BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE EDUCATION

BACHELOR OF SCIENCE EDUCATION HONOURS DEGREE IN MATHEMATICS

APPLICATION OF THE APOS THEORY IN TEACHING AND LEARNING OF GEOMETRICAL TRANSFORMATION AT ORDINARY LEVEL. A CASE STUDY OF DETE CLUSTER IN HWANGE DISTRICT

BY

SIBANDA PRIVILEDGE

REG NO. B225501B

A DISSERTATION SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS OF THE BACHELOR OF SCIENCE HONORS DEGREE IN MATHEMATICS EDUCATION

JUNE 2024

RELEASE FORM

Title of the dissertation: application of the APOS theory in teaching and learning of Geometrical Transformation at Ordinary level. A case study of Dete cluster in Hwange District.

1. **To be completed by the student**

   I certify that this dissertation is in conformity with the preparation guidelines as presented in the Faculty Guide and Instructions for Typing dissertations.

   30/06/2024

   (Signature of student) (Date)

2. **To be completed by the supervisor**

   This dissertation is suitable for submission to the Faculty. This dissertation should be checked for conformity with Faculty guidelines.

   13 July 2024

   (Signature of Supervisor) (Date)

3. **To be completed by the chairperson of the department**

   I certify to the best of my knowledge that the required procedures have been followed and the preparation criteria have been met for this dissertation

Dr. Z Ndemo

pp 10/10/24

(Signature of the chairman) (Date)

APPROVAL FORM

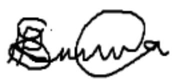
Name of the student: SIBANDA PRIVILEDGE

Registration Number: B225501B

Dissertation Title: Application of the APOS theory in teaching and learning of Geometrical Transformation at Ordinary level. A case study of Dete cluster in Hwange District

Degree Title: Bachelor of Science Honors Degree in Mathematics Education.

Year of completion: 2024



Signed .. Date 30/06/2024

Permanent Address

6772 Nketa 9

P O Box Nkulumane

Bulawayo

AKNOWLEDGEMENTS

My sincere gratitude to first and foremost my supervisor Dr Mukavhi for his support and invaluable guidance throughout this research. I am also greatly indebted to colleagues at Nechilibi High School for material and moral support. My thanks also goes to my family for their prayers and financial support. I also thank the administrators of schools in Dete cluster for allowing me to carry out the researches at their institutions.

Thank you a lot more for making this dream come true.

DEDICATION

This study is dedicated to my lovely wife Febbie who encouraged me through all circumstances to reach beyond the stars I am so thankful. To my mother Mrs Kasongola and family members for your financial support. To my dear friends and colleagues I am thankful for all the encouragement you have given me.

ABREVIATIONS/ACCRONYMS

APOS- Action Process Object and Schema

GT- Geometrical Transformation

HBSCED-   Honours Bachelors Science Education Degree

ABSTRACT

The project sought to explore the application of Action, Process, Object and Schema (APOS) theory in teaching and learning geometrical transformations at the Ordinary level in Dete cluster in Hwange District, with a focus on addressing the challenges identified in ZIMSEC reports and other sources. The study aims to investigate how APOS theory can guide instructional strategies to enhance students' understanding and performance in geometrical transformations and to identify common misconceptions and challenges faced by students in the learning process and explore teachers' perceptions and practices.

The descriptive survey method was employed. Interviews, questionnaires and lesson observations were used to collect both qualitative and quantitative data. The population was made up of teachers and students from 9 secondary schools in Dete cluster in Hwange district. A sample of 5 teachers and 20 learners was drawn, simple random sampling was used to select the 5 teachers and 20 learners.

The data collected was presented and analysed. The study looked into how the APOS theory might be used to teach geometric transformations to maths students at the ordinary level. Students gained a stronger knowledge of geometrical transformations as they moved through the action, process, object, and schema stages of the APOS framework, the APOS theory provides an organised method for addressing

misconceptions.

It has been shown that using the APOS theory to teach geometrical transformations improves students' comprehension and skill level with this mathematical idea and improves learning results from the use of a variety of instructional tactics, such as practical exercises, visual aids, and real-world applications, which meet the requirements and preferences of different learners.

RELEASE FORM i
APPROVAL FORM ii
AKNOWLEDGEMENTS iii
DEDICATION iv
ABREVIATIONS/ACCRONYMS v
ABSTRACT vi
1.1 Abstract: .... **Error! Bookmark not defined.**
CHAPTER 1 4
1.1 INTRODUCTION TO THE STUDY 4
1.2 BACKGROUND OF THE STUDY 4
1.3 PURPOSE OF THE STUDY 7
1.4 STATEMENT OF THE PROBLEM 7
1.5 RESEARCH AIM 8
1.6 RESEARCH QUESTIONS 8
1.7 RESEARCH OBJECTIVES 9
1.8 SIGNIFICANCE OF THE STUDY 9
1.9 LIMITATIONS TO THE STUDY 11
1.10 DELIMITATIONS TO THE STUDY 12
1.11 DEFINITION OF TERMS 12
1.12 ORGANIZATION OF THE STUDY 14
1.13 SUMMARY OF THE CHAPTER 14
CHAPTER 2 17
2.1 INTRODUCTION 17
2.2 APOS Theory in Geometrical Transformation 17
2.3 Challenges Faced by Learners in Teaching and Learning Geometrical Transformation 18
2.4 Relationship between Challenges and Mathematical/Geometrical Thinking 20
2.5 Reasons for Learners' Misconceptions in Geometrical Transformation 22
2.6 Application of APOS Theory to Address Learners' Misconceptions 24
2.5 RESEARCH GAPS 26
2.6 CHAPTER SUMMARY 27

CHAPTER 3 29
RESEARCH METHODOLOGY 29
3.1 Introduction 29
3.2 Design of the Research 29
3.2.1 Qualitative Phase 30
3.2.2 Quantitative Phase 30
3.3 Research Methods 31
3.3.1 Population and Sampling 31
3.3.2 Data Collection Methods/Instruments 32
3.3.3 Data Analysis Methods/Instruments 34
3.5 Ethical Issues 35
3.5.1 Approval to Conduct Research. 35
3.5.2 Informed Consent 35
3.5.3 Anonymity of Informants 35
3.5.4 Confidentiality 36
3.6 Chapter Summary 36
CHAPTER 4 37
DATA PRESENTATION, ANALYSIS AND DISCUSSION/ INTERPRETATION 37
4.1 INTRODUCTION 37
4.2 Data Presentation, analysis and discussion/interpretation 38
Fig. 1 Demographic Information 38
Fig. 2 Distribution by highest professional qualification 39
a) Challenges Faced by Learners in the Teaching and Learning of Geometrical Transformation 39
b) Relationship of Challenges to Mathematical Thinking and Geometrical Thinking 41
c) Application of APOS Theory to Address Learners' Misconceptions on Geometrical Transformation 42
d) Reasons for Learners' Misconceptions in the Teaching and Learning of Geometrical Transformation 45
Fig. 3, a chart on learners misconceptions 45
TABLE 3: learner challenges 46
Discussion 48

4.3 Chapter Summary 50
Chapter 5 51
SUMMARY, CONCLUSION AND RECOMMENDATIONS 51
5.1 Introduction 51
5.1.1 Application of APOS Theory in Teaching Geometrical Transformations 51
5.2 Summary of the Study 52
5.3 Conclusion 53
5.4 RECOMMENDATIONS 54

CHAPTER 1

1.1 INTRODUCTION TO THE STUDY

In the implementation of mathematics education, effective teaching and learning strategies are essential for fostering students' understanding and mastery of geometric concepts. One such strategy that has gained attention in recent years is the Application of Action, Process, Object and Schema (APOS) theory. This theory offers a framework for understanding how learners develop mathematical concepts through concrete actions, mental processes, and abstract reasoning. In the context of geometrical transformation instruction at the Ordinary level, the application of the APOS theory holds promise for enhancing students' comprehension and problem-solving abilities.Zimbabwean school have experienced low performances of learners in Mathematics at ordinary level. An analysis of internal examination results at school level in terms of topics answered in Mathematics Paper section B showed that most of the students did not tackle or failed to answer questions on transformation geometry that is there is evidence learners have challenges in understanding and interrelating Geometric Transformation concepts as they are in Mathematics

1.2 BACKGROUND OF THE STUDY

Zimbabwe, like many countries, places significant emphasis on mathematics education as a foundation for academic success and future career opportunities. The Zimbabwe

Schools Examination Council (ZIMSEC) oversees the national examination system and sets standards for mathematics education at the Ordinary level. However, reports from ZIMSEC (2019-2023) and other sources indicate persistent challenges in students' performance in geometrical transformations among other components. These challenges may stem from factors such as curriculum implementation issues, inadequate teacher training, and limited access to resources. The teaching and learning of geometrical transformations at Ordinary level are crucial in developing students' spatial reasoning skills and understanding of fundamental geometric concepts. Geometrical transformations encompass translation, rotation, reflection, and dilation, which are essential for solving problems in various fields such as mathematics, engineering, and computer science. To enhance the effectiveness of teaching geometrical transformations, educators often employ various instructional theories and approaches. One such theory is the APOS (Action, Process, Object, and Schema) theory, which provides a framework for understanding how students construct mathematical knowledge and concepts.

