.039	9.6710	0.917
		20	-0.012	-0.011	9.6925	0.941
		21	0.012	-0.011	9.7129	0.960
		22	-0.021	-0.071	9.7767	0.972
		23	0.001	-0.011	9.7769	0.982
		24	0.012	0.019	9.7986	0.988
		25	-0.017	-0.040	9.8453	0.992
		26	-0.021	0.013	9.9146	0.995
		27	-0.004	0.016	9.9177	0.997
		28	-0.019	-0.011	9.9726	0.998
		29	-0.015	0.020	10.010	0.999
		30	-0.008	0.010	10.021	0.999
		31	-0.032	-0.030	10.185	1.000
		32	-0.071	-0.053	11.014	0.999
		33	-0.004	-0.017	11.018	1.000
		34	0.010	-0.012	11.033	1.000
		35	-0.013	-0.011	11.064	1.000
		36	-0.028	-0.052	11.199	1.000

Tentative Adjusted ARMA(16,20) +AR(18)

Dependent Variable: R1 RATES
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/28/22 Time: 09:50
Sample: 2 120
Included observations: 119
Convergence achieved after 182 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.018463	0.025467	0.724992	0.4699
AR(16)	0.213164	0.048891	4.359978	0.0000
AR(18)	0.157594	0.277166	0.568592	0.5708
MA(20)	-0.214769	0.132410	-1.621999	0.1076
SIGMASQ	0.006090	0.000313	19.47658	0.0000

R-squared	0.126427	Mean dependent var	0.017945
Adjusted R-squared	0.095775	S.D. dependent var	0.083846
S.E. of regression	0.079729	Akaike info criterion	-2.160896
Sum squared resid	0.724672	Schwarz criterion	-2.044126
Log likelihood	133.5733	Hannan-Quinn criter.	-2.113480
F-statistic	4.124640	Durbin-Watson stat	1.779178
Prob(F-statistic)	0.003700		

Inverted AR Roots	.94	.87+.34i	.87-.34i	.68+.63i
	.68-.63i	.40+.81i	.40-.81i	.09+.83i
	.09-.83i	-.09+.83i	-.09-.83i	-.40+.81i
	-.40-.81i	-.68-.63i	-.68+.63i	-.87-.34i
	-.87+.34i	-.94		
Inverted MA Roots	.93	.88+.29i	.88-.29i	.75-.54i
	.75+.54i	.54-.75i	.54+.75i	.29+.88i
	.29-.88i	.00-.93i	-.00+.93i	-.29+.88i
	-.29-.88i	-.54+.75i	-.54-.75i	-.75-.54i
	-.75+.54i	-.88-.29i	-.88+.29i	-.93

Appendix F

Tentative Adjusted ARMA(16,20) +MA(18)

Dependent Variable: R1 RATES
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/28/22 Time: 09:52
Sample: 2 120
Included observations: 119
Convergence achieved after 184 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.018192	0.026023	0.699075	0.4859
AR(16)	0.210749	0.049736	4.237374	0.0000
MA(20)	-0.242113	0.218902	-1.106032	0.2710
MA(18)	0.210746	0.535229	0.393749	0.6945
SIGMASQ	0.006004	0.000348	17.24841	0.0000

R-squared	0.138768	Mean dependent var	0.017945
Adjusted R-squared	0.108549	S.D. dependent var	0.083846
S.E. of regression	0.079164	Akaike info criterion	-2.169362
Sum squared resid	0.714435	Schwarz criterion	-2.052592
Log likelihood	134.0770	Hannan-Quinn criter.	-2.121945
F-statistic	4.592112	Durbin-Watson stat	1.777673
Prob(F-statistic)	0.001785		

Inverted AR Roots	.91	.84+.35i	.84-.35i	.64+.64i
	.64+.64i	.35-.84i	.35+.84i	-.00+.91i
	-.00-.91i	-.35-.84i	-.35+.84i	-.64+.64i
	-.64+.64i	-.84-.35i	-.84+.35i	-.91
Inverted MA Roots	.88	.87+.25i	.87-.25i	.77+.52i
	.77-.52i	.58-.75i	.58+.75i	.31-.91i
	.31+.91i	.00-.96i	-.00+.96i	-.31-.91i
	-.31+.91i	-.58+.75i	-.58-.75i	-.77+.52i
	-.77-.52i	-.87+.25i	-.87-.25i	-.88

Correlogram of the residual Adjusted ARMA(16,20) +AR(18)

Date: 11/28/22 Time: 10:25
Sample: 1 150 IF RATES<=142.4237
Q-statistic probabilities adjusted for 3 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.111	0.111	1.5027	
		2	-0.024	-0.037	1.5742	
		3	0.044	0.052	1.8156	
		4	0.107	0.097	3.2569	0.071
		5	0.058	0.039	3.6878	0.158
		6	-0.023	-0.029	3.7526	0.289
		7	0.051	0.053	4.0929	0.394
		8	-0.036	-0.065	4.2596	0.513
		9	-0.039	-0.033	4.4572	0.615
		10	-0.027	-0.025	4.5568	0.714
		11	-0.051	-0.054	4.9080	0.767
		12	-0.054	-0.040	5.3002	0.807
		13	-0.054	-0.031	5.6891	0.841
		14	-0.054	-0.043	6.0841	0.868
		15	0.027	0.056	6.1832	0.907
		16	0.031	0.039	6.3162	0.934
		17	-0.000	0.009	6.3162	0.958
		18	-0.021	-0.008	6.3786	0.973
		19	-0.010	-0.015	6.3921	0.983
		20	0.015	-0.001	6.4263	0.990
		21	0.009	0.002	6.4386	0.994
		22	-0.041	-0.056	6.6843	0.996
		23	-0.009	-0.007	6.6975	0.998
		24	0.025	0.020	6.7941	0.999
		25	-0.023	-0.032	6.8759	0.999
		26	-0.012	0.006	6.8985	1.000
		27	0.010	0.017	6.9127	1.000
		28	-0.012	-0.016	6.9343	1.000
		29	-0.003	0.017	6.9359	1.000
		30	0.005	0.005	6.9393	1.000
		31	-0.023	-0.039	7.0286	1.000
		32	-0.056	-0.050	7.5398	1.000
		33	-0.010	-0.006	7.5569	1.000
		34	-0.001	-0.010	7.5570	1.000
		35	-0.011	-0.001	7.5793	1.000
		36	-0.015	-0.008	7.6177	1.000

