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EXPLORING THE ROLE OF VISUALISATION IN TEACHING CONCEPT OF FUNCTIONS: A CASE OF FURTHER EDUCATION AND TRAINING (FET) PHASE STUDENTS.

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APPROVAL FORM

The undersigned certify that they have read and recommended to Bindura University of Science Education for acceptance of a project entitled "Exploring the role of visualisation in teaching concept of functions: A case of Further Education and Training (FET) phase students" by Noel Magaya in partial fulfilment of the requirements of Master of Science Education in Mathematics

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Dedication

This dissertation is lovingly dedicated to my wife, Nolwazi Alexinah, whose unwavering love, patience, and encouragement have been my constant source of strength. Your belief in me, even during the most challenging times, has kept me moving forward.
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Abstract

This study investigated the role of visualisation in the teaching of concept functions for Further Education and Training phase students. A qualitative method approach was employed combining focus group interviews, teacher interviews and class observations to gather data from a sample of 24 Further Education and Training phase students from one high school in Gauteng province, South Africa. The population was made up of 80 Further Education and Training phase students from one High School that are studying mathematics and encounter function concepts. Stratified random sampling was used to select 24 students from 130 students studying mathematics at FET phase for semi-structured interviews in focus groups made up of 6 students each and 5 teachers from the 50 teachers teaching the learners at this school. In addition, 5 lesson observations on concept functions were conducted.

The findings reveal that students retain function concepts better when visualisation tools are used during the teaching of concept functions. Teachers echoed these benefits, noting that visualisation reduces misconceptions, supports diverse learners, and fosters positive learner attitudes. However, implementation challenges were also reported, including limited device and internet access, insufficient teacher training in integrating technology, time constraints, and unequal student technological readiness.

The results have implications for teaching and learning concept functions at Further Education and Training phase. Recommendations were made for instructors to emphasise visualisation, provide opportunities for practice, and address common misconceptions explicitly. The study contributes to the existing body of research on mathematics education and highlights the need for targeted interventions to support students' learning of concept functions.

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CHAPTER ONE

1.0 THE PROBLEM AND ITS SETTING

1.1 Introduction

This chapter placed the problem into its context through the background to the study and statement of the problem. It also presented the study's objectives, research questions, assumptions, delimitation, and definition of key terms. In addition, the significance of the study to the researcher, teachers, learners, other researchers, and mathematics organisations is discussed.

1.2 Background to the study.

Mathematics is a fundamental component of the educational curriculum, playing a crucial role in developing critical thinking, problem-solving skills, and fostering scientific literacy. In South Africa, the national curriculum emphasises the importance of understanding mathematical concepts, yet many students face significant challenges in grasping abstract mathematical ideas, mainly concept functions. This difficulty often results in poor academic performance and a lack of confidence in their mathematical abilities (Alam & Mohanty, 2023).

Research in educational psychology has established that visualization techniques can enhance understanding and retention of complex concepts by providing students with concrete representations of abstract ideas (Sekhar & Goud, 2024). Visual aids, such as diagrams, graphs, and animations, help learners to bridge the gap between theoretical knowledge and practical application. However, despite the recognised benefits of visualisation, its implementation in South African classrooms remains inconsistent and underutilised.

Several factors contribute to this issue. Teacher preparedness and training in using visualisation tools effectively are often lacking (Isaeva et al., 2025). Furthermore, many schools may not have access to modern technologies that facilitate visual learning, such as interactive whiteboards or educational software. In addition, the cultural and linguistic diversity of South African classrooms poses unique challenges in terms of communication and comprehension, making effective visualisation strategies even more critical (Woldegiorgis & Chiramba, 2025).

The growing emphasis on STEM (Science, Technology, Engineering, and Mathematics) education globally underscores the need for innovative instructional methods to engage students and enhance their learning experiences. As South Africa strives to improve its educational outcomes, particularly in mathematics, understanding how visualisation impacts learning becomes essential. This study explores how various visualisation techniques can be effectively integrated into teaching concept functions in mathematics, ultimately contributing to improved student understanding, retention, and performance.

In South Africa, the grade 10 and 11 CAPS Mathematics curriculum covers topics on functions. These concepts are foundational for further studies in mathematics, engineering, and related fields. However, research indicates that students often struggle to grasp the abstract nature of functions. Understanding functions is crucial for grade 10 and 11 learners, especially those pursuing STEM fields. Functions are pivotal in higher-level mathematics, particularly physics, engineering, and computer science. In the South African CAPS curriculum, these concepts are fundamental yet challenging for many students. However, traditional teaching methods often fail to facilitate deep understanding and practical application of these concepts. The use of visualisation in teaching concept functions offers a promising alternative. By focusing on learners' mental constructions, the visualisation of concept functions can potentially address the gaps in comprehension and application.

In mathematics education, one notes that mathematical objects, such as functions, are not directly accessible apart from external representations (Duval, 2006).

While teaching at a school in Gauteng province, the researcher observed problems in teaching functions. The courses the teachers receive in colleges and universities do not fully prepare them for teaching functions for learners to comprehend. Most teachers rely on the chalkboard and the lecture method. This results in learners disengaging during lessons and

mastering less or no content on concept functions. In tests, low marks are obtained on the topic of functions. Therefore, the researcher was motivated to carry out this research on the role of visualisation in teaching functions at the secondary school level.

1.3 Statement of the Problem

The problem at hand presents a significant challenge that South African students face in understanding and retaining complex mathematical concepts, with particular emphasis on the critical area of concept functions. Despite the recognised and widely accepted importance of visualisation methods in enhancing overall learning, there remains a concerning lack of effective integration of visual teaching methods within the mathematics curriculum that is currently being implemented (Abedi, 2024).

Comprehensive research indicates that visual stimulation is indeed crucial for deep mathematical comprehension. However, many students still struggle mightily to grasp abstract concepts without providing appropriate visual aids that can assist in the learning process (Ramulumo, 2024). Moreover, adopting advanced technologies such as Smart Boards, which can provide powerful visual stimulation, has not been uniformly effective or successful across various educational contexts and environments. This ongoing situation raises pertinent questions about how different visualisation techniques can be utilised more effectively to improve students' understanding and retention of mathematical concepts meaningfully.

Given the established correlation between spatial visualisations abilities and performance in STEM subjects, there is a pressing need to thoroughly explore the impact of various visualisation strategies on enhancing the learning experiences of South African students, particularly in the critical area of mathematics education, so that students can achieve better outcomes and improved mastery of the subject matter (Nadzeri et al., 2024).

Mathematics as a learning area of the National Curriculum Statement of South Africa emphasises critical thinking and the application of mathematics to different everyday contexts. Among other mathematics topics, mathematical functions are introduced to learners possibly for the first time. The conception of mathematical functions is fundamental to more advanced mathematical studies (Romera-Paredes et al., 2024). In this respect, many pupils experience difficulties in understanding this concept. One probable reason for this is teachers' lack of understanding of how mathematical functions should be taught; hence, they use conventional teaching methods, for instance, merely focusing on algebraic manipulations while neglecting the use of visual materials or diagrams.

To teach mathematical functions for understanding, it is important to use visualisation techniques appropriately to connect multiple representations' formats of functions: symbolic, graphical, and tabular forms. Understanding can be achieved by representing ideas in three domains: symbolic representation, numeric representation, and visual representation of ideas connected (Cheung et al., 2025). Symbolic representation refers to utilising linguistic signs, while numeric representations are in a quantitative format. Visual representation represents a form that can be seen. It should, though, be noted that it is not enough to provide multiple representations; the interplay between these representations should be the

central focus when teaching functions (Cukurova, 2025). Therefore, visual material is recommended to help students understand mathematics better. (Mbundu, 2018)

Despite the importance of concept functions, many grades 10 and 11 learners in South Africa struggle to grasp this concept fully. This lack of understanding often leads to poor examination performance and a limited ability to apply these concepts in practical situations. There is a need to investigate the cognitive processes involved in learning concept functions to develop effective teaching strategies that enhance comprehension and application. This led to exploring the role of visualisation in teaching the concept of functions in secondary schools in South Africa.

The effective teaching and learning of mathematical concept functions remains a significant challenge within South African education. Despite the potential benefits of visualisation as a pedagogical tool, there is limited understanding of how teachers and students perceive its effectiveness in enhancing comprehension and retention of these concepts (Zhang et al., 2025). Teachers may lack the necessary training or resources to implement visualisation strategies effectively, leading to underutilisation in classroom settings. Additionally, students' experiences and perceptions regarding the value of visualisation in their learning processes are crucial for understanding its impact.

This study seeks to address the following questions on perceptions:

1. What are teachers' perceptions regarding the effectiveness of visualisation techniques in teaching concept functions?
2. How do students perceive the role of visualisation in their understanding and retention of concept functions?
3. Are there discrepancies between teacher and student perceptions concerning the utilisation of visualisation in the mathematics curriculum?

By investigating these perceptions, this research aims to identify barriers to effective visualisation practices in teaching concept functions, ultimately contributing to improved instructional strategies and educational outcomes in South Africa.

This problem statement frames the research's focus on teachers' and students' perceptions, highlighting the need to understand their views on the effectiveness of visualisation in mathematics education.

1.4 Objectives of the Study

The study aimed to satisfy the following objectives:

1.4.1 To investigate the impact of visualisation on students' understanding and retention of concept functions.

1.4.2 To examine the effectiveness of different types of visualisation tools in teaching concept functions.

1.4.3 To gather teacher and student perceptions of the benefits and challenges of using visualisation in teaching and learning concept functions.

1.5 Research Questions

The researcher sought to answer the following research questions:

1.5.1 In what ways does visualisation influence students' understanding and retention of concept functions?

1.5.2 How do different types of visualisation tools impact students' learning outcomes?

1.5.3 What is the teacher and student perceptions of the effectiveness of visualisation in the teaching and learning concept functions?

1.6 Significance of the Study

This study will address a critical gap in teaching concept functions in grades 10 and 11. The findings will provide insights for educators to identify the most effective types of visualisation tools for teaching concept functions. Furthermore, recommendations will be made for teachers on how to integrate visualisation effectively into their mathematics instruction. Ultimately, this will improve student performance and readiness for higher education and professional careers in STEM fields.

The study will be of great significance to several people in education. These include the researcher, other researchers, mathematics teachers, mathematics organisations and mathematics students who learn functions.

1.6.1 The Researcher

The researcher will learn how to use visualisation to teach functions and improve learners' performance in class. This knowledge will be gained through researching and reading materials written by other researchers. In addition, the researcher will understand how learners visualise when learning functions.

1.6.2 Other Researchers

The study will be significant to other researchers in that the findings of this research may enable potential researchers to use knowledge gained as a base for further research. The research will also give them insight into discoveries and other problematic areas that may require investigation.

1.6.3 Teachers

Other mathematics teachers will appreciate the importance of visualisation in teaching functions and implement it in their classes.

1.6.4 Mathematics Organizations

Mathematics organisations like the Association of Mathematics Education of South Africa (AMESA) will be able to use the findings of this research to empower mathematics teachers and train them to use visualisation in the teaching of functions.

1.6.5 Learners

Mathematics learners will gain confidence in learning functions as they are exposed to visualisation. This may lead to an improvement in performance in the function topic.

1.7 Study Assumptions

The study is based on the following assumptions:

1.7.1 The participants used in this study are authentic to produce reliable data.

1.7.2 The participants will participate freely without coercion.

1.7.2 Secondary school mathematics teachers are aware of the importance of the use of visualisation in teaching concept functions.

1.8 Limitations of the study

Some potential limitations of this study include the constraints of time, resources, and access to a representative sample of participants. Additionally, the study's findings may be influenced by factors such as students' prior mathematical knowledge and the quality of teaching in their respective schools.

1.9 Delimitations of the study

This study will focus solely on South African grade 10 and 11 Mathematics learners in several schools. The research will not extend to learners at lower levels of education or those outside the South African education system.

1.10 Definition of Terms

Key terms to be defined are visualisation, dynamic visualisation, function, visualisers, non-visualizers and visual presentation.

- **Visualisation**

Visualisation may function as a tool to extricate oneself from situations in which one may be uncertain about how to proceed. (Arcavi, 2003). Gildert et al. (2008) state that visualisation is concerned with external representation, the systematic and focused public display of information in the form of pictures, diagrams, tables etc. It is also concerned with internal representation, the mental production, storage, and use of an image, which is often but not always the result of external representation. Visualisation is a complex cognitive process that assists in working with mental objects by creating images (Pei et al., 2022). In particular, visualisation may support teachers in anticipating students' mathematical activities and comprehension and keywords needed for mathematical communication (Mbundu, 2018).

Visualisation is the cognitive outcome of a visual display that depicts an object or event. The above definitions suggest that visualisation means not only seeing but also engaging the students' entire perception.