Several factors contribute to the difficulties students encounter in learning geometrical transformations. Firstly, the abstract nature of transformational geometry concepts can pose challenges for students, particularly those who have difficulty visualizing geometric figures and understanding spatial relationships (Sago et al, 2011). Additionally, limited access to technology and manipulative may hinder students' ability to explore and engage with geometrical transformations in a hands-on manner (Van de Walle et al, 2010). Teacher preparedness and pedagogical approaches play a significant role in students' understanding of geometrical transformations. Teachers in the Dete cluster may face challenges in effectively conveying abstract mathematical concepts and providing meaningful learning experiences for their students. Inadequate training in instructional strategies and a lack of familiarity with theories such as the APOS theory may further exacerbate these challenges.

Despite these obstacles, research suggests that implementing APOS-informed instructional strategies can enhance students' understanding and performance in geometry. For example, Jones and Pratt (2013) found that APOS theory provided valuable insights into students' learning processes and helped identify effective teaching strategies for geometry instruction. Additionally, Zhang and Zhang (2015) demonstrated the effectiveness of APOS theory in addressing students' misconceptions and promoting conceptual understanding in mathematics education.However, despite the popularity and importance of Geometrical Transformation education, ZIMSEC Reports (2019-2023) show that among other components of mathematics, the topic has the least pass rate at O level in Zimbabwe and the researcher discovered that students/ learners had misconceptions that transformation geometry is a difficult topic. An analysis of results in terms of topics in examinations showed that transformation geometry is one of the topics pupils have not been performing very well as compared to other geometry topics.

In Hwange District, learners face challenges in dealing with transformation geometry, they prefer answering other topics in Mathematics Paper 2. These challenges may breed negative attitudes towards the topic, which may culminate into phobia. Haggart (2002) concurs with Jaji (1990) who affirms that learners' challenges with Geometrical Transformation are merely pedagogical, and failure can be attributed to the teaching approaches employed and availability of teaching facilities.

The Ministry of Primary and Secondary Education in Zimbabwe and other stakeholders are of the view that an intervention to enhance the learners' performance in the subject is necessary. This discovery led to the review of the Zimbabwean mathematics curriculum so that application of APOS can be integrated into the mainstream curriculum which will improve the learner's interest and performance. The purpose of this study is therefore to explore the effects of using APOS in teaching and learning Geometrical Transformation at Ordinary level in Dete cluster in Hwange District

1.3 PURPOSE OF THE STUDY

The purpose of this study is to explore the application of Action, Process, Object and Schema (APOS) theory in teaching and learning geometrical transformations at the Ordinary level in Dete cluster in Hwange District, with a focus on addressing the challenges identified in ZIMSEC reports and other sources. Specifically, the study aims to investigate how APOS theory can guide instructional strategies to enhance students' understanding and performance in geometrical transformations. By examining the effectiveness of APOS-informed instruction, the study seeks to identify the impact of sequential learning experiences that transition from concrete actions to mental processes and abstract reasoning. Additionally, the study aims to identify common misconceptions and challenges faced by students in the learning process and explore teachers' perceptions and practices regarding the implementation of APOS theory in the classroom. Ultimately, the study aims to provide insights and recommendations for improving the teaching and learning of geometrical transformation using APOS theory, thereby informing educational practice and policy in Dete cluster in Hwange District.

1.4 STATEMENT OF THE PROBLEM

Despite efforts to improve mathematics education in Zimbabwe, ZIMSEC reports indicate persistent challenges in students' performance in geometrical transformations at the Ordinary level. These challenges may include low pass rates, high rates of misconceptions, and limited proficiency in applying geometric principles to problem-solving tasks. Furthermore, ZIMSEC reports suggest that traditional instructional approaches may not adequately address these issues, highlighting the need for innovative teaching strategies informed by research-based theories such as APOS. By addressing these challenges through the application of APOS theory, this study aims to contribute to the enhancement of mathematics education practices in Zimbabwean schools. However, by trying to align teaching practices with ZIMSEC recommendations and addressing the specific needs of students in geometrical transformations, educators can better prepare students for success in mathematics examinations and future academic pursuits.

1.5 RESEARCH AIM

The aim of this research is to explore the effects of the application of the Action, Process, Object and Schema (APOS) theory in teaching and learning geometrical transformations at the Ordinary level in the Dete cluster, Hwange District. Specifically, the study seeks to explore the effectiveness of APOS-informed instructional strategies in enhancing students' understanding and performance in geometrical transformations.

1.6 RESEARCH QUESTIONS

a) What are challenges faced by learners in the teaching and learning of geometrical transformation?

b) How are these challenges related to Mathematical thinking and geometrical thinking?

C) How can the APOS theory be applied to address the learners' misconception on geometrical transformation?

d) Why do learners have misconception in the teaching and learning of geometrical transformation?

1.7 RESEARCH OBJECTIVES

a) To identify challenges faced learners in answering transformation geometry questions,

b) To identify challenges related to Mathematical thinking and geometrical thinking.

c) To apply the APOS theory in addressing the learners misconception on geometrical transformation.

d) To find out the reasons why learners have misconceptions in the teaching and learning of geometrical transformation.

1.8 SIGNIFICANCE OF THE STUDY

The significance of applying t (APOS) theory in teaching and learning geometrical transformations at the Ordinary level in the Dete cluster, Hwange District is essential to teachers, legislators and other stakeholders who work in Mathematics education. The study's findings could help in the designing focused interventions and policies that would improve Mathematics in a broad scope. It extends to several key areas:

1.8.1 Improving Student Achievement

By implementing APOS-informed instructional strategies, educators can enhance students' understanding and mastery of geometrical transformations. Research has shown that APOS theory provides valuable insights into students' cognitive processes and can help identify effective teaching methods that promote deeper learning (Jones & Pratt, 2013).

1.8.2 Addressing Misconceptions

Geometrical transformations are abstract concepts that students may struggle to comprehend fully. APOS theory offers a framework for identifying and addressing students' misconceptions by guiding educators in scaffolding students' learning experiences from concrete actions to abstract understanding (Zhang & Zhang, 2015).

1.8.3 Enhancing Teaching Practices

Educators in the Dete cluster could benefit from incorporating APOS theory into their teaching practices. By aligning instructional strategies with APOS principals, teachers can create meaningful learning experiences that engage students and foster conceptual understanding of geometrical transformations (Dubinsky & McDonald, 2001).

1.8.4 Informing Curriculum Development

The findings of this study could inform curriculum development efforts aimed at improving mathematics education in Zimbabwe. By highlighting the effectiveness of APOS-informed instructional strategies, curriculum developers can integrate these approaches into curriculum frameworks and educational policies to support teachers in delivering high-quality instruction in geometrical transformations.

1.8.2 Contributing to Educational Research

Research on the application of APOS theory in geometry instruction in the Dete cluster adds to the body of knowledge in mathematics education. By documenting the effectiveness of APOS-informed strategies in a specific educational context, this study contributes valuable insights that can inform future research and educational practices (Lappan et al., 2015). The significance of this study lies in its potential to improve mathematics education practices, enhance student learning outcomes, and contribute to the advancement of educational research in Zimbabwe and beyond. By embracing APOS theory as a guiding framework for teaching geometrical transformations, educators can empower students with the mathematical knowledge and skills needed for success at the ordinary level and beyond.

1.9 LIMITATIONS TO THE STUDY

1.9.1 Sample Size

One potential limitation of this study is the size of the sample population. Due to logistical constraints and resource limitations, the study may only involve a limited number of schools or students in the Dete cluster. As a result, the findings may not be fully representative of the entire population and may lack generalizability to other contexts.

1.9.2 Time Constraints

Another limitation is the time frame allocated for data collection and analysis. Conducting comprehensive research within a limited time frame may restrict the depth and scope of the study. Consequently, certain aspects of the research, such as longitudinal data analysis or extensive qualitative interviews, may not be feasible within the allotted time.

1.9.3 Resource Availability

The availability of resources, including funding, technology, and personnel, may pose challenges to the implementation of the study. Limited resources may impact the researcher's ability to conduct extensive data collection, access specialized equipment or software, or recruit participants from diverse backgrounds.

1.10 DELIMITATIONS TO THE STUDY

- Geographical Scope

The study focuses specifically on the Dete cluster within the Hwange District of Zimbabwe. While this geographic delimitation allows for a more focused investigation of local educational contexts and challenges, it may limit the generalizability of the findings to other regions or districts.

- Grade Level

The study targets students at the Ordinary level, typically in the lower secondary grades. While this grade level is of particular interest due to its importance in the Zimbabwean education system, the findings may not be applicable to students at other educational levels, such as primary or advanced levels.

- Instructional Context

The study examines the application of APOS theory within the context of geometrical transformation instruction. While this instructional focus allows for a detailed analysis of specific teaching strategies and learning outcomes, it may overlook other aspects of mathematics education or curriculum implementation in the Dete cluster.

1.11 DEFINITION OF TERMS

Geometrical Transformation

Geometrical transformation refers to the process of changing the position, orientation, or size of geometric figures in a plane or space. It includes operations such as translation (movement), rotation (turning), reflection (flipping), and dilation (resizing), which preserve the shape and properties of the original figure (Stewart, 2008). The study of Geometrical Transformations allows learners to recognise and perform changes in the coordinates of objects in relation to their images regarding their position, orientation, direction and size, (Kekana, 2016).

Mathematics

Mathematics is the systematic study of quantity, structure, space, and change. It encompasses a broad range of concepts, including numbers, patterns, shapes, and relationships, and provides tools and methods for solving problems, making predictions, and understanding the world around us (Stewart, 2008).