Ljung Box-Test Adjusted ARMA(16,20) +AR(18)

Date: 11/28/22 Time: 10:24
Sample: 1 150 IF RATES<=142.4237
Included observations: 119

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.003	-0.003	0.0009	0.976
		2	-0.012	-0.012	0.0172	0.991
		3	-0.012	-0.012	0.0337	0.998
		4	0.001	0.001	0.0339	1.000
		5	-0.006	-0.006	0.0377	1.000
		6	-0.012	-0.012	0.0554	1.000
		7	-0.007	-0.007	0.0619	1.000
		8	-0.013	-0.013	0.0832	1.000
		9	-0.013	-0.013	0.1038	1.000
		10	-0.013	-0.014	0.1260	1.000
		11	-0.012	-0.013	0.1445	1.000
		12	-0.011	-0.012	0.1617	1.000
		13	-0.011	-0.012	0.1792	1.000
		14	-0.010	-0.012	0.1939	1.000
		15	-0.006	-0.007	0.1990	1.000
		16	0.126	0.124	2.4026	1.000
		17	-0.011	-0.011	2.4182	1.000
		18	-0.013	-0.012	2.4427	1.000
		19	-0.012	-0.011	2.4648	1.000
		20	-0.011	-0.013	2.4824	1.000
		21	-0.013	-0.014	2.5069	1.000
		22	-0.011	-0.010	2.5257	1.000
		23	-0.011	-0.012	2.5443	1.000
		24	-0.013	-0.012	2.5707	1.000
		25	-0.013	-0.012	2.5976	1.000
		26	-0.014	-0.013	2.6264	1.000
		27	-0.004	-0.004	2.6290	1.000
		28	-0.004	-0.003	2.6313	1.000
		29	-0.004	-0.004	2.6338	1.000
		30	-0.004	-0.004	2.6365	1.000
		31	-0.004	-0.005	2.6387	1.000
		32	0.001	-0.018	2.6388	1.000
		33	-0.004	-0.004	2.6417	1.000
		34	-0.004	-0.004	2.6447	1.000
		35	-0.004	-0.004	2.6478	1.000
		36	-0.004	-0.004	2.6506	1.000

Appendix G

Tentative GARCH(1,1)

Dependent Variable: R1 RATES
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 11/28/22 Time: 13:53
Sample: 1 150 IF RATES<=142.4237
Included observations: 118
Convergence achieved after 21 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.016431	0.033583	0.489270	0.6247
R1 RATES(-1)	0.107192	0.907471	0.118122	0.9060
Variance Equation				
C	0.004381	0.004690	0.934238	0.3502
RESID(-1)^2	-0.016021	0.000641	-24.98971	0.0000
GARCH(-1)	0.587529	0.444149	1.322819	0.1859

R-squared	0.012521	Mean dependent var	0.018040
Adjusted R-squared	0.004009	S.D. dependent var	0.084197
S.E. of regression	0.084028	Akaike info criterion	-2.106731
Sum squared resid	0.819036	Schwarz criterion	-1.989329
Log likelihood	129.2971	Hannan-Quinn criter.	-2.059062
Durbin-Watson stat	1.981105		

Tentative GARCH(1,2)

Dependent Variable: R1 RATES
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 11/28/22 Time: 14:07
Sample: 1 150 IF RATES<=142.4237
Included observations: 118
Convergence achieved after 17 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1) + C(6)*GARCH(-2)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.020603	0.029416	0.700396	0.4837
R1 RATES(-1)	0.096128	0.777784	0.123592	0.9016
Variance Equation				
C	0.004333	0.006426	0.674324	0.5001
RESID(-1)^2	-0.014866	0.038883	-0.382332	0.7022
GARCH(-1)	0.523018	1.609488	0.324959	0.7452
GARCH(-2)	0.034447	1.272450	0.027071	0.9784

R-squared	0.009701	Mean dependent var	0.018040
Adjusted R-squared	0.001164	S.D. dependent var	0.084197
S.E. of regression	0.084148	Akaike info criterion	-2.045181
Sum squared resid	0.821376	Schwarz criterion	-1.904299
Log likelihood	126.6657	Hannan-Quinn criter.	-1.987979
Durbin-Watson stat	1.954746		

Tentative GARCH(2,1)

Dependent Variable: R1 RATES
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 11/28/22 Time: 14:02
Sample: 1 150 IF RATES<=142.4237
Included observations: 118
Failure to improve Likelihood after 13 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.015973	0.031610	0.505299	0.6133
R1 RATES(-1)	0.128885	0.908569	0.141855	0.8872
Variance Equation				
C	0.004732	0.005874	0.805600	0.4205
RESID(-1)^2	0.025938	0.087831	0.295317	0.7678
RESID(-2)^2	-0.028830	0.054738	-0.526681	0.5984
GARCH(-1)	0.528567	0.591117	0.894184	0.3712