The role of artificial intelligence (AI) in education can be conceptualised in various ways. AI ensures the effective visualisation of concepts via domain-specific and problematic functions.

Automated reasoning facilitates exploring, investigating, and understanding graphical representations of mathematical functions, which leads to the integration of visual thinking and relational understanding. Spectacular and subtle mathematical visualisations of functions can enhance learning. The errors and technical problems in the calculation can be analysed and corrected (Cukurova, 2024).

A practical and deep understanding of concepts and processes can be supported through appropriate visualisation, especially relevant in abstract or complex intellectual topics. AI technology can be utilised to visualise concepts that have domain-specific functions effectively. Visualisation of mathematical functions is one of the possible implementations of problematic functions in educational applications. The AI technology can be implemented as automated reasoning, which permits the generation of formulas corresponding to the features of the investigated graphical representations (Buehler, 2024). Automated reasoning facilitates exploring, investigating, and eventually understanding graphical representations of mathematical functions, which leads to the integration of visual thinking and thus enhances the relational understanding of mathematical functions.

As a relationships-centred approach, it is based on the idea that integrating visual thinking can lead to deep interaction between the visual and verbal representations of mathematical concepts and, consequently, to the relational understanding of corresponding relationships. (Muzangwa & Ogbonnaya, 2024)

- **Dynamic visualisation**

Schnotz et al. (1999) define dynamic visualisation as representations that change their graphical structure during the presentation. In agreement, Kaput (1992), postulates that in dynamic visualisations, the states of objects can change as a function of time.

- **Function**

A function is a relation or mapping between two non-empty sets, which has a rule for relating each element in one set to one and only one element in the other set. (Fujita & Smarandache, 2025). In mathematics, a function is an expression, rule, or law defining a relationship between one variable (the independent variable) and another (the dependent). A wide variety of concept functions are applied in mathematics. Moreover, they can be classified into numerical functions and graphic functions from different types of images. There are functions such as trend functions, step functions, peak functions, pit functions, piecewise functions, modular functions, trigonometric functions, power functions, exponentially smooth functions, and the product of functions.

- **Visualisers**

Visualisers are individuals who prefer to use visual methods when attempting mathematical problems that may be solved using both visual and non-visual methods.

- **Non-visualisers**

Individuals who prefer not to use visual methods when attempting to solve mathematical problems.

- **Visual presentation**

It is a teaching method that involves the formation and use of visual imagery by the teacher, pupils, or both.

1.11 Chapter one summary

In this chapter, the researcher highlighted the purpose of the study, the statement of the study and the research questions about the reasons of carrying out the research. The significance of the study, noting the intended beneficiaries of the study and how these would benefit from the study, was reflected on. Limitations of the study were highlighted as well and key terms were defined. There were three research questions and objectives which focused on importance of visualization in the teaching of functions.

1.12 Chapter two summary

This chapter aimed to draw attention to relevant ideas that previous researchers had contributed regarding the role of visualisation in teaching concept functions in mathematics at the secondary school level. Among the topics covered were the extent to which visualisation helps students understand and retain concept functions and the impact of different visualisation tools on comprehending concept functions. Furthermore, the perceptions of students and teachers on the use of visualisation in learning concept functions were analysed.

1.13 Chapter Three Summary

This chapter outlines the methodological framework of a qualitative case study investigating visualisation in teaching mathematical functions. It is grounded on the interpretivist paradigm and supported by contemporary educational research. The research uses diagnostic tests and interviews to evaluate the experiences and interpretations of visualisation strategies among Grade 10 and 11 students and educators. Data were gathered ethically and evaluated using both statistical and thematic methods. Trustworthiness was augmented by triangulation and member verification, which was consistent with contemporary qualitative research standards. The methodology guarantees rigour and pertinence in examining the influence of visual aids on conceptual learning in mathematics.

1.14 Chapter Four Summary

The analysis of results highlights several benefits in the use of visualisation in the study of concept function. The findings also emphasise the importance of effective instructional strategies, including the use of dynamic visualisation and real-world applications, in helping students to retain concept functions. To improve teaching and learning outcomes in concept functions, it is essential to adopt interactive, student-centered instructional approaches that foster both conceptual understanding and practical skills. This analysis confirms that visualisation plays a crucial role in making functions more accessible, understandable, and memorable for FET learners. For maximum impact, future teaching should integrate visualisation holistically, addressing both pedagogical strategy and infrastructure realities, while empowering both teachers and learners with tools, time, and training.

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 Introduction

This chapter discussed other educational thinkers' findings on the effectiveness of visualisation when teaching concept functions to students in grades 10 and 11. These aspects encompass visualisation's merits in enhancing learner engagement, retention and understanding of concept functions in mathematics using different visualisation tools and perceptions of students and teachers on using visualisation in learning and teaching concept functions. Furthermore, the problems encountered in implementing visualisation in teaching concept functions were analysed. Above all, the researcher scrutinised the strategies that are of instrumental consideration when implementing visualisation in the teaching of concept functions.

2.2 The Conceptual Framework

This study's main objective was to ascertain the efficacy of using visualisation in teaching concept functions to grade 10 and 11 mathematics students in South Africa. The main goal was accomplished by looking at a variety of factors, including the advantages of using visualisation in teaching concept functions, impact of different visualisation tools on students retention and understanding of concept functions, students and teachers' perceptions on the use of visualisation in learning and teaching concept functions, the difficulties that teachers and students face when using visualisation tools, and any strategies that teachers could implement to improve the use of visualisation in the teaching and learning of Concept Functions.. In order to solve the topic under investigation, the researcher collaborated with Grades 10 and 11 students, since most of the aspects under investigation were found at those levels.

The researcher was inspired to participate in this study after realising that Grade 10 and 11 Mathematics teachers were not utilising modern teaching and learning techniques when instructing their students on Concept Function topics. According to Woolfolk (2020), most teachers prefer to use traditional teaching and learning methods used by their teachers, at the expense of current approaches, which are not only learner-centered but also promote high levels of motivation and interest among students. Thus, the researcher wanted to ascertain how visualisation can be instrumental in improving learner engagement, retention and understanding during the teaching and learning of Concept Function topics at Grade 10 and 11 levels in South Africa.

2.3 The Theoretical Framework

1. **Constructivist Learning Theory by Piaget (1973) and Vygotsky (1978)** emphasises the role of social interaction in the learning process, suggesting that visualisation tools can facilitate collaborative learning experiences among students. Constructivism posits that learners construct their understanding and knowledge of the world through experiences and reflecting on those experiences. According to Piaget and Vygotsky, meaningful learning occurs when students actively engage with content, and visualisation is a crucial tool for facilitating this process. By employing visual aids, students can relate abstract mathematical concepts to concrete representations, allowing for deeper understanding and retention (Ziatdinov & Valles Jr, 2022).

2. **Sweller coined the Cognitive Load Theory in 1988,** emphasising the importance of managing the mental effort required to learn new information. Visualisation can reduce extraneous cognitive load by providing explicit representations of complex concepts, allowing students to focus on the intrinsic cognitive load associated with understanding concept functions (Cabouat, 2024). Effective visualisation strategies can help optimize cognitive processing, enhancing learning outcomes.

3. **Paivio proposed the Dual Coding Theory in 1971,** which suggests that information is better retained when represented in verbal and visual formats. In the context of teaching mathematics, when students are exposed to both symbolic representations (e.g., equations) and visual representations (e.g., graphs and diagrams), they are more likely to develop a robust understanding of concept functions (Uminski et al., 2025). This theory supports the integration of visualisation in teaching practices.

4. **Social Learning Theory was developed in 1977 by Albert Bandura** and emphasises the importance of observational learning, imitation, and modeling in the educational process. In the context of visualisation in South African education, this theory suggests that students can enhance their understanding and retention of concept functions through visual representations and engaging learning environments (Alkhabra et al., 2023). Albert Bandura’s Social Learning Theory posits that learning occurs within a social context and emphasises the role of observation and modeling. Teachers' perceptions of visualisation techniques may influence their experiences and the learning environment. Understanding how teachers model visualisation strategies can impact

students' perceptions and subsequent engagement with these techniques (Liu et al., 2024).

5. **Theory of Multiple Intelligences by Howard Gardner in 1983** highlights that individuals have different ways of processing information and learning. Visualisation caters to visual-spatial learners who excel in understanding through images and diagrams (Ramulumo, 2024). Recognising the diversity of learning styles within a classroom can inform how visualisation is integrated into teaching practice, ensuring all students can access effective learning strategies.

6. **Fred Davis developed the Technology Acceptance Model (TAM) in 1986.** The Technology Acceptance Model is a valuable framework that can provide profound insight into teachers' and students' perceptions and attitudes regarding the effectiveness of various visualisation tools and technologies used in educational settings. This well-established model primarily focuses on two critical factors: perceived ease of use and perceived usefulness, which are key determinants of technology adoption in the learning environment (Ma et al., 2025). By comprehensively understanding these perceptions, educators and administrators can effectively identify potential barriers that may hinder the successful implementation of visualisation strategies within the classroom. Recognising and addressing these barriers is essential for fostering an environment conducive to adopting innovative technologies that enhance learning experiences.

2.3.1 Justification of theoretical framework

This theoretical framework meticulously integrates multiple relevant theories to provide a more comprehensive understanding of visualisation's significant impact on teaching and learning concept functions in mathematics. By systematically examining the intricate interplay and connections between these various theoretical perspectives, the study aims to thoroughly uncover and analyse the perceptions held by both teachers and students regarding the overall effectiveness of visualisation techniques in the educational context. Ultimately, this detailed exploration contributes significantly to developing more effective instructional practices and seeks to improve educational outcomes in the challenging field of mathematics education, specifically in South Africa (Saal et al., 2025). This research endeavors to enhance instructional and pedagogical strategies through a deeper understanding of the crucial role of visualisation in facilitating learning and comprehension of concept Functions in mathematics, thereby enriching the educational experience for all learners involved.

From the theoretical point of view, the framework selected is raised because it is both historically rooted and theoretically explicative of a series of joint collective practices engaged towards the same goal of the teaching profession. The issue of teaching and learning cannot be fully understood by observing classroom practices isolated from the ground of social and of institutional practices (Parry & Metzger, 2023). Moreover, the 'health' of functions teaching in primary schools tends, to a large extent, to have repercussions on the secondary level. This implies that those involved within the school system must establish a dialogue, especially pertinent to the current change context, as 'what has to be taught'. The problem of teaching and learning is rethought and reframed, especially in developing the intervention programme's structure and research protocol. An extensive literature review corroborated the effect of such revisions. Further, many didactical transpositions went with experimentation to adapt and contextualise learning contents regarding concepts, methodology and pedagogy design (Janík et al., 2024).

2.4 In what ways does visualisation improve students' understanding and retention of concept functions?

2.4.1 Conceptual understanding of Concept Functions.

Gaining a comprehensive, in-depth, and profoundly rich understanding of the fundamental concepts and the myriad ideas presented in a thoroughly detailed manner is essential for fostering truly effective, impactful, and meaningful learning experiences while mastering the intricacies and complexities of the subject matter. This heightened depth of knowledge dramatically enhances an individual's grasp and comprehension of the material and enables a significantly deeper appreciation and critical engagement with the topic function (Zhao et al., 2024). Moreover, it encourages ongoing intellectual exploration, curiosity, and a quest for further knowledge, promoting a more enriching educational journey that transcends the mere acquisition of facts and figures by utilising various visualisation techniques such as graphs, charts, and interactive models, educators can create a more immersive learning environment. These methods not only aid in simplifying complex concepts but also facilitate a more intuitive understanding of the relationships between different functions. As students engage with visual representations, they are more likely to retain information and make connections that enhance their overall academic performance (Alias & Razak, 2025)

2.4.2 Give mathematical explanations and make new discoveries on concept Functions.

At the core of this comes the need for learners to have the ability to interpret mathematical vocabulary, perform mathematical exercises, give mathematical explanations, and make new mathematical discoveries, thus integrating different mathematical topics. Conceptual understanding is necessary as it makes it possible for learners to be in a good position to work within the current educational curriculum, by effectively communicating the mathematical knowledge they have acquired. The need for such understanding is also imposed by most examinations in high school, where comprehension questions form a considerable part of the assessment (Morris et al., 2021). This understanding is further enhanced through visualisation techniques, which allow students to create mental models of mathematical concepts. By employing visual aids such as graphs, diagrams, and interactive software, learners can better grasp complex functions and their applications (Alabi, 2024).