Teaching

Teaching refers to the process of facilitating learning, imparting knowledge, and guiding students in acquiring skills, understanding concepts, and developing attitudes. It involves the deliberate arrangement of instructional activities and resources to support students' learning goals and objectives (Ormrod, 2016).

Learning

Learning is the process of acquiring knowledge, skills, behaviors, or attitudes through experience, study, instruction, or reflection. It involves the active engagement of learners in making connections, constructing meaning, and internalizing new information or concepts (Ormrod, 2016).

APOS Theory

The APOS (Action, Process, Object, and Schema) theory is a constructivist learning theory that describes the stages through which students develop mathematical understanding. It posits that learning progresses from concrete actions to abstract processes, culminating in the formation of mental schemas or cognitive structures (Dubinsky & McDonald, 2001).

1.12 ORGANIZATION OF THE STUDY

The study account is organized into five chapters. Chapter 1 provides an introduction to the study, including background information, the rationale for the research, objectives, and scope. Chapter 2 reviews relevant literature on geometrical transformations, the APOS theory, and instructional approaches in mathematics education. Chapter 3 describes the methodology employed in the study, including research design, data collection procedures, and data analysis techniques. Chapter 4 presents the findings of the study, including an analysis of the effectiveness of instructional strategies informed by the APOS theory. Finally, Chapter 5 offers conclusions, implications, and recommendations for future research.

1.13 SUMMARY OF THE CHAPTER

The overview of the study's focus on exploring the application of the (APOS) theory in the teaching and learning of geometrical transformation in mathematics at Ordinary Level in Dete cluster in Hwange District is given in this chapter. This chapter introduced the study by unpacking the research problem and its context through the background to the study and statement of the problem. The primary issues which were discussed in this chapter were the research problem in the background of the study, the statement of the problem, significance of the study, assumption of the study, limitations of the study as well as the delimitation of the study. Key terms were also defined. This opened the door for literature review in chapter two.

CHAPTER 2

2.1 INTRODUCTION

In this chapter, we explore the theoretical framework guiding the teaching and learning of geometrical transformation at the Ordinary level in the Dete cluster, Hwange District. Specifically, we examine the Application of Action, Process, Object, and Schema (APOS) theory, which provides a structured approach to understanding how students develop mathematical concepts through sequential cognitive stages.

2.2 APOS Theory in Geometrical Transformation

The Application of Action, Process, Object, and Schema (APOS) theory, proposed by Dubinsky and McDonald (2001), outlines a developmental framework for understanding how students learn mathematical concepts. In the context of geometrical transformation, APOS theory suggests that students' progress through four cognitive stages: action, process, object, and schema. The APOS theory provides a structured framework for guiding instructional practices in teaching geometrical transformation. By understanding the cognitive stages through which students' progress, educators can design instruction that effectively scaffolds students' learning experiences, addresses misconceptions, and promotes deep conceptual understanding (Dubinsky & McDonald, 2001; Zhang & Zhang, 2015).

Firstly, the action stage, students engage in physical manipulation of geometric shapes to explore transformational concepts. This hands-on experience allows students to develop an initial understanding of how transformations affect geometric figures. As students continue to explore geometrical transformations, they begin to develop mental processes to understand the underlying principles and procedures involved. They learn to articulate and apply transformational rules and procedures, such as the rules for reflecting or rotating shapes. In the object stage, students construct mental objects to represent transformed geometric figures. These mental objects serve as internal representations that allow students to visualize and manipulate transformed shapes mentally. Students develop the ability to mentally visualize the outcome of transformations without physical manipulation. Finally, in the schema stage, students integrate their understanding of geometrical transformations into abstract schemas. They can apply transformational concepts in problem-solving contexts, generalize their knowledge to new situations, and articulate the underlying principles of geometrical transformation.

2.3 Challenges Faced by Learners in Teaching and Learning Geometrical Transformation

Students encounter various challenges when learning geometrical transformations. One common challenge is the abstract nature of the concepts involved, such as translation, rotation, reflection, and dilation. These transformations require students to understand spatial relationships and visualize geometric figures in different orientations, which can be difficult for some learners (Seago et al., 2011). Another challenge is the complexity of transformation notation and terminology, which may confuse students and hinder their ability to apply the correct transformations to geometric figures (Alkhateeb &

Qaryouti, 2014). Additionally, limited access to resources, such as manipulative and technology tools, can restrict students' opportunities for hands-on learning experiences and exploration of transformational geometry concepts (Van de Walle et al., 2010).

One of the primary challenges faced by learners is the abstract nature of geometrical transformation concepts. ZIMSEC reports indicate that students may struggle to grasp abstract mathematical concepts such as translation, rotation, reflection, and dilation, which are fundamental to geometrical transformation (ZIMSEC, n.d.). The abstract nature of these concepts can make it difficult for learners to visualize and understand the effects of transformations on geometric figures.

Another challenge identified in ZIMSEC reports is the complexity of transformation notation and terminology. Students may find it challenging to interpret and apply transformational notation correctly, leading to errors in their mathematical reasoning and problem-solving skills (ZIMSEC, n.d.). Additionally, inconsistencies in terminology and notation across different resources or instructional materials can further confuse learners.

Access to resources, such as manipulative and technology tools, is another significant challenge faced by learners in geometrical transformation instruction. ZIMSEC reports highlight disparities in resource availability across schools, with some learners lacking access to essential tools for hands-on learning and exploration of transformational geometry concepts (ZIMSEC, n.d.). Limited access to resources can hinder students' ability to engage with and internalize geometrical transformation concepts effectively.

ZIMSEC reports also point to inadequate teacher preparedness as a contributing factor to learners' challenges in geometrical transformation. Teachers may lack the necessary content knowledge, pedagogical skills, and resources to effectively teach transformational geometry concepts (ZIMSEC, n.d.). Furthermore, variations in teacher quality and instructional practices across schools can impact students' learning experiences and outcomes.

Cultural and contextual factors can also influence learners' experiences and perceptions of geometrical transformation. Socio-economic disparities, language barriers, and cultural differences may affect students' access to educational opportunities and support networks, potentially exacerbating challenges in learning geometrical transformation (Chinamasa, 2017).

Addressing these challenges requires a multifaceted approach that involves improving teacher training and support, enhancing resource provision, and promoting culturally responsive teaching practices. By addressing these challenges, educators can better support learners in developing a deeper understanding of geometrical transformation concepts and improving their mathematical proficiency.

2.4 Relationship between Challenges and Mathematical/Geometrical Thinking

The challenges faced by learners in geometrical transformations are closely related to both mathematical and geometrical thinking. Mathematical thinking involves reasoning, problem-solving, and making connections between mathematical concepts. Learners may struggle with geometrical transformations because they require mathematical thinking skills such as spatial visualization, pattern recognition, and logical reasoning

(Hiebert & Grouws, 2007). Geometrical thinking, on the other hand, involves the ability to visualize, analyze, and manipulate geometric shapes and structures. Learners' difficulties in understanding and applying geometrical transformations may stem from limitations in their geometrical thinking abilities, such as difficulties in mentally rotating shapes or visualizing transformations in multiple dimensions (Clements & Battista, 1990).

Geometrical thinking plays a crucial role in learners' ability to understand and apply geometrical transformations. One aspect of geometrical thinking is spatial visualization which is the ability to mentally manipulate geometric shapes and understand spatial relationships. Learners who struggle with spatial visualization may find it challenging to visualize the effects of transformations on geometric figures, leading to difficulties in understanding transformational geometry concepts (van Garderen & Montague, 2003). For example, students may have difficulty visualizing the result of a rotation or reflection without physically manipulating the figure or using visual aids. Geometrical thinking also involves pattern recognition and logical reasoning skills, which are essential for understanding transformational geometry concepts. Learners must recognize patterns in transformational processes and logically deduce the effects of different transformations on geometric figures (Hoyles et al., 2010). However, students who lack strong pattern recognition or logical reasoning skills may struggle to identify transformational patterns or make connections between different types of transformations.

Mental rotation is another aspect of geometrical thinking that influences learners' understanding of geometrical transformations. Mental rotation involves mentally rotating geometric figures to visualize them from different perspectives (Hegarty & Waller, 2004). Learners with poor mental rotation abilities may have difficulty visualizing transformations involving rotation or reflection, hindering their ability to comprehend these concepts. For example, consider a student attempting to understand the concept of a reflection across a line. To comprehend this transformation, the student must mentally visualize the original shape and its mirror image across the line of reflection. However, if the student struggles with spatial visualization skills or mental rotation abilities, they may find it challenging to visualize the transformation accurately. As a result, the student may struggle to grasp the concept of reflection and apply it correctly in problem-solving tasks.

Learners' challenges in teaching and learning geometrical transformation are closely related to their geometrical thinking abilities, including spatial visualization, pattern recognition, logical reasoning, and mental rotation. Educators must consider these factors when designing instructional strategies and supporting learners in developing a deeper understanding of transformational geometry concepts. By addressing learners' geometrical thinking challenges, educators can help students overcome obstacles to learning geometrical transformations and improve their mathematical proficiency.

2.5 Reasons for Learners' Misconceptions in Geometrical Transformation

Learners may have misconceptions in the teaching and learning of geometrical transformations for several reasons. One reason is the abstract nature of the concepts, which can lead to misunderstandings or misinterpretations of transformational geometry principles. Geometrical transformations involve abstract concepts such as translation, rotation, reflection, and dilation, which can be challenging for Ordinary Level learners to grasp. Without concrete examples or hands-on experiences, learners may struggle to understand the underlying principles of transformations (Seago et al., 2011). For example, learners may have difficulty visualizing the effect of a rotation on a geometric shape without physically rotating the figure or using visual aids.