R-squared	0.012268	Mean dependent var	0.018040
Adjusted R-squared	0.003753	S.D. dependent var	0.084197
S.E. of regression	0.084039	Akaike info criterion	-2.031889
Sum squared resid	0.819247	Schwarz criterion	-1.891007
Log likelihood	125.8814	Hannan-Quinn criter.	-1.974687
Durbin-Watson stat	2.022597		

Tentative GARCH(2,2)

Dependent Variable: R1 RATES
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 11/28/22 Time: 14:06
Sample: 1 150 IF RATES<=142.4237
Included observations: 118
Failure to improve Likelihood after 13 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2 + C(6)*GARCH(-1)
+ C(7)*GARCH(-2)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.016337	0.033220	0.491795	0.6229
R1 RATES(-1)	0.126051	0.294046	0.428678	0.6682
Variance Equation				
C	0.004688	0.009050	0.517956	0.6045
RESID(-1)^2	0.020737	0.070558	0.293902	0.7688
RESID(-2)^2	-0.024389	0.088233	-0.276411	0.7822
GARCH(-1)	0.476478	2.127893	0.223920	0.8228
GARCH(-2)	0.036468	1.501061	0.024295	0.9806

R-squared	0.012320	Mean dependent var	0.018040
Adjusted R-squared	0.003806	S.D. dependent var	0.084197
S.E. of regression	0.084036	Akaike info criterion	-2.025939
Sum squared resid	0.819203	Schwarz criterion	-1.861576
Log likelihood	126.5304	Hannan-Quinn criter.	-1.959203
Durbin-Watson stat	2.017122		

Appendix H

GARCH(1,1) Gaussian Normal Distribution

Dependent Variable: R1 RATES
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 11/28/22 Time: 13:53
Sample: 1 150 IF RATES<=142.4237
Included observations: 118
Convergence achieved after 21 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.016431	0.033583	0.489270	0.6247
R1 RATES(-1)	0.107192	0.907471	0.118122	0.9060
Variance Equation				
C	0.004381	0.004690	0.934238	0.3502
RESID(-1)^2	-0.016021	0.000641	-24.98971	0.0000
GARCH(-1)	0.587529	0.444149	1.322819	0.1859

R-squared	0.012521	Mean dependent var	0.018040
Adjusted R-squared	0.004009	S.D. dependent var	0.084197
S.E. of regression	0.084028	Akaike info criterion	-2.106731
Sum squared resid	0.819036	Schwarz criterion	-1.989329
Log likelihood	129.2971	Hannan-Quinn criter.	-2.059062
Durbin-Watson stat	1.981105		

GARCH(1,1) Gaussian Normal Distribution Residual Diagnostics at Lag 1

Heteroskedasticity Test: ARCH

F-statistic	0.000567	Prob. F(1,115)	0.9811
Obs*R-squared	0.000576	Prob. Chi-Square(1)	0.9808

Test Equation:
Dependent Variable: WGT RESID^2
Method: Least Squares
Date: 11/28/22 Time: 14:32
Sample: 1 150 IF RATES<=142.4237
Included observations: 117

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.668820	0.541662	1.234756	0.2194
WGT RESID^2(-1)	-0.002220	0.093250	-0.023804	0.9811

R-squared	0.000005	Mean dependent var	0.667339
Adjusted R-squared	-0.008691	S.D. dependent var	5.795051
S.E. of regression	5.820178	Akaike info criterion	6.377485
Sum squared resid	3895.564	Schwarz criterion	6.424702
Log likelihood	-371.0829	Hannan-Quinn criter.	6.396654
F-statistic	0.000567	Durbin-Watson stat	2.000044
Prob(F-statistic)	0.981050		

GARCH(1,1) Gaussian Normal Distribution Residual Diagnostics at Lag 36

Heteroskedasticity Test: ARCH

F-statistic	218.5277	Prob. F(36,45)	0.0000
Obs*R-squared	81.53362	Prob. Chi-Square(36)	0.0000

Test Equation:
Dependent Variable: WGT RESID^2
Method: Least Squares
Date: 11/28/22 Time: 14:34
Sample: 1 150 IF RATES<=142.4237
Included observations: 82

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.005979	0.007948	0.752171	0.4559
WGT RESID^2(-1)	-0.023579	0.145124	-0.162477	0.8717
WGT RESID^2(-2)	0.700590	0.145018	4.831058	0.0000
WGT RESID^2(-3)	0.009462	0.176614	0.053575	0.9575
WGT RESID^2(-4)	-0.314020	0.176457	-1.779590	0.0819
WGT RESID^2(-5)	0.002226	0.181599	0.012260	0.9903
WGT RESID^2(-6)	0.066510	0.181667	0.366109	0.7160
WGT RESID^2(-7)	0.006955	0.176358	0.039435	0.9687
WGT RESID^2(-8)	0.034424	0.176091	0.195492	0.8459
WGT RESID^2(-9)	-0.029673	0.144746	-0.204999	0.8385
WGT RESID^2(-10)	-0.067624	0.144107	-0.469260	0.6411
WGT RESID^2(-11)	0.002254	0.001920	1.173868	0.2466
WGT RESID^2(-12)	0.000789	0.001321	0.597049	0.5535
WGT RESID^2(-13)	0.000273	0.001327	0.205548	0.8381
WGT RESID^2(-14)	-0.000488	0.001320	-0.369463	0.7135
WGT RESID^2(-15)	-0.000327	0.001154	-0.282919	0.7785
WGT RESID^2(-16)	0.000425	0.001150	0.369375	0.7136
WGT RESID^2(-17)	0.000693	0.001145	0.605082	0.5482
WGT RESID^2(-18)	0.038524	0.000904	42.61222	0.0000
WGT RESID^2(-19)	0.000590	0.005652	0.104311	0.9174
WGT RESID^2(-20)	0.040985	0.005645	7.260147	0.0000
WGT RESID^2(-21)	0.000515	0.008215	0.062716	0.9503
WGT RESID^2(-22)	-0.035369	0.008214	-4.305739	0.0001
WGT RESID^2(-23)	-0.000104	0.009668	-0.010747	0.9915
WGT RESID^2(-24)	0.018545	0.009651	1.921577	0.0610
WGT RESID^2(-25)	0.000132	0.009945	0.013228	0.9895
WGT RESID^2(-26)	-0.005587	0.009931	-0.562551	0.5765
WGT RESID^2(-27)	0.000102	0.009922	0.010248	0.9919
WGT RESID^2(-28)	0.000358	0.009924	0.036096	0.9714
WGT RESID^2(-29)	0.002495	0.009846	0.253429	0.8011
WGT RESID^2(-30)	0.004482	0.009804	0.457203	0.6497
WGT RESID^2(-31)	-0.000273	0.000916	-0.298346	0.7668
WGT RESID^2(-32)	-9.93E-05	0.000898	-0.110549	0.9125
WGT RESID^2(-33)	-0.000249	0.000896	-0.277721	0.7825
WGT RESID^2(-34)	0.000174	0.000893	0.194967	0.8463
WGT RESID^2(-35)	0.000362	0.000893	0.405983	0.6867
WGT RESID^2(-36)	-0.001340	0.000887	-1.510946	0.1378