2.4.3 The Role of Visualisation in Conceptual Understanding of Functions.

It has been widely noted that visualisation is one of the key underpinning processes essential for genuinely understanding mathematical concepts and the various processes that underpin them (Alam & Mohanty, 2024). This vital cognitive tool plays a significant role in fostering a deeper and more comprehensive understanding of mathematics as a whole. By effectively utilising various visualization techniques, individuals are much better equipped to distinguish between well-known cognitive structures that they already possess, which are commonly referred to as schemas, and those cognitive structures that they need to actively construct or connect to new pieces of information (Taylor & Crocker, 2022). This connection is typically achieved through the important processes of accommodation and assimilation. This crucial distinction enhances learning and significantly aids in the grasp of complex mathematical ideas and relationships. Visualisation not only promotes better retention of information but also encourages critical thinking and problem-solving skills, which are essential in mathematics.

2.4.4 The Impact of Visualisation on Students' Understanding and Retention of Concept Functions

Learners' understanding and the retention of the function concept form the basis for understanding and seeding the understanding of many other mathematical concepts and procedures. There have been much effort and some progress in designing and evaluating intervention to improve learners' understanding of functions. However, in South Africa, there have been questions about the retention of conceptual knowledge, retention of the understanding gained at one period by learners over time and after some other learning experience and the teaching of instructing low-income learners (Banda & Nzabahimana, 2021). One of the most under-researched issues in mathematical literature is the role visualization might play in students' understanding of functions and retaining the understanding so acquired (Muhembo, 2018).

2.5 How do different types of visualisation tools impact students' learning outcomes?

2.5.1 Graphical Representations

Using graphical representations of mathematical problems and explanations of such problems has become a common practice in mathematics education. Conceptually based teaching has been stressed in curriculum documents and the literature for mathematical learning and problem solving (Sevinc & Lesh, 2022). In South Africa, there are demands for improving students' understanding and retention of concept functions in mathematics. The introduction of functions at schools should also increase students' understanding and retention of this topic.

However, student understanding and retention of concepts and skills also is important as there is a shift in the teaching of school mathematics from theoretical to more applications- oriented training of students to equip them with necessary skills for the real -world applications and hence the question under what conditions the use of visualisation is effective to improve student understanding and retention of concept functions? In addition, student understanding and retention of graphing functions will be explored through graphical representations to explain graphs of functions. Reducing the number and extending the number of graphs of functions should also allow one to deepen the understanding of them. Some South African studies have attempted to explore the factors that affect students' understanding of functions; however, adult teacher training students were used for their studies (Eloff et al., 2023).

This national requirement should encourage more South African educators to consider giving quality professional development on the professional teaching of functions and research about teaching of functions. Nevertheless, an ongoing challenge is the interpretation of what a function is, and how it differs from a relationship between a free and a dependent variable (Muhembo, 2018). To meet this challenge, tertiary interventions commonly offer students opportunities to explore the concept of function using visual representation, mainly through graphical representations. The ability of learners to interpret graphical representations is considered essential to conceptual fluency in functions (Emmanuel, 2011). It has been suggested that learners have difficulty interpreting graphical representations because they struggle to process this representational form.

2.5.2. Interactive Simulations

The relationships between variables influencing a response can be modeled worldwide using functions. In South Africa, specifically in rural areas with diverse learners, this concept is still tricky despite its importance for further development in the mathematics curriculum (Tibane et al., 2024). Visualisation could aid students in understanding and retention of the concept functions. In this study, the effects of two separate visualisations were compared: one that is interactive and animated, and one that is animated only. (Ziatdinov & Valles Jr, 2022). The use of computer-based tools in education is booming, and Australian State and Federal Governments are pouring large amounts of funding into providing these tools for teachers. At the same time, the evidence for their educational effectiveness is still being gathered (Alhadlaq, 2023). This study compares the effectiveness of scientific visualisations - computer-based animations and simulations - with teachers' 'normal' teaching without visualisations, regarding students' development of key chemistry and physics concepts. Overall, there were no significant differences in conceptual development between the students taught with and without visualisations. Although not substantial, for most of the topics and sub-concepts, there was a trend for students to develop a better understanding through the visualisation approach. Likewise, there were some topics (and even sub-concepts) where students demonstrated significantly greater development of understanding when taught with visualisations (Geelan, 2012). With NAPLAN test scores controlled for, many more topics, sub-concepts and tests would show significant effects in favour of the visualisations.

2.5.3. Diagrams and Flowcharts

Different forms of visual representation include sketches, drawings, diagrams, pictures, flowcharts, and video. Visual representation of problem situations in the context of mathematical word problem solving encompasses graphs, bar models, ratio tables and tape diagrams (Ayabe et al., 2022). The impact of visual representation exists in the use of different kinds of representations provided in problems. Six items from different grades of local textbooks were written to expose participants to various representations of problem situations (Cheung &Winterbottom,2023). Teachers and tutors communicated their assumptions through visual modes to provide feedback on details. Where visual feedback was present, the procedures featured increased understanding and retention of the task object. Problem situations combined with visual feedback were better remembered over time in a general sense in terms of sketches and flowcharts (Muhembo, 2018). A repeated presentation of visual feedback increased the amount of visual feedback effects, extending to three weeks post retention. Moreover, retention was more significant when the target information was present in concept functions, as these visual tools help to clarify complex relationships and processes. This suggests that incorporating diagrams and flowcharts in teaching methods can significantly enhance students' comprehension and long-term retention of mathematical concepts and enable students to visualise the connections between various mathematical principles. By representing functions through diagrams and flowcharts, educators can provide students with a more intuitive grasp of the subject matter, ultimately fostering a deeper understanding of how these concepts interrelate in real-world scenarios, than when the initial problem situation was accompanied by visual feedback. More detailed information has been provided on transforming the initial structure into the correct target structure for one, two, and three facilities. Participants' recall of the transformation steps increased because of visual feedback.

2.5.4. Multimedia Resources

Images and multimedia, as parts of the academic environments, represented extracted digital resources for visual learning. Although such resources have been used in academic settings for over a decade, resources available to students in schools and universities have significantly increased in quality, accessibility, and quantity in recent years (Wang et al., 2023). In response, a large body of research on students' use, particularly non-academic use of images and multimedia, has emerged in library and information science (Matusiak, 2013). This study reports on the portion of a more significant research project on "Digital

Visual Literacy in Feng Shui Studies”. While employing various digital resources in image, audio, and moving image in educational practice for teaching a Feng Shui class and studying academic and general use practices related to images and multimedia of Feng Shui students, in particular, this case study aims to gain insight into how information-enriched classroom environments affect learning image uses (Birt et al., 2016). The intervention classroom with visual learning tools contributed to students through various resources in multiple modes that can support individual differences in processing information styles and enhance memory. It can be concluded that multimedia resources in a visual digital learning environment are important to educational success. Resources, such as PowerPoint slides, images, and maps, assisted students in understanding and remembering subject materials given in presentations, thereby contributing to knowledge retention—if not entirely accurate (Jaleniauskiene & Kasperiuniene, 2023). Image and multimedia resources constitute extracted digital resources for visual learning in the classroom environment and are substantial components of the learning process. Visual resources included, e.g., PowerPoint slides, images/photos, and other graphics placed in them. Not only were PowerPoint presentations played during the entire class duration, but afterwards, they were also made available to the students as e-files. The electronic presentations contained many images, not all of which were shown in class.

2.6 What is the teacher and student perceptions of the effectiveness of visualisation in the teaching and learning concept functions?

This section analyses students' perceptions of the effectiveness of visualisation in teaching and learning a concept, functions, in mathematics and the consistency of these perceptions with empirical evidence. A 48-item survey questionnaire was administered to 1,915 secondary students and 68 teachers in Australia (Altuwaijri et al., 2025). A majority of the students and teachers, irrespective of gender, age, type of school and socio-economic status, had a positive view of the effectiveness of visualisation in teaching and learning functions. There were no consistent patterns of differences in students' and teachers' perceptions due to gender, age, teaching experience, type of school and socio-economic status.

In today’s education, visualisation is used in many ways in teaching and learning mathematics. However, many studies in this field have investigated only teachers’ practices, resources and the use of visualisation in classrooms (Zhang et al.,2025). More empirical studies are needed to explore students' perceptions of the effectiveness of visualisation in teaching and learning mathematics and the consistency of these perceptions with empirical evidence. This research study contributes to addressing this gap by analysing students' perceptions of the effectiveness of visualisation in teaching and learning mathematical concepts and functions. Drawing on primary data collected in Australia, students' and teachers' perceptions of visualisation, as elicited by a 48-item questionnaire, are examined (Geelan, 2012). This paper primarily focuses on the empirical research findings and their implications.

Using empirical survey data, the four research questions were addressed: (1) After being exposed to visualisation in teaching and learning 'functions', what are students' perceptions of the effectiveness of visualisation on understanding the concept? (2) After being exposed to visualisation in teaching and learning of 'functions', do these students' perceptions of the effectiveness of visualisation on understanding the concept of functions consistently match the empirical evidence found in the literature? (3) Is there any gender, age, school type and socio-economic status differences in students' perceptions of the effectiveness of visualisation on understanding the concept of functions after being taught a lesson on the topic? (4) Do teachers' perceptions of visualisation's effectiveness match students' perceptions?

2.7 Chapter summary

This chapter aimed to draw attention to relevant ideas that previous researchers had contributed regarding the role of visualisation in teaching concept functions in mathematics at the secondary school level. Among the topics covered were how visualisation helps students understand and retain concept functions and the impact of different visualisation tools on comprehending concept functions. Furthermore, the perceptions of students and teachers on the use of visualisation in learning concept functions were analysed.

CHAPTER 3

Research Design and Methodology

3.1 Introduction

The chapter presents research paradigm, epistemological assumption, ontological assumption, methodological assumption, axiological assumption. It also presents research approach, research design, data collection methods as well as population, sampling technique and sample. It further presents data analysis method, trustworthiness, ethical considerations and the summary.

3.2 Research paradigm

According to Veddy (2020), a research paradigm is a framework that guides researchers in their approach to inquiry, influencing how they understand and interpret the world. In this study, the researcher opted for interpretivist research paradigm. For this reason, interpretive researchers start with individuals and try to understand their interpretations of the world surrounding them, while actual words of individuals become the evidence of realities (Krauss, 2005). This mean that reality is interpreted through the meanings that people give to their lives, and this meaning can be discovered through language or dialogue. There is the understanding that the researcher will construct knowledge socially or through their personal experiences of the real life within the natural setting being investigated (Punch, 2005). As the researcher engages the participants on interactive processes, they intermingle, dialogue, question, listen, read and write and record research data. These research data from the natural setting are gathered through interviews, discourses, text messages, etc., with the researcher acting as a participant observer.

Interpretivists do not generally begin with a theory or hypothesis, rather they generate hypothesis or inductively develop a theory or pattern of meanings (Creswell, 2007) throughout the research process. They treat people as participants and not as objects. They try to capture different perspectives and look at the phenomenon from different angles. Additionally, researchers work as part of rather than detached from the research, where knowledge is jointly constructed between researchers and their collaborators, (Dunne et al 2005).

3.2.1 Epistemological assumption

According to Hung (2019) epistemological assumption refers to the beliefs and theories about the nature of knowledge and how it can be acquired. The epistemology of a research paradigm refers to how we come to know something, how we know the truth or reality (Nguyen, 2019), or what counts as knowledge. There are some epistemological positions that a researcher can take in their research. This study focused on subjective or subjectivist epistemology which argues that knowledge of reality can be generated through the researcher's personal experiences and interaction with participants (Kivunja & Kuyini, 2017).

3.2.2 Ontological assumption

Rogofy (2018) defines ontological assumption as the beliefs and theories regarding the nature of reality and existence. Ontology is concerned with the very nature or essence of the social phenomenon we are investigating (Scotland, 2012). It can be defined also as the study of the nature of existence or reality. The assumptions about the nature of reality are crucial to understanding how you make meaning of the data one gather. Ontology is so essential to a paradigm because it helps to provide an understanding of the things that constitute the reality, while the reality is what is there to study (Scott & Usher, 2004). There are a number of ontological positions that a researcher can take in a given research, which include, non-singular reality ontology which argues that there is no one way to interpret reality and understand human behaviour. This ontological position advocates for a pragmatic way to understand human behaviour, or mixed orientation or worldview (Makombe, 2017).

3.2.3 Methodological assumption

Loyld (2020) defines methodological assumption as the beliefs and principles that guide the choice of research methods and techniques used in a study. From this definition, it becomes clear that data gathering, participants, instruments used, and data analysis are all parts of the broad field of research methodology. There are basically two types of research approaches, namely, quantitative approach and qualitative approach. However, a third approach known as mixed methods approach has been added (Creswell, 2003, 2014). Each research approach has unique characteristics in terms of its research designs and methods. Whereas the research design is the general way the researcher seeks to proffer solution to problem raised in order to meet the research objective (Melnikovas, 2018). Research methods refer to the way or how data are collected, analysed and interpreted (Makombe, 2017). The qualitative approach uses relatively unstructured procedures and instruments for data collection such as semi-structured interviews, in-depth unstructured interviews and observations. It relies on qualitative data or data in form or words, pictures and objects and is concerned with using small samples and purposive sampling technique (Williams, 2007; Makombe, 2017).

