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Use of dynamic Asset Allocation during inflationary periods using Hidden Markov Models to Maximise Risk Adjusted returns in Zimbabwe.

BY

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APPROVAL FORM

I Chatikobo Molety thus certifies that, except the citations of other authors' work that have been recognized by Bindura University of Science Education this contribution is my own work research topic entitled as Use of dynamic Asset Allocation during inflationary periods using Hidden Markov Models to Maximise risk-adjusted returns in Zimbabwe. I hereby state that neither the entirety nor a portion of this work has ever been submitted as part of a degree at this university.

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DEDICATION

This project is dedicated to my parents, my brothers Clive and Rodney, who have always supported me financially and morally, taken care of all my needs as I worked hard to achieve the best, and taught me that even the most challenging tasks can be completed if they are carried out step by step. Above all, however, I want to give God the glory for His guidance throughout the entire process.

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ABSTRACT

Static asset allocation involves maintaining fixed asset allocations which are not responsive to time-varying expected returns and risk. The research intends to analyse the significance of using dynamic asset allocation that incorporates time-varying expected return and risk, to maximise risk-adjusted returns during inflationary periods in the Zimbabwean Markets. The dynamic allocation decisions are based on the inflation regimes identified by using Hidden Markov Models (HMMs). The study used samples of two HMMs on a dynamic portfolio and compared the performance of the portfolio to the static portfolio. The two methods using HMMs had better risk-adjusted returns than the static portfolio for the period under study. The analysis was mainly on Zimbabwean inflation, interest rates and All-share Index data. The findings for the Zimbabwean market were consistent with other research on international markets such as the S&P 500 in America.

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LIST OF ACRONYMS

HMM:	Hidden Markov Models
SAA:	Strategic Asset Allocation
DAA:	Dynamic Asset Allocation
SAA:	Static Asset Allocation
CAPM:	Capital Asset Pricing Model
MVPT:	Mean Variance Portfolio Theory
CPI:	Consumer Price Index
GDP:	Gross Domestic Product
CSV:	Comma-Separated Values
AIC:	Akaike Information Criterion
BIC:	Bayesian Information Criterion

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CHAPTER 1: INTRODUCTION

1.1 Introduction

This research addresses how investors can maximise asset returns during inflationary periods using active portfolio strategies. Investors must decide how much to invest in the financial markets and how to allocate financial resources among the many available financial securities.

Inflation reduces the real returns of financial assets and may result in negative real returns for some portfolios. In this project, we investigate how dynamic asset allocation can minimise the effect of rising inflation on portfolio returns. Asset allocation refers to the process of deciding how to distribute an investor's wealth among different asset classes for investment purposes (Reilly, Brown, & Leeds, 2019).

A dynamic asset allocation strategy would be preferable in an uncertain market as seen in the past two decades in Zimbabwe. This would vary on the respective company's investment objectives and its own Investment Policy Statement. However, most financial institutions have maximising shareholder value as their main objective and as such active portfolio strategies would propel institutions in the right direction to achieve this objective.

One way to improve on the traditional Markowitz asset allocation Model is to introduce a Markov model that has different states that trigger the use of certain assumptions and parameters in the traditional model. This project aims to analyse to what extent this approach can improve the real returns of a portfolio. To classify the static portfolio theory the investment must know the expressions for an optimal portfolio with different horizons and attributes towards risk and deal with it realistically with both the price and return characteristics of long-term bonds as well as with inflation.

This chapter outlines the background information on asset allocation by asset investors in Zimbabwe during inflationary periods. Furthermore, it outlines the research problem together with objectives and research questions.

This project comprises five chapters which are laid out as follows: Chapter 1 consists of an introduction, background of the study, problem statement, research objectives, research questions, the significance of the study, limitations of the study, delimitations of the study, and project layout. Chapter 2 consists of a literature review that gives a detailed description of the basic concepts of study-related information, and it also describes how to maximise returns. The methodology which in part explains the data employed, statistical software used and application of learned theoretical methods in chapter 3. The analysis and results of the study are discussed in Chapter 4 of the project. Finally, chapter 5 contains the study's conclusion and recommendation.

1.2 Background of the study.

Inflation refers to the rising price levels within an economy (Sloman & Jones, 2020). The expected inflation rate is a key component of pricing assets as investors would be interested in returns above the rate of inflation. As such, unexpected change in inflation is what cause headaches for investors. If inflation is worse than expected after one year, then investors' returns are wiped out by inflation. There are two main explanations for causes of inflation which are demand-pull and cost-push factors. In demand-pull factors, an increase in consumer demand may overtake aggregate supply. The excess demand would pull prices higher usually as workers earn more during economic expansion. On the other hand, on cost-push factors, we observe that an increase in the cost of inputs or factors of production is pushed onto consumers which fuels an increase in prices (Sloman & Jones, 2020).

During inflationary periods in a country, it is observed that the real returns of fixed-income assets and stocks decline as expected inflation is lower than actual inflation. For investments in stocks, returns vary depending mainly on the industry and how big the company is. Periods of Hyperinflation create uncertainty in the growth of companies as the cost of inputs increases at a different rate that companies can compete with by adjusting their prices. This would reduce the investor's expected returns from these companies.

Inflation has been an issue in Zimbabwe for several years for many different reasons. The country has seen a reboot of its financial system more than three times in the past two decades and inflation has been at the heart of the issues. A ZWE dollar today is worth less in two years in periods of Hyperinflation as seen in the periods 2006-2008 and 2020-2022. Many financial institutions rely on the returns of financial assets in their business model to settle their liabilities as they fall due and make a profit (Pensions and Insurance companies), whilst Banks and Investment companies acting as intermediaries in the financial system, rely on returns in the form of Interest Income from lending and fixed income securities. Retail Investors are not exempt from such risks, there are likely the most affected.

There have been several ways to address these issues but in the Zimbabwean markets, they seem not good enough. Financial Institutions can hedge their portfolios against the effects of inflation, they can adopt variable interest rates for fixed-income assets and invest in securities issued in a different currency which is perceived to be relatively stable.

The Zimbabwean financial market is not deep enough to provide sufficient liquid assets that hedge against inflation at all tenors. There is no developed market for derivatives. This makes this option challenging for most investors. Investors that held fixed-income securities had Government-issued securities which did not allow for interest rate adjustments. This made it difficult for companies holding statutory securities to adjust their investments in line with the prevailing level of inflation, only institutions like Banks and Microfinance banks had this liberty in their retail lending portfolios, even though it was limited as per regulations.

The dollarisation of the Zimbabwean economy because of high inflation, negative interest rates on domestic assets and frequent exchange rate devaluations contributed to the rise of the ratio between forex deposits to total deposits, which motivated domestic banks to lend in forex (Delgado, Kanda, & Casselle, 2000). Zimbabwean Forex deposits to total deposits were at 60.68% as of July 2022 (Bank Supervision Division, 2022). Forex securities are exposed to inflation and forex risk. Forex securities can import inflation from other countries. For instance, if an investor receives fixed USD cashflows over one year and the inflation rate in USD terms has been increasing over the period then we would expect the real returns of the investment to decline during the year. Moreover, the Zimbabwean Government has been changing the forex regulations frequently which increases the uncertainty of forex cashflows.

Other Institutions like Pension funds may decide to accept the risk of inflation and transfer it to the customers by paying benefits lower than anticipated. This would affect the welfare of the individuals in Zimbabwe as their purchasing power is lost through years of hyperinflation and the reputation of pension funds and the financial system is damaged (Njagu, 2019).

Furthermore, there is a question of whether companies hedge against inflation by using different asset allocation strategies if they are doing it at the optimum level possible, maximising the possible returns at a given level of risk the portfolio may be exposed to. This could be also interpreted as targeting an expected return by minimising the risk exposure. Most companies have passive investment strategies and ideally hold the assets up to maturity. Traditional allocation strategies would not be good enough as the portfolio of investments would need to be rebalanced in response to inflationary conditions within the economy.

The practice of distributing an investment portfolio among several asset classes, such as fixed-income bonds, shares, cash, and government bills and bonds, is known as asset allocation.

(Bodie, Kane, & Marcus, 2018). This would be mainly influenced by Investor preferences, liabilities, risk tolerance, liquidity, and tax treatments. Asset allocation makes it possible for investors to diversify their investments to reduce the risk of large losses.

Dynamic asset allocation is an extension of asset allocation that would consider the portfolio weights based on which assets have higher expected returns in the current economic environment. This is an active portfolio strategy that aims to generate higher returns by relocating financial resources especially when markets deviate from fair value. Markets are not completely efficient in practice and not all investors act rationally, then there are opportunities in the short term to capitalise and earn an extra return above traditional strategy.

Markov models can assist in dynamic asset allocation when it allows the investor to apply specific allocation strategies based on the state of the economy. For instance, an investor can have a Markov Switching model or Markov Hidden Model that specifies different states of inflation within an economy and each state of inflation would have its allocation strategies. Using this we have a probability distribution of changes in returns based on inflation. This also helps the investor in setting up capital for his investment.

The project aims to address these problems of not achieving an optimum expected real return and minimising portfolio risk by incorporating dynamic asset allocation strategies to maximise risk-adjusted returns during inflationary periods and transition periods. The main goal of many financial institutions is maximising returns while taking the least possible risk. For most financial institutions this is necessary to preserve their capital and remain solvent to cover the company's financial liabilities as they fall due.

1.3 Problem statement

Inflation erodes most of the returns earned. The recently issued risk-free 270-day Zimbabwean Treasury Bill on the 3rd of July 2020 was at 19.13% and had real returns of approximately -25%. As inflation increase and investors continue to forecast rising inflation, then we would expect depressed returns from this asset class (Old Mutual Securities Zimbabwe, 2021). This shows the recent effects of inflation on government fixed-interest securities.

The returns of the All-Share price index were 235% in the year 2021 and 63% in the year 2022 whilst the inflation rate was 98% and 193% in 2021 and 2022 respectively. Investors perceived the returns as better off compared to fixed-income securities as they try to avoid the effects of inflation on their portfolios. In 2021 most investors allocated most of their financial resources towards Equities. The problem is how effective was the allocation strategy in maximising the portfolio risk-adjusted returns during inflationary periods.

Most investors would be keen to adapt their portfolios based on the current economic conditions. There would be a follow-up question “What is the scientific way or process of linking asset returns to changes in inflation with a Zimbabwean economy”. If a process or method exists, then how feasible is it to draw a probability distribution of the returns in periods of transitions among inflation states?

1.4 Research Objectives

Given the problem statement outlined above, the specific objectives of the research are:

1. To investigate the impact of dynamic asset allocation using Hidden Markov Models on portfolio returns during inflationary periods.
2. To develop and evaluate a Hidden Markov model for determining the probability of changes in inflation states from one state to another.
3. To compare the performance of dynamic asset allocation using Hidden Markov models to traditional static allocation strategies during inflationary periods.

1.5 Research Questions

1. How does the use of Hidden Markov models during inflationary periods influence portfolio returns?
2. Can Markov models be used to measure the probability of inflation transitions?
3. How does dynamic asset allocation using Markov models compare to traditional static allocation strategies in terms of return maximisation during inflationary periods?

1.6 Significance of the Study

The study provides an evaluation of the importance of asset allocation in achieving business financial objectives for financial Institutions in Zimbabwe. The use of Markov models provides a recent additional approach to discovering an optimum portfolio that maximises returns in inflationary or recession periods and stable economic conditions whilst minimising risk.

To Investors

The study would assist investors in allocating their financial resources efficiently to achieve an expected return in line with their investment needs. Both Retail and Institutional investors would be able to create a suitable portfolio of assets from the market that helps them in absorbing inflationary shocks and able to retain value for their stakeholders.

The Government and Central Bank.

The more individual households rely on the financial system, the more they trust the financial system. This is important for the government as it can have confidence in enacting its policies. It is expected that the monetary policy and fiscal policy implementation would be effective especially when the financial system is vibrant and active. For example, a decision to increase interest rates to address inflation by the Central Bank (RBZ) would likely be more effective as it would affect many institutions and households that are participating in the larger financial market.

To the financial system

If many investors would adopt this approach, we would expect an increase in the market working efficiently in allocating financial resources among different parties based on different investor needs. There is a possible increase in the consumer confidence in the financial system. Economic agents would be able to transfer risk to parties that bear it. For instance, A bank can be able to securitise its risky lending portfolio and issue it on the capital markets because there would be interested buyers. Companies like Pension Funds would be able to find assets in the market that match their needs and liability structure.