R-squared	0.994312	Mean dependent var	0.104520
Adjusted R-squared	0.989762	S.D. dependent var	0.538163
S.E. of regression	0.054452	Akaike info criterion	-2.680615
Sum squared resid	0.133426	Schwarz criterion	-1.594656
Log likelihood	146.9052	Hannan-Quinn criter.	-2.244619
F-statistic	218.5277	Durbin-Watson stat	1.998600
Prob(F-statistic)	0.000000		

GARCH(1,1) Gaussian Normal Distribution Test for serial Auto correlation at lag 1

Date: 11/28/22 Time: 14:36
Sample: 1 150 IF RATES<=142.4237
Q-statistic probabilities adjusted for 1 dynamic regressor

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.093	0.093	1.0356	0.309
		2	-0.040	-0.049	1.2357	0.539
		3	0.035	0.044	1.3889	0.708
		4	0.022	0.013	1.4514	0.835
		5	0.032	0.032	1.5783	0.904
		6	-0.030	-0.037	1.6947	0.946
		7	0.040	0.049	1.9001	0.965
		8	-0.039	-0.055	2.0972	0.978
		9	-0.026	-0.011	2.1872	0.988
		10	-0.029	-0.035	2.3011	0.993
		11	-0.017	-0.008	2.3405	0.997
		12	-0.038	-0.041	2.5325	0.998
		13	-0.046	-0.030	2.8124	0.999
		14	-0.047	-0.047	3.1124	0.999
		15	0.023	0.038	3.1837	0.999
		16	0.216	0.213	9.6602	0.884
		17	0.032	0.003	9.8080	0.911
		18	0.157	0.186	13.314	0.773
		19	-0.029	-0.084	13.431	0.816
		20	-0.199	-0.203	19.155	0.512
		21	0.017	0.015	19.199	0.572
		22	-0.022	-0.057	19.268	0.629
		23	0.001	-0.005	19.268	0.686
		24	-0.017	0.028	19.309	0.735
		25	-0.034	-0.033	19.483	0.773
		26	-0.008	0.018	19.494	0.815
		27	-0.013	0.023	19.519	0.850
		28	-0.014	-0.011	19.550	0.880
		29	-0.015	0.016	19.584	0.905
		30	-0.008	0.014	19.594	0.927
		31	-0.011	-0.028	19.614	0.944
		32	-0.011	-0.067	19.634	0.957
		33	0.009	-0.025	19.648	0.968
		34	0.052	-0.053	20.106	0.972
		35	-0.021	0.006	20.182	0.979
		36	-0.067	0.001	20.959	0.979

*Probabilities may not be valid for this equation specification.

Appendix I

GARCH(1,1) Gaussian Normal Distribution Test for serial Auto correlation at lag 36

Date: 11/28/22 Time: 14:37
Sample: 1 150 IF RATES<=142.4237
Included observations: 118

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.002	-0.002	0.0006	0.981
		2	-0.010	-0.010	0.0126	0.994
		3	-0.012	-0.012	0.0292	0.999
		4	-0.012	-0.012	0.0469	1.000
		5	-0.010	-0.011	0.0602	1.000
		6	-0.012	-0.013	0.0797	1.000
		7	-0.008	-0.009	0.0886	1.000
		8	-0.013	-0.013	0.1091	1.000
		9	-0.013	-0.014	0.1326	1.000
		10	-0.013	-0.014	0.1541	1.000
		11	-0.012	-0.014	0.1744	1.000
		12	-0.012	-0.014	0.1944	1.000
		13	-0.012	-0.013	0.2131	1.000
		14	-0.010	-0.012	0.2262	1.000
		15	-0.010	-0.012	0.2414	1.000
		16	0.064	0.062	0.8088	1.000
		17	-0.012	-0.014	0.8304	1.000
		18	0.025	0.025	0.9202	1.000
		19	-0.013	-0.013	0.9442	1.000
		20	0.054	0.055	1.3722	1.000
		21	-0.014	-0.014	1.4017	1.000
		22	-0.014	-0.012	1.4297	1.000
		23	-0.014	-0.013	1.4582	1.000
		24	-0.014	-0.013	1.4897	1.000
		25	-0.013	-0.013	1.5171	1.000
		26	-0.004	-0.003	1.5197	1.000
		27	-0.004	-0.004	1.5222	1.000
		28	-0.004	-0.003	1.5247	1.000
		29	-0.004	-0.003	1.5275	1.000
		30	-0.004	-0.004	1.5306	1.000
		31	-0.004	-0.004	1.5337	1.000
		32	-0.005	-0.010	1.5372	1.000
		33	-0.005	-0.004	1.5410	1.000
		34	-0.002	-0.007	1.5414	1.000
		35	-0.005	-0.004	1.5451	1.000
		36	0.000	-0.010	1.5451	1.000