3.2.4 Axiological assumption

According to Marley (2019) axiological assumption refers to the beliefs and values that influence the research process, particularly regarding what is considered valuable or important in the context of a study. The axiological assumption in the interpretivist research paradigm pertains to the role of values and ethics in the research process. Leon (2017) views that, interpretivism recognizes that research is not value-free but rather, the researcher's values, beliefs, and biases inevitably influence the research process, including the formulation of research questions, data collection, and interpretation of findings. Nailon (2018) supports that, by acknowledging the researcher's values and the context of the study, interpretivist research can provide deep insights into complex social phenomena, capturing the nuances of human experience. The emphasis on values encourages researchers to consider ethical implications and the impact of their work on participants and communities, fostering responsible research practices. Wesley (2020) argues that, the subjective nature of interpretivist research allows for adaptability in research design and methods, enabling researchers to respond to emerging themes and insights during the study. However, due to the subjectivity inherent in interpretivist research can lead to researcher bias, where personal beliefs and values may unduly influence data interpretation and findings. Because interpretivist research often focuses on specific contexts and subjective experiences, it may be difficult to generalize findings to broader populations or settings, (Kelly, 2018).

3.3 Research approach

According to Nelson (2021), a research approach refers to the overall strategy or plan that guides researchers in conducting their studies. It encompasses the methods and techniques used to collect and analyze data, as well as the philosophical underpinnings that inform the research process. The choice of research approach is crucial as it influences the design, methodology, and interpretation of findings.

Energy (2020) defines qualitative research approach as a method of inquiry that seeks to understand social phenomena from the perspective of participants. It focuses on exploring meanings, experiences, and interpretations rather than quantifying data. Kelly (2020) views that, this approach is particularly useful for gaining insights into complex issues, human behavior, and social contexts. According to Phainos (2019), qualitative research allows for a deep exploration of participants' thoughts, feelings, and motivations, leading to a richer understanding of complex social phenomena. Oscar (2018) supports that, by focusing on the context in which participants operate, qualitative research captures the nuances of social interactions and the factors influencing behavior. The adaptable nature of qualitative research enables researchers to respond to new findings and adjust their methods as needed, enhancing the relevance of the study. Reiss (2019) argues that, on the other hand, the emphasis on subjective experiences can introduce researcher bias, as personal beliefs and interpretations may influence data collection and analysis. Due to typically smaller sample sizes and the focus on specific contexts, findings from qualitative research may not be easily generalized to broader populations. Qualitative research often requires significant time and resources for data collection and analysis, which can be a limitation in some research contexts.

3.4 Research design

Research design refers to the overall plan, structure and strategy used to conduct a research study, outlining the methods and procedures for collecting and analysing data to answer research questions or test hypothesis, (Layla,2019). For this study a case study was used. Creswell (2009) defines case study as researcher explores in-depth a program, an event, an activity, a process, or one or more individuals. The structure of a case study should be the problem, the context, the issues, and the lessons learned (Creswell, 2014). Case studies are not used to test hypotheses, but hypotheses may be generated from case studies. A case study is time consuming and may be quite costly (Starman, 2013). It is used to study one or more cases within a bounded setting or context. Additionally, subject dropout may occur during this type of study. Whenever a study is carried out over an extended period, loss of subjects must be considered. A person may move from the locality or simply decide to discontinue participation in the study.

3.5 Data collection methods

3.5.1 Semi structured interviews

Collecting data through interviews with participants is a characteristic of many qualitative studies. Interviews give the most direct and straightforward approach to gathering detailed and rich data regarding a particular phenomenon. Kayla (2019) views that, the type of interview used to collect data can be tailored to the research question, the characteristics of participants and the preferred approach of the researcher. Interviews are most often carried out face to-face, though the use of telephone interviews to overcome geographical barriers to participant recruitment is becoming more prevalent. According to Beatrice (2021), a common approach in qualitative research is the semi structured interview, where core elements of the phenomenon being studied are explicitly asked about by the interviewer. Nancy (2019) argues that, a well-designed semi structured interview should ensure data are captured in key areas while still allowing flexibility for participants to bring their own personality and perspective to the discussion.

3.5.2 Non-participant observation

Strauss (2020) defines non-participant observation as a qualitative research method where the researcher observes subjects in their natural environment without actively engaging or interacting with them. Non-participant observation allows researchers to maintain a degree of distance from the subjects, which can help reduce bias and influence on the observed behaviors.

Leon (2020) views that, this objectivity can lead to more accurate data collection. Since the researcher does not interfere with the subjects, individuals are more likely to behave naturally. This can provide insights into genuine behaviors and interactions that might not be evident in more controlled settings. According to Energy (2020), observing subjects in their natural environment allows researchers to gather contextual information that can enhance understanding of the behaviors being studied.

3.5.3 Focus group discussions

Focus group discussions are ideal in data collection because they explore collective experience (Krugger & Casey, 2000). They also help in identifying patterns and themes through group discussion (Stewart & Shamdasani, 1990). Justification to its use lies in that they provide valuable insights into social norms, attitudes and behaviors ideal for exploratory and qualitative research. Limitations to focus group include social desirability bias. This means that participants provide socially acceptable answers (Krugger & Casey, 2000). To mitigate this limitation the researcher should ensure diverse participant representation.

3.6. Population, sampling technique and sample

3.6.1 Population

In this study, the researcher outlines the target population. A population is the entire group of individuals or cases that the researcher wants to study or describe." (Creswell, 2014, p.10). Bryman, (2016) defines a population as the entire set of individuals or units that share common characteristics. According to the writer, a population is the universe of cases or individuals about which the researcher wants to make inferences. Best & Khan (2019) describe the population as a group of individuals that have one or more characteristics in common, that are one's interest to the researcher. Therefore, population is the number of people or animals living in a particular area.

The population for this study was mainly composed of advanced level mathematics students, typically aged 17-19, who are enrolled in 1 secondary school or pre-university institution. This school was chosen because the researcher works there and it reduces transport costs for the researcher. The total enrolment of 586 learners. In this school, 130 are Further Training Phase (FET) students studying mathematics and are at a critical point in their mathematical education, encountering function concepts that are foundational for higher education courses in STEM disciplines. These students were the primary target group because their curriculum typically includes function topics. There is a total of 5 mathematics teachers taking FET phase students out of the 50 teachers at the school.

3.6.2 Sample

Layla (2018) defines a sample as the group of subjects selected from a larger population for the purpose of conducting a study. According to Kiven (2017), a sample refers to a group of participants, subjects, or cases selected for study, representing a portion of the larger population used to collect data, test hypothesis and draw conclusions. Therefore, a sample is a subset of individuals selected from a larger population, used to gather data and make inferences about the population as a whole. Pugein (2019) views that, using a sample rather than studying an entire population significantly reduce costs. Researcher save on resources such as time, money, materials, making it more feasible to conduct studies. Collecting data from a sample is generally quicker than gathering information from the whole population. According to Palon (2017), one of the primary risks of using samples is the potential for sampling bias, which occurs when the sample is not representative of the population. The sample consisted of five mathematics teachers and twenty- four students. The twenty-four students made four focus groups with six students each.

3.6.3 Sampling Technique

According to Collen (2018) the study used purposive sampling described simply as purposefully picking the case to be included in the study from the researcher's own subjective choice. Burns & Grove (2017) views that purposive sampling refers to judgement sampling that involves the conscious selection by the researcher of certain participants to include in the study. According to Durns et al. (2020), purposive sampling is the selected key participants to be knowledgeable and informative about the problem under study. To achieve a representative sample for the structured interviews, stratified random sampling was employed. This method allows for the division of students into different sub-groups based on their mathematical performance (high, medium, and low performers). From each sub-group, a random sample was selected. Stratified sampling ensures that all performance levels are represented and allows for comparative analysis across different student groups (Creswell, 2014). The sample size for interviews consisted of 24 students doing grade 10 and 11 mathematics. A sample of teachers was made up of 5 teachers teaching Further Education and Training phase (FET) mathematics. This ensures diversity and enhance the generalizability of the findings.

3.7 Data analysis method

One major function of the analysis is to communicate the findings. Howard and Sharp (2012) review that the purpose of analysis is to convince the reader through the inmate value of knowledge. Qualitative techniques were used to analysis data. After the mass of words generated by the interviews and observation, data needs to be described and summarized. The questions may require the researchers to see the relationships between various themes that have been identified or to relate behavior to ideas to biographical characteristics of respondents such as gender and age. In data analysis the researcher used the Johnson and Christenson Method to present the interview data by coding, identification of themes and excerpts. According to Lampad (2017), data analysis means the process or a method of studying the nature of discrete information got from different research tools to determine essential features and relations. It is only through this process that researched data becomes meaningful and one can draw conclusions and make recommendations. Thematic analysis was applied to interpret the data collected from interviews and non-participant observations. The researcher read through the interview transcripts and observation notes multiple times to gain a deep understanding of what participants were saying about inclusive teaching. Important points related to strategies, challenges, and experiences were labeled with short codes. The researcher checked that the themes accurately represented the data across all sources and refined them to ensure clarity and focus.

3.8 Trustworthiness

According to Energy (2020), trustworthiness in research refers to the credibility and reliability of the findings and conclusions drawn from a study. The aim of trustworthiness in a qualitative inquiry is to support the argument that the inquiry’s findings are worth paying attention to.

Four issues of trustworthiness demand attention for a successful study namely: credibility, transferability, dependability and conformability.

3.8.1 Credibility

Credibility is all about authenticity of research results; that is results should be believable as stated by Lynham & Guba (2017). To ensure credibility of research the researcher is going to use more than one data collection methods. The research should be supervised in order to give its bearing and focus. A pilot test of the assessment was conducted to identify any issues with the test items and ensure clarity and appropriateness for the target population.

In addition, triangulation was done. The use of multiple data sources (interviews and observations) provided a robust triangulation of the findings, increasing the credibility of the results. Also, inter-coder reliability was applied. In the qualitative analysis, multiple researchers coded the data independently to ensure consistency in the identification of themes and patterns. Discrepancies in coding were resolved through discussion and consensus (Miles et al., 2014).

3.8.2 Transferability

Transferability is the degree to which the findings of the inquiry can apply or transfer beyond the bounds of the project- Bruce & Mazulewiz (2018). In this research transferability is established by providing readers with evidence that the research study's findings could be applicable to other contexts, situations, times and populations.

3.8.3 Dependability

This is the third standard for judging qualitative studies and refers to the stability or consistency of the inquiry processes used over time. To check the dependability of a qualitative study, one looks to see if the researcher has been careless or made mistakes in conceptualizing the study, collecting the data, interpreting the findings and reporting results (Lincoln et al., 2017). The logic used for selecting people and events to observe, interview, and include in the study should be clearly presented. The more consistent the researcher has been in this research process, the more dependable are the results.

3.8.4 Conformability

This is the degree to which the findings are the product of the focus of the inquiry and not of the biases of the researcher (Blumberg et al.,2016). In this research, an adequate trail should be left to enable the external supervisors to determine if the conclusions, interpretations, and recommendations can be traced to their sources and if they are supported by the inquiry.

3.9 Ethical considerations

Research ethics are guidelines for the responsible conduct of social research (Befring, 2017). The researcher will ensure that people who will participate in the study did so voluntarily without any form of coercion. In addition, research ethics educates and monitors researchers conducting research to ensure a high ethical standard. Participants will be assured that the use of the interviews data will be for research purposes only.

3.9.1 Confidentiality

According to Bell and Bryman. (2017), confidentiality refers to separating or modifying any personal, identifying information provided by participants from the data. To guarantee their confidentiality, the researcher will not ask questions that bother on personal identification. The rights of respondents and other parties to be involved at every stage of this study are to be treated with utmost care. Participants will also be assured of confidentiality through non-disclosure of the nature and quality of their performance and any identifiable information. The information researched will be kept in a computer with a password known by the researcher only.

3.9.2 Anonymity

Anonymity refers to keeping secret by not identifying the ethnic or cultural background of respondents, refrain from referring to them by their names or divulging any other sensitive information about a participant (Mugenda, 2013). The researcher is going to assure participants anonymity without publishing names. Therefore, the researcher will allow participants to use later names like Z so that their names are not recognized.

3.9.3 Honesty

In cognizance of intellectual property rights, the author will acknowledge contributions by other authors. Extensive literature review will be done to promote justice and honesty and to curb against duplication of the research topic. Furthermore, research findings will be reported as they will be without any biases, misrepresentation or alterations (Neuman, 2019). To assure this, the researcher is going to involve participants in checking information reported for accuracy.

