BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE EDUCATION

DEPARTMENT OF EDUCATIONAL FOUNDATIONS

AN INVESTIGATION ON THE ERRORS AND MISCONCEPTIONS MADE BY LEARNERS IN ADDITION AND SUBTRACTION OF DIRECTED NUMBERS AT SEKE 2 HIGH SCHOOL

BY

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A RESEARCH PROJECT SUBMITTED TO THE BINDURA UNIVERSITY OF SCIENCE EDUCATION IN PARTIAL FULFILMENT OF THE REQUIREMENTS OF THE HONOURS BACHELOR OF SCIENCE EDUCATION, MATHEMATICS

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DEDICATION

My research is dedicated to my family members who gave me unconditional love and support throughout the program.

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ABSTRACT

The high failure rate in Mathematics at Ordinary Level is a cause for concern. The aim of this study was to investigate the challenges faced by form 1 students in learning directed numbers. The research was a case study of Seke 2 High School in Chitungwiza District in Harare Province. The study adopted the descriptive research design which affords the researcher the chance to use questionnaires for data collection to provide primary data. The target populations consisted of eight mathematics teachers and learners in form 1, all taken from Seke 2 high School. The sample group, which was accessed through the random sampling technique, consisted of thirty form 1 students. The collected data was presented through the effective use of frequency table and a pie chart before it was analyzed. The study established that directed numbers are a difficult topic which poses problems to both teachers and pupils in the mathematics syllabus. The most challenging aspects of the topic were those operations involving negative numbers. Key findings reveal that procedural errors, conceptual misunderstandings, and misapplications of mathematical properties are prevalent among learners. Specifically, students frequently misinterpret the rules for adding and subtracting negative numbers and struggle to visualize operations on a number line. The main cause of errors and misconceptions is superficial understanding, which was most probably due to teachers rushing to complete the extensive syllabus, and consequently, students resorted to memorizing rules because of surface understanding. Teaching episodes were found to lack multiple-representation, creativity, as well as cooperative learning and active learning. The study contributes to existing literature on mathematical errors and misconceptions, providing valuable insights for educators and curriculum developers.

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CHAPTER 1

1.1 INTRODUCTION

As students advance in their mathematical education, they must comprehend directed numbers. More complex mathematical ideas, such as algebra, calculus, and beyond, are based on operations employing directed numbers, especially addition and subtraction. Nonetheless, studies have repeatedly demonstrated that a large number of students have difficulty with these operations, which can result in recurring errors and misconceptions that can impair their confidence and mathematical ability.

The purpose of the study is to investigate the particular errors and misconceptions that students make when adding and subtracting directed numbers. By recognizing these issues, the study aims to offer knowledge that can guide better teaching practices and raise student achievement. Given the importance of directed numbers in mathematics, it is imperative that researchers, educators, and curriculum developers all comprehend the nature of these errors.

This chapter will outline the background of study, problem statement, significance of study, the research questions guiding the study and an overview of the organization of the research.

1.2 BACKGROUND OF STUDY

Directed numbers are a fundamental concept in mathematics which is foundational for various mathematical concepts and operations, serving as the building block for more advanced topics such as algebra, calculus and beyond and it is introduced to learners in early secondary education thus it is a critical component of mathematical education. Mastery of operations involving directed numbers particularly addition and subtraction, is essential for learners to succeed in their mathematical journey. However, research has consistently shown that learners struggle with operations involving directed numbers particularly addition and subtraction (Harnett, 2011:

Smith, 2015). These difficulties often stem from misconceptions and errors that hinder learners' understanding and application of directed number concepts.

Due to the encouragement of the integration of subjects such as the physical sciences, social sciences and mathematics by the Department of Education, one needs to have a good background of mathematics to do well in these subjects. STEM (Science, Technology, Engineering and Mathematics) Education requires the integration of science and mathematics to help in solving real world problems (Bashan and Marino, 2013). For example in science there are problems that one requires to apply mathematical knowledge to solve them. In such cases, learners may be able to work out the scientific part but when it comes to the mathematical part, they fail to manipulate the calculations especially those that involve directed numbers.

Directed numbers is one of the topics that make up the foundational mathematics background knowledge required for algebra and advanced mathematics. Since nearly all computations require directed numbers, it is a topic that, if not adequately grasped at the form 1 level, may lead to subpar performance in mathematics and other courses as well. Negative numbers are thought of as an extension of natural numbers. They can be used to indicate credits in bookkeeping and to depict values on a scale that go below zero, like temperature. Using both positive and negative numbers is part of bookkeeping. Red numbers indicate amounts owed, or a number in parenthesis can be used as an alternate notation for negative values.

Makara and Tau (2019) as well as Makonye and Fakude (2016), supported that directed numbers are useful for temperatures less than zero, and owing in the bank is designated by the negative numbers. The directed numbers lay foundation for most of the topics in mathematics, for example, algebraic expression, sequences, vectors, matrices, transformation and equations, to mention but just a few.