GARCH(1,1) Student t's Distribution

Dependent Variable: R1 RATES
Method: ML ARCH - Student's t distribution (Marquardt / EViews legacy)
Date: 11/28/22 Time: 14:37
Sample: 1 150 IF RATES<=142.4237
Included observations: 118
Failure to improve Likelihood after 35 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.007754	0.023423	0.331027	0.7406
R1 RATES(-1)	0.123203	0.334197	0.368654	0.7124
Variance Equation				
C	0.004467	0.007256	0.615608	0.5382
RESID(-1)^2	-0.010962	0.029463	-0.372066	0.7098
GARCH(-1)	0.400234	0.986673	0.405640	0.6850
T-DIST. DOF	15.24315	2.793323	5.456996	0.0000

R-squared	0.003126	Mean dependent var	0.018040
Adjusted R-squared	-0.005468	S.D. dependent var	0.084197
S.E. of regression	0.084427	Akaike info criterion	-2.659348
Sum squared resid	0.826829	Schwarz criterion	-2.518466
Log likelihood	162.9015	Hannan-Quinn criter.	-2.602146
Durbin-Watson stat	1.992988		

GARCH(1,1) Gaussian Normal Distribution Test for serial Auto correlation at lag 36

Date: 11/28/22 Time: 14:37
Sample: 1 150 IF RATES<=142.4237
Included observations: 118

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.002	-0.002	0.0006	0.981
		2	-0.010	-0.010	0.0126	0.994
		3	-0.012	-0.012	0.0292	0.999
		4	-0.012	-0.012	0.0469	1.000
		5	-0.010	-0.011	0.0602	1.000
		6	-0.012	-0.013	0.0797	1.000
		7	-0.008	-0.009	0.0886	1.000
		8	-0.013	-0.013	0.1091	1.000
		9	-0.013	-0.014	0.1326	1.000
		10	-0.013	-0.014	0.1541	1.000
		11	-0.012	-0.014	0.1744	1.000
		12	-0.012	-0.014	0.1944	1.000
		13	-0.012	-0.013	0.2131	1.000
		14	-0.010	-0.012	0.2262	1.000
		15	-0.010	-0.012	0.2414	1.000
		16	0.064	0.062	0.8088	1.000
		17	-0.012	-0.014	0.8304	1.000
		18	0.025	0.025	0.9202	1.000
		19	-0.013	-0.013	0.9442	1.000
		20	0.054	0.055	1.3722	1.000
		21	-0.014	-0.014	1.4017	1.000
		22	-0.014	-0.012	1.4297	1.000
		23	-0.014	-0.013	1.4582	1.000
		24	-0.014	-0.013	1.4897	1.000
		25	-0.013	-0.013	1.5171	1.000
		26	-0.004	-0.003	1.5197	1.000
		27	-0.004	-0.004	1.5222	1.000
		28	-0.004	-0.003	1.5247	1.000
		29	-0.004	-0.003	1.5275	1.000
		30	-0.004	-0.004	1.5306	1.000
		31	-0.004	-0.004	1.5337	1.000
		32	-0.005	-0.010	1.5372	1.000
		33	-0.005	-0.004	1.5410	1.000
		34	-0.002	-0.007	1.5414	1.000
		35	-0.005	-0.004	1.5451	1.000
		36	0.000	-0.010	1.5451	1.000

GARCH(1,1) Student t's Distribution Residual test at lag 36

Heteroskedasticity Test: ARCH

F-statistic	166.0281	Prob. F(36,45)	0.0000
Obs*R-squared	81.38725	Prob. Chi-Square(36)	0.0000

Test Equation:
Dependent Variable: WGT RESID^2
Method: Least Squares
Date: 11/28/22 Time: 14:39
Sample: 1 150 IF RATES<=142.4237
Included observations: 82