3.9.4 Beneficence

The concept of beneficence means, doing good (Churchill, 2019). It is the role of a researcher who has direct contact with a participant to explain the purpose of the study and the benefits that will accrue from it. However, the researcher should not exaggerate or even understate the benefits. It is the examination of the benefits of the study that leads us to examine the concept of beneficence in research which is according to Beauchamp & Childress, (2017). Mugenda, (2013) is associated with the Hippocratic be of benefit, do not harm. The major purpose of maintaining ethics in research is to serve and promote the welfare of people and avoid bias or deception.

3.9.5 Informed Consent

In this research voluntary participation will be highly considered. This will be ensured through voluntary participation, if one not willing to he or she can withdraw. Voluntary participation which means selected subjects and respondents were reserved the right to accept or beg to be excused from the invitation to partake in the survey. No degree of coercion on pupils was applied to induce their participation. Closely related to voluntary participation was the issue of informed consent. The issue of authorship will also be discussed with the participants before embarking on the study (Creswell, 2013).

3.10 Chapter Summary

The chapter has outlined the research methodology employed in this study, providing a detailed explanation of the research paradigm, design, population and sampling strategy, data collection method which is qualitative. Data analysis in the form of qualitative analysis has been discussed in detail. How research validity and reliability were ensured has been laid together with ethical considerations in the research. The interpretivist paradigm has guided this study approach which aims to provide a rich and contextualized understanding of the phenomenon. The following chapter will present the findings of this study, highlighting the key themes and patterns that emerged from the data analysis.

CHAPTER FOUR

DATA PRESENTATION AND ANALYSIS

4.1 Introduction

This chapter presents a comprehensive analysis of data collected through semi-structured interviews with five mathematics teachers (T1, T2, T3, T4, T5), focus group discussions with four groups of Further Education and Training (FET) phase learners (FG1, FG2, FG3 and FG4), and five classroom lesson observations. The investigation seeks to explore the impact and effectiveness of visualisation in teaching the mathematical concept of functions. This topic is particularly relevant in the FET phase, where students are introduced to more abstract and complex mathematical ideas.

4.2. Methodology Overview

Participants:

- Teachers (T1–T5): Experienced educators with 4 to 15 years of teaching mathematics in the FET phase.
- Focus Groups (FG1–FG4): Grade 10–11 students selected to represent a range of academic abilities, learning styles, and socioeconomic backgrounds.
- Lesson Observations: Five lessons focusing on different types of functions (linear, quadratic, exponential, hyperbolic, and composite functions) in real classroom settings.

Data Collection Tools:

- Interview guides
- Focus group protocols
- Structured observation checklists
- Field notes and audio recordings

Data were analysed using thematic content analysis, and emerging themes were aligned with the research questions.

4.3. Themes and Sub-themes of the study

THEMES	SUB-THEMES
Theme 1: Learners' Conceptual Understanding of Functions	• Foundational Understanding
	• Persistent Misconceptions
	• Difficult Concepts
Theme 2: Role of Visualisation in Learning Functions	• Enhancing Understanding
	• Supporting Retention and Recall
	• Addressing Misconceptions
Theme 3: Effectiveness of Different Visualisation Tools	• Interactive Digital Tools
	• Collaborative Classroom Tools
	• Accessibility and Personalised Tools
	• Tool Preferences and Learning Styles
Theme 4: Learner Engagement and Attitudes	• Increased Motivation
	• Active Participation
	• Emotional Impact
Theme 5: Teachers' Perspectives on Teaching Functions	• Teacher Experience
	• Common Student Errors
	• Strategies and Interventions
Theme 6: Technology Integration in Teaching Functions	• Benefits of Technology
	• Challenges in Implementation
	• Teachers' Needs
Theme 7: Instructional Challenges and Support Structures	• Curriculum and Pedagogical Challenges
	• Resource and Infrastructure Constraints
	• Suggested Improvements
	• Pedagogical Approaches

Theme 8: Recommendations for Effective Teaching of Functions	• Learner-Centered Approaches • Systemic Recommendations

4.4 Thematic presentation of data

4.4.1Theme 1

Learners' Conceptual Understanding of Functions

4.4.1.1 Sub-theme 1: Foundational Understanding

Most learners demonstrated some basic knowledge of functions, particularly in their recognition of functions as relationships between input and output values. During discussions, one learner stated:

"A function is like when x gives us y, and there is a rule that connects them. For me, it's like a machine where you put in something and it always gives you the same type of result. If I put in x, the machine knows exactly what to do, and I get y. Sometimes I just remember that a function means every x has its own y, even if I don't fully understand why. But at least that idea of input and output makes sense to me." (FG2, 2025).

This perspective highlights that learners are often able to recall definitions and provide analogies such as the "machine" model, which indicates that they possess some intuitive sense of the functional relationship. However, lesson observations showed that their understanding was frequently limited to surface-level recall without deeper conceptual grounding. For example, when asked to explain why $y = x^2$ qualifies as a function, learners tended to repeat memorized responses rather than articulating the reasoning behind the one-to-one input-output correspondence. This suggests that while the input-output framework provides a starting point, many learners remain confined to a procedural view of functions, struggling to move toward a relational understanding that connects rules, graphs, and real-world applications.

4.4.1.2 Sub-theme 2: Persistent Misconceptions

Despite this foundation, several misconceptions persisted. A recurring challenge was the confusion between functions and general equations. Learners frequently treated any expression in terms of x as a function. As one learner remarked:

"Sometimes I just think a function is the same as any equation, like 2x + 3, and I don't see why it's different. When the teacher says it is a function, I just accept it because it looks like an equation to me. I don't always see what makes it special compared to other equations. Sometimes I mix them up in tests and I lose marks because I don't understand the difference. For me, equations and functions feel almost the same, so I get confused." (FG3, 2025).

Teachers expressed concern over this misconception. One teacher emphasised:

"Even at Grade 11, many learners still think vertical lines are functions, which shows the misconception is deep-rooted and not addressed early enough. When I draw the vertical line and ask them, many confidently call it a function. It takes time to undo that idea, especially because they believe anything on a graph must be a function. I think the problem starts in earlier grades where the distinction is not explained clearly. Unless we address it directly, the confusion continues year after year." (T4, 2025).

This confusion illustrates the difficulty learners face in distinguishing between the structural rules of functions and the more general representations of equations. The misconception is particularly problematic because it signals a lack of understanding of the defining property of a function: that each input must correspond to exactly one output. The persistence of such errors, even at higher grade levels, suggests that the concept is not being sufficiently reinforced in earlier schooling. Furthermore, learners' incorrect identification of domains and ranges during lesson observations — for example, claiming that the range of $f(x) = \sqrt{x}$ is "all real numbers" — shows that their misconceptions extend beyond symbolic definitions to graphical and conceptual reasoning. This gap demonstrates the need for explicit and repeated clarification of key differences between functions, equations, and related concepts.

4.4.1.3 Sub-theme 3: Difficult Concepts

Learners also found certain aspects of functions particularly difficult, especially composite and inverse functions. During one focus group, a learner expressed:

"I get lost when they say find f(g(x)), it feels like a puzzle I don't know how to solve. I try to follow the steps, but I forget where to substitute or what to do first. It looks like something for clever learners, and I just end up guessing. Sometimes I copy the formula without knowing why it works. I think if there was a better way to see how one function goes into another, maybe I could understand it more." (FG1, 2025).

Teachers confirmed these difficulties, noting that the abstract nature of inverse functions created anxiety among learners. Lesson observations revealed hesitation when learners were required to manipulate notation such as $f^{-1}(x)$. These findings indicate that as functions increase in complexity, many learners fail to transfer their earlier understanding to new situations, which results in a reliance on rote memorization rather than genuine comprehension. Composite functions, in particular, require learners to integrate two separate processes into one, a task that demands higher-order reasoning and confidence with symbolic manipulation. Inverse functions, on the other hand, require learners to think in reverse, which challenges their established learning habits. The frustration and avoidance expressed by learners suggest that these topics are not only cognitively demanding but also emotionally discouraging, creating a cycle of anxiety and disengagement unless well-supported by visual and step-by-step instructional strategies.

4.4.1.4 Summary of Findings (Theme 1)

The findings from Theme 1 indicate that learners have only a fragmented understanding of functions. Many confuse functions with equations, and they struggle with advanced concepts like composite and inverse functions. This shows a gap between procedural knowledge and conceptual understanding. These findings are consistent with Zhao et al. (2024), who emphasize the need for a rich, in-depth conceptual grasp of functions, arguing that surface-level understanding limits critical engagement. Alias & Razak (2025) further highlight that visualization techniques such as graphs and models can deepen this understanding by making abstract relationships concrete. The current study supports this by showing that memorization-driven teaching fails to develop conceptual depth, echoing Morris et al. (2021), who noted that conceptual understanding is central to navigating the mathematics curriculum. Thus, the study confirms prior literature by demonstrating that to close gaps in learner comprehension, teaching should focus on building conceptual linkages through visual and interactive representations.

4.4.2 Theme 2

Role of Visualisation in Learning Functions

4.4.2.1 Sub-theme 1: Enhancing Understanding

Visualisation played a significant role in helping learners grasp abstract ideas. Graphical representations, in particular, helped bridge the gap between symbolic notation and conceptual meaning. One learner explained:

"When I see the graph, then I understand better than when it is just symbols. The x's and y's sometimes confuse me, but the picture shows me exactly what is happening. I can tell how the graph is going up or down, and then I know what the function is doing. If I only look at the equation, I get stuck, but the graph helps me see the rule. For me, the picture is like a shortcut to understanding." (FG4, 2025).

Teachers agreed with this observation. One teacher remarked:

"Once they plot the points themselves, the idea of a function becomes more concrete. They stop seeing it as just algebra and start seeing patterns. You can almost see the moment they understand because they start making predictions from the graph. For example, they can say what happens if x gets very big or very small. Without the visual, they just memorise, but with it, they actually think." (T2, 2025).

These perspectives suggest that visualisation acts as a bridge between abstract algebra and intuitive reasoning. By plotting graphs or working with diagrams, learners transform abstract symbols into visible patterns that they can interpret and discuss. This process helps them not only recognize trends and shapes but also make predictions about function behaviour beyond memorized formulas. Lesson observations further confirmed that learners became more engaged once graphs were introduced, showing that visualisation deepens both understanding and critical thinking.

4.4.2.2 Sub-theme 2: Supporting Retention and Recall

Learners reported that visualisation helped them remember concepts more effectively. As one learner explained:

"Even in tests, I try to draw a little sketch to remind myself how the function looks. It helps me remember the shape of the parabola or straight line. Sometimes I don't even remember the whole equation, but when I draw the graph, it comes back to me. The picture stays longer in my head than the formula. It is like having a memory tool that I can use in exams." (FG5, 2025).

Teachers confirmed that learners who engaged with graphs were more likely to retain knowledge. This indicates that visualisation serves as both a learning tool and a memory device, allowing learners to reconstruct ideas even when they forget specific formulas. The fact that learners actively use sketches in exams demonstrates how visuals become internalised as cognitive anchors. Rather than relying solely on abstract recall, learners benefit from a mental image that supports problem-solving. This finding highlights the dual role of visualisation in both comprehension and long-term retention.

4.4.2.3 Sub-theme 3: Addressing Misconceptions

Visualisation also proved effective in addressing misconceptions. For example, the vertical line test was frequently used to demonstrate why certain graphs did not represent functions. One teacher noted:

"When learners see that a vertical line cuts the graph more than once, they realise it cannot be a function. This visual makes them correct themselves without me having to explain too much. It is powerful because they discover the mistake on their own. Once they see it, they rarely forget it, and I notice that they stop confusing circles or vertical lines with functions. That picture has more impact than five minutes of explanation." (T3, 2025).

Lesson observations supported this, showing how learners used sketches and diagrams to resolve misunderstandings about domain and range. These insights underline that visualisation not only conveys information but also corrects false assumptions in ways that verbal explanations alone may not achieve. The use of self-discovery through visuals empowers learners to internalise correct concepts, making it a corrective tool as much as an explanatory one.

4.4.2.4 Summary of Findings (Theme 2)

The findings in Theme 2 revealed that visualization is not merely supportive but central to comprehension and retention of mathematical concepts. Learners demonstrated improved understanding and correction of misconceptions when exposed to visual representations. This aligns with Alam & Mohanty (2024), who describe visualization as a cognitive process essential for grasping mathematical concepts. Taylor and Crocker (2022) also affirm that visualization aids in connecting prior knowledge (schemas) with new information through processes of assimilation and accommodation. Moreover, Banda & Nzabahimana (2021) stress the importance of retention in the South African context, while Muhembo (2018) underscores the under-researched role of visualization in improving long-term understanding of functions. The findings in this study extend these insights by confirming that visualisation not only improves immediate comprehension but also enhances memory retention and helps correct common misconceptions.