Additionally, learners may have preconceived notions or intuitive beliefs about geometric transformations that are inconsistent with formal mathematical definitions and properties (Kaur & Parveen, 2018). Furthermore, learners' misconceptions may arise from instructional practices that emphasize procedural fluency over conceptual understanding. When students focus solely on memorizing transformation rules without understanding the underlying mathematical principles, they are more likely to develop misconceptions or make errors in applying transformations to geometric figures (Sowder et al., 2018). Some O'level learners may rely on memorization of transformation procedures without understanding the underlying concepts. This superficial understanding can lead to misconceptions and errors in applying transformational rules (Seago et al., 2011). For instance, learners may memorize the steps for performing a translation without understanding how the coordinates of points change during the transformation. Spatial visualization skills are crucial for understanding geometrical transformations. O'level learners who lack strong spatial visualization abilities may struggle to mentally manipulate geometric shapes and visualize the outcomes of transformations (van Garderen & Montague, 2003). Students may find it challenging to visualize the result of a reflection across a line without physically drawing or manipulating the figure.

Moreso, the notation used to represent geometrical transformations can be complex and confusing for O'level learners. Misinterpretation of transformation notation may lead to misconceptions about the direction or nature of transformations (Seago et al., 2011). Learners may confuse the notation for a reflection across the x-axis with a reflection across the y-axis, leading to incorrect interpretations of transformational processes. O'level learners may again struggle with understanding and applying transformation terminology, such as translation, rotation, and reflection. Without a clear understanding of these terms, learners may misinterpret instructions or make errors in problem-solving tasks (Alkhateeb & Qaryouti, 2014).To thus challenge, students may confuse the terms "rotation" and "reflection" or use them interchangeably, leading to misunderstandings about the nature of transformations.

2.6 Application of APOS Theory to Address Learners' Misconceptions

Addressing the reasons for misconceptions in geometrical transformations requires targeted instructional strategies that promote conceptual understanding, provide hands-on learning experiences, and scaffold learners' progression through the cognitive stages outlined in the Application of Action, Process, Object, and Schema (APOS) theory (Dubinsky & McDonald, 2001). The Application of Psychological and Ontological Structure (APOS) theory offers a framework for addressing learners' misconceptions in geometrical transformations. According to APOS theory, learning progresses through four stages: action, process, object, and schema. By guiding students through concrete actions, identifying underlying processes, and developing mental objects and schemas, educators can help learners overcome misconceptions and deepen their understanding of geometric transformations (Zhang & Zhang, 2015). For example, educators can use concrete manipulatives or visual representations to engage students in hands-on activities that demonstrate geometrical transformations. By allowing students to physically manipulate shapes and observe the effects of transformations, educators can help students build a solid foundation of understanding before moving to more abstract representations and symbolic notation (Dubinsky & McDonald, 2001). In a classroom activity, students are given transparent overlays to physically reflect geometric shapes across a line. By observing the changes in the shape's orientation and position, students gain a concrete understanding of reflection.

Addressing these misconceptions in the learning of geometrical transformations requires a multifaceted approach that integrates mathematical thinking, geometrical thinking, and APOS-informed instructional strategies. By recognizing the interconnectedness of these factors and implementing effective teaching practices, educators can support learners in developing a deep and meaningful understanding of transformational geometry concepts. Geometrical transformation is a fundamental topic in mathematics education, yet learners often encounter various challenges when trying to understand and apply these concepts.

Moving to the process stage, educators guide students' exploration of transformational concepts through structured activities. Encouraging students to articulate the steps and reasoning behind transformations solidifies their understanding. For instance, prompting students to explain how translations affect the coordinates of points fosters deeper comprehension (Zhang & Zhang, 2015). During a group discussion, students are asked to describe the process of performing a rotation. By verbalizing the steps involved, students deepen their understanding of rotational transformations.

As students’ progress to the object stage, they construct mental objects to represent transformed geometric figures. Educators can support this process by providing visual representations and encouraging mental visualization. For instance, using interactive software or virtual manipulative aids students in mentally visualizing transformations (Zhang & Zhang, 2015). Students use virtual manipulative software to explore transformations. They manipulate geometric shapes on a computer screen, mentally visualizing the outcomes of different transformations. Lastly, in the schema stage, students integrate their understanding into abstract schemas for problem-solving. Educators can foster abstract reasoning by presenting diverse problem-solving tasks. For instance, challenging students to apply transformational principles in novel contexts promotes schema integration (Dubinsky & McDonald, 2001). Students are given a set of transformational tasks involving multiple steps. They must sequence the transformations correctly to achieve a desired outcome, demonstrating their ability to apply abstract reasoning skills.

By employing instructional strategies grounded in APOS theory, educators can effectively address learners' misconceptions in geometrical transformations. Providing concrete experiences, guiding exploration and process understanding, facilitating mental visualization, and promoting abstract reasoning all contribute to scaffolding students' progression through the cognitive stages outlined in APOS theory and fostering deep conceptual understanding (Zhang & Zhang, 2015).

2.5 RESEARCH GAPS

Many studies on the application of APOS theory in mathematics education focus on general principles rather than specific contexts like teaching geometrical transformation at the O level in rural areas such as the Dete cluster, Hwange District. There is a need for research that examines the effectiveness of APOS-informed instructional strategies in diverse educational settings to address the unique challenges faced by students and educators (Zhang & Zhang, 2015). While APOS theory offers a promising framework for enhancing mathematics instruction, there is limited research on the preparation and professional development of teachers to effectively implement APOS-informed instructional strategies. Investigating the training needs and professional development opportunities for educators in rural areas like the Dete cluster can provide insights into supporting teachers in integrating APOS theory into their teaching practices (Jones & Pratt, 2013).

More so, study on the application of APOS theory often focuses on instructional strategies and student learning processes, with less emphasis on assessment and measurement of learning outcomes. There is a gap in understanding how to assess students' progression through the cognitive stages outlined in APOS theory and evaluate the effectiveness of APOS-informed instruction in improving learning outcomes in geometrical transformation (Zhang & Zhang, 2015)

With the 21st Century skills as requirements and increasing use of technology in education, there is a need to explore how APOS theory can be integrated with digital tools and resources to enhance teaching and learning of geometrical transformation. Research on the development and implementation of technology-enhanced APOS-informed instructional materials and software can provide valuable insights into leveraging technology to support students' cognitive development in rural contexts like the Dete cluster (Jones & Pratt, 2013).Lastly, While this study provides insights into students' progression through the cognitive stages of APOS theory, there is a lack of longitudinal studies that track students' development over time. Longitudinal research can provide a deeper understanding of how students' understanding of geometrical transformation evolves and how instructional interventions impact their learning trajectories in the O level curriculum (Zhang & Zhang, 2015).

Addressing these research gaps can contribute to a more comprehensive understanding of the application of APOS theory in teaching geometrical transformation at the O level in rural contexts like the Dete cluster, Hwange District. Conducting research that addresses these gaps can inform the development of effective instructional strategies, teacher preparation programs, assessment practices, technological interventions, and longitudinal studies to support student learning and achievement in geometrical transformation.

2.6 CHAPTER SUMMARY

This chapter explored the Application of Action, Process, Object, and Schema (APOS) theory in teaching geometrical transformation at the O level in the Dete cluster, Hwange District. It emphasized employing APOS-informed instructional strategies, such as providing concrete manipulative, guiding exploration, facilitating mental visualization, and promoting abstract reasoning, to address learners' misconceptions. Additionally, the chapter identified research gaps, including the need for context-specific studies, teacher professional development, assessment practices, integration with technology, and longitudinal research. Understanding and addressing these gaps can enhance the effectiveness of APOS theory in teaching geometrical transformation in the Dete cluster, Hwange District.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Introduction

This chapter covered the research approach used to explore the application of (APOS) theory in the teaching and learning of geometrical transformation at Ordinary Level in Dete cluster, Hwange District. This chapter included an overview of the research design, methodologies, ethical considerations, reliability and validity, and a summary of the chapter.

3.2 Design of the Research

In essence, the research design is the blueprint for organising and presenting all of the data that has been gathered for the study. A research design is the overarching plan that is implemented in an appropriate and methodical manner in order to conduct a significant research investigation. Research design, according to Thomas [2013], is the science and art of organising studies to identify the most reliable sources. According to Magenta and Mugenda (2015), research design is the basic structure or method of the analysis and the reasoning behind it. According to Cohen and Manion (2013), some of the research designs included case studies that were developmental, historical, experimental, ethnographic, and descriptive survey designs. In order to get comprehensive understanding of the incorporation of digital technology for teaching mathematics, a mixed methods approach was employed for this project. The research paradigm is interpretivist because it acknowledges the subjectivity of human experience and the importance of understanding the perspectives of educators and learners. Within a single study, a mixed research design incorporates components of both qualitative and quantitative research approaches. To gain a thorough grasp of the topic of obstacles and potential in incorporating digital technology for teaching Mathematics to O' Level students in the Dete cluster, a mixed study design may prove advantageous. Here is a possible structure for such a design.