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.006272	0.012019	0.521837	0.6043
WGT RESID^2(-1)	-0.042257	0.144423	-0.292593	0.7712
WGT RESID^2(-2)	0.541212	0.143673	3.766964	0.0005
WGT RESID^2(-3)	0.173648	0.163177	1.064173	0.2929
WGT RESID^2(-4)	-0.075513	0.165390	-0.456579	0.6502
WGT RESID^2(-5)	-0.116075	0.165371	-0.701908	0.4864
WGT RESID^2(-6)	-0.065157	0.165784	-0.393024	0.6962
WGT RESID^2(-7)	-0.004220	0.165759	-0.025461	0.9798
WGT RESID^2(-8)	0.052684	0.163442	0.322342	0.7487
WGT RESID^2(-9)	0.090749	0.144109	0.629724	0.5321
WGT RESID^2(-10)	-0.002603	0.143987	-0.018076	0.9857
WGT RESID^2(-11)	0.000926	0.001888	0.490540	0.6261
WGT RESID^2(-12)	0.001063	0.001468	0.723953	0.4728
WGT RESID^2(-13)	0.000971	0.001472	0.659817	0.5127
WGT RESID^2(-14)	-0.000417	0.001477	-0.282121	0.7791
WGT RESID^2(-15)	-0.000630	0.001333	-0.472826	0.6386
WGT RESID^2(-16)	-0.000650	0.001336	-0.486790	0.6288
WGT RESID^2(-17)	2.70E-05	0.001333	0.020238	0.9839
WGT RESID^2(-18)	0.042203	0.000965	43.75481	0.0000
WGT RESID^2(-19)	0.001539	0.006166	0.249535	0.8041
WGT RESID^2(-20)	0.038263	0.006133	6.239012	0.0000
WGT RESID^2(-21)	-0.004929	0.008259	-0.596815	0.5536
WGT RESID^2(-22)	-0.030103	0.008240	-3.653478	0.0007
WGT RESID^2(-23)	-0.005911	0.009325	-0.633874	0.5294
WGT RESID^2(-24)	0.007267	0.009310	0.780585	0.4391
WGT RESID^2(-25)	0.007480	0.009310	0.803388	0.4260
WGT RESID^2(-26)	0.001746	0.009396	0.185833	0.8534
WGT RESID^2(-27)	-0.004048	0.009018	-0.448935	0.6556
WGT RESID^2(-28)	-0.003182	0.008958	-0.355181	0.7241
WGT RESID^2(-29)	-0.005407	0.008869	-0.609656	0.5452
WGT RESID^2(-30)	0.000110	0.008864	0.012387	0.9902
WGT RESID^2(-31)	-0.000166	0.000969	-0.170948	0.8650
WGT RESID^2(-32)	-0.000166	0.000966	-0.171611	0.8645
WGT RESID^2(-33)	-0.000210	0.000966	-0.217184	0.8290
WGT RESID^2(-34)	1.94E-05	0.000965	0.020048	0.9841
WGT RESID^2(-35)	7.64E-05	0.000965	0.079232	0.9372
WGT RESID^2(-36)	-0.001680	0.000965	-1.741732	0.0884

R-squared	0.992527	Mean dependent var	0.134057
Adjusted R-squared	0.986549	S.D. dependent var	0.744259
S.E. of regression	0.086317	Akaike info criterion	-1.759199
Sum squared resid	0.335278	Schwarz criterion	-0.673241
Log likelihood	109.1272	Hannan-Quinn criter.	-1.323203
F-statistic	166.0281	Durbin-Watson stat	2.009290
Prob(F-statistic)	0.000000		

Appendix J

GARCH(1,1) Student t's Distribution Serial correlation test at lag 1

Date: 11/28/22 Time: 14:40
Sample: 1 150 IF RATES<=142.4237
Q-statistic probabilities adjusted for 1 dynamic regressor

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.075	-0.075	0.6759	0.411
		2	-0.045	-0.051	0.9237	0.630
		3	0.031	0.023	1.0392	0.792
		4	0.015	0.017	1.0663	0.900
		5	0.038	0.043	1.2437	0.941
		6	-0.038	-0.032	1.4282	0.964
		7	0.048	0.046	1.7245	0.974
		8	-0.033	-0.032	1.8664	0.985
		9	-0.019	-0.019	1.9133	0.993
		10	-0.023	-0.032	1.9799	0.996
		11	-0.010	-0.012	1.9920	0.999
		12	-0.029	-0.036	2.1042	0.999
		13	-0.038	-0.037	2.3035	1.000
		14	-0.042	-0.053	2.5419	1.000
		15	0.032	0.028	2.6847	1.000
		16	0.211	0.219	8.8854	0.918
		17	-0.044	0.001	9.1599	0.935
		18	0.161	0.189	12.809	0.803
		19	0.013	0.029	12.833	0.847
		20	-0.199	-0.209	18.573	0.550
		21	0.026	-0.039	18.669	0.606
		22	-0.017	-0.056	18.711	0.663
		23	0.006	-0.037	18.716	0.718
		24	-0.007	0.031	18.723	0.767
		25	-0.025	-0.010	18.818	0.806
		26	-0.004	0.006	18.821	0.844
		27	-0.010	0.036	18.837	0.876
		28	-0.011	0.004	18.858	0.902
		29	-0.013	0.012	18.885	0.924
		30	-0.005	0.019	18.889	0.942
		31	-0.010	-0.036	18.907	0.956
		32	-0.010	-0.073	18.924	0.968
		33	0.010	-0.032	18.940	0.976
		34	0.054	-0.061	19.427	0.979
		35	-0.019	-0.004	19.490	0.984
		36	-0.065	0.004	20.215	0.984

*Probabilities may not be valid for this equation specification.

GARCH(1,1) Student t's Distribution serial correlation at 36

Date: 11/28/22 Time: 14:41
Sample: 1 150 IF RATES<=142.4237
Included observations: 118

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.004	-0.004	0.0023	0.962
		2	-0.010	-0.010	0.0139	0.993
		3	-0.011	-0.011	0.0291	0.999
		4	-0.011	-0.012	0.0453	1.000
		5	-0.010	-0.010	0.0569	1.000
		6	-0.013	-0.013	0.0781	1.000
		7	-0.007	-0.008	0.0846	1.000
		8	-0.013	-0.014	0.1069	1.000
		9	-0.014	-0.014	0.1312	1.000
		10	-0.013	-0.014	0.1531	1.000
		11	-0.013	-0.014	0.1742	1.000
		12	-0.013	-0.014	0.1959	1.000
		13	-0.012	-0.014	0.2159	1.000
		14	-0.011	-0.013	0.2320	1.000
		15	-0.010	-0.012	0.2452	1.000
		16	0.068	0.066	0.8831	1.000
		17	-0.012	-0.014	0.9044	1.000
		18	0.029	0.028	1.0219	1.000
		19	-0.013	-0.013	1.0464	1.000
		20	0.048	0.048	1.3748	1.000
		21	-0.013	-0.013	1.4010	1.000
		22	-0.014	-0.012	1.4301	1.000
		23	-0.014	-0.014	1.4583	1.000
		24	-0.014	-0.013	1.4894	1.000
		25	-0.014	-0.013	1.5186	1.000
		26	-0.004	-0.003	1.5213	1.000
		27	-0.004	-0.004	1.5239	1.000
		28	-0.004	-0.003	1.5267	1.000
		29	-0.004	-0.003	1.5296	1.000
		30	-0.004	-0.004	1.5328	1.000
		31	-0.004	-0.004	1.5361	1.000
		32	-0.005	-0.010	1.5395	1.000
		33	-0.005	-0.004	1.5430	1.000
		34	-0.001	-0.007	1.5433	1.000
		35	-0.005	-0.004	1.5470	1.000
		36	0.000	-0.010	1.5470	1.000

*Probabilities may not be valid for this equation specification.