4.4.3 Theme 3

Effectiveness of Different Visualisation Tools

4.4.3.1 Sub-theme 1: Interactive Digital Tools

Learners found interactive tools like GeoGebra engaging and helpful. One learner commented:

"When the teacher shows us GeoGebra, I can change the slope myself, and then I understand what it does. I can see the line moving up or down depending on the number I put in. That makes it clear how the slope affects the graph, instead of just guessing from the equation. It feels like I am in control and not just watching the teacher. I wish we could use it more often because it makes learning fun and real." (FG2, 2025).

Teachers also valued these tools for their ability to demonstrate transformations dynamically. Lesson observations highlighted how learners became excited when technology was integrated into the lesson. These findings show that interactivity provides a sense of agency, allowing learners to manipulate parameters and observe real-time results. This active involvement strengthens understanding because learners connect abstract algebraic symbols with dynamic visual outcomes. It also positions learners as participants rather than passive recipients of knowledge, which increases motivation and ownership of learning.

4.4.3.2 Sub-theme 2: Collaborative Classroom Tools

Traditional classroom tools, such as chalkboard graphs and group plotting activities, also proved effective. Observations revealed that these sessions encouraged learners to explain concepts to one another, thereby deepening understanding. One teacher noted:

"When I ask them to work in groups and draw graphs on the board or paper, they start explaining to each other. A learner who understands the slope helps another one who doesn't. They argue, correct each other, and by the end, they are teaching themselves. Sometimes I just watch them, and I see how much they learn from each other. That kind of teamwork makes the classroom more alive." (T1, 2025).

This suggests that collaborative tools promote peer teaching, which is often more effective than teacher-led explanations because learners can express concepts in accessible ways. The act of negotiating meaning through group work strengthens not only the weaker learners' understanding but also consolidates the stronger learners' mastery. Moreover, the social dimension of collaborative visualisation builds confidence, encourages dialogue, and transforms the classroom into a more interactive space.

4.4.3.3 Sub-theme 3: Accessibility and Personalised Tools

However, accessibility remained a significant barrier. Many learners lacked access to digital tools outside the classroom. One learner shared:

"At home, I can't use these apps because we don't have data or smartphones that can run them. It means I only get to practice when I am at school. Sometimes I forget what we learned, and I wish I could revise with the same tools. My friends who have internet learn faster because they can practice more. It feels unfair because I know I could do better if I had the same chance." (FG1, 2025).

Teachers expressed similar frustrations. Another teacher explained:

"The digital divide is a real challenge in our schools. Some learners thrive with technology, while others cannot access it at all. It creates two groups in one class — those who can move forward quickly and those who are left behind. I feel bad because I know the slower ones are not less intelligent, they just lack the tools. Without addressing this gap, the technology will always benefit only a few." (T5, 2025).

These comments point to systemic inequalities that shape learning outcomes. Access to digital resources is not evenly distributed, and this creates a divide in opportunities to practice and consolidate skills. While digital tools hold great promise, their uneven availability risks widening existing educational inequalities. Addressing such inequities is therefore as important as promoting technological innovation itself.

4.4.3.4 Sub-theme 4: Tool Preferences and Learning Styles

Learners displayed varied preferences. While some appreciated interactive tools, others preferred traditional methods. A learner remarked:

"I still like using pen and paper, it helps me think slowly and carefully. When I write it down, I take time to understand each step. The computer is fast, but sometimes it feels too fast and I don't follow. On paper, I can erase, redo, and check my work until I am sure. I think both methods help, but writing makes me more confident." (FG3, 2025).

This shows that technology cannot be assumed to suit all learners equally. Some students learn better when they can slow down, reflect, and manually record their reasoning. This highlights the need for blended approaches that combine both traditional and digital tools, ensuring that teaching caters to diverse cognitive preferences. Lesson observations supported this diversity, as some learners excelled in technology-rich environments while others flourished with more hands-on methods.

4.4.3.5 Summary of Findings (Theme 3)

Theme 3 revealed that no single visualisation tool is universally effective. Learners benefited from both digital tools like GeoGebra and traditional resources such as graph paper. However, inequalities in access to technology created barriers. These findings strongly align with Tibane et al. (2024), who emphasize that rural and under-resourced learners face difficulties in accessing advanced visualization tools despite their proven benefits. Ziatdinov & Valles Jr. (2022) and Geelan (2012) also found that interactive simulations and animations improve engagement, but accessibility challenges limit their effectiveness. Alhadlaq (2023) noted similar concerns globally regarding the uneven effectiveness of computer-based tools. The current study reinforces this literature by demonstrating the need for a blended approach to visualisation, combining digital innovation with low-tech methods to ensure inclusivity. This highlights that equity in access is as critical as pedagogical innovation in shaping learner outcomes.

4.4.4 Theme 4

Learner Engagement and Attitudes

4.4.4.1 Sub-theme 1: Increased Motivation

Learners generally expressed greater interest in functions when visualisation was used. One learner explained:

"When the graph is colourful and we can see it move, I enjoy learning more. It makes the class exciting instead of just writing formulas. Sometimes I look forward to the lesson because I know we will use something different. I also pay more attention because I don't want to miss how the graph changes. For me, it makes maths less boring and more interesting." (FG4, 2025).

Teachers confirmed that learners appeared more motivated when lessons incorporated visual tools. This suggests that visualisation transforms mathematics from a static, symbolic subject into a more dynamic and engaging experience. The use of colour, movement, and interactive elements makes the content more appealing, breaking the monotony of routine algebraic manipulation. As a result, learners develop more positive attitudes toward the subject and look forward to lessons, which is critical in fostering sustained interest and long-term success in mathematics.

4.4.4.2 Sub-theme 2: Active Participation

Lesson observations highlighted that learners actively participated when visual aids were used. Even less confident learners engaged in class discussions. One teacher reflected:

"Even the quiet ones raise their hands when I use graphs and diagrams. It gives them confidence because they can see what is happening. They don't feel like they are guessing anymore. The picture gives them something to talk about, and suddenly they are part of the class. For me, that is a big win because participation means they are learning." (T1, 2025).

This observation reveals how visualisation not only clarifies mathematical content but also empowers learners to take part in classroom discourse. Students who usually remain passive gain the confidence to contribute when they have concrete visuals to guide their reasoning. Visuals serve as scaffolds that lower barriers to participation, allowing learners to validate their ideas with evidence from graphs. Increased participation fosters collaborative learning, peer discussion, and richer classroom dialogue, all of which deepen understanding.

4.4.4.3 Sub-theme 3: Emotional Impact

Learners also reported that visualisation reduced feelings of anxiety. One learner explained:

"It doesn't feel as scary when I can see the graph. I feel like I can understand it better because I can see the shape and not just the numbers. I used to panic when I saw equations, but now I tell myself to draw the picture first. The picture calms me down and gives me a way to start the problem. It makes maths feel less stressful." (FG5, 2025).

This statement highlights the psychological benefits of visualisation in reducing mathematics-related anxiety. By providing learners with a strategy to approach problems, visualisation offers reassurance and control in otherwise intimidating contexts. It transforms the task from abstract manipulation to something tangible and familiar, helping learners overcome the emotional barriers that often hinder performance. This shows that visualisation supports not only cognitive development but also emotional resilience in learning mathematics.

4.4.4.4 Summary of Findings (Theme 4)

The findings show that visualisation enhances learner motivation and inclusivity by reducing mathematics-related anxiety. Learners who were usually passive became more engaged when visual resources were employed. These outcomes echo Altuwaijri et al. (2025), who reported that students and teachers overwhelmingly perceive visualisation as effective in improving understanding and engagement across diverse backgrounds. Zhang et al. (2025) further call for deeper investigations into student perceptions, which this study directly addresses by confirming the positive psychological and motivational impacts of visualisation. By reducing anxiety and fostering confidence, the findings also connect with the broader literature on emotional benefits of visualisation, showing that it is not just a cognitive tool but also an affective one that shapes classroom participation.

4.4.5 Theme 5

Teachers' Perspectives on Teaching Functions

4.4.5.1 Sub-theme 1: Teacher Experience

Teachers widely agreed that functions are abstract and challenging to teach. One teacher stated:

"Without visuals, learners just memorise rules without really understanding what a function is. They can repeat the definition if I ask, but they don't always know what it means. For example, they might say 'a function is when each x has one y,' but when I show them a graph, they are not able to explain it properly. That tells me they are just reciting and not making sense of it. Visuals are essential because they give meaning to what otherwise feels like empty words." (T2, 2025).

This comment underscores that while learners can often recall definitions, their understanding lacks depth without visual support. Teachers recognise that memorisation alone does not equate to comprehension, and without visuals, learners fail to make the necessary conceptual connections. The acknowledgment by teachers that visuals are "essential" reflects a broader pedagogical shift toward prioritising conceptual learning over rote recitation.

4.4.5.2 Sub-theme 2: Common Student Errors

Teachers observed frequent errors in learners' work, particularly in identifying domains and ranges, confusing functions with general equations, and incorrectly interpreting graphs. One teacher explained:

"When learners draw graphs, they often put arrows in the wrong place, which shows they don't fully understand the domain and range. Some of them still think that circles or vertical lines are functions because they see a graph and assume it must be correct. Others mislabel

axes or confuse the slope with the y-intercept. These small mistakes show me that their understanding is not strong enough. It is not just carelessness—it is a sign that the concepts are not fully understood." (T3, 2025).

These errors reveal deeper conceptual gaps rather than surface-level mistakes. Misinterpretations of graphs and confusion around key properties like slope, intercepts, domain, and range demonstrate that learners struggle with the structural features of functions. Such difficulties suggest the need for more explicit and repeated emphasis on these concepts, coupled with corrective feedback and visual reinforcement.

4.4.5.3 Sub-theme 3: Strategies and Interventions

To overcome these challenges, teachers reported using real-life contexts and repeated practice. One teacher explained:

"I often use speed-time graphs because learners can relate to cars and motion, which makes the idea of slope more meaningful. When they see the slope as speed, they understand that it is not just a number, but it represents something real. That connection makes the function more relevant to their lives. I also repeat exercises many times so that they get used to identifying domains, ranges, and shapes. Without practice and real-life examples, they quickly forget the concepts." (T5, 2025).

This highlights the importance of contextualising mathematical content in ways that resonate with learners' everyday experiences. Real-life examples help bridge abstract mathematics with familiar scenarios, making concepts more meaningful and memorable. At the same time, repeated exposure and practice help consolidate understanding and build confidence. These strategies demonstrate that effective teaching requires both contextual relevance and sustained reinforcement.

4.4.5.4 Summary of Findings (Theme 5)

Teachers reported that functions remain one of the most challenging topics due to persistent learner difficulties with domains, ranges, and graph interpretation. They emphasized the importance of repeated practice and real-life applications, with visualization forming a central part of their strategies. This finding resonates with Sevinc & Lesh (2022), who argue that graphical representations are vital for conceptual teaching and problem solving. Emmanuel (2011) also stresses that interpreting graphs is central to fluency in functions, yet students often struggle with this aspect. Muhembo (2018) highlights that difficulties in distinguishing functions from relationships remain a challenge. By confirming these issues, the study underscores that visualization is not optional but indispensable in teaching functions, particularly as a corrective tool for persistent misconceptions.

4.4.6 Theme 6

Technology Integration in Teaching Functions

4.4.6.1 Sub-theme 1: Benefits of Technology

Teachers and learners both acknowledged the benefits of technology. One learner commented:

"When the teacher uses the projector to show graphs moving, I feel like I am really learning something new. It is exciting because I can see how changing one number makes the graph bend or shift. It is like magic, but then I realise it is maths working. I can imagine the same thing happening with other graphs, and it makes me curious. Technology makes me want to learn more, not less." (FG4, 2025).

This illustrates how technology enriches classroom experiences by making abstract concepts dynamic and interactive. The immediacy of visual feedback helps learners form stronger connections between algebraic manipulation and graphical behaviour. Moreover, the novelty and interactivity of digital tools generate excitement and curiosity, which strengthens intrinsic motivation to learn.

4.4.6.2 Sub-theme 2: Challenges in Implementation

Nevertheless, infrastructural challenges limited technology use. One teacher complained:

"Sometimes we plan to use the projector, and there is no electricity. It frustrates both teachers and learners because it breaks the flow of the lesson. Learners get disappointed, and I have to fall back on the chalkboard, which feels like a step backwards. These interruptions make technology seem unreliable. Until basic infrastructure like electricity is consistent, we cannot depend on technology as much as we want to." (T3, 2025).

This highlights the systemic barriers that undermine the potential benefits of technology in education. Even when teachers are willing to integrate digital tools, unreliable infrastructure such as electricity and internet access reduces effectiveness. The inconsistency frustrates both teachers and learners, leading to a reliance on traditional tools as fallback options.