3.2.1 Qualitative Phase

Perform semi-structured interviews with educators, administrators, and students to investigate their perspectives, experiences, and attitudes regarding the application of the (APOS) theory. These interviews can offer comprehensive insights into the obstacles encountered, opportunities recognised, and tactics utilised in the application of the theory in geometrical transformation in mathematics education.

3.2.2 Quantitative Phase

Surveys should be given to a wider group of educators and learners in order to collect quantitative information about the theory's applicability, difficulties, and levels of participation. In addition to qualitative information, surveys can offer statistical data that supports comparisons and generalisations.

To objectively evaluate the application and efficacy of the (APOS) theory in mathematics education, make observations in the classroom. Empirical information on instructional strategies and student engagement levels can be obtained through observations.

All things considered, a mixed study design provides a strong and thorough technique of investigating complicated educational phenomena like technology integration, enabling researchers to take advantage of the advantages of both qualitative and quantitative methodologies to learn more about the subject.

3.3 Research Methods

3.3.1 Population and Sampling

O'Level students and maths teachers in the Dete cluster of the Hwange District make up the population of interest. Polit and Beck (2017) state that the term "population" encompasses all of the examples in which the researcher is concerned. The Dete cluster of secondary schools in the Hwange District is referred to as "this" in this context. Polit and Beck (2017) state that in qualitative studies, the population to be included is chosen based on determining which particular people are qualified to take part in the research rather than on the intention of generalising the results. Therefore, administrators, teachers, and students of mathematics are included in the study.

According to Sharma (2017), sampling is the process by which researchers methodically select a smaller number of representative objects or people from a designated population. The goals of the study are then followed while using these chosen subjects for experimentation or observation. Purposive sampling was used by the researcher in this investigation. Purposive sampling is a kind of sampling approach that excludes probability, according to Black (2010). It is applied when the researcher uses their own discretion to choose sample items. This approach, according to Cresswell and Clark (2011), comprises locating and choosing people or organisations that have extraordinary knowledge or experience on a given area of interest.A purposive sampling technique were employed to select 5 facilitators and 20 students from Dete cluster schools, who have experience in teaching and learning Mathematics. The utilization of the sample technique offers numerous benefits to the researcher. This is as a result of the method's time and cost effectiveness. Sharma (2017) states that this sampling method can be helpful in examining anthropological contexts where an intuitive approach can lead to significant discoveries. The researcher intuitively gathered information from the subjects by using this method. Purposive sampling is the best approach, in Sharma's opinion (2017), when primary data sources that can add to the study are scarce. The sampling procedure is constrained by its vulnerability to mistakes resulting from the researcher's judgement when choosing participants. Cresswell (2014) claims that there is a high degree of bias and a low degree of reliability in the technique.

3.3.2 Data Collection Methods/Instruments

Data were gathered using both quantitative and qualitative techniques. Semi-structured interviews with maths teachers and student observations were used to collect qualitative data. To gain comprehensive insights into the experiences, perspectives, and attitudes of facilitators and education administrators regarding the integration of digital technology in mathematics education, semi-structured interviews were carried out. In order to see first-hand how instructors incorporate digital technology into their teaching of mathematics and to spot any possibilities or problems that may come up throughout the teaching process, the researcher also conducted observations in classrooms.

In order to get more comprehensive insights, questionnaires with quantitative data were given to a larger sample of students. To get quantitative data on the usage patterns, preferences, and perceived efficacy of digital technology in mathematics education, survey a sample of teachers and students.

Interviews

Interviews were used by the researcher. Interviews are a sort of consultation in which the researcher seeks further information about a certain subject from the viewpoint of the person being questioned, according to Aldhabi and Anomie (2017). The investigator designed and carried out in-depth interviews with the participants. Using these instruments was essential to the research. According to Charmaz (2017), interviewers can increase participants' comfort level by developing a relationship with them. This enabled the study participants to provide more thoughtful responses, especially when talking about challenging topics. As a result, this suggests that the researcher can interact with participants and come up with more questions during the interview. According to Adhabi and Anozie (2017), interviewers are better able to ask follow-up questions and go into more details, and revisit crucial issues during the interview process, thereby fostering a comprehensive comprehension of attitudes, perspectives, and motivations. An inherent drawback of in-depth interviews is their considerable time consumption, since it necessitates the transcription, organization, analysis, and reporting of the interviews.

3.3.3 Data Analysis Methods/Instruments

Data analysis, according to Ghauri and Gronhaug (2002:137), is the methodical process of classifying, categorising, and assigning importance to a wide range of collected data.

The collected data was analysed qualitatively. The data was edited to make sure it was complete and consistent before it was processed. After editing, the data was encrypted to make it easier to classify the responses into several groups. This methodology enabled the methodical collection and categorization of data into discrete groups. The findings were described in order to clarify and communicate the facts in a qualitative way. We'll make sure that no information that has been collected is lost by doing this just as it is.

Qualitative Analysis

Coding or thematic analysis was used to examine qualitative data and find recurrent themes, patterns, and connections. The experiences and viewpoints of participants were richly described and explained through qualitative analysis.

Quantitative Analysis

3.4 Reliability and Validity

The trustworthiness indicators for qualitative research that were sustained by member verification, triangulation, and keeping an audit record of the research process included credibility, transferability, dependability, and conformability. In quantitative research, validity and reliability are proven, survey instrument pilot testing is done, and data correctness and consistency are guaranteed.

3.5 Ethical Issues

All during the research procedure, ethical issues were of the utmost importance. This included securing the required study permits, getting participants' informed consent, protecting participant privacy, and handling any possible conflicts of interest. The appendices contain copies of consent forms, letters, and research permits.

3.5.1 Approval to Conduct Research.

Research authorization According to Grey (2009), it is imperative for a researcher to adhere to ethical principles while conducting research. The researcher received a letter of recommendation from the university. The letter was utilized to request permission from the Ministry of Primary and Secondary Education to carry out the research. Consent was also requested from the respective schools.

3.5.2 Informed Consent

The informed consent principle was another ethical guideline that was adhered to. Creswell (2013) asserts that there are defined protocols that researchers need to follow. This entails explaining the goals to the participants so that they may make informed decisions throughout the study. The researcher made it clear that participants could leave the study at any time in accordance with this ethical guideline.

3.5.3 Anonymity of Informants

Anonymity is another ethical issue that was considered. The researcher ensured that the subject's identity is untraceable with regard to their personal responses by using this technique. Ford and Reutter (2010) state that it is illegal to modify identifying information and use false identities when transcribing interview recordings.

3.5.4 Confidentiality

Maintaining privacy is essential when doing research. Levine (2019) defines confidentiality as the ability of an individual to decide how much information they wish to reveal to or withhold from a specific person. The researcher in this study made sure that all of the audio recordings and papers used in the research remained private. The investigator made it clear to the participants that no information would be shared with any parties outside the parameters of the study.

3.6 Chapter Summary

The study techniques utilised to investigate the use of the APOS theory in teaching geometrical transformation to Level students of mathematics were comprehensively summarised in the chapter above. The mixed methods approach, which combines qualitative and quantitative procedures, ensures rigour and ethical standards are upheld while providing a comprehensive understanding of the phenomenon under study.

CHAPTER 4

DATA PRESENTATION, ANALYSIS AND DISCUSSION/ INTERPRETATION

4.1 INTRODUCTION

During the study, the researcher carried out a research on the application of the APOS theory in teaching geometrical transformation. This chapter aims to explore the challenges in the application of the APOS theory in teaching geometrical transformation in Mathematics at Ordinary Level. The evaluation provided insights into the current performance of students and identify areas of strength and weakness, and propose recommendations for improving approaches. A number of quantitative and qualitative research instruments were applied. Participants responded to items on questionnaires, interviews developed by the researcher and validated by the tutor. The items were divided into sections according to sub research questions. The findings of the study were presented, analyzed, discussed, interpreted and summarized in this chapter.

4.2 Data Presentation, analysis and discussion/interpretation

Table 1 Demographic Information

Category	Learners	Teachers
Sex	Female = 10	Female = 2

	Male = 10	Male = 3
Age	18 years- Female = 1 , Male = 1 17years - Female = 3, Male = 3 16years - Female =7 Male = 4	35years -Female = 1 34 years - Male = 1 , Female = 1 39 years – Male = 1 41 years – Male = 1
Qualifications		The teachers had a diploma in Education except for one female who had a Bachelor of Education degree in Mathematics.
Experience		Male: 7years = 1 Male: 12 years = 1 Female :2 years = 1 Male : 8years = 1 Female : 4years = 1

This diversity of teaching experiences—from more recent graduates who could be more conversant with cutting-edge instructional strategies to more seasoned instructors who offer a multitude of useful classroom insights—provides a well-rounded viewpoint. Table 2's participant ages reveal a gender disparity between the male and female teachers in the sample that was chosen.

Table 2 Distribution by highest professional qualification

Highest Professional qualification	Frequency	Percentage
Diploma in Education	4	80%
Bachelor of Education	1	20%
Master of Education Degree	0	0%

Although the teachers in this study do not use a range of teaching methods, the researcher has found that the teachers are sufficiently competent and experienced, thus these factors do not contribute to the students' low performance. According to Tichapondwa (2011), having appropriately qualified teachers increases the amount of time students spend in class, which has a good impact on student performance. This suggests that educators from the chosen schools possess all the necessary pedagogical abilities to apply the APOS theory.

a) Challenges Faced by Learners in the Teaching and Learning of Geometrical Transformation

Teacher A

- The challenge of visualising changes in a coordinate plane.
- Misunderstanding the differences between various transformations (such as rotation versus reflection).