GARCH(1,1) Gaussian Error Distribution

Dependent Variable: R1 RATES
Method: ML ARCH - Generalized error distribution (GED) (Marquardt / EViews legacy)
Date: 11/28/22 Time: 14:47
Sample: 1 150 IF RATES<=142.4237
Included observations: 118
Convergence achieved after 45 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000199	4.53E-05	4.397539	0.0000
R1 RATES(-1)	0.111586	0.018938	5.892033	0.0000
		Variance Equation		
C	0.000477	0.000168	2.835268	0.0046
RESID(-1)^2	12.26906	10.20546	1.202206	0.2293
GARCH(-1)	0.002646	6.61E-13	4.00E+09	0.0000
GED PARAMETER	0.284956	0.025561	11.14789	0.0000

R-squared	-0.023184	Mean dependent var	0.018040
Adjusted R-squared	-0.032005	S.D. dependent var	0.084197
S.E. of regression	0.085533	Akaike info criterion	-6.299215
Sum squared resid	0.848651	Schwarz criterion	-6.158332
Log likelihood	377.6537	Hannan-Quinn criter.	-6.242012
Durbin-Watson stat	1.920052		

GARCH(1,1) Gaussian Error Distribution Residual Diagnostics at Lag 1

Heteroskedasticity Test: ARCH

F-statistic	0.011690	Prob. F(1,115)	0.9141
Obs*R-squared	0.011892	Prob. Chi-Square(1)	0.9132

Test Equation:
Dependent Variable: WGT RESID^2
Method: Least Squares
Date: 11/28/22 Time: 14:51
Sample: 1 150 IF RATES<=142.4237
Included observations: 117

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	13.45729	12.43232	1.082444	0.2813
WGT RESID^2(-1)	-0.010082	0.093246	-0.108120	0.9141

R-squared	0.000102	Mean dependent var	13.32297
Adjusted R-squared	-0.008593	S.D. dependent var	133.2319
S.E. of regression	133.8031	Akaike info criterion	12.64756
Sum squared resid	2058875.	Schwarz criterion	12.69478
Log likelihood	-737.8824	Hannan-Quinn criter.	12.66673
F-statistic	0.011690	Durbin-Watson stat	2.000190
Prob(F-statistic)	0.914089		

Appendix k

GARCH(1,1) Gaussian Error Distribution Residual Diagnostics at Lag 36

Heteroskedasticity Test: ARCH

F-statistic	324.0068	Prob. F(36,45)	0.0000
Obs*R-squared	81.68486	Prob. Chi-Square(36)	0.0000

Test Equation:
Dependent Variable: WGT RESID^2
Method: Least Squares
Date: 11/28/22 Time: 14:52
Sample: 1 150 IF RATES<=142.4237
Included observations: 82

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.091660	0.089147	1.028185	0.3094
WGT RESID^2(-1)	-0.038675	0.122036	-0.316912	0.7528
WGT RESID^2(-2)	0.241853	0.118638	2.038591	0.0474
WGT RESID^2(-3)	0.125534	0.122320	1.026269	0.3102
WGT RESID^2(-4)	-0.022256	0.123077	-0.180826	0.8573
WGT RESID^2(-5)	-0.031923	0.122575	-0.260436	0.7957
WGT RESID^2(-6)	0.092457	0.122675	0.753671	0.4550
WGT RESID^2(-7)	-0.044224	0.123174	-0.359037	0.7212
WGT RESID^2(-8)	-0.039901	0.122454	-0.325844	0.7461
WGT RESID^2(-9)	0.217438	0.118806	1.830189	0.0738
WGT RESID^2(-10)	0.020536	0.122151	0.168119	0.8672
WGT RESID^2(-11)	7.89E-05	0.000585	0.134886	0.8933
WGT RESID^2(-12)	7.09E-05	0.000583	0.121557	0.9038
WGT RESID^2(-13)	-0.000249	0.000583	-0.427298	0.6712
WGT RESID^2(-14)	3.67E-05	0.000583	0.062874	0.9501
WGT RESID^2(-15)	2.79E-05	0.000580	0.048146	0.9618
WGT RESID^2(-16)	-0.000734	0.000575	-1.275644	0.2086
WGT RESID^2(-17)	-0.000122	0.000583	-0.210015	0.8346
WGT RESID^2(-18)	0.045496	0.000450	101.0294	0.0000
WGT RESID^2(-19)	0.001687	0.005564	0.303255	0.7631
WGT RESID^2(-20)	0.006225	0.005409	1.150805	0.2559
WGT RESID^2(-21)	-0.005130	0.005457	-0.940029	0.3522
WGT RESID^2(-22)	-0.003263	0.005487	-0.594732	0.5550
WGT RESID^2(-23)	-0.000790	0.005481	-0.144086	0.8861
WGT RESID^2(-24)	-0.003732	0.005480	-0.680960	0.4994
WGT RESID^2(-25)	0.002441	0.005499	0.444011	0.6592
WGT RESID^2(-26)	0.000113	0.005477	0.020636	0.9836
WGT RESID^2(-27)	-0.009244	0.005318	-1.738393	0.0890
WGT RESID^2(-28)	-0.000302	0.005454	-0.055400	0.9561
WGT RESID^2(-29)	-0.003809	0.002100	-1.813936	0.0764
WGT RESID^2(-30)	-0.000420	0.002157	-0.194660	0.8465
WGT RESID^2(-31)	-4.76E-05	0.000451	-0.105464	0.9165
WGT RESID^2(-32)	-1.52E-06	0.000451	-0.003364	0.9973
WGT RESID^2(-33)	-5.88E-05	0.000451	-0.130304	0.8969
WGT RESID^2(-34)	-3.00E-05	0.000451	-0.066501	0.9473
WGT RESID^2(-35)	-7.82E-05	0.000451	-0.173314	0.8632
WGT RESID^2(-36)	-0.002118	0.000450	-4.702034	0.0000