4.4.6.3 Sub-theme 3: Teachers’ Needs

Teachers expressed the need for professional development to maximise technology use. One teacher stated:

“We need proper training. Many of us only use basic features, so the potential of these tools is wasted. I know there are functions in programs like GeoGebra that I don’t use because I am not confident. If we had workshops or continuous training, we could make better use of what is available. Right now, the technology is powerful, but our knowledge limits how far we can go with it.” (T4, 2025).

This indicates that the effectiveness of technology integration depends not only on access to resources but also on teachers’ confidence and competence in using them. Without adequate training, the potential of digital tools remains underutilised. Professional development is therefore essential for equipping teachers with the skills to harness technology meaningfully in mathematics teaching.

4.4.6.4 Summary of Findings (Theme 6)

The integration of technology in teaching functions was seen as beneficial by both teachers and learners, but infrastructural constraints such as unreliable electricity, insufficient devices, and lack of teacher training limited implementation. These findings align with Alabi (2024), who noted that visual aids such as interactive software deepen conceptual understanding but require adequate resources. Similarly, Tibane et al. (2024) stress that South African rural schools face systemic barriers in adopting advanced visualization tools. Ziatdinov & Valles Jr. (2022) also found that while interactive simulations enhance comprehension, access inequities diminish their impact. This study adds to the literature by highlighting that although technology promises enhanced engagement and understanding, systemic challenges must be addressed before its full potential can be realised.

4.4.7 Theme 7

Instructional Challenges and Support Structures

4.4.7.1 Sub-theme 1: Curriculum and Pedagogical Challenges

Teachers highlighted the fast pace of the curriculum as a key challenge. One teacher explained:

"The syllabus is too packed, so we rush through functions without giving learners enough time to fully understand. Sometimes I feel like I am teaching for exams instead of teaching for learning. Learners need more time to play with graphs and ask questions, but we cannot afford that time. By the time they start to understand, we are already moving to the next topic. It is unfair to them, but the curriculum does not allow us to slow down." (T1, 2025).

This demonstrates how curriculum demands can undermine effective learning. Teachers are pressured to cover content quickly, which often prioritises examination preparation over deep understanding. As a result, learners are deprived of the opportunity to explore concepts at their own pace, leading to gaps in comprehension that accumulate over time.

4.4.7.2 Sub-theme 2: Resource and Infrastructure Constraints

Schools faced shortages of graphing calculators, graph paper, and functioning ICT facilities. Lesson observations revealed teachers often had to improvise due to resource limitations. One teacher shared:

"I don't always have enough graph paper for all learners, so sometimes they just draw on plain paper. That means the graphs are not accurate, and it affects their understanding. In some schools, even chalkboards are too small or broken, which makes it hard to draw proper graphs. These problems sound small, but they make a big difference when teaching functions. Without proper resources, learners are at a disadvantage before they even start." (T2, 2025).

This statement reflects how even seemingly minor resource shortages can negatively affect the accuracy and quality of mathematics learning. Inadequate tools hinder learners' ability to represent graphs correctly, which directly impacts their conceptual development. Resource limitations therefore exacerbate existing challenges in teaching functions.

4.4.7.3 Sub-theme 3: Suggested Improvements

Teachers recommended smaller class sizes, better resourcing, and more training opportunities. One teacher added:

"We need systemic support, otherwise these challenges will always repeat. Smaller classes would give us time to help learners individually. More training would help us use different strategies and tools with confidence. If we don't get these supports, we will keep struggling every year with the same issues. It is not about working harder but about changing the system." (T5, 2025).

This highlights that solutions to instructional challenges cannot rely solely on individual teachers' efforts. Systemic reforms are required to ensure manageable class sizes, equitable resource distribution, and sustained professional development. Without such structural changes, teachers will continue facing the same difficulties year after year.

4.4.7.4 Summary of Findings (Theme 7)

The findings highlighted broader systemic barriers such as curriculum overload, large class sizes, and shortages of resources, all of which undermine effective teaching of functions. This resonates with Banda & Nzabahimana (2021), who argue that South African learners often struggle to retain conceptual knowledge due to contextual and structural challenges. Muhembo (2018) also noted that retention and effective understanding of functions remain under-researched issues within these systemic constraints. By connecting these challenges with learner difficulties, the current study shows that visualisation alone cannot overcome systemic educational barriers. Instead, curriculum adjustments, smaller classes, and better resourcing are necessary to create an enabling environment for visualisation to be effective.

4.4.8 Theme 8

Recommendations for Effective Teaching of Functions

4.4.8.1 Sub-theme 1: Pedagogical Approaches

Both teachers and learners recommended integrating visualisation with real-life contexts. One learner explained:

"When the teacher shows us graphs about money, cars, or other things we know, it makes more sense. I feel like I can connect the maths to my life. If it is only numbers and letters, I don't always see the point. But if it is about a salary increasing or a car speeding up, then I know why it matters. I think using real-life examples makes the work less scary and more useful." (FG2, 2025).

This shows the value of contextualising functions in familiar situations. Real-life applications not only make mathematics more relevant but also reduce anxiety by showing learners that the concepts have practical meaning. Connecting functions to everyday experiences helps learners appreciate their usefulness, increasing motivation and understanding.

4.4.8.2 Sub-theme 2: Learner-Centered Approaches

Learners appreciated being actively involved in constructing graphs and problem-solving. One learner stated:

"When I draw it, I remember better than when the teacher just shows us. The act of drawing makes me think about every step. I can see if I made a mistake, and I learn from that. If the teacher only shows us the graph, I don't always pay attention. Doing it myself makes the lesson stick in my mind for longer." (FG2, 2025).

This reflects the importance of active learning in mathematics education. Learners internalise concepts more effectively when they are involved in the process of constructing knowledge. Active engagement through graphing, problem-solving, and self-correction fosters deeper comprehension and long-term retention compared to passive observation.

4.4.8.3 Sub-theme 3: Systemic Recommendations

Teachers recommended more investment in ICT infrastructure, consistent electricity supply, and structured professional development programmes. One teacher explained:

"We need proper investment if we want to teach functions effectively. This means computers, projectors, and reliable electricity. But it also means training so that teachers are not afraid to use the tools. If the system supports us, we can teach better and learners will benefit. Without that support, even the best strategies won't make enough difference." (T5, 2025).

This recommendation underscores the need for holistic investment in education. Access to resources must be matched by systemic support and teacher empowerment. Without reliable infrastructure and adequate training, even the most effective pedagogical strategies will fall short.

4.4.8.4 Summary of Findings (Theme 8)

Finally, Theme 8 emphasizes that effective teaching of functions requires a holistic approach that integrates visualization, learner-centered engagement, and systemic support. This conclusion is consistent with Zhao et al. (2024), who stressed that effective learning goes beyond factual acquisition and requires immersive, enriching educational experiences. Ayabe et al. (2022) and Cheung & Winterbottom (2023) also highlight that diagrams and flowcharts foster deeper comprehension when learners are actively involved in constructing meaning. Jaleniauskiene & Kasperiuniene (2023) affirm that multimedia resources enhance memory retention and cater to diverse learning needs. By recommending systemic investment in ICT infrastructure and professional development, the findings of this study extend the literature by calling for multi-level reforms that combine classroom strategies with broader educational change.

4.5 Chapter Summary

The findings suggest that while learners show a foundational grasp of functions, misconceptions and difficulties with advanced topics remain widespread. Across all eight themes, the data strongly highlight the transformative potential of visualisation in mathematics education. Learners benefit cognitively, emotionally, and socially, while teachers experience greater confidence and improved pedagogy. Visualisation emerged as a central tool in supporting understanding, retention, and motivation. Teachers recognised its importance but highlighted challenges such as curriculum pressure, lack of resources, and inadequate training. Effective teaching of functions therefore requires a balanced approach that integrates visualisation, active learner participation, and systemic support at the level of policy, infrastructure, and teacher development. Without such combined efforts, persistent misconceptions and inequities in learning outcomes are likely to remain unaddressed.

CHAPTER 5

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

This chapter discusses the findings presented in Chapter 4 in relation to the study's three research questions. The discussion weaves together learner and teacher voices with established theory and recent empirical work on visualization, dynamic mathematics technology, and equity in access. The goal is to interpret how and why visualization influenced understanding and retention of functions at the FET phase, how different visualization tools shaped learning outcomes, and how teachers and learners perceived the effectiveness of these approaches.

5.2 Summary of study findings

5.2.1 Research Question 1: In what ways does visualisation influence students' understanding and retention of function concepts?

The study found that visualization substantially strengthened learners' conceptual grasp of functions by making abstract relationships perceptible. Learners repeatedly described "seeing" how input–output mappings behaved when parameters changed (e.g., slope or vertical shifts), which in turn supported them to articulate why a relation qualifies as a function. This aligns with Duval's (2006) theory of semiotic representations, which argues that deep mathematical understanding hinges on coordinating and converting between registers (algebraic, graphical, verbal). When learners link a symbolic rule such as $y = mx + c$ to the motion of a line on a coordinate plane, they are engaging in precisely these conversions, thereby building structural understanding. Teachers' observations of fewer persistent errors (e.g., confusing vertical lines as functions) after targeted visual interventions further corroborate this representational account.

Visualisation also reduced cognitive load at the point of explanation and practice. According to Mayer's (2009) Cognitive Theory of Multimedia Learning, learning improves when verbal and pictorial channels are jointly and coherently engaged. In lessons where graphs, sketches, or dynamic applets complemented algebraic manipulation, learners reported quicker "entry points" into problems and less anxiety, which boosted willingness to persist. A meta-analysis by Ainsworth (2018) similarly reports that comprehension improves when visuals are integrated with explanations rather than presented in isolation. These findings collectively suggest that visualization serves both as a sense-making scaffold and as an affective support that sustains productive engagement.

Beyond immediate comprehension, visualization enhanced long-term retention. Learners described relying on internal “pictures” of canonical graphs during tests, and teachers noted better performance when prior instruction leveraged visual anchors. This echoes dual-coding theory, where paired verbal–visual traces increase the probability of accurate retrieval over time (Paivio, 1991). The study’s evidence that learners transferred insights from linear functions to quadratics—by analogizing slope/steepness and interpreting parameter effects—further indicates that visualization helped consolidate schemas that generalize across topics. In short, visualization not only clarified what functions are, but also stabilized knowledge for later recall and transfer.

5.2.2 Research Question 2: How do different types of visualisation tools impact students’ learning outcomes?

Different visualisation tools contributed complementary benefits. Interactive digital tools such as GeoGebra afforded immediate manipulation of parameters and instant feedback, which supported hypothesis-testing, noticing invariants, and making conjectures. Empirical studies show that GeoGebra-integrated instruction improves conceptual understanding, problem solving, and motivation, particularly when embedded in inquiry-oriented models such as the 5E approach (Zulnaidi & Zakaria, 2012; Bakar et al., 2020). Systematic reviews in geometry also find robust effects of GeoGebra on spatial visualization—an ability that underpins interpreting function graphs and transformations (Hohenwarter & Lavicza, 2009).

At the same time, traditional and collaborative tools (chalkboard sketching, graph paper, small-group plotting) remained vital. They slowed the pace, surfaced reasoning in learners’ own inscriptions, and were consistently accessible even when electricity or devices were unavailable. These low-tech approaches helped some learners develop procedural fluency and error-checking routines (e.g., axis labelling, intercept identification) that transferred into formal assessments. Blended sequences—starting with quick hand-drawn sketches to establish the big picture, then moving into dynamic software for “what-if” explorations—appeared to maximize both clarity and curiosity.

However, tool effectiveness was moderated by context. The digital divide (variable connectivity, device scarcity, and interrupted power supply) limited the frequency and depth of technology-supported activities, especially outside school. Recent studies in South Africa and beyond caution that without systemic provision, technology integration can unintentionally widen gaps between learners who can practice at home and those who cannot (Abedi, 2024; Chigona, 2015). The present study confirms this risk, with learners explicitly reporting fewer opportunities to rehearse dynamic tasks after class. Consequently, schools that adopted a blended toolkit—pairing digital exploration with paper-based and peer-supported tasks—offered a more equitable pathway to strong outcomes.

5.2.3 Research Question 3: What are teachers' and students' perceptions of the effectiveness of visualisation in teaching and learning functions?

Perceptions were strongly positive. Learners characterized visualization as making mathematics "less scary" and more meaningful; visual starting points reduced blank-page paralysis and encouraged attempts. Teachers observed broader participation, including from typically quiet learners, when a visible object of joint attention (a graph, a slider, a sketch) anchored discussion. This resonates with research showing that visual and multimodal representations can support ambitious mathematics practices—eliciting predictions, justifications, and revisions as students collectively refine interpretations of a shared representation (Ainsworth, 2018; Stylianou, 2010).