- Misreading the indication for transformation.

- A poor comprehension of the characteristics of transformed shapes.

Teacher B

- Having difficulty seeing changes, particularly in a coordinate plane.

- Combining various transformations, such as rotation and reflection.

- Having trouble correctly comprehending transformation notation.

- A poor understanding of shape attributes following alterations.

Teacher C

- Difficulties with imagining how shapes shift in orientation or position.

- Bewilderment about several kinds of transformations.

- The intricacy of applying and understanding transformation notation.

Insufficient comprehension of the properties of shape following transformations.

Teacher D

- Having trouble seeing changes on a coordinate plane.

- Misunderstanding rotational and reflectional transformations.

- The transformation notation is misinterpreted.

- Inability to understand shape attributes after transformation.

Teacher E

-failing to plot the given points on the Cartesian plane

-combining various transformation such as rotation and reflection

The respondents listed several difficulties they encountered, such as having trouble visualising transformations, being confused about the various kinds of transformations, misreading transformation notation, and not knowing what attributes remain after transformation. These difficulties show up as erroneous converted point charting, improper transformation rule application, reliance on visual aids, and misconceptions that show up in test answers and homework assignments, among other issues, in classroom activities and assessments.

b) Relationship of Challenges to Mathematical Thinking and Geometrical Thinking

Teacher A

-Students' difficulties visualising transformations restrict their ability to use spatial reasoning, which is essential for thinking geometrically.

- Students' capacity for logical reasoning and problem-solving, two essential aspects of mathematical thinking, is impacted by misinterpreting transformation notation.

Teacher B

-Students' comprehension of mathematical relationships is hampered by difficulty visualising transformations, which affects geometrical reasoning.

- Errors in problem-solving arise from a lack of understanding of various transformation types, which impairs mathematical reasoning abilities.

Teacher C

-Students' geometrical thinking is impacted by difficulties visualising transformations since it hinders their comprehension of geometric properties.

-Students' mathematical understanding is impacted when they misunderstand transformation notation because it makes it more difficult for them to correctly apply transformation rules.

Teacher D

- Students' geometrical thinking is impacted when they have trouble visualising transformations since it hinders their comprehension of spatial relationships.
- Misunderstanding the various transformations causes mistakes when using geometric principles, which impairs one's ability to think mathematically.

Teacher E

- Students' geometrical thinking is impacted by difficulties visualising transformations because some of them even fail to plot points on Cartesian plane.

These problems are directly associated with geometrical and mathematical reasoning. Students' capacity to solve problems in geometry is hampered by their inability to comprehend geometrical transformations, which results in deficiencies in their ability to develop logical arguments and proofs. These difficulties also have an impact on students' comprehension of spatial characteristics and relationships, which are crucial elements of geometrical and mathematical reasoning. In general, difficulties with geometrical transformations can lead to poorer performance in other mathematical domains where spatial reasoning is necessary.

c) Application of APOS Theory to Address Learners’ Misconceptions on Geometrical Transformation

Teacher A

- Action Stage: Explain changes with practical exercises.
- Process Stage: Assist pupils in creating mental images.
- Object Stage: Assist children in creating imaginary objects with altered shapes.
- Schema Stage: Promote using abstract ideas to address issues.

Teacher B

- Action Stage: Give pupils practical exercises to help them comprehend fundamental transformation acts.

- Process Stage: Walk pupils through sequential steps and mental images.

- Object Stage: Help pupils comprehend transformations as whole entities on a deeper level.

Schema stage: Include transformations in more extensive mathematical settings.

Teacher C

- Action Stage: Introduce transformation acts with tangible examples and manipulative.

- Process Stage: Assist pupils in mentally visualising changes and comprehending the order in which they occur.

- Object Stage: Assist pupils in realising that transformations are entire entities with properties that remain constant.

- Schema Stage: Link ideas of transformation to more general mathematical settings.

Teacher D

- Action Stage: To introduce fundamental change acts, begin with practical exercises.

- Process Stage: Walk pupils through sequential steps and mental models.

- Object Stage: Assist pupils in realising that transformations are entire entities with distinct characteristics.

- Schema Stage: Make connections between transformation ideas and other mathematical ideas as well as with actual situations.

Teacher E

- Action Stage: To introduce fundamental change acts, begin with practical exercises.

- Process Stage: Assist pupils in mentally visualising changes and comprehending the order in which they occur.

- Object Stage: Assist pupils in realising that transformations are entire entities with distinct characteristics.

- Schema Stage: Promote using abstract ideas to address issues

At the Action stage, students engage in hands-on activities to understand basic actions involved in transformations. In the Process stage, they progress to visualizing transformations mentally and understanding the step-by-step processes involved. The Object stage deepens their understanding of transformations as complete entities, while the Schema stage integrates their understanding into broader mathematical contexts. By guiding students through each stage of the APOS framework, teachers can help address misconceptions and promote deeper learning.

d) Reasons for Learners' Misconceptions in the Teaching and Learning of Geometrical Transformation

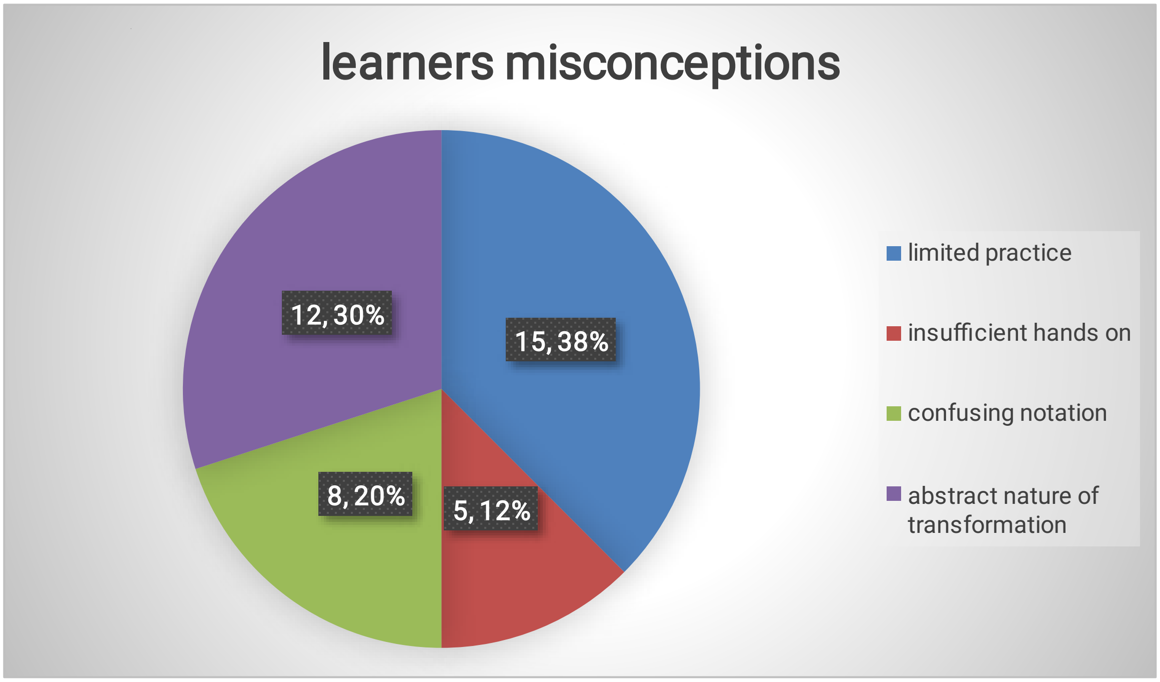


Figure. 3: Learners misconceptions

The respondents listed a number of factors that contributed to learners' misunderstandings, including as the abstract character of transformations, unclear terminology and notation, a lack of practical experience, poor step-by-step training, and little opportunity for practice and feedback.

Analysis of Student Interview Responses

Challenges

Table 3: Learner Challenges

CHALLENGE	NUMBER	PERCENTAGE
Visualizing transformation in a coordinate plane	14	70%
Confusion on relation vs. rotation	13	62%
Misinterpreting transformation notation	20	100%
Insufficient grasp of shape properties	6	24%
Lack of comprehension	16	80%

The answers from the learners were not all the same. Some thought the Action stage was the easiest since it was simple, while others liked the Practice, Object, or Schema phases because they involved more practice, a deeper knowledge, or connections. All of the stages' difficulties were mentioned: the Action stage's initial bewilderment, the Process stage's requirement for significant practice, the Object stage's complexity of general comprehension, and the Schema stage's difficulty in integrating concepts. Regarding how students perceive and approach learning geometric transformations (rotations, dilations, reflections, and translations). Diverse opinions regarding the issue were demonstrated by the responses, which ranged from finding it extremely interesting and delightful to finding it challenging and not very engaging. A few students brought up interactive tools, visual aids, step-by-step explanations (Action, Process, and Object), and links to previously taught topics (Schema), while others indicated no exposure to APOS theory. This shows variability in teaching methods.

Common Misconceptions Identified

I. Transformations Modify Dimensions

ii. Transformational Order

iii. Translation Effects

vi. Fixed Rotation Centre

Learning Geometrical Transformations

Using software or interactive tools, visualising transformations, practicing on paper or with models, and adhering to teacher demonstrations were some of the responses. This suggests different methods of first learning. Through group projects, online tests, textbook issues, and practical exercises, students gained experience. This illustrates a variety of approaches to strengthening comprehension. The ability to communicate it to others, apply it to various situations, visualise without the need for assistance, and solve challenging challenges were among the responses. These answers emphasise the signs of in-depth comprehension.