The Households

If investors can achieve their goals by using the approach in this study, it can be observed that the stakeholders of the respective companies would benefit from the value created by the company and the returns generated by the company. For instance, the customers of Pension funds would be getting valuable benefits and the shareholders would be achieving better returns, and customers of Banks would see lower transaction costs and charges as the company relies more on its net interest income.

1.7 Limitations of the study

1. There are Assets in the Zimbabwean economy that cannot be sold easily. This makes the use of active portfolio strategies such as frequent tactical asset allocation difficult.
2. The cost of switching between assets using dynamic allocation strategies could be high, because of commissions, taxes, broker fees, charges, and penalties.

3. The analysis period includes the 2021 Covid pandemic period. This was an unlikely event and could have affected the data used.

1.8 Delimitation of the study.

The study confines to the following parameters:

1. The research assumes that all financial assets can be bought and sold in the Zimbabwean markets.
2. The research assumes that the costs of switching between assets are insignificant.
3. The research assumes that the data for the Covid period maintained the fundamental relationships among the key variables of interest for the study.

1.8.1 Period of the study

This study uses the monthly asset returns data from January 2018 to December 2022 for Zimbabwe's main asset classes.

1.8.2 Data sources

The Reserve Bank of Zimbabwe, the Zimbabwe Stock Exchange, and financial institutions are only a few of the public sources that provided the data for this study, which was conducted using only publicly available information.

1.9 Definition of terms

Asset allocation

How a portfolio is built, or which key asset classes, such as stocks, bonds, and cash, are included in the creation of the portfolio, is generally determined by asset allocation. (Bodie et al., 2008)

Dynamic Asset Allocation

Macro, or top-down, portfolio management is referred to as dynamic asset allocation (DAA). When capital markets vary from "fair value," the process seeks to increase returns or reduce portfolio risks by reallocating capital (First state Investments, 2014).

Hidden Markov chain

The hidden Markov chain attempts to explain the observed sequence by the underlying unobservable process having an emission probability. The idea behind the term "hidden" is that it is impossible to correctly infer the hidden states from the observed data.(Ramaprasad & Shigeyuki, 2004).

Inflation

In a market economy, prices for goods and services are constantly subject to inflation. Some costs increase, while others decrease. When the cost of products and services rises broadly rather than just for a few specific commodities, it is said to be inflation. This means that now you can buy less for $1 than you did yesterday. Inflation generally over time decreases the value of the currency (Sloman & Jones, 2020).

1.10 Summary

This chapter introduced how Zimbabwean investors can maximise risk-adjusted portfolio returns during inflationary periods using active portfolio strategies such as dynamic asset allocation. Chapter two (2) presents the literature review on the research topic.

CHAPTER 2: LITERATURE REVIEW

2.0 Introduction

In this chapter, the researcher focused on reviewing the literature of other authors on topics of dynamic asset allocation using Hidden Markov Models during inflation periods to minimise risk. Numerous studies have come through intending to maximise risk-adjusted returns in different market scenarios. The pursuit of returns by retail and institutional investors has led them to take on market risks. Investors must consider ways to manage this risk to reduce exposure to their capital. Inflation risk is one of the market risks that investors aim to manage.

2.1 Theoretical Framework

2.1.0 Dynamic asset allocation

Reilly, Brown and Leed (2019) defined asset allocation as the process of choosing how to divide an investor's wealth among various nations and asset classes for the goal of investing. Active portfolio management includes dynamic asset allocation (DAA). The First State Research (2004) paper defines Dynamic asset allocation (DAA) as the process that aims to generate additional returns, or abate portfolio risks, by reallocating capital when capital markets deviate from ‘fair value’. When the process is done more frequently in response to changes in the investment environment then it is said to be dynamic.

DAA is an example of an active asset allocation technique in which the investor continuously modifies the asset mix in response to market fluctuations and changes in the strength and weakness of the economy (Magadi & Harinangoni, 2015). Lustig (Multi-asset Investing: A practical guide to modern portfolio management, 2013) Recall that, in contrast to a constant-weighting strategy, dynamic asset allocation involves selling assets that are falling in value and buying ones that are rising in value. Lustig proceeded by saying that, for instance, if the stock market is poor, you should sell stocks in anticipation of future declines, and if the market is strong, you should buy stocks in anticipation of additional market gains.

Marston (Portfolio Design: A Modern Approach to Asset Allocation, 2011) identified different aims of asset allocation depending on the investor’s objectives. Kaplan (Frontiers of modern asset allocation, 2011) backed the aforementioned statement by stating that while investors in retirement frequently want to burrow down to ensure that no irrational losses occur, younger investors may wish to increase wealth over time, taking risks that might not be sensible for an older investor. Overall, the assets that these two investors select might not be very different, but the relative weights that each asset is assigned in the portfolio undoubtedly be very different. A younger investor, for instance, maintains a higher percentage of stocks than an investor who is getting ready to retire. Institutional investors' investment approaches also vary. Marston (2011) discussed one endowment; perhaps a family foundation might prefer to retain money if future possibilities for raising additional funds are limited. Kaplan (2011) further said.

Portfolio management has centred on finding techniques to maximize returns for any given level of risk since Markowitz's work in the 1950s. The best portfolios for that degree of risk should aim to have the highest returns. But minimizing risk for a certain goal return is as crucial. Although the target return may come from a variety of asset types, the risk is typically reduced by a portfolio's combination of these assets. According to Richard Marston, to create such portfolios, investors must have accurate predictions of asset returns. The estimates of risk and correlation are additionally significant, if not more so. It is not sufficient to just take long-run averages of each asset type to obtain these.

2.1.1 The value of Asset allocation

Reilly, et al. (2019) highlight the two main advantages of Asset allocation. He pointed out that Allocation allows the investor to attain his objectives and reduce the variability of his returns. Asset allocation allows the investor to select asset classes that he or she believes leads to the attainment of the investment objectives. The process lets the investor attach appropriate weights to the chosen asset classes (Reilly, Brown, & Leeds, 2019). The addition of investment strategies that outperform the rest of your portfolio, such as those with low correlation, can lower the volatility of the entire portfolio. This is due to the volatility of particular asset classes, but the presence of other assets in a well-built portfolio largely counters this volatility on both the upside and downside, resulting in a more stable return pattern.

The second reason asset allocation is crucial is that it aids investors in maintaining a long-term view and preventing emotional responses. Investors frequently seek out the market's top-performing sectors and a well-balanced portfolio. Other investments help create a steadier return pattern by somewhat offsetting that volatility, both upward and downward.

2.1.2 Active and Passive Portfolio Management.

The antithesis of passive investing is passive portfolio management (or passive portfolio management). The returns of a price index, such as a recognized market benchmark, are tracked by a passive method, claims Peterson (2002). It is normally carried out by holding all the securities that make up each index in accordance with how they are represented in the index. (Peterson, 2012). Maintaining a passive investment strategy requires no trading in the absence of changes in index composition.

A return above the index return, which is often attained by passive techniques, is what active portfolio management strategies strive to achieve, according to Peterson (2012). The returns above the benchmark index, are also known as "alphas". This can be represented easily using the CAPM single index model (Peterson, 2012).

$$r_i = \alpha + \beta_i r_m + \epsilon$$

Where r_i is the return on the investment in question, r_m is the return on the market portfolio Then excess returns over the benchmark are,

$$r_i - r_m = \alpha + \beta_i r_m + \epsilon - r_m$$

Taking the expectation of both sides, we have

$$r_i - r_m = \alpha + (\beta_i - 1) r_m$$

We then observe that the return above α is due to the portfolio Beta being greater than one. This would be achieved by taking on more incremental systematic risk or exposure to market risk. Active management strategies assist investors in achieving the extra return. However, as shown before this may be costly to some investors.

Asset class behaviour can vary significantly over shifting economic scenarios. No single asset class dominates under all economic conditions. Traditional asset allocation makes no effort to adapt to such economic shifts (Dahlquist & Harvey, 2001). If economic conditions are persistent and strongly linked to asset class performance, then the DAA strategy should add value over the use of static weights.

2.1.3 Factors affecting Asset Allocation.

Marston (2011) suggested that modern investors seek to develop diversified portfolios with positive returns to reduce portfolio risks. For equities, the style of the investor would lead the investor in choosing between value and growth stocks. According to Marston, the nature of the liabilities of the institutional investor contributes to the investor’s choice among different asset classes.

2.2 Hidden Markov Models

Because different probability distributions can be applied to the returns for each underlying state, Markov chains are effective for simulating the changing conditions of financial markets.(Nikolaj & Dimitrij, 2012). In hidden Markov models, the state of an underlying and unseen Markov process affects the probability distribution that results in an observation. (Nystrup, 2014). A sequence of discrete random variables $\{S_t : t \in N\}$ is said to be a first-order Markov chain if, for all $t \in N$, it satisfies the Markov property:

$$\Pr\,(S_t + 1 | S_t, \ldots, S_1) \;=\; \Pr\,(S_t + 1 | S_t)$$

If the Markov chain $\{S_t\}$ has m states, then the bivariate stochastic process $\{S_t, X_t\}$ is called an m-state Hidden Markov Model. Let $\mathbf{X^t}$ and $\mathbf{S^t}$ represent vectors with values from 1 to t. then we have:

$$\Pr(S_t |\ \mathbf{S^{t-1}}) = \Pr(S_t |\ S_{t-1}), t = 1,2,3 \ldots$$

$$\Pr(X_t |\ \mathbf{X^{t-1}}, \mathbf{S^t}) = \Pr(X_t |\ S_t), \qquad t \epsilon \mathbb{N}$$

When S_t is known then, the Distribution of X_t depends only on S_t. This causes autocorrelation of $X_t$to be strongly dependant on the persistence of S_t (Nystrup, 2014).

Another illustration of a two-state chain model is when a trader might view one state as a bullish (upward trending) market and the other as a bearish (downward progressing) market. Consequently, a bull market would be depicted by a Gaussian distribution with a positive mean and low variance, whereas a bear market would have a negative mean and high volatility.(Nikolaj & Dimitrij, 2012):

$$\text{Bull regime} \sim N\left(\mu_{\text{bull}}, \sigma^2_{\text{bull}}\right)$$

$$\text{Bear regime} \sim N\left(\mu_{\text{bear}}, \sigma^2_{\text{bear}}\right)$$

The distribution of the states doesn't have to be gaussian. Then for some observation O_t and regime R_t we have a two-state HMM as,

$$O_t | R_t \sim N(\mu_{R_t}, \sigma^2_{R_t})$$

$$O_t = \mu_{R_t} + \sigma_{R_t}\epsilon_t, \qquad \epsilon_t \sim N(0,1)$$

$\mu_{R_t} = \begin{cases}\mu_1 \text{ if } R_t = 1 \\ \mu_2 \text{ if } R_t = 2\end{cases}$, and $\sigma_{R_t} = \begin{cases}\sigma_1 \text{ if } R_t = 1 \\ \sigma_2 \text{ if } R_t = 2\end{cases}$, with transition matrix $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22}\end{bmatrix}$

Using the model, we can then estimate the transition probabilities by estimating the most likely sequence of R_t

Figure 2.1: Hidden Markov Models Explanation Diagram

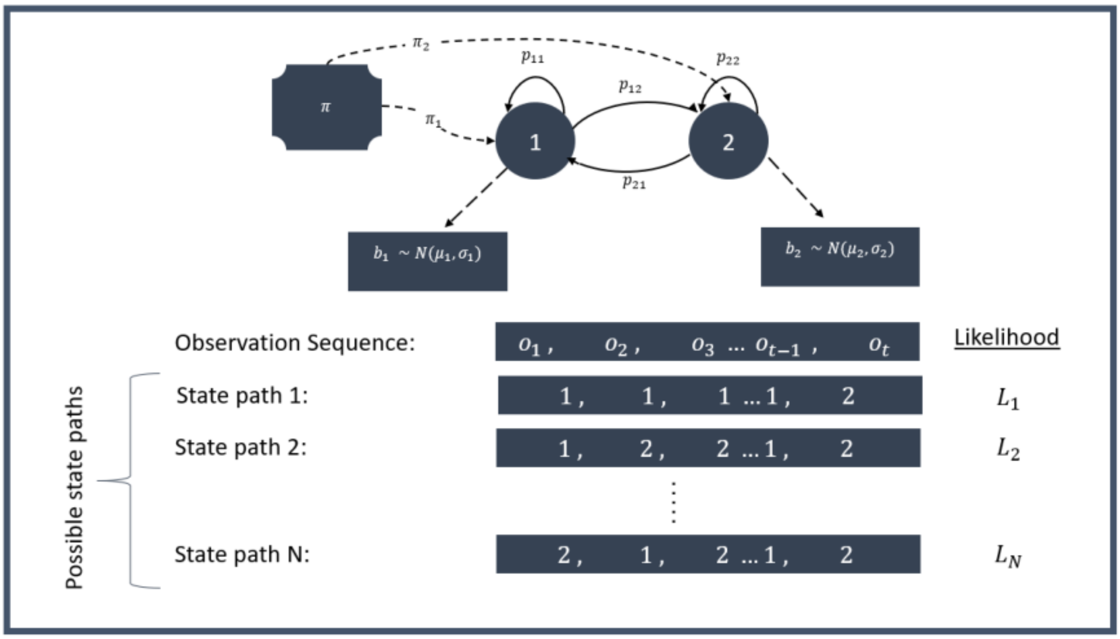


Figure 2.1 shows the parameters of an HMM. δ is the start probability, while circles 1 and 2 represent the distinct states, from which emissions follow Gaussian distributions b_j and p_ij are transition probabilities. The goal is to discover the sequence that maximizes by fitting the most likely sequence Qt to the observation sequence Ot.L_j. Source: (Nikolaj & Dimitrij, 2012)

2.3 Empirical Literature Review.

In this section, the author evaluates the literature on the use of Hidden Markov models in dynamic asset allocation to minimise portfolio risk and maximise portfolio risk-adjusted returns. Firstly, the research looks at the development of HMMs in finance. The review then focuses on the available literature on using HMMs in asset allocation. Lastly, the section focuses on the Literature Gaps.