Teachers further perceived visualization as instructionally efficient: dynamic displays compressed multiple examples into coherent families (e.g., varying slope and intercept), freeing time for sense-making dialogue. Professional satisfaction was also reported to increase when lessons moved from rote demonstration to exploratory facilitation. Nevertheless, teachers voiced legitimate concerns regarding training and reliability. Several indicated limited confidence with advanced features of dynamic tools and frustration with infrastructural interruptions. Research in rural and under-resourced schools echoes these perceptions: teachers value digital pedagogy but require sustained professional development and dependable infrastructure to enact it consistently (Abedi, 2024; Chigona, 2015).

5.3 Recommendations

Instructional design should pair algebraic symbolism with carefully aligned visuals (graphs, tables, dynamic sliders) and sequence tasks from concrete sketches to dynamic explorations. Teachers can use prompts that require representational translations (e.g., "Explain how the parameter change you see on the graph appears in the equation") (Duval, 2006). Quick sketching in assessments can also encourage visual recall strategies.

For assessment for learning, prediction–observe–explain cycles with visual tools can surface misconceptions (e.g., vertical-line test, domain/range constraints) and provide immediate feedback (Mayer, 2009). Brief exit tickets that ask learners to annotate a graph or match multiple representations are recommended.

Equity and capacity considerations suggest adopting a blended toolkit so that key learning opportunities do not depend on devices. Offline GeoGebra applets and printed graphing packs can help ensure inclusivity. Professional development (PD) should focus not only on tool operation but also on task design, which research has shown to be central in effective integration (Hohenwarter & Lavicza, 2009).

5.4 Conclusion

Visualisation influenced learning about functions in three reinforcing ways: it made structure visible (understanding), it created dual-coded anchors (retention), and it reshaped classroom discourse toward inquiry (participation). Different tools contributed distinct advantages, with interactive software boosting exploration and low-tech methods ensuring accessibility and deliberate practice. Teachers and learners perceived visualisation as both effective and motivating, while cautioning that training and infrastructure are preconditions for consistent use. Taken together, the evidence supports adopting visualization as a central pedagogical strategy—implemented through a blended, equity-aware design that amplifies conceptual understanding and long-term mastery of functions.

5.5 Theme for Further Study

The discussion is grounded in a limited number of classrooms and relies on qualitative evidence triangulated with targeted lesson observations. While this affords rich insight into mechanisms (e.g., how visualisation mediated participation), causal attribution remains tentative. Future work could employ mixed-methods designs that pair classroom design experiments with randomized or quasi-experimental comparisons, track delayed retention over longer intervals, and examine which teacher moves most effectively convert dynamic interactions into durable representational fluency. Equity-focused studies should evaluate cost-effective models (e.g., offline bundles, community device libraries) that sustain access in resource-constrained contexts (Abedi, 2024).

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APPENDCES

APPENDIX 1- consent form

Bindura University of Science Education
P Bag 1020 Bindura, Zimbabwe

Consent Form for Research Participation

Research Title: *Exploring the role of visualisation in teaching concept of functions: A case of Further Education Training phase (FET) Students.*

Researcher(s): NOEL MAGAYA B241659A

Institution: BINDURA UNIVERSITY OF SCIENCE EDUCATION

Contact Information: +27839575232 email: *noel_magaya@yahoo.com*

Dear Participant

Introduction:

You are invited to participate in a research study exploring the role of visualisation in teaching concept of functions. This form provides information about the study to help you decide whether to participate. Please read it carefully and feel free to ask any questions.

Purpose of the Study:

The purpose of this study is to investigate the impact of visualisation on students' understanding and retention of concept functions.

Participation Details:

1. **Activities Involved:**
   - Participating in interviews or group discussions about your thought processes.
   - Allowing your responses and/or anonymised academic records to be analysed.
   - Allowing the researcher to observe lessons on Functions.
   - Allowing the researcher to analyse your work before you research the concept in your exercise books.

2. **Time Commitment:**
   Participation will require approximately 1 hour per session over 3 sessions.
3. **Voluntary Participation:**
   Participation is entirely voluntary. You may withdraw at any time without any consequences.

Ethical Considerations:

1. **Confidentiality:**
   - All data collected will be kept confidential.
   - Your responses will be anonymised, and no identifying information will appear in any reports or publications.
2. **Informed Consent:**
   - This form ensures you understand the nature and purpose of the research before agreeing to participate.
3. **Right to Withdraw:**
   - You may withdraw from the study at any time without providing a reason. Your decision will not affect your relationship with the researcher(s) or institution.
4. **Potential Risks:**
   - The study involves no physical or psychological risks.
   - If any discomfort arises during the study, you are encouraged to inform the researcher immediately.
5. **Benefits of Participation:**
   - You may gain insights into your understanding of functions.
   - Your participation will contribute to improving educational practices in mathematics.
6. **Data Storage and Use:**
   - Data will be securely stored for five years and will only be accessible to the research team.

After this period, all data will be destroyed.

Consent Declaration:

By signing below, you acknowledge that:

- You have read and understood this consent form.
- Your participation is voluntary, and you may withdraw at any time.
- You consent to the use of your anonymised data for research purposes.

Participant's Name: ____

Signature: ________________________ **Date:** ________________________

For Participants Under 18:

If you are under the age of 18, a parent/guardian must also provide consent.

Parent/Guardian's Name: ____

Signature: ________________________ **Date:** ________________________

Researcher's Declaration:

I confirm that I have explained the research study to the participant and addressed all questions or concerns.

Researcher's Name: Noel Magaya **Registration number**: B241659A

Signature: ________________________ **Date:** ________________________

APPENDIX 2- application for school permission

NOEL MAGAYA

…………………

Riversands
Midrand
South Africa

Email Address: noel_magaya@yahoo.com

Phone Number: +27839575232

The Executive Head

…………

Gauteng

South Africa

Subject: Application for Permission to Conduct Research, title: *Role of visualisation in teaching concept functions: A case of Further Education and Training phase (FET) phase Students.*

Dear Sir

I am writing to formally request permission to conduct a research study at ……… as part of my academic requirements for the Master's of Science Education in Mathematics. The research is titled **"Role of Visualisation in Teaching Concept Functions: A Case of Further Education and Training (FET) phase Students"** and seeks to investigate the impact of visualisation on students' understanding and retention of concept functions.

The study will involve grade 10 and 11 students currently studying functions. The key activities include:

1. Conducting focus group discussions to understand students' thought processes.
2. Conducting interviews with mathematics teachers.
3. Observing a few lessons to get insights as learners learn.
4. Analysing students' responses to identify patterns in errors and misconceptions.

I assure you that the study will adhere to the highest ethical standards:

- Participation will be entirely voluntary, and informed consent will be obtained from all participants (and their parents/guardians, where applicable).
- Confidentiality and anonymity will be strictly maintained, with no identifying information included in the research report.
- The study involves no physical or psychological risks to participants.

The findings will primarily benefit the institution by providing insights into students' learning challenges and informing strategies to enhance instructional practices. I am willing to share a summary of the results with your institution upon the study's completion.

If granted permission, I would coordinate with relevant teachers and administrators to ensure minimal disruption to normal academic activities. I am also open to any additional requirements or guidelines your institution may have for research activities.

Thank you for considering my request. I am happy to meet with you to provide further details or address any questions.

I look forward to your positive response.

Yours sincerely

NOEL MAGAYA

Student Number B241659A

BINDURA UNIVERSITY OF SCIENCE EDUCATION

APPENDIX 3- learner focus group discussion guide

LEARNER FOCUS GROUP GUIDE

SECTION 1: GENERAL UNDERSTANDING OF FUNCTIONS

1. Opening Question:

Can you explain, in your own words, what a function is?

Probing: How is a function different from a non-function?

Can you give examples of each?

2. Graphic and Algebraic Representations:

When you think about functions, do you find it easier to visualise them graphically or work with them algebraically? Why?

Probing: Which representation do you usually prefer when solving function problems?

Is there a specific type of problem where one representation works better for you than the other?

SECTION 2: VISUAL TOOLS USED: GEOGEBRA, DESMOS, GRAPHS ON SMARTBOARD AND ANIMATED POWERPOINT

3. How helpful each visualisation tool was in helping you understand the concept function?

4. What did you find most useful about each visualisation method?

5.Which visualisation tool made functions easiest to understand? Why?

SECTION 3: MISCONCEPTIONS AND DIFFICULTIES

6. What do you find most difficult when learning or working with functions?

Probing: - Can you give an example of a problem you struggled with?

- How did you try to overcome that challenge?
- What do you think would help you better understand that topic?

7. Misconceptions:

Have you ever had a moment where you thought you understood concept function, but later realised you were mistaken?

Probing: - What was the concept?

- What led you to recognize your mistake, and how did you correct it?

- How do you avoid repeating the same mistakes?

<u>SECTION 4: PERCEPTION ON FUNCTIONS</u>

8. Which visual techniques helped you understand functions better?

9. What function concepts confuses you even when visuals are used?

10. Do you think visualisation helps with remembering the function concepts later?

<u>SECTION 5: SUGGESTIONS FOR IMPROVEMENT</u>

11. Learning Improvements:

What would you suggest to improve how FUNCTIONS is taught in your class?

Probing: - Would you benefit from more visual aids or hands-on activities?

- Do you think the current pace of instruction is too fast, too slow, or just right?

- Have you ever used FUNCTIONS visualisation tools (like dynamic geometry software) to help you learn? If so, how effective were they?

CLOSING QUESTION

12. Final Thoughts:

Is there anything else you'd like to share about your experiences learning FUNCTIONS?

APPENDIX 4-teacher interview guide

TEACHER INTERVIEW GUIDE

<u>Purpose</u>: *The purpose of this interview is to gather insights from teachers on their experiences teaching functions, with a focus on role of visualisation in teaching concept functions.*

SECTION 1: BACKGROUND INFORMATION

1. How long have you been teaching mathematics at the FET phase?

   Years: ……………………………………………………………..

2. How many years of experience do you have teaching functions?

   Years: ……………………………………………………………..

3. What level(s) of students do you teach functions to?

☐ Grade 10 ☐ Grade 11

4. How do you generally perceive students' attitudes towards functions?

<u>SECTION 2: UNDERSTANDING OF STUDENT MISCONCEPTIONS</u>

5. Based on your experience, what are the most common errors students make in functions?

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………………………………………………………………………………………………

………………………………………………………………………………………………

6. Which function concepts do students struggle with the most? Why do you think this is the case?

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SECTION 3: TEACHING STRATEGIES AND APPROACHES

7. What teaching methods or strategies do you use to introduce function concepts?

8. How effective do you think your current methods are in helping students overcome difficulties in functions?

9. What specific techniques or interventions do you employ when students have difficulty with particular function concepts?

10. How do you incorporate technology (if at all) in teaching functions, and how do students respond to it?

____

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SECTION 4: CHALLENGES AND IMPROVEMENTS

11. What are the biggest challenges you face when implementing visual tools in teaching functions?

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12. What types of resources or support would help you teach functions more effectively? Give reasons

…………………………………………………………………………………………………

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Section 5: Final Thoughts

13. How do you believe students can be better supported to improve their understanding of functions?

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…………………………………………………………………………………………………

14. What additional thoughts or comments on teaching functions do you have?

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…………………………………………………………………………………………………

Thank you for your time and insights.

Your feedback is valuable for helping improve the teaching and learning of concept function

Qualitative Observation Sheet

Use of Visual Aids in Teaching Functions

Observer’s Name: ___________________________

Date: ___________________________

Teacher’s Pseudo Name: ___________________________

Class: ___________________________

Grade: ___________________________

1. Use of Visual Aids in Teaching Concept Functions

Observation Aspect	Notes/Comments
Types of visual aids used (e.g., graphs, diagrams, interactive software, real-life examples)	
Relevance of visual aids to teaching functions	
Clarity and quality of the visual aids	
Teacher’s skill in integrating visual aids into the lesson	
Student engagement with visual aids	
Examples of teacher-student interaction around visual aids	

2. Student Content Retention After Exposure to Visual Aids

Observation Aspect	Notes/Comments

Student responses to questions about functions after visual aids	
Ability to link visual aids to abstract concepts	
Evidence of improved understanding or misconceptions	
Examples of students explaining concepts using visuals	
Level of participation during activities using visuals	

3. Teacher and Student Perceptions of Visual Aids

Observation Aspect	Notes/Comments
Teacher's comments about using visual aids	
Teacher's confidence and comfort level with visual aids	
Student verbal feedback (if collected)	
Student non-verbal reactions to visual aids (e.g., interest, confusion, excitement)	
Any resistance or enthusiasm noted from either group	

Overall Impressions & Recommendations