Recommendations

More visual and interactive tools, real-world examples, a variety of practice challenges, and group activities were among the suggestions. These suggestions are meant to improve comprehension and involvement. Students emphasised the necessity for varied and interesting tools by suggesting interactive software, manipulative, visual aids, and online quizzes.

.

Discussion

These elements play a part in creating misconceptions regarding transformations, such as thinking that a transformation changes its size or shape or that the transformations happen in a certain order. In order to dispel these myths, educators must offer students real-world examples, concise explanations, lots of practice chances, and continuous feedback to make sure they have mastered every stage of the learning process. According to the results, students have a difficult time comprehending geometric transformations, which are related to mathematical and geometrical reasoning. These difficulties may impair pupils' capacity for logical thought, problem-solving skills, and general mathematical ability. However, by systematically leading students through the learning process, the implementation of the APOS theory gives a structured strategy to overcome these issues. Through the integration of interactive exercises, visual aids, and practical scenarios, educators enable learners to surmount misunderstandings and enhance their comprehension of geometric transformation.

The replies from the students offer insightful information about their experiences applying the APOS theory and mastering geometrical transformations. The demographic data presents a heterogeneous student body with differing degrees of subject familiarity and interest. The answers demonstrate how students interact with each of the four stages of the APOS framework—Action, Process, Object, and Schema. Students' reactions to geometrical transformations revealed a range of attitudes and learning styles, from enjoyment to difficulty.

Different teachers used different teaching strategies and applied APOS theory in their lessons. Some students received interactive tools and step-by-step explanations, while others did not.

Students' descriptions of their various methods for learning and applying transformations highlight the value of a variety of teaching pedagogies. All phases of the APOS framework were found to present challenges, highlighting the necessity of individualised guidance to assist students in moving forward. Students' suggestions indicate that adding additional interactive resources, real-world examples, and group projects can enhance instruction. They also emphasised how crucial it is to have a variety of resources to promote comprehension.

Overall, the analysis highlights how crucial it is to teach geometrical transformations using a variety of methods, including the APOS theory, interactive resources, and real-world applications, in order to improve student comprehension and engagement.

Learners face significant challenges in understanding geometrical transformations due to their abstract nature, difficulties in spatial reasoning, challenges with spatial understanding, and the intricacy of relating geometric operations to symbolic representations. These problems stem from basic characteristics of geometrical and mathematical thinking, which both call for a high level of abstraction, reasoning, and visualisation. Inadequate visualisation abilities, inconsistencies with past knowledge, and occasionally inefficient teaching strategies are the causes of misconceptions. By guiding students through phases from actual actions to abstract comprehension, the APOS theory provides an organised method to address these mistakes and aids in their internalisation and integration of geometric transformations into larger mathematical frameworks. Good teaching techniques should help students visualise concepts more clearly, provide them hands-on experiences, and help them build a thorough mental knowledge of geometric transformations. With the aid of this thorough method, students can get past misconceptions and develop a deeper comprehension of geometric transformations.

4.3 Chapter Summary

The research study has revealed that visualisation, misinterpretation and lack of shape knowledge as well as the learner's negative attitudes are a challenge to the application of the APOS theory in learning Mathematics at ordinary level. The next chapter will present summary, conclusions and recommendations of the study.

Chapter 5

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

This chapter examines how geometrical transformations are taught at the Ordinary Level of mathematics using the APOS (Action, Process, Object, and Schema) theory. Fundamental ideas in geometry, such as translations, rotations, reflections, and dilations, must be taught to pupils in an efficient manner in order for their mathematical development to progress. The APOS theory offers a framework for comprehending how students pick up mathematical ideas and can direct teaching techniques to improve student performance.

5.1.1 Application of APOS Theory in Teaching Geometrical Transformations

Stage of Action: During this phase, students work through practical exercises to comprehend the fundamental movements involved in geometric transformations. Teachers illustrate translations, rotations, reflections, and dilations using tangible manipulative like graph paper and real models.
- Example: To illustrate translations, students physically move objects on a grid. To comprehend rotations, they rotate shapes with a compass.

Process Stage: During this phase, learners advance to mentally visualise changes and comprehend the sequential procedures required. At this point, interactive tools and dynamic geometry software are useful resources because they let students alter shapes digitally and observe transformations in action.
- As an example, students rotate and reflect shapes using software, noting how the shapes alter in position and orientation.

Object stage: Students gain a more thorough comprehension of transformations as whole entities at this phase. Without the use of physical aids, they are able to visualise changes and understand the invariant features of transformed shapes.
As an illustration, students who are well-versed in the transformation process may describe and mentally picture the result of a change without actually doing it.
Schema stage: During this phase, students use their knowledge of geometric transformations to more extensive mathematical situations. They build links between

coordinate geometry and algebra, solve problems in the real world, and hone their problem-solving abilities.

For example, students examine geometric transformations in map navigation systems or architectural plans and use what they learn to solve real-world issues.

5.2 Summary of the Study

This study looked into how the APOS theory might be used to teach geometric transformations to maths students at the ordinary level. Students gained a stronger knowledge of geometrical transformations as they moved through the action, process, object, and schema stages of the APOS framework. Every level of the APOS framework was covered through practical exercises, visual aids, interactive tools, and real-world applications that helped to support learning. Geometrical transformations create a number of obstacles for learners because of their abstract character, issues with spatial reasoning, and intricate symbolic representations. The core ideas of geometrical and mathematical reasoning underlie these difficulties. By leading students through phases from actual actions to abstract comprehension and eventually assisting them in integrating transformations into a more comprehensive mathematical framework, the APOS theory provides an organised method for addressing misconceptions. Inadequate visualisation abilities, inconsistencies with past knowledge, and occasionally inefficient teaching strategies are the causes of misconceptions. Good teaching techniques should help students visualise concepts more clearly, provide them hands-on experiences, and help them build a thorough mental knowledge of geometric transformations.

5.3 Conclusion

It has been shown that using the APOS theory to teach geometrical transformations improves students' comprehension and skill level with this mathematical idea. Through the action, process, object, and schema stages, teachers may help students get a deeper knowledge of concepts and learn more effectively. Improved learning results result from the use of a variety of instructional tactics, such as practical exercises, visual aids, and real-world applications, which meet the requirements and preferences of different learners. Educators can apply this theory by providing hands-on activities for concrete exploration. Encouraging mental visualization and discussion of processes. Guiding students from concrete actions to abstract thinking. Incorporating transformations into more extensive applications of mathematics. Due to the abstract character of the concepts, problems with spatial reasoning, and the difficulty of making the connection between symbolic representations and geometric operations, learners have substantial difficulties while attempting to understand geometrical transformations. These problems are fundamentally based in geometrical and mathematical thinking since they both call for a high level of abstraction, reasoning, and visualisation. Inadequate visualisation abilities, inconsistencies with past knowledge, and occasionally inefficient teaching strategies are the causes of misconceptions. By guiding students through an organised path from actual actions to abstract knowledge, the APOS theory can assist solve these fallacies and promote deeper conceptual comprehension and integration into larger mathematical schemas. Enhancing visualisation abilities, offering tangible experiences, and assisting students in internalising and abstracting geometric changes are all important components of effective teaching methods.

5.4 RECOMMENDATIONS

Based on the study's findings, the following suggestions for teachers were put forth:

• Use interactive resources and practical exercises to get students interested in the action stage of learning geometric transformations.

• Make use of visual aids and dynamic geometry tools to make the process stage transformations easier to see.

• Give students the chance to apply transformation ideas to practical issues, establishing links with other mathematical topics.

• Provide support and instruction that is differentiated to meet the needs and preferences of different learners.

• Regularly evaluate students' comprehension of the APOS framework at each level and give timely comments to help with learning.

By putting these suggestions into practice, teachers can make the most of the APOS theory in order to teach geometric transformations to students at the Ordinary Level in mathematics and encourage deeper learning.

REFERENCES

Alkhateeb, F., & Qaryouti, M. (2014). Challenges in Learning Transformations: The Case of Jordanian Eighth Grade Students. Journal of Mathematical Sciences & Mathematics Education.

Dubinsky, E., & McDonald, M. (2001). APOS: A constructivist theory of learning in undergraduate mathematics education research. In D. Holton (Ed.), the teaching and learning of mathematics at university level: An ICMI study .Kluwer Academic Publishers.

Dreyfus, T.Hershkowitz, R., & Schwarz, B. B. (2013). The role of definitions in mathematical and pedagogical discourses. Educational Studies in Mathematics.

Elrand, K. & Pratt, D. (2013). Using the APOS framework to assess learning and teaching of proof at the tertiary level. ZDM Mathematics Education.

Kaur, J. & Parveen, M. (2018). The challenges of learning geometrical transformations. Journal on Mathematics Education

Kilpatrick, J. (2013). What does it mean to understand mathematics? The Handbook of Mathematics Education.

Lappan, G. Fey et al (2015). Curriculum and evaluation standards for school mathematics. In W. J. Masalski & P. C. Elliott (Eds.), Reform in school mathematics and authentic assessment.

Ormrod, J. E. (2016). Essentials of Educational Psychology: Big Ideas to Guide Effective Teaching (4th Ed.). Pearson.