2.3.1 Development of Hidden Markov chains (HMMs) in the field of finance.

Regimes refer to different states in the Markov Model. Quandt (1958) pioneered the use of Hidden Markov Models, by using them in the estimation of regression parameters using two regimes. After several years, the implementation of Hidden Markov chains was in the field of signal processing. It was when Hamilton (1989) introduced the concepts in the field of finance and business. He first initiated the use of HMMs in finance focusing on autoregressive models with Markov-switching parameters. Amy and Bekaert (2002) also considered the impact of regime shifts on asset allocation (Nystrup, 2014). They modelled Germany, UK and USA equity returns during the period 1970 to 1997 as a multivariate regime-switching process with two states. In their subsequent studies, they found that regime-switching strategies dominated static strategies for out-of-sample global equities.

Nystrup (2014) further developed Hamilton's ideas to their use in continuous time using different states and probability distributions. He proposed two main factors to consider when using the HMM. The HMMs emphasised the need to have a choice of the probability distribution for the observed data and the distribution of the duration spent in each state. Ryden, et al. (1998) used the HMMs to reproduce the observed temporal properties of the Standard and Poor (S&P 500) for the sample period 1928 to 1990. Bulla, et al. (2011) went further with Ryden, et al. (1998) findings to suggest that HMMs with t-distributed components have a higher likelihood of replicating the historical moments of the S&P. The t-distributed components showed a higher state persistence relative to HMMs with normally distributed mixtures.

Guidolin and Timmermann (2006) suggested the use of at most 4 states in HMMs to capture the joint distribution of Bonds and stock returns. They used 4 states which are "crash", "bull", "recovery" and "slow growth". The optimal asset allocations varied across the 4 states. Nystrup (2014) warned that the use of more states may lead to better distributions but may result in overfitting. Kritzman, et al. (2012) employed a two-state HMM to forecast regimes in market turbulence, inflation, and economic growth. They found out that the dynamic process outperformed static asset allocation, especially for investors who seek to avoid large losses. Their findings showed a lower downside risk and improved value at risk compared to a static strategy.

If the investor manages to identify different market regimes for example regimes of low inflation, moderate inflation and hyperinflation, and measure important parameters to be used in the allocation strategies he or she can employ a Markov model that allows for asset allocation in each regime (Nikolaj & Dimitrij, 2012). For example, key parameters in the MVPT are the mean and variance of the assets and hence of the portfolio. These would be different in each regime and using Markov state transitions between each regime we can optimise our portfolio to improve the overall returns of the investor.

The investor may optimise the parameters to be used in the asset allocation strategies based on an HMM that does not explicitly identify the regimes but rather creates regimes based on the intervals of the parameters (Kritzman, Sébastien, Turkington, & David, 2012). The reason for this could be that the investor cannot come up with the necessary regime states for the model.

2.3.2 Value of Dynamic Asset allocation based on the HMM regimes.

Kim (2002) research highlighted how regime detection can be used to minimise capital losses in equity and commodity markets. They proposed that periods of a market crash can be identified with high probabilities using regime detection analysis. In his conclusions, he encouraged the combination of regime detection analysis with traditional optimisation-based asset allocation strategies to achieve stable capital growth.

Another research by Bae, et al. (2014) indicated that the Downside risk was reduced during left tail events when using HMMs with traditional optimisation asset allocation strategies as mean-variance portfolio theory (MVPT) models. Bae, et al. (2014) developed stochastic MVPT models based on the identified regimes for the stock, bond, and commodity markets. The HMM model identified 4 regimes and the author observed that their investment strategy performed remarkably well during market crashes.

Wang, et al (2020) used the HMM models to identify different market regimes for Equities in the US market. The author wanted to use different investment strategies based on factor models in the modelled regimes. In the research, the author used S&P 500 market data as observations in the HMM model. After back testing, the author discovered that the Investment strategies in different regimes produced higher absolute returns compared to individual factor models.

Furthermore, research done by Bulla, et al. (2011) which compared the regime-based allocation strategy to the buy-and-hold strategy showed that the strategy was profitable after taking transaction costs into account. The analysis was done using daily returns of indices from the USA, Japan, and Germany. For these markets, the research showed that the volatility of returns of the regime-based allocation strategy used was reduced by 41%.

Kritzman, et al. (2012) developed an HMM that identified two regimes using observed data for financial market turbulence, inflation and economic growth. Again they also came to the conclusion that regime switching allocation strategy significantly improves portfolio performance with unconditional static strategies.

The literature showed that during forms of market crisis, HMMs resulted in better asset allocation that reduced the portfolio risk and improved portfolio returns under those circumstances.

2.3.3 Criticisms of dynamic asset allocation under HMMs.

One of the problems that dynamic asset allocation introduces when using regime-switching models is that of parameter uncertainty (Guidolin & Timmermann, 2006). Parameter uncertainty exists in the sense that the investor cannot easily know the appropriate parameters to use in the future. However, Hartman & Heaton (2011) suggested that to solve this problem, one must adopt a Bayesian approach to the HMMs to account for the uncertainty.

At times the models used present computational complexity (Zheng, Xu, & Zhang, 2021). Choosing the appropriate model from many choices can be difficult. The implementation of the model in practice may also be expensive and require the hiring of experts by the companies (Petre, A case for dynamic asset allocation for long term investors, 2015). If an error is made in the assumptions, then there is a higher chance of making subsequent errors which would affect the asset allocation of the company, and this may be costly.

2.3.4 Research Gap.

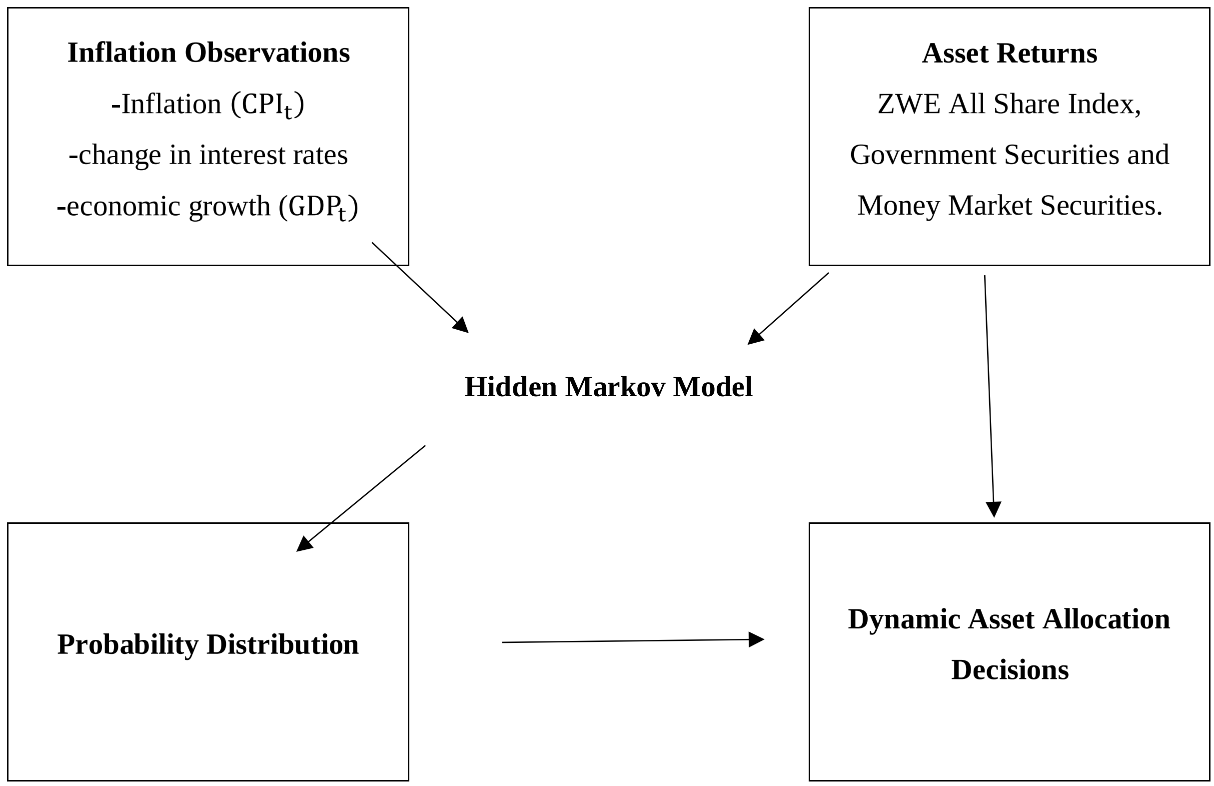
Petre (2015) highlighted that for long-term investors the issues related to the interaction between the funding level and the overall risk appetite of the company need to be researched further. He suggested that there may exist governance issues for the long-term institutional investor which can affect the implementation of DAA strategies.

Nikolaj and Dimitrij (2012) encouraged further research into the implementation of HMMs in different countries with different asset classes. The author pointed out that HMMs strategies strive if there is a clear sign of persistent cyclicality in the asset universe based on economic fundamentals and behavioural factors. For instance, this research looks at the implementation of HMMs in the Zimbabwean market.

2.4 Conceptual framework.

From the literature discussed earlier, we have noted that the use of dynamic asset allocation has notable merits for all types of investors over static passive strategies. Long-term institutional investors such as pension funds, and life insurance companies can also adopt DAA strategies to improve their investment gains while minimising investment risks. Short-term investors can also use the DAA strategies in tactical allocation strategies.

Figure 2.2: Conceptual framework



One of the main concerns in the allocation problem is the issue of choosing a strategy that allows the business to be solvent and continue to exist during a market or economic crisis. From previous studies of HMMs, the author observed seen how important it is for companies to minimise risk in risky market conditions like periods of inflation.

The main goal of the research is to measure how valuable is Dynamic Asset allocation using Hidden Markov Models to the investor during periods of high inflation. The Research compares the risk-adjusted outcomes of the model to a static buy-and-hold passive strategy.

2.5 Summary

The ideal asset allocation ceiling for inflation is still up for debate in terms of investor objectives. However, research has shown that employing HMMs typically boosts risk-adjusted returns in a range of financial crises. In conclusion, the size of the assets under management, the amount of knowledge, and the evaluation of risk and return all have an impact on the distribution of investments in assets. Additionally, other elements including liquidity risk, the accessibility of a variety of asset classes, and corporate governance influence the final asset allocation cut-off point for dynamic asset allocation. The research methodology is described in Chapter 3 (3).

CHAPTER 3: RESEARCH METHODOLOGY

3.1 Introduction.

The third chapter provides a summary of the research philosophy used in the study's methodology, along with information on the strategy, sampling design, and data sources. The chapter also discusses research limitations on the study's scope in addition to ways for modification and analysis of data, Study design, population and sample method, data sources, data collection, data credibility, research limits and the chapter summary.

3.2 Research Design

An experimental research design is employed in the study. The research design, which describes the stages and procedures used in the systematic data collection exercise as well as the guiding philosophy, is essentially the blueprint for the study (Strauss & Corbin, 1998). The thrust of the research was to determine the value of using Markov Hidden Models for dynamic asset allocation during inflationary periods. The study focused on how this approach compares to traditional approaches in maximising risk-adjusted returns. The portfolio risk and return were assumed to be a function of the asset allocation strategy employed and the other factors. Thus, the independent variable in the study was the allocation strategy and the dependent variables were the portfolio standard deviation as a measure of risk and mean portfolio return. The moderator variables were the portfolio weights. The study maintained all other factors as constant or as a control and did not influence the outcome of the experiment. Other factors that influence returns could be company-specific factors or general economic factors which include exchange rates, GDP, and recessions.