R-squared	0.996157	Mean dependent var	1.226373
Adjusted R-squared	0.993082	S.D. dependent var	7.722724
S.E. of regression	0.642316	Akaike info criterion	2.254909
Sum squared resid	18.56563	Schwarz criterion	3.340868
Log likelihood	-55.45126	Hannan-Quinn criter.	2.690905
F-statistic	324.0068	Durbin-Watson stat	2.059593
Prob(F-statistic)	0.000000		

GARCH(1,1) Gaussian Error Distribution Test for serial Auto correlation at lag 1

Date: 11/28/22 Time: 14:52
Sample: 1 150 IF RATES<=142.4237
Q-statistic probabilities adjusted for 1 dynamic regressor

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.018	-0.018	0.0375	0.847
		2	-0.042	-0.043	0.2566	0.880
		3	-0.004	-0.005	0.2581	0.968
		4	-0.013	-0.015	0.2786	0.991
		5	-0.009	-0.010	0.2894	0.998
		6	-0.019	-0.020	0.3334	0.999
		7	0.032	0.030	0.4629	1.000
		8	-0.018	-0.019	0.5038	1.000
		9	-0.015	-0.013	0.5317	1.000
		10	-0.016	-0.018	0.5635	1.000
		11	-0.009	-0.011	0.5744	1.000
		12	-0.027	-0.029	0.6718	1.000
		13	-0.032	-0.034	0.8097	1.000
		14	-0.019	-0.026	0.8610	1.000
		15	-0.015	-0.019	0.8927	1.000
		16	0.065	0.062	1.4882	1.000
		17	-0.000	-0.001	1.4882	1.000
		18	0.184	0.189	6.2747	0.995
		19	-0.012	-0.006	6.2956	0.997
		20	-0.139	-0.125	9.0850	0.982
		21	-0.006	-0.013	9.0911	0.988
		22	-0.017	-0.026	9.1345	0.993
		23	-0.007	-0.014	9.1417	0.995
		24	-0.003	-0.001	9.1431	0.997
		25	-0.024	-0.041	9.2311	0.998
		26	-0.005	-0.004	9.2347	0.999
		27	-0.007	0.003	9.2429	0.999
		28	-0.007	-0.005	9.2517	1.000
		29	-0.008	-0.004	9.2620	1.000
		30	-0.003	0.003	9.2636	1.000
		31	-0.008	-0.002	9.2740	1.000
		32	-0.001	-0.006	9.2741	1.000
		33	-0.000	-0.007	9.2741	1.000
		34	0.014	-0.021	9.3095	1.000
		35	-0.002	-0.010	9.3104	1.000
		36	-0.019	-0.041	9.3697	1.000

*Probabilities may not be valid for this equation specification.

GARCH(1,1) Gaussian Error Distribution Test for serial Auto correlation at lag 1

Date: 11/28/22 Time: 14:53
Sample: 1 150 IF RATES<=142.4237
Included observations: 118

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.010	-0.010	0.0123	0.912
		2	-0.009	-0.009	0.0230	0.989
		3	-0.010	-0.010	0.0354	0.998
		4	-0.010	-0.011	0.0485	1.000
		5	-0.010	-0.011	0.0618	1.000
		6	-0.010	-0.011	0.0758	1.000
		7	-0.008	-0.008	0.0829	1.000
		8	-0.011	-0.011	0.0976	1.000
		9	-0.011	-0.012	0.1126	1.000
		10	-0.011	-0.012	0.1277	1.000
		11	-0.011	-0.012	0.1428	1.000
		12	-0.011	-0.012	0.1584	1.000
		13	-0.011	-0.012	0.1743	1.000
		14	-0.011	-0.013	0.1909	1.000
		15	-0.011	-0.013	0.2080	1.000
		16	-0.003	-0.005	0.2092	1.000
		17	-0.011	-0.013	0.2259	1.000
		18	0.035	0.033	0.3947	1.000
		19	-0.011	-0.013	0.4135	1.000
		20	0.006	0.005	0.4186	1.000
		21	-0.011	-0.013	0.4378	1.000
		22	-0.012	-0.013	0.4581	1.000
		23	-0.012	-0.013	0.4787	1.000
		24	-0.012	-0.013	0.4994	1.000
		25	-0.012	-0.014	0.5209	1.000
		26	-0.003	-0.005	0.5222	1.000
		27	-0.003	-0.005	0.5235	1.000
		28	-0.003	-0.005	0.5249	1.000
		29	-0.003	-0.005	0.5264	1.000
		30	-0.003	-0.005	0.5280	1.000
		31	-0.003	-0.005	0.5297	1.000
		32	-0.003	-0.005	0.5314	1.000
		33	-0.003	-0.005	0.5333	1.000
		34	-0.003	-0.005	0.5349	1.000
		35	-0.004	-0.005	0.5370	1.000
		36	-0.003	-0.007	0.5391	1.000

*Probabilities may not be valid for this equation specification.