Stewart, J. (2008). Calculus: Concepts and Contexts (4th Ed.). Brooks/Cole.

SUNY Press.Liang, Y. & Qi, C. (2016). Research on misconceptions in learning transformation geometry and their resolutions. Journal of Educational Science.

Seago, N. Chamberlin, S., Driscoll, M., Hudson, R., Marinas, J., Moyer, J., & Williams, J. (2011). Defining and assessing geometric knowledge for teaching. The Mathematics Enthusiast, 8(1&2).

Zhang, D. & Zhang, Q. (2015). Application of APOS theory in research on mathematics education. Educational Research and Experiment.

All communications should be addressed to
"The Provincial Education Director"
Tele-Fax: 67574
E-mail: matnorth12@gmail.com

ZIMBABWE

Ministry of Primary and Secondary Education
Matabeleland North Province
P O Box 555
Bulawayo
Zimbabwe

17 May 2024

Nechilibi High school
P.O. Box 21
Dete
Hwange

REQUEST FOR PERMISSION TO CARRY OUT A RESEARCH PROJECT AT NECHILIBI HIGH SCHOOL: HWANGE DISTRICT MATABELELAND NORTH PROVINCE: SIBANDA PRIVILEDGE STUDENT NO. B225501B

Reference is made to your letter dated 07 May 2024, requesting for permission to enable you to carry out a research project on **"Mathematics"** Hwange District Matabeleland North Province.

You are hereby granted permission to carry out your research at Nechilibi High School. However, your research should not in any way disturb the smooth running of teaching and learning activities in schools.

You will be required to furnish the Province with a copy of your findings after the research.

N.B: Before proceeding into schools, please ensure that you pass through the District Education Office - Hwange

DEP. PROV. EDUC. DIR (SECNFE)
MIN. OF PRY AND SEC. EDUC
(MAT. NORTH PROVINCE)
17 MAY 2024

Mathe P . (Mr)
A/Provincial Education Director - Matabeleland North.
CC - District Schools Inspector - Hwange

Schools Insp.-(Hwange North Circuit)
MIN. OF PRY. & SEC. EDUCATION
HWANGE DISTRICT OFFICE
24 MAY 2024
P.O. BOX 230, HWANGE
ZIMBABWE TEL: 0281-32368

Teacher Questionnaire

I Sibanda Priviledge, a Bindura University Student carrying a research on integration of digital technology in Mathematics. Your input will help me understand the challenges and opportunities in the application of the APOS theory in teaching geometrical transformation in Mathematics education to O' Level students in Dete cluster. Please answer the following questions to the best of your ability.

a) Demographic Information

1. Name: [Optional]…………………………………………………………………………….

2. School Name: [Optional]………………………………………………………………….

3. Years of Teaching Experience:……………………………………………………………

Less than 5 years ………… 5-10 years

11-15 years

b) Challenges faced by learners in the teaching and learning of geometrical transformation.

1. What are the most common difficulties your students encounter when learning geometrical transformations?...

2. How do these challenges typically manifest in classroom activities or assessments?

3. What strategies or resources do you think could help address these challenges

effectively?

C) **Relationship of challenges to Mathematical thinking and geometrical thinking.**

1. How do difficulties in understanding geometrical transformations affect students' overall mathematical thinking skills?

2. Can you provide examples where challenges in geometrical thinking hinder students' problem-solving abilities in geometry?

3. How do you integrate the development of both mathematical thinking and geometrical thinking in your lessons?

d) Application of APOS Theory to address the learners' misconception on geometrical transformation.

1. How have you implemented the APOS theory in your teaching of geometrical transformations?...

..

2. Which specific aspects of the APOS theory do you find most effective in addressing students' misconceptions?..

3. Can you share any examples or case studies where the APOS theory helped improve students' understanding of geometrical transformations?..

e) **Learners' misconceptions in the teaching and learning of geometrical transformation.**

1. What are the most frequent misconceptions your students have about geometrical transformations?...

2. How do you identify and address these misconceptions during your lessons?...

3. What strategies do you use to prevent the formation of misconceptions in your teaching practice?...

....

Student Questionnaire

Thank you for taking the time to participate in this survey. Your feedback will help me understand your experiences and perceptions regarding the application of the APOS theory in teaching geometrical transformation in Mathematics education to O' Level students in Dete cluster. Please answer the following questions honestly.

a) Challenges faced by learners in the teaching and learning of geometrical transformation.

1. How familiar are you with the APOS theory?

2. What do you find most challenging about learning geometrical transformations?

3. Are there specific topics or types of problems in geometrical transformations that you find particularly difficult?

b) **Relationship of challenges to Mathematical thinking and geometrical thinking.**

1. How do difficulties with geometrical transformations affect your understanding of other math topics?

2. Do you think improving your understanding of geometric concepts could help you in

other areas of math? How?

c) **Application of APOS Theory to address the learners' misconception on geometrical transformation.**

1. Have you tried learning geometrical transformations by breaking them into smaller steps (like actions, processes, and understanding the whole object)? Did this approach help you?

2. Do you think practicing each step of a geometrical transformation can help you understand it better? Why or why not?

d) Learners' misconceptions in the teaching and learning of geometrical transformation.

1. What parts of geometrical transformations do you often get confused about or misunderstand?

2. Why do you think some concepts in geometrical transformations are easy to get wrong? What could help make them clearer for you?

Interview for Teachers

Demographic Information

1. Name(optional)……………………………………………………………………………

2. Age:……………………………………………………………………………………….

3. Gender:…………………………………………………………………………………….

4. Years of teaching experience:…………………………………………………………………………………

5. Educational background:………………………………………………………………..

6. How familiar are you with the APOS theory in the context of teaching mathematics?

Very familiar   Somewhat familiar   Not familiar

7. Have you received any professional development or training on APOS theory? If yes, please describe the training.

8. Describe how you introduce the concept of geometrical transformations (translations, rotations, reflections, and dilations) to your students.

9. How do you incorporate the APOS framework in your teaching of geometrical transformations? Please provide specific examples.

10. Can you describe an instance where you guided students from the 'Action' stage to the 'Process' stage while teaching a geometrical transformation?

11. How do you help students transition from understanding geometrical transformations as processes to perceiving them as objects?

12. What strategies do you use to develop students' overall schema for geometrical transformations?

13. How do you assess students' understanding at each stage of the APOS framework?

Challenges

14. In your experience, what are the benefits of using the APOS theory in teaching geometrical transformations?

15. What challenges have you encountered when applying the APOS theory in your teaching?

16. How do your students generally respond to the APOS-based teaching methods for geometrical transformations?

17. In what ways do you believe the APOS framework impacts students' overall understanding and performance in geometry?

Recommendations

18. What recommendations would you provide for other teachers who are considering using the APOS theory in their teaching?

19. Are there any resources or support systems that you think would help in better implementing APOS theory in teaching geometrical transformations?

Interview for Students

General Information

1. Name(optional)

2. Age:

3. Gender:

4. Current grade level:

5. How long have you been studying geometrical transformations?

6. How do you feel about learning geometrical transformations (translations, rotations, reflections, and dilations)?

7. Have your teachers mentioned the APOS theory or used a particular method for teaching geometrical transformations? If yes, can you describe it?

8. Can you describe what you do when you first learn a new type of geometrical transformation (Action stage)?

9. How do you practice and become more familiar with geometrical transformations (Process stage)?

10. When do you feel you truly understand a geometrical transformation as a whole concept (Object stage)?

11. How do you connect different geometrical transformations to each other and other areas of mathematics (Schema stage)?

Challenges

12. Which stage of learning geometrical transformations do you find the easiest? Why?

13. Which stage do you find the most challenging? Why?

14. How do your teachers help you move from one stage of understanding to the next?

15. Can you give an example of a geometrical transformation that you found difficult at first but understand well now? How did you reach that understanding?

RECOMMENDATIONS

16. How do you think the teaching methods for geometrical transformations can be improved?

17. Are there any tools, resources, or activities that you think would help you better understand geometrical transformations?

18. How do you feel about the overall approach your teacher uses to teach geometrical transformations? What do you like or dislike about it?

19. What advice would you give to other students who are learning geometrical transformations using the APOS theory?

THANK YOU

LESSON OBSERVATION GUIDE

Application of the APOS theory in teaching geometrical transformation in Mathematics

Observer: ____________________________ Date: _______________________________

Class_________________________ Teacher: _____________________________

Introduction

Identify the purpose of the lesson observation.

-Confirm with the teacher the objectives and tools to be used during the lesson.

Observation Focus Areas

1. Instructional Content

Is the APOS theory being effectively applied to support the teaching of geometrical transformation in Mathematics.

How does the teacher apply the theory (action, process, object and schema) to enhance student understanding?

2. Student Engagement

Observe the level of student engagement during learning activities.

Note any instances of active participation, collaboration, or disengagement.

3. Theory Application

Assess the teacher's proficiency in using the APOS theory.

Note any technical issues or challenges encountered during the lesson.

4. Differentiation and Personalization

Identify strategies used to cater to diverse learning needs and preferences.

Note any attempts to personalize learning experiences using the theory.

Reflection and Feedback

Discuss observations with the teacher immediately after the lesson.

Provide constructive feedback on strengths and areas for improvement in integrating digital technology for Mathematics instruction.

Collaborate with the teacher to brainstorm ideas for enhancing future lessons through effective technology integration.

Conclusion

Summarize key observations and insights from the lesson.

Express appreciation to the teacher for the opportunity to observe the class.