The research collected data mainly from secondary sources to achieve its objectives. Secondary data can be defined as the data that was collected for a different purpose other than this research. The study collected Archival data for Consumer Price Index (CPI), money market securities, property prices, equities, government bills and bonds and cash. The data was retrieved from the Reserve Bank of Zimbabwe, the Zimbabwean Stock Exchange, the Financial Securities Exchange, and Property Agencies. The data was in CSV, Excel formats and accessible on their websites. It was cheaper to acquire the data. The data was used to create a hypothetical portfolio using one SAA strategy the MVPT and DAA strategy that uses HMMs. The manipulation, calculation of returns, creation of portfolios and creation of the HMM models were all done in

R and Excel. The R code can be found in the appendix. The emphasis was on testing the performance of the models in maximising risk-adjusted returns, and hence speculative strategies were ignored as they expose most institutional investors in Zimbabwe to risks in their portfolios. The data collected comprised equity indices, property indices, government securities rates and money market rates for the past 5 years. The past 5 years were used as the sample of one of the inflation cycles in the Zimbabwean economy.

3.3 Research Philosophy

The positivist paradigm, which has a focus on using objective quantitative research techniques to uncover the justification for using HMMs in DAA strategies for the Zimbabwean market during inflationary periods, served as the foundation for the research methodology. The paradigm used by the author allows for the development of intricate and highly organized deductive research. Additionally, it permitted the use of an inferential approach to extrapolate a sample's characteristics to the target population. The positivist method is predicated on the idea that assertions are supported by facts, and as such, hypotheses are evaluated considering these facts (Robson, 2002).

3.3.1 Research Approach

The research study used a quantitative technique in keeping with the positivist research philosophy. This strategy involved the utilization and gathering of numerical data for analysis, whose outcomes could be summarised in the form of tables, graphs, and statistics (Creswell, 1994). The goal of the quantitative method in this research was to produce generalizable data about the appraisal of the value of HMMs employing DAA tactics in the Zimbabwean market, as well as to evaluate the planned hypothesis in Chapter 1 (Marshall, 1996). The research's hypotheses were put to the test:

Major Null Hypothesis (H0):

The research assumes that Dynamic asset allocation (DAA) using HMMs results in a low-risk portfolio in inflationary periods. Hence let for the standard deviation of portfolio returns σ_p for Dynamic asset allocation (DAA) using HMMs and Static (passive) Asset Allocation (SAA), we have:

If $\sigma_p = f(X, Y_1 \dots Y_n)$, where $X = \text{allocation strategy}$ and $\mathbf{Y} = \text{other factors}$ then the hypothesis is

$$H_0 : \sigma_p(X = \text{DAA}, \bar{\mathbf{y}}) < \sigma_p(X = \text{SAA}, \bar{\mathbf{y}})$$

Minor Null Hypothesis (H0):

The initial assumption is that there exists risk-adjusted value above the traditional asset allocation methods. Hence let for expected portfolio returns μ for Dynamic asset allocation (DAA) using HMMs and Static Asset Allocation (SAA), we have:

If $\sigma_p = f(X, Y_1 \dots Y_n)$, $\mu_p = G(X, Y_1 \dots Y_n)$, where $X =$ allocation strategy and vector, $\mathbf{Y} =$ other factors, then the hypothesis is:

$$H_0: \frac{\mu_p(X = DAA, \bar{\mathbf{y}})}{\sigma_p(X = DAA, \bar{\mathbf{y}})} > \frac{\mu_p(X = SAA, \bar{\mathbf{y}})}{\sigma_p(X = SAA, \bar{\mathbf{y}})},$$

3.4 Population and Sampling Techniques

The next sections explain the targeted demographic and sampling method used to acquire the data.

3.4.1 Population

According to Frey (2022), a population is the entire collection of entities the researcher seeks to understand, study, and draw inferences from. For this research, the population consisted of all financial returns from asset classes in the Zimbabwean market that retail and institutional investors can invest in. This encompassed property investments, equities, money market securities, and government instruments. The study also collected CPI data which is the primary input into the calculation of inflation in Zimbabwe. CPI is a weighted index that measures the changes in the Prices of Goods in an economy (Gillespie, 2019). The overall population consists of data from 1980. However, there was a standstill in the collection of data after the 2007-2008 economic collapse in the country. Also, there was a 6-month hiatus from mid-2019 in the collection of CPI data by the Reserve Bank. The research adjusted the base value of the CPI value to reflect the reported inflation figure by the Reserve Bank of Zimbabwe. The movements in the property market and equities market were summarised by using the property market indices, all share indices, and industrial and manufacturing equities index data.

3.4.2 Sampling: Purposive Sampling Technique.

The research opted for non-probability or non-random sampling on the collection of data from an inflationary economic cycle. It is a purposive type of sampling technique to reveal the value of DAA using HMMs during inflationary economic cycles. Purposive sampling is you need to use your judgement to select cases that best enable you to answer your research question(s) and to meet your objectives (Saunder, Lewis, & Thornhill, 2019). All the data could not be collected from the target population because of the availability and costs of doing so. The study was focused on the use of mainly secondary data and as such the extent of the availability of quality data influenced the inflationary period the researcher focused on. It was an interval time-based sampling. The data was collected for a recent economic cycle, the 5 years from January 2018 to December 2022. One data point in each variable reflects a monthly period. As such there are 60 data points for each variable. The process involved selecting all necessary archival data that was mainly available online, determining the variables from the data and using the data in the asset allocation and HMM model. The gap after the 2008 economic crisis made the researcher opt out of the period and choose the most recent inflationary cycle in the Country because of the availability of data and the cost of acquiring the data. Past inflationary cycles data was also used for illustrations only.

3.4.3 Experiment Variables and Procedure

The main independent variables chosen in the study are the allocation weights of a portfolio. The weights of the portfolio represent the investor's allocation strategy. The weights would then be assessed in how they influence two main dependent variables which are the portfolio variance and portfolio mean. This is done with all other factors maintained as constant. The other main variable in the research was the stock market indices returns and the government securities returns. The variable would allow the researcher to compute the portfolio returns during the period of study. The returns were regarded as a moderator and remained constant during the experiment. The CPI data was the final key variable that allows the user to calculate the inflation of the country. The calculated inflation was used to determine and infer the two regimes from the data. The regimes involved Markov states of risky, Stable, and low inflationary periods. The variables determined from the experiment would then be used for statistical analysis to check the research hypothesis.

Calculated independent variables from collected data.

1. Inflation is calculated as shown in Equation 3.1. CPI is the consumer price index at time t

$$\mathbf{I_t} = \mathbf{ln}\left(\frac{\mathbf{CPI_t}}{\mathbf{CPI_{t-1}}}\right) \qquad \text{Equation 3.1: Inflation calculation}$$

2. Asset returns. There are calculated as $r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$, P_t is the index or asset value at time t.
3. Portfolio weights. These are based on allocation and are a vector $\bar{\mathbf{w}} = (w_1 \dots w_n), \sum w_i = 1$
4. The government bonds interest rate δ_t.
5. Use of HMM models and Hamilton Hidden Markov AR(1) model (Hamilton J. , 1989) for inflation regimes.

The dependent variables are:

1. Portfolio standard deviation σ_p.
2. Portfolio expected return μ_p

3.4.4 Research Instruments

The research collected secondary data from the Central Bank, Zimstat, Property ZW, Zimbabwe Stock Exchange, and Financial Securities Exchange. The main instrument was the researcher's computer which allowed the author to collect the electronic data. The use of the Excel Office software package was important in accessing the data since the data was in electronic format mainly Excel and CSV formats. This was important because the data was to be used tools used in the analysis and manipulation of data were Excel, and R-software. This reduced the use of physical paper and was cheaper to access and in conduction the research. The R code used in the model can be found in the Annexure section.

3.5 Data collection procedure

The collection of data, the conducting of the research and the output of the experiment were conducted electronically using computer software. The data processing was done in Excel and R. The mathematical models were computed in R and Mathematica.

3.5.1 Data Analysis Technique

3.5.1.1 Descriptive Statistics

The key statistics for the research output were the standard deviation and mean of portfolio returns for both DAA and SAA allocation strategies.

3.5.1.2 Model Diagnostics

Excel and R Studio were used in analysing data in R which uses the R programming language and was used to calculate descriptive statistics and visualization. The data is presented in tables and graphs. Data analysis was done using R programming whereby inferential statistics were applied to the portfolio performance of SAA and DAA. Analysis of The Bayesian Information Criterion (BIC) was used. It also applies to models fitted using the framework of maximum likelihood estimation and adds a penalty dependent on the number of parameters being estimated in the model for probability and inference. A mathematical tool for assessing how well a model matches the data it was derived from is the Akaike information criterion (AIC). AIC is used to evaluate various potential models and choose the one that best fits the data by comparing them (Schwars, 1978).

3.5.1.3 Analytical Model

The author used two theoretical portfolios to represent both Static and Dynamic strategies. The performance of the two portfolios is compared using risk-adjusted measures in Table 3.2. The Portfolio Returns were defined in Equation 3.2

$$\mathrm{r_i} = \sum_{\mathrm{t=1}}^{\mathrm{T}} \sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{w_{i,t}}(1 + \alpha_{\mathrm{i,t}})$$

Equation 3.2: Total portfolio Returns

With $\mathrm{w_{i,t}}$ as weights in an asset i at time t and $\alpha_{\mathrm{i,t}}$ as returns on the asset I at time t.

The static portfolio involves the investor buying equities that represent the all-share index and debt and money market instruments that the investor holds onto until maturity. The investor could also hold property and private equity as a means of portfolio diversification. In this research, the author used a static portfolio of holding an all-share index and money market instruments. There were missing data in other instruments data. Equities represented the risky investment.

The author assumed a Dynamic portfolio that allowed the investor to switch between equities and money market assets in response to perceived inflation regimes in the market. The data on the performance of the two portfolios was collected and analysed.

Table 3.1: Risk-adjusted measures

Measure	Formula	Variables
Return ($\boldsymbol{R_p}$) per risk ($\boldsymbol{\sigma_p}$)	$\frac{R_p}{\sigma_p}$	R_p=portfolio returns, σ_p=standard deviation of returns

Return per Value at Risk	$\frac{R_p}{VaR(\alpha)}, VaR(\alpha) = F_r^{-1}(1-\alpha)$	α%= confidence level, F_r^{-1}= inverse cumulative distribution of the returns
Return per portfolio Beta	$\frac{R_p}{\beta_p}, \beta = \frac{cov(r_i, r_m)}{\sigma_m^2}$	$cov(r_i, r_m)$=covariance of the market and portfolio returns σ_m^2=variance of market returns
Return per Conditional VaR	$\frac{R_p}{CVaR(\alpha)}$ $CVaR(\alpha_2) = F_{r\|r<VaR}^{-1}(1-\alpha_2)$	C-Var = average of negative returns worse than the VaR
Return per drawdown	$\frac{R_p}{\max(\|r_i\|)}, r_i < 0$	

3.6 Data Analysis and data credibility

The R-language and Excel were used to undertake the statistical analyses of the collected data. The data was used to calculate the variables as highlighted above and CPI was fitted in a Hidden Markov model to identify regimes. The credibility of the sources of the data depends on the credibility of the Government institutions that collect the data. It can be said that the secondary data represent the economic variables within Zimbabwe. Other asset classes included missing data for some months as such the research focused on the use of The All-share index and interest rates during the three periods.

3.6.1 Data validation

The extent to which a research tool does or measures what it is intended to. (Thanasegaran, 2009).The validity of the research tool was assessed.

3.6.2 Content validity

The data used was valid for the analysis conducted by the study. The research used extensively the all-share index data, Consumer Price Index (CPI) data and interest rate data in the calculations. In addition, content validity was ensured throughout the modelling process.

3.7 Research limitations

The research was confined to the factors noted in the literature review and research limitations. Hence, it is only applicable to Investments in Zimbabwe. The data also included the Covid 19 period which may result in different results in a world that was not affected by the pandemic. Also, good past financial performance does not guarantee good future financial performance.

The financial performance may not represent real-world performance because of the transaction costs involved, taxation and ease of buying and selling securities in the Zimbabwean market. The use of HMM also requires that the investor has human capital that is equipped with the necessary skills and expertise. This may be costly for the company in the initial stages.

3.8 Summary

Through the research philosophy chosen, the data collection technique employed, and the method utilized to analyse the data, the chapter outlines the dissertation's research design. The chapter also describes the pilot study that was conducted. R was used to analyse the research's data, which was gathered via ZIMSTATS and RBZ. The methods employed to assure data credibility, validity, and reliability were highlighted. The data analysis, presentation, and interpretation are covered in Chapter 4.

CHAPTER 4: DATA PRESENTATION AND ANALYSIS

4.1 Introduction

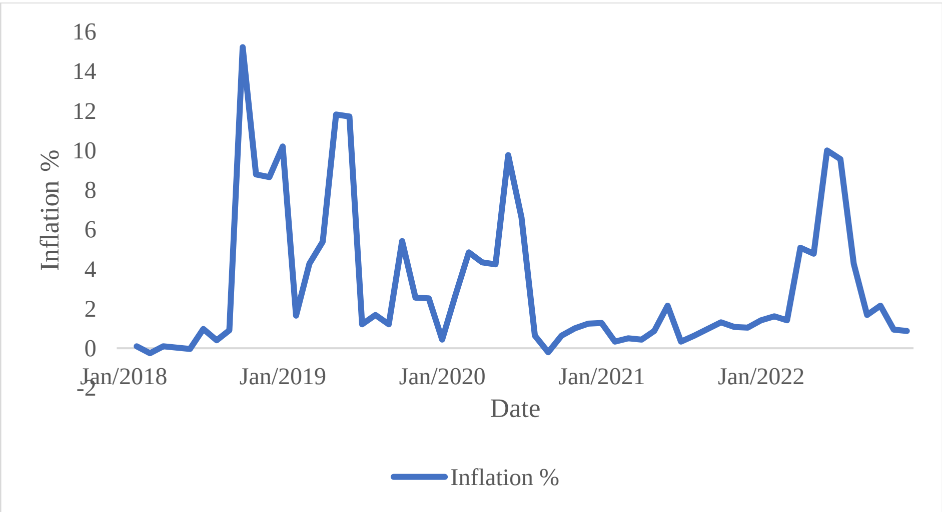
This chapter focused on the analysis of the data and results from the use of HMM in dynamic asset allocation in Zimbabwean. The data include the returns of the assumed portfolios on main three main asset classes in Zimbabwe and CPI as a measure of inflation. The first part describes the features of inflation data and asset returns, followed by the analysis of the portfolio returns of the Market portfolio as a static portfolio (benchmark portfolio) for the passive investor and the Dynamic asset allocation portfolio that responds to prevailing inflation states. Inflation states were modelled using the Hamilton inflation regime switching model and a two-state HMM. The asset returns are calculated in the ZWL currency and not the USD. The passive portfolio would be compared to the portfolio of the dynamic asset allocation. The findings on the performance of the two portfolios would be assessed at the end.

4.2 Data analysis of Inflation and three main asset classes.

The main aim of this section is to describe and analyse the features of data observed by the author in the research process on main assets available to institutional investors in Zimbabwean markets and comment on the insights of the data.

4.2.1 Inflation Data

Figure 4.1: Monthly inflation

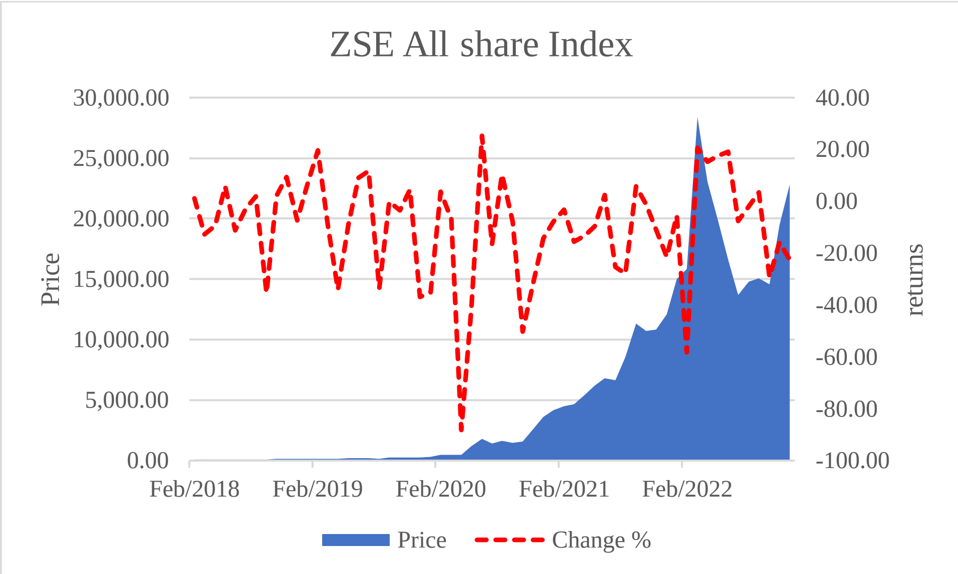


Monthly Inflation was calculated using the CPI data and was expressed as a percentage. Inflation was slightly negative at the start of 2018 and was positive up to the end of 2022.

The biggest rise in prices was in October 2018. The major range of inflation each month was between 1% and 12%. This meant that the price of goods was mostly increasing each month for the period of analysis.

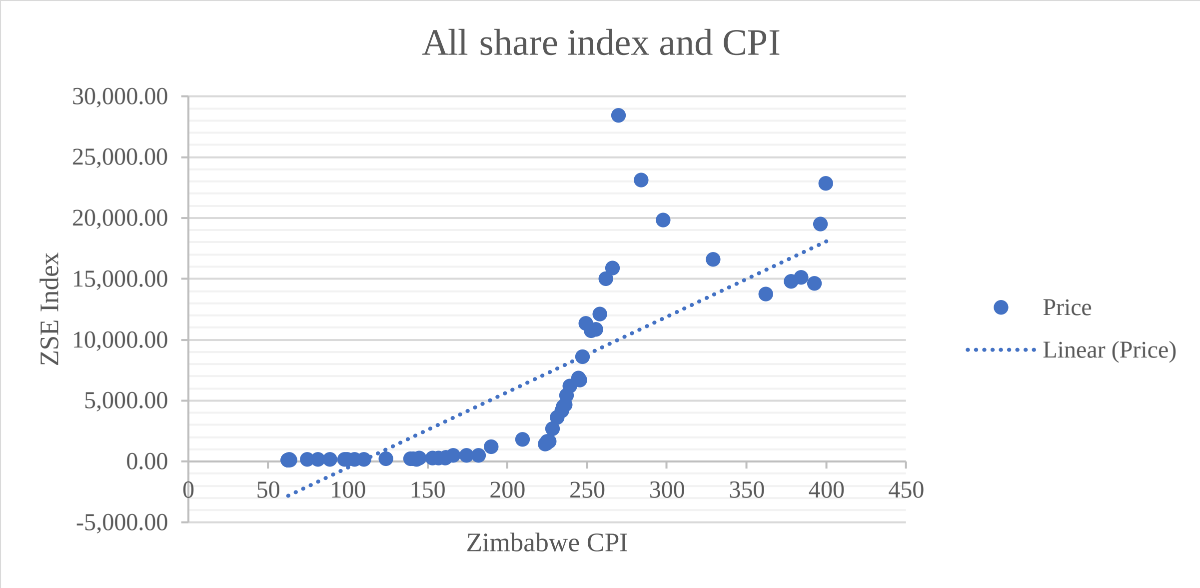
4.2.2 Zimbabwe All Share Index (Passive Portfolio)

Figure 4.2: All Share Index returns



The all-share index summarises the movement of all equities on the ZSE. The data showed that as the price of goods increased also did the value of the All-share index. From the literature, it was noted that almost every rational investor would want to achieve returns which are at least above the benchmark portfolio returns.

Figure 4.3: All-share index and CPI relationship



There was a high correlation between the movement of Equities and the prices of goods and services with a Pearson correlation between the two of 80.66%. The other non-linear correlation measures in Table 4.1 also showed that there was a high degree of co-movement between the two variables. This observation was consistent with most literature on the relationship between prices of goods and share prices.

Table 4.1: Correlation between equities and price of goods

Method	**Pearson**	**Kendall**	**Spearman**
Correlation	0.8065529	0.9111631	0.9851549

4.2.3 Interest Rates in Zimbabwe

Figure 4.4: Zimbabwean Interest rates for the past 3 years

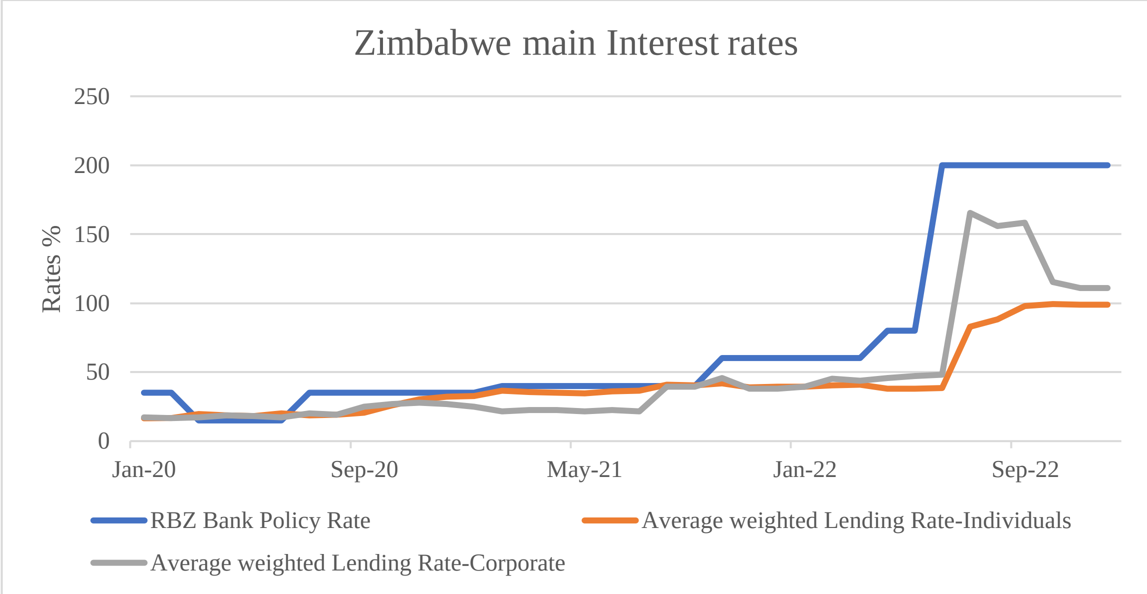
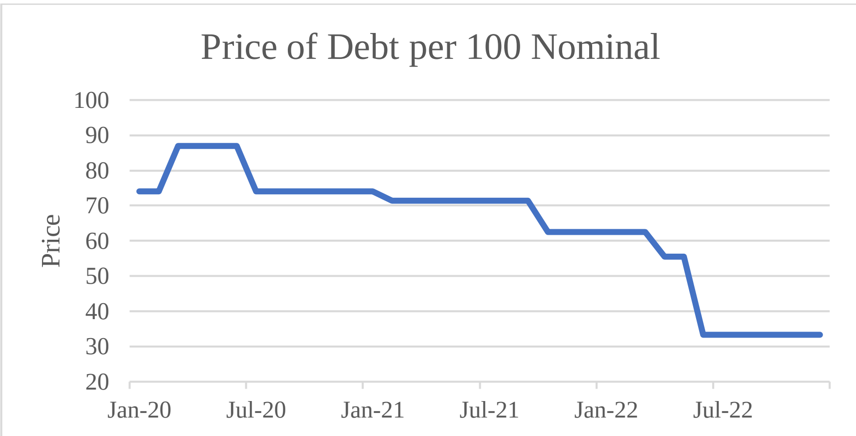


Figure 4.4 shows a time series of monthly interest rates over 3 years. Interest rates reflect the inflation expectations in pricing. From the research, the Pearson correlation between interest rates and prices was 94.29%. This could be explained by including inflation expectation in the pricing of debt and hence as interest rates increase, the price of debt would also decline as shown in Figure 4.5.

Figure 4.5: Price of Debt



4.3 Static portfolio and dynamic portfolio strategies used in the research experiment.

The research collected data for Property prices, Term deposits, Treasury bills, All-Share Index, and corporate bonds. The author defined the static(passive) portfolio as a portfolio that maintains a buy-and-hold strategy on the all-share index plus a combination of the other assets. The dynamic portfolio was defined as a strategy in which an investor alters the asset allocations based on the prevailing inflation states. Based on the data collected, features and constraints of the different asset classes in the five years, the research focused on three main asset classes which are cash, term deposits and all-share index in creating the static and dynamic portfolios.

For the static portfolio, the investor would purchase the all-share index and maintain a contingent cash balance at the Bank. The portfolio would be maintained from January 2018 to December 2022. On the other hand, the dynamic portfolio would alternate between cash as a term deposit at the bank and the all-share index based on prevailing inflation states. These two portfolios would represent SAA and DAA strategies respectively. The performance of the two portfolios would be compared using the metrics in Table 3.2.

4.4 Hidden Markov Models for Identifying Inflation States

4.4.1 Regime switching model with Hamilton AR(1) model.

This section analyses the performance of the DAA strategy against the SAA strategy based on inflation states of the Hamilton Regime Switching model. The author used the auto-Arima function in R to find the best ARIMA time series model for the inflation data to confirm the findings of Hamilton. The best model fitted was $AR(1)$ which confirmed Hamilton's assumption. Figure 4.6 shows the findings of the model.

Figure 4.6: Hamilton $AR(1)$ Inflation model

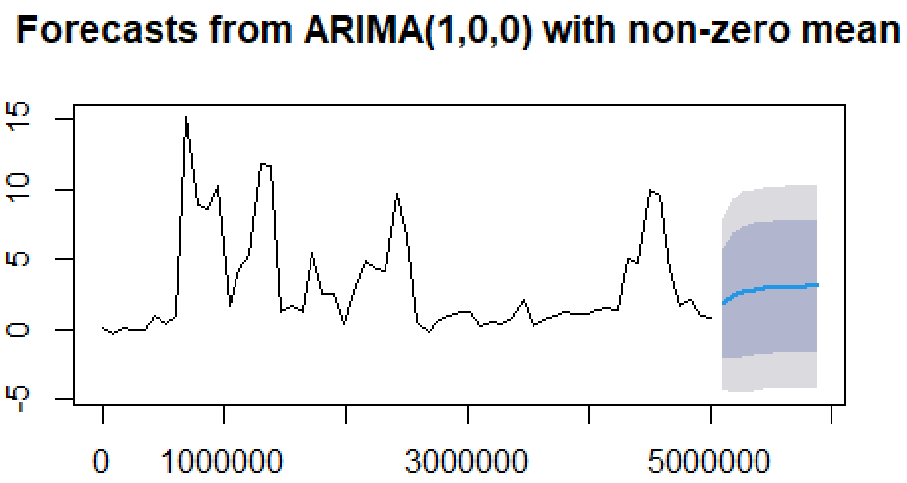


Figure 4.7: $\boldsymbol{AR}(\mathbf{1})$ Model Outputs

```
Series: inflation
ARIMA(1,0,0) with non-zero mean

Coefficients:
         ar1    mean
      0.5518  3.0345
s.e.  0.1075  0.8608

sigma^2 = 9.445:  log likelihood = -149.12
AIC=304.25   AICc=304.68   BIC=310.48
```

The AR(1) model for the inflation data using Auto Arima was $y_t = 3.0345 + 0.5518y_{t-1} + \epsilon_t$. The AR(1) is considered a standard inflation model as proposed by Hamilton (1989).

The research also used the method of regressing the lagged terms as an approximation to the AR(1) model to find better estimates of the coefficients. Regressing the lag terms as $Y_t = \beta_0 + \beta_1 Y_{t-1} + \epsilon$ we had the following output:

Figure 4.8: Linear regression inflation model output

```
Call:
lm(formula = y ~ y1, data = df)

Residuals:
   Min      1Q         Median      3Q     Max
-6.7017    -1.4060    -0.7575  0.5026   13.2819

Coefficients:
               Estimate  Std. Error      t value       Pr(>|t|)
(Intercept)  1.4318     0.5366          2.668    0.00996 **
y1             0.5541     0.1108          5.001    0.00000598 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3.082 on 56 degrees of freedom
Multiple R-squared:  0.3087,    Adjusted R-squared:  0.2964
F-statistic: 25.01 on 1 and 56 DF,  p-value: 0.000005976
```

The parameter estimates were significant at the 5% level. The study used the estimated linear model with Markov regime-switching model. The Markov model had two regimes (states). The output of the model is presented in Figure 4.8.

Figure 4.9: Markov Switching Model Output.

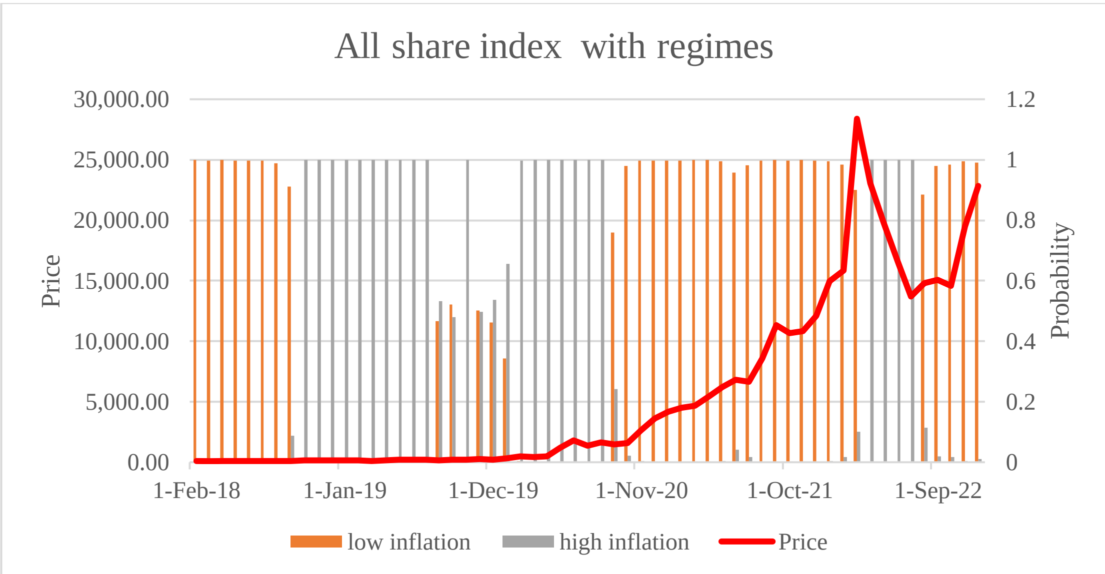
```
Markov Switching Model

Call: msmFit(object = lrm, k = regime_states, sw = rep(T, mv), p = 0,
    control = list(parallel = FALSE))

       AIC          BIC       logLik
  231.1483   255.6318    -111.5741

Coefficients:

Regime 1
---------
                   Estimate    Std. Error   t value         Pr(>|t|)
(Intercept)(S)   0.5288     0.1196      4.4214     0.000009806344617***
y1(S)               0.3824     0.0549      6.9654     0.000000000003275***

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.536499
Multiple R-squared: 0.6033

Standardized Residuals:
          Min                     Q1                  Med                Q3
-1.01082274530 -0.03585332677   0.00005304817   0.17620701756
          Max
 1.25935659156

Regime 2
---------
                      Estimate    Std. Error   t value     Pr(>|t|)
(Intercept)(S)   4.8169     1.5635      3.0808      0.002064 **
y1(S)                0.2068     0.2077      0.9957      0.319396
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.848874
Multiple R-squared: 0.04424

Standardized Residuals:
       Min           Q1               Med        Q3            Max
-6.0237875 -1.0340623    -0.2794414    -0.1667085   10.2141046

Transition probabilities:
                Regime 1       Regime 2
Regime 1   0.90131427    0.1266158
Regime 2   0.09868573    0.8733842
```

The two states identified by the model are high and low inflation states as shown by the estimates of the intercept or the mean of 4.8169 and 0.5288 respectively. The model also provides transition probabilities of moving from one inflation state to another. Over the five years, there was a higher chance of moving from low to high inflation than from high to low inflation, but there were high chances of remaining in each regime. Regime two estimations were not good as they had a low R-squared. The results meant that an HMM can assist an investor in Zimbabwe to quantify the chances of moving from one inflation state to another.

4.4.1.1 Performance comparison between DAA and SAA strategies using Hamilton HMM.

The researcher evaluated the performance of the DAA strategy that alters its portfolio allocation based on predictions of the Hamilton inflation model and SAA strategy. The dynamic two-asset portfolio allowed an investor in Zimbabwe to invest in the stock market and Bank term deposits ignoring tax and transaction costs and assuming that shares are tradable. The investor can then purchase equivalent shares to the benchmark market portfolio the All-Share Index when inflation is in state 1 and move money from shares to bank term deposit for a month when inflation is in state 2 with anticipation of taking advantage of changes in interest rates because of inflation spikes.

Figure 4.10: All-share Index with Regime Probabilities



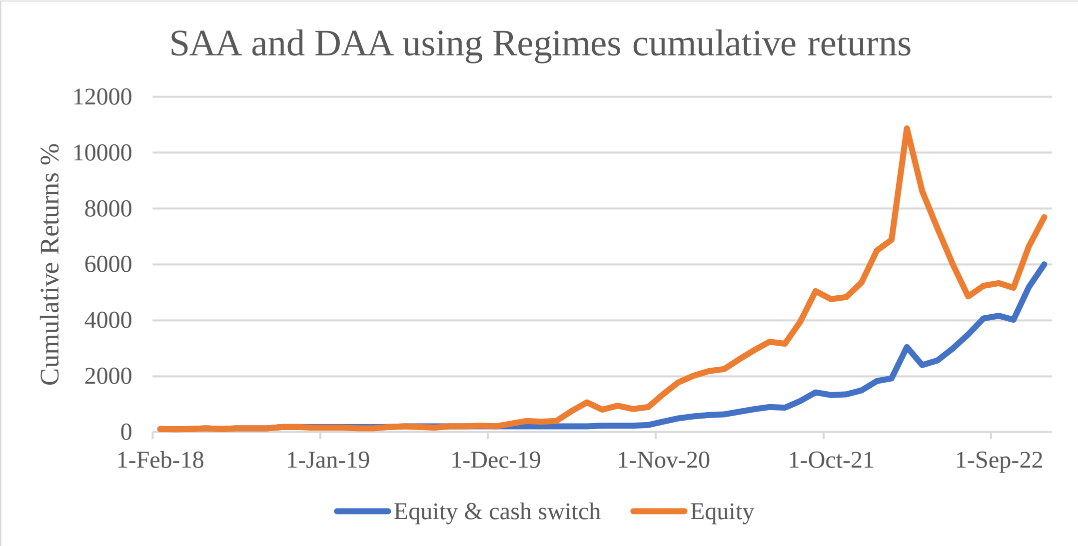
Using the inflation regimes, table 4.2 compares the result of the cumulative returns over five years.

Table 4.2: DAA and SAA Portfolio performance measures.

Portfolio	Std	Expected Returns	5% VaR	5% C-VaR	Drawdown
Benchmark/static	20.99%	9.58%	-19.21%	-21.90%	-25.28%
Equity and Cash (dynamic)	11.54%	7.06%	-4.86%	-11.75%	-23.25%

The drawdown did not affect the dynamic portfolio as much as the static portfolio as shown in Figure 4.11.

Figure 4.11: SAA and DAA portfolio comparison using Hamilton regimes



The returns of the All-Share Index and term deposit portfolio were lower than the returns of the static portfolio. However, the static portfolio had high-risk measures as shown in Table 4.2. The risk-adjusted returns were lower on the dynamic portfolio. The dynamic portfolio had more returns per risk on the standard deviation (Std), VaR and C-VaR risk measures.

Table 4.3: SAA and DAA risk-adjusted portfolio returns

Risk-Adjusted Returns	**Std**	**5% VaR**	**5% C-VaR**	**Drawdown**
Benchmark/static	45.65%	49.87%	43.76%	37.91%
AR(1) Equity and Cash	61.15%	145.4%	60.06%	30.36%

4.4.1.2 Testing the Major Hypothesis.

If $\sigma_p = f(X, Y_1 \dots Y_n)$, where $X =$ allocation strategy and $\mathbf{Y} =$ other factors then the hypothesis is

$$H_0: \sigma_p(X = DAA, \bar{\mathbf{y}}) \leq \sigma_p(X = SAA, \bar{\mathbf{y}})$$

$$H_1: \sigma_p(X = DAA, \bar{\mathbf{y}}) > \sigma_p(X = SAA, \bar{\mathbf{y}})$$

Figure 4.12: F test results on the comparison of portfolio returns

```
> var.test(one$dynamic,one$static, alternative = "greater")

        F test to compare two variances

data:  one$dynamic and one$static

F = 0.30254, num df = 57, denom df = 57, p-value = 1
alternative hypothesis: true ratio of variances is greater than 1

95 percent confidence interval:
 0.194967      Inf
sample estimates:
ratio of variances
        0.3025415
```

From the F-test done in R, comparing the two variances there is no evidence to reject H_0. This showed that there was no evidence to reject the hypothesis that the DAA portfolio variance is lower than the SAA portfolio variance.

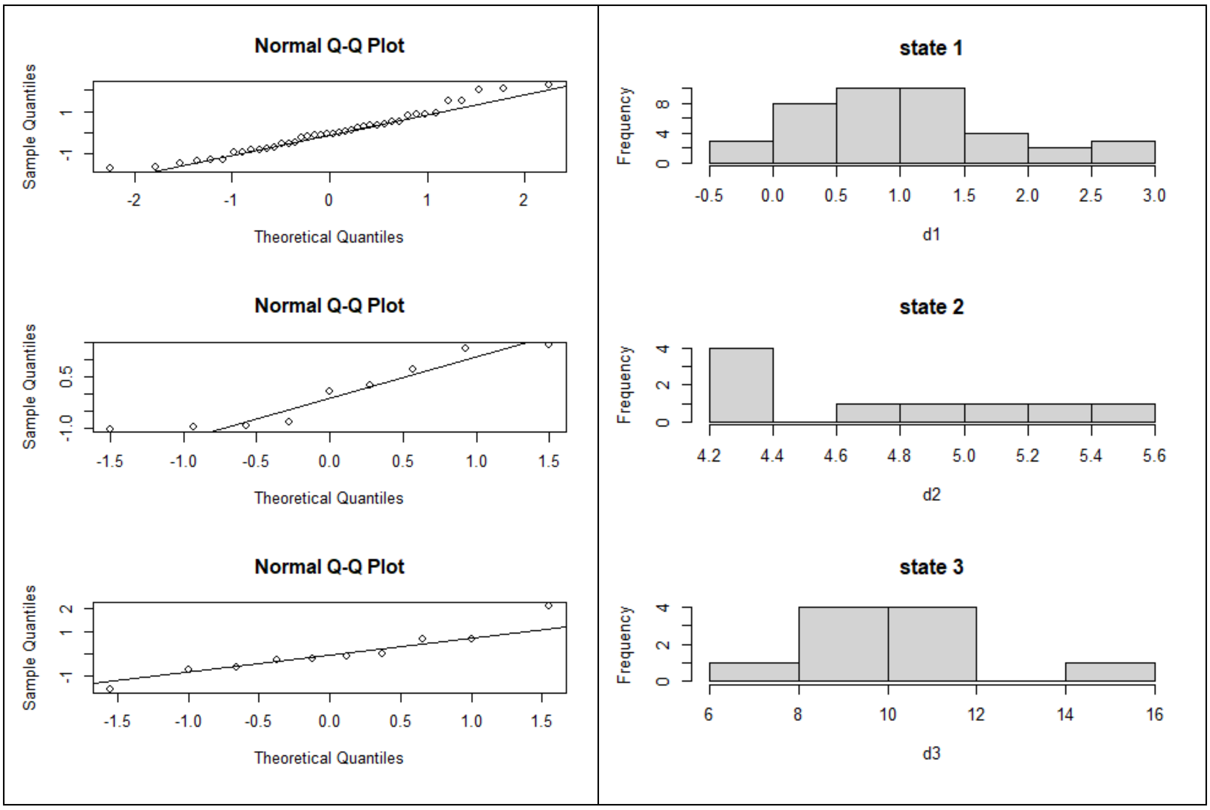
4.4.2 Three-State Hidden Markov Model (HMM)

The researcher used three-state Hidden Markov Model to estimate inflation states. The Algorithm that solves the states is the Baulm-Welch Algorithm and Viterbi Algorithm. The performance of the static and dynamic portfolios was then assessed.

4.4.2.1 Inflation Density Function.

The distribution of the inflation data was modelled as a Gaussian Mixture Distribution with three states. The data analysis produced the QQ-Plots and the Histogram of the 3 states on the inflation data is shown in Figure 4.13. Each state had its fitted normal distribution with different parameters.

Figure 4.13: Normal QQ-plot and histogram for observations in each of the three states



The three hidden states can be observed as low, medium, and high inflation states. The Figure below shows the transition matrix denoted by the matrix Pi. For each of the states, the inflation is modelled as a normal random variable with the mean and standard deviations shown in the vectors pm$mean and pm$sd respectively.

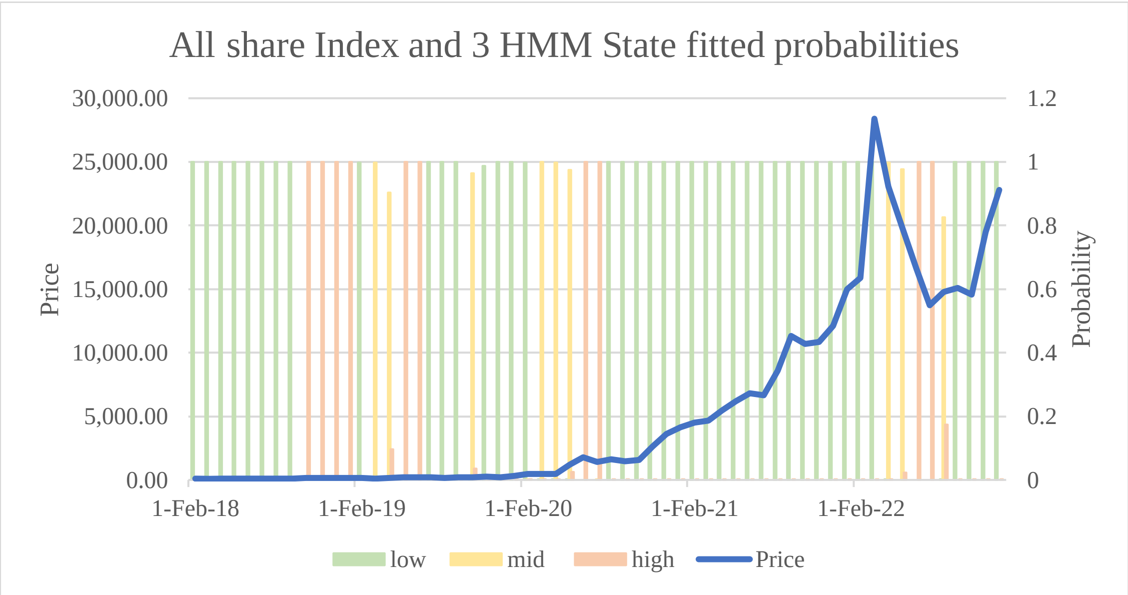
Figure 4.14: Three-state HMM fitted results on inflation data

```
$Pi
          [,1]       [,2]       [,3]
[1,] 0.8717744  0.10149614  0.02672943
[2,] 0.2058035  0.44527795  0.34891855
[3,] 0.3103471  0.08041838  0.60923449

$nonstat
[1] TRUE

$distn
[1] "norm"

$pm
$pm$mean
[1]  1.009699  4.737789 10.024993

$pm$sd
[1] 0.7398132 0.4496550 2.4013768
```

4.4.2.2 DAA and SAA Portfolio Performance Evaluation

Figure 4.15 shows the time series of the Market portfolio (All-share index) with probabilities of the 3 hidden states of inflation.

Figure 4.15: All-share index and state probabilities



The static portfolio involves buying assets that replicate the movement of the index or investing in the index. The Dynamic portfolio would involve the investor buying the assets that replicate the index when in state 1 (low), reducing the investment in equity assets by 50% when in state 2 and the other 50% in a bank term deposit, and 100% in a bank term deposit when in state 3(high).

Comparing the static portfolio and the dynamic portfolio that uses the identified inflation states the author came up with the results shown in Figure 4.16 and Table 4.4 and Table 4.5.

Figure 4.16: Comparison of SAA and DAA returns

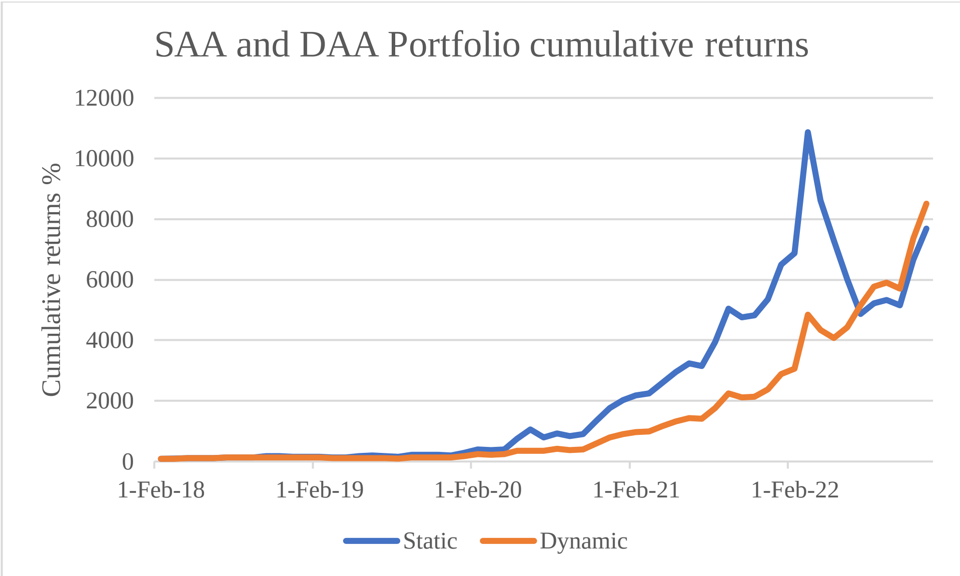


Table 4.4: DAA and SAA portfolio results

Portfolio	Expected Returns	Std	5%VaR	5% C-VaR	Max Drawdown
Benchmark (static)	9.58%	20.99%	-19.21%	-21.90%	-25.28%
Equity and Cash (Dynamic)	7.66%	13.44%	-10.44%	-11.47%	-12.51%

The dynamic portfolio had a return of 7.66% compared to the return of 9.58% on the static portfolio over the 5 years. However, the dynamic portfolio was less risky compared to the static portfolio as shown by the risk-adjusted measures of returns shown below.

Table 4.5: Risk-adjusted returns of DAA and SAA based on 3 states hmm

Risk-Adjusted Returns	Std	5% VaR	5% C-VaR	Max Drawdown
Benchmark(static)	45.65%	49.87%	43.76%	37.91%
Equity and Cash (Dynamic)	57.02%	73.37%	66.81%	61.24%

The 3-state HMM model resulted in better returns and risk-adjusted returns than the Hamilton AR(1) model.

4.4.2.3 Testing the Major Hypothesis.

If $\sigma_p = f(X, Y_1 \ldots Y_n)$, where $X =$ allocation strategy and $\mathbf{Y} =$ other factors then the hypothesis is

$$H_0: \sigma_p(X = DAA, \bar{\mathbf{y}}) \leq \sigma_p(X = SAA, \bar{\mathbf{y}})$$

$$H_1: \sigma_p(X = DAA, \bar{\mathbf{y}}) > \sigma_p(X = SAA, \bar{\mathbf{y}})$$

Figure 4.17: F-test results on the DAA and SAA portfolio variance

```
> var.test(two$Dynamic,two$Static, alternative = "greater")

        F test to compare two variances

data:  two$Dynamic and two$Static
F = 0.40977, num df = 57, denom df = 57, p-value =
0.9995
alternative hypothesis: true ratio of variances is greater than 1

95 percent confidence interval:
 0.2640697       Inf
sample estimates:
ratio of variances
         0.4097722
```

The F-test on the null hypothesis of the two variances resulted in a p-value of 0.9995 and thus there is insufficient evidence to reject the null hypothesis H_0 at the 5% level.

4.5 Discussion of Research outcomes

The research discovered that the use of HMMs with a dynamic asset allocation (DAA) strategy is likely to reduce the portfolio risks compared to a static asset allocation (SAA) portfolio strategy within the Zimbabwean financial markets. This finding is consistent with other research and literature done for other international markets. Kim (2012) in his research concluded that the HMM is a good starting point in asset allocation strategies. In his research, HMM strategies performed better than static strategies. The DAA strategy had high risk-adjusted returns. The portfolio returns on the DAA strategy was lower compared to the returns on the SAA.

Private and public equities and property prices had higher returns compared to fixed interest instruments This was largely because during inflation the returns on public equities, private equities and property increased in line with the rising prices. So, this would favour a buy-and-hold passive investor that had high investments in equities which have returns that are expected to increase over time compared to debt instruments (Kim, 2012). However, as shown by the research the investor would be exposed to high drawdowns from the equities market and or high delays in selling property. when we compare the risk-adjusted returns of the two strategies DAA had better returns per risk taken.

From the research process, the author noted that the HMM provided transition probabilities that showed that there was a higher chance of remaining in one inflation state than moving to another state. This was also consistent with literature for other markets in the international spheres (Wang, Lin, & Mikhelson, 2020). The HMM transition probabilities were a guide to an investor that makes it possible for the investor to adjust their expected asset returns based on different inflation states.

As seen from the research the process of identification of states requires technical expertise and experience to be able to come up with accurate models. There can be numerous parameters involved that can present model risks for the investor.

4.6 Conclusion

The HMM models used from the family of HMM models on inflation data and the dynamic asset allocation strategy of holding a replicating portfolio of the market index and bank deposit, the research observed that the use of HMM in making rational allocation decisions resulted in better risk-adjusted returns than the static portfolio. An investor can also include other assets in the portfolio that optimise the returns as per their risk appetite and the dynamic portfolio could perform better by using the hidden states in the inflation data in making allocation decisions.

CHAPTER 5: SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

This chapter provides a conclusion and a discussion of the main findings of the research. It addresses the research objectives and provides recommendations.

5.2 Summary of findings

The study concluded that dynamic asset allocation using HMMs improves the risk-adjusted returns of a portfolio strategy. The extent to which the use of HMM improves risk-adjusted performance depends on the strategy adopted by the investor. However, using HMM did not guarantee superior returns above the static portfolio.

The two main asset class types in Zimbabwe were Equities and Fixed income securities such as bank term deposits and treasury bills and bonds. The research showed that equity returns are expected to perform better in the long term compared to fixed-interest assets, but there are risks of large drawdowns from unexpected market shifts and irrational exuberance in market prices that threaten portfolio performance. The research discovered that the HMM can smoothen this drawdown by recognising the country's inflation state and responding accordingly based on investor strategy.

The research observed that HMM models are also a scientific way to explain and justify the need to change portfolio allocations based on different regimes in the state of the economy. The models provided transition probabilities between inflation states. This allows investors to maintain a record of past technical assumptions and performance which can be incorporated into future models. This gives an audit record for the investor.

5.3 Research Conclusions

Hidden Markov models are a good scientific tool for identifying inflation regimes within the Zimbabwean economy. The models can assist in making investment decisions and asset management. They provide information to aid in the investment process of the company or for the individual retail investor. The research showed that an investor can improve their risk-adjusted returns, through the use of HMMs in their allocation decisions.

5.4 Recommendations

1. The research recommends the use of HMMs as a tool in asset allocation for investors that employ static investment strategies. The tool provides better risk-adjusted performance on the investor's investment strategy.
2. There was no evidence by the research that using HMM results in higher returns. As such it is recommended that active investors seeking higher returns resort to other techniques if they want to achieve a higher alpha.
3. The research recommends that the investor collects sufficient data to model HMMs. As shown by the research the modelling process requires the sufficient data to be available.

5.5 Areas of further research

The following are the areas that would require further research.

1. Comparison of HMMs with other alternative scientific models in identifying inflation states. There is a need to compare the effectiveness of HMMs with other models such as Deep learning models and ARIMA and GARCH models in dynamic asset allocation during inflationary periods to improve risk-adjusted returns.
2. Sensitivity of HMMs to asset allocation strategies. One could research how sensitive the HMMs are to outcomes of various asset allocation strategies using different parameters.
3. Behavioural aspects of investors using HMMs. There is a need for research that focuses on the uptake and use of the HMM models in the Zimbabwean market by investors and how the use of the models would influence their biases towards investment performance.
4. Integration of HMMs in portfolio optimisation. There is a need for further research on the use of different portfolio optimisation techniques alongside HMMs. The subsequent research could discuss the implications of the main optimisation techniques on HMM portfolios.

5.6 Conclusion

In conclusion, dynamic asset allocation is important in making efficient portfolios. Inflation HMMs assist in making asset allocation decisions in constructing efficient portfolios. This is important to institutional investors that have been using static asset allocation strategies that intend to minimise risk in the pursuit of good returns. With HMM inflation regime dynamic asset allocation investors adjust their portfolios less frequently compared to active and tactical allocation strategies but allow for adjustments that are manageable for the passive investors.

Appendix

6.1 R-code for computations

```
library(MSwM)
library(quantmod)
library(forecast)
#import data
CPI <- readxl::read_excel("CPI_Adjusted.xlsx")
ZIndex <- readxl::read_excel("ZSE_Index.xlsx")

# Calculate monthly inflation rates
inflation <- ((log(CPI$CPI[-1])-log(CPI$CPI[-60])))*100
inflation  <- rev(inflation)
dates <- rev(c(CPI$Date[-1]))
d = data.frame(Date=dates,inflation)
inflation = as.xts(d$inflation, order.by = d$Date)
# plot inflation
plot(inflation, type = "l", col="blue",
     main = "ZWE Inflation Rate", ylab = "%")
#time series model
t_model = auto.arima(inflation)
f = forecast(t_model)
plot(f)
# fit a two-regime AR(1) model
# Define the model
size = length(inflation)
regime_states = 2
y0 <- inflation[2:size]
y1 <- inflation[1:(size-1)]
df <- data.frame(y = y0, y1 = y1)
lrm <- lm(y ~y1,df)
mv <- 3 # means of 2 variables + 1 for volatility
summary(lrm)
rsm <- msmFit(lrm, k = regime_states, sw = rep(T,mv), p=0,
                control=list(parallel=FALSE))
summary(rsm)
plot(rsm, type = "states")

# graphs with regime probabilities:
x11(); plotProb(rsm,which=1) # filtered/smoothed prob
x11(); plotProb(rsm,which=2) # regime 1 prob & shaded area
x11(); plotProb(rsm,which=3) # regime 2 prob & shaded area

#ZSE Index
dates2 <- rev(c(ZIndex$Date[-1]))
ZSE <- rev(c(ZIndex$Price[-1]))
z = data.frame(Date=dates2,ZSE)
ZSE = as.xts(z$ZSE, order.by = z$Date)
# plot ZSE Index
plot(ZSE, type = "l",
     main = "ZWE All share Index", ylab = "Price")
#ZSE Index
dates2 <- rev(c(ZIndex$Date[-1]))
ZSE_returns <- rev(c((ZIndex$Price[-1]-ZIndex$Price[-60])/ZIndex$Price[-60]))
z_r = data.frame(Date=dates2,ZSE_returns)
ZSE_returns = as.xts(z_r$ZSE_returns, order.by = z$Date)


# plot ZSE Index
plot(ZSE_returns, type = "l",
     main = "ZWE All share Index Returns", ylab = "Price")
```

6.2 Baulm-Welch Algorithm (Discrete Time)

```
EMNt = function(x, mu, sigma, Gamma, delta, nu, maxiter = 500,
                tol = 1e-8, print = FALSE){
  n = length(x)
  m = length(mu)
  mu.next = mu
  sigma.next = sigma
  Gamma.next = Gamma
  delta.next = delta
  nu.next = nu
  u = numeric(n)
  llk.prev = 0
  for (iter in 1:maxiter){
    lallprobs = matrix(NA, n, m)
    for (i in 1:n){
      for (j in 1:m){
        if (j < m)
          lallprobs[i, j] = dnorm(x[i], mean = mu.next[j],
                                  sd = sigma.next[j], log = T)
        else
          lallprobs[i, j] = dtmod(x[i], mu.next[j], sigma.next[j],
                                  nu.next, log = T)
      }
    }
    fb = lalphabeta(x, mu.next, sigma.next, Gamma.next,
                    delta.next, nu.next)
    la = fb$la
    lb = fb$lb
    c = max(la[, n])
    llk = c + log(sum(exp(la[, n] - c)))
    for (j in 1:m){
      for (k in 1:m)
        Gamma.next[j, k] = Gamma[j, k]*sum(exp(la[j, 1:(n-1)] +
                                   lallprobs[2:n, k] + lb[k, 2:n] - llk))
      if (j < m){
        mu.next[j] = exp(la[j, ] + lb[j, ] - llk)%*%x/
          sum(exp(la[j, ] + lb[j, ] - llk))
        sigma.next[j] = sqrt((exp(la[j, ] + lb[j, ] - llk)*
                           (x - mu.next[j]))%*%(x - mu.next[j])/
                          sum(exp(la[j, ] + lb[j, ] - llk)))
      }
      else{
        u = (nu.next + 1)/(nu.next + (x - mu.next[j])*
                          (x - mu.next[j])/sigma.next[j]^2)
        w = (exp(la[j, ] + lb[j, ] - llk)*u)%*%x
        z = exp(la[j, ] + lb[j, ] - llk)%*%u
        mu.next[j] = w/z
        w = (exp(la[j, ] + lb[j, ] - llk)*u*(x - mu.next[j]))%*%
          (x - mu.next[j])
        sigma.next[j] = sqrt(w/z)
        w = exp(la[j, ] + lb[j, ] - llk)%*%(log(u) - u)
        z = sum(exp(la[j, ] + lb[j, ] - llk))
        estimator = function(nu, frac){
          -digamma(exp(nu)/2)+log(exp(nu)/2) + 1 + frac +
            digamma((exp(nu) + 1)/2) - log((exp(nu) + 1)/2)
        }
        nu.next = exp(uniroot(estimator, frac = w/z, lower =
                           log(0.01), upper = log(500))$root)
      }
    }
    Gamma.next = Gamma.next/apply(Gamma.next, 1, sum)
    delta.next = exp(la[, 1] + lb[, 1] - llk)
    delta.next = delta.next/sum(delta.next)
    crit = llk - llk.prev
    if(crit < tol){
      np = (m + 2)*m
      AIC = -2*(llk - np)
      BIC = -2*llk + np*log(n)
      return(list(mu = mu, sigma = sigma, Gamma = Gamma,
                  delta = delta, nu = nu, iterations = iter,
                  mllk = llk, AIC = AIC, BIC = BIC))
    }
    mu = mu.next
    sigma = sigma.next
    Gamma = Gamma.next
```

```
    delta = delta.next
    nu = nu.next
    llk.prev = llk
    if(print == T)
      print(paste('Iteration', iter, 'LogLik', round(llk, 4)))
  }
  print(paste('No convergence after', maxiter, 'iterations'))
  return(list(mu = mu, sigma = sigma, Gamma = Gamma,
              delta = delta, nu = nu, iterations = iter, mllk = llk))
}

lalphabeta = function(x, mu, sigma, Gamma, delta, nu){
  n = length(x)
  m = length(mu)
  lalpha = lbeta = matrix(NA, m, n)
  P = rep(NA, m)
  for (j in 1:m){
    if (j < m)
      P[j] = dnorm(x[1], mean = mu[j], sd = sigma[j])
    else
      P[j] = dtmod(x[1], mu[j], sigma[j], nu)
  }
  foo = delta*P
  sumfoo = sum(foo)
  lscale = log(sumfoo)
  foo = foo/sumfoo
  lalpha[, 1] = log(foo) + lscale
  for (i in 2:n){
    for (j in 1:m){
      if (j < m)
        P[j] = dnorm(x[i], mean = mu[j], sd = sigma[j])
      else
        P[j] = dtmod(x[i], mu[j], sigma[j], nu)
    }
    foo = foo%*%Gamma*P
    sumfoo = sum(foo)
    lscale = lscale + log(sumfoo)
    foo = foo/sumfoo
    lalpha[, i] = log(foo) + lscale
  }
  lbeta[, n]
  = rep(0, m)
  foo = rep(1/m, m)
  lscale = log(m)
  for (i in (n - 1):1){
    for (j in 1:m){
      if (j < m)
        P[j] = dnorm(x[i + 1], mean = mu[j], sd = sigma[j])
      else
        P[j] = dtmod(x[i+1], mu[j], sigma[j], nu)
    }
    foo = Gamma%*%(P*foo)
    lbeta[, i] = log(foo) + lscale
    sumfoo = sum(foo)
    foo = foo/sumfoo
    lscale = lscale + log(sumfoo)
  }
  list(la = lalpha, lb = lbeta)
}

dtmod = function(x, mu = 0, sigma = 1, nu = 30, log = FALSE){
  den1 = try(sigma*beta(1/2, nu/2))
  num1 = try(nu^(-1/2))
  den2 = try(nu*sigma^2)
  num2 = try((x - mu)^2)
  dtmod = try(num1/den1*(1 + num2/den2)^(-1*(nu + 1)/2))
  if (log == TRUE)
    dtmod = log(dtmod)
  return (dtmod)
}
```

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