The researcher also attended several workshops with teachers and seminars with learners where issues on how to improve pass rates at Ordinary level were being discussed and one of the issues mentioned was the issue of directed numbers. According to records of analysis of math test and exam scripts conducted by mathematics departmental heads in schools, students have trouble manipulating directed numbers. The pass rate for mathematics in schools excluding private candidates has remained at slightly below 20 percent (https://www.chronicle.co.zw). The report read that mathematics is arguably the most dreaded subject in the country with statistics showing that less than a fifth of candidates manage to pass it. The statistics recorded were as in 2019; 19.23 percent of public candidates passed O- level mathematics and the figure went down to 18, 21 in 2020 and slightly up to 18.71 percent in 2021. In 2022; 18, 03 percent passed mathematics.

Failing to pass mathematics has reduced the chances of many as they are deemed unqualified to venture into careers in Sciences. Since accounts need the use of both positive and negative numbers, the majority of firms are worried that the matriculates they hire for their business sectors would have difficulty with accounting calculations (Makonye & Fakude, 2016). The researcher's experience has shown that it is difficult for an ordinary level mathematics learner who fails to grasp the concept of directed numbers at Zimbabwe Junior Certificate Level (ZJC) to pass at Ordinary Level since the topic of directed numbers appears in almost every topic in mathematics. The topic of directed numbers is the topic that constitutes fundamental mathematics background knowledge needed in algebra and higher mathematics hence the study to investigate on the errors and misconceptions made by learners in addition and subtraction of directed numbers.

1.3 STATEMENT OF THE PROBLEM

Directed numbers serve as the foundation for the majority of mathematical disciplines, and mathematics is a spiral in nature. As a result, it is necessary to be understood by the students and to be effectively presented to them. The success of algebra is based on the directed numbers (Khalid & Embong, 2020). It has been observed that the majority of students fail mathematics at ordinary level not that because the subject is hard, but rather because they are unable to understand the concept of directed numbers at Z.J.C., which is present in practically every topic. Masers (2008) states that negative numbers darken the whole doctrine of equations and make dark of the things which are their nature extensively obvious and simple. Therefore, it is necessary to investigate the mistakes and misunderstandings made when working with directed numbers, particularly addition and subtraction, since if students are unable to comprehend directed numbers at a lower level, it will impact their comprehension and performance in other areas of mathematics, ultimately resulting in failure of the subject.

In order to improve instructional tactics that improve learners' conceptual comprehension and decrease errors in directed number arithmetic operations, this research attempts to uncover these problems.

1.4 RESEARCH OBJECTIVES

By the end of the research, the researcher should be able to:

1. Identify the errors and misconceptions made by form 1learners when adding and subtracting directed numbers.
2. Identify the causes of the errors and misconceptions.
3. Identify effective strategies in teaching and learning of directed numbers which help learners to grasp the concept.

1.5 RESEARCH QUESTIONS

The following research questions serve as the study's framework for direction:

1. What are the types of errors and misconceptions that the learners have in the addition and subtraction of directed numbers in form 1?
2. What explanations may account for those errors and misconceptions?
3. What are the most effective strategies for teaching addition and subtraction of directed numbers?

1.6 SIGNIFICANCE OF STUDY

The study seeks to establish the errors and misconceptions that learners make in the mastery of concepts in the topic of directed numbers and find effective and everlasting solutions. The study will help the teachers to plan and deliver lessons that clearly bring out to learners the understanding of the operations of directed numbers particularly addition and subtraction. The study will help in the mastery of the topic of directed numbers that will be potential benefit to the pupils as the concept eases complex multiplication, division, addition and subtraction throughout the study of mathematics. The nation will benefit from this study as the concept of directed numbers is useful in Science and Technology. The study will help teachers to identify grey areas in teaching and learning of directed numbers and strategies to clear those grey areas. The study will help curriculum planners and teachers to provide strategies to improve the teaching and learning of mathematics in secondary schools. The findings can also inform the refinement of mathematics curricula, ensuring that directed numbers are taught effectively. This research will also help the researcher to apply appropriate teaching methods when teaching directed numbers so that learners will understand the concept.

1.7 DELIMITATIONS

The research is limited to form 1students at Seke 2 high in Chitungwiza only limiting the findings and applicability to other age groups. The researcher chose Seke 2 High School for the research because the school was convenient to carry out the research since the teacher was working at that school by the time of research. So learners and teachers were likely to provide true and useful information since the researcher was familiar to them and the proximity of the school would easily allow the researcher to get the information needed from the research participants who were teachers and learners of Seke 2 High School.

The study is also limited directed number operations which are addition and subtraction only again limiting the applicability to other operations.

1.8 LIMITATIONS

This study has several limitations. Firstly the sample size is limited to thirty participants which may not be representative of the larger population. Additionally the study is relying on learners' self-reported data which may be subject to biases which may not accurately reflect their actual understanding. The researcher is also self-sponsoring on any costs incurred like photocopying and printing of worksheets and questionnaires which may limit the results of the research as there are insufficient resources. Time frame also limits the generalizability of results of the research as the learners have other topics to do and not only directed numbers.

1.9 DEFINITION OF TERMS

Terms to be defined are directed numbers, misconceptions, errors, teaching and learning.

Directed numbers are defined as integers that can be negative as well as positive. (https://maths.olympiadsuccess.com/definitions/directed-numbers). The Merriam-Webster dictionary defines a directed number as a number that is preceded by a plus or minus sign.

Directed numbers are numbers with both size and direction of which one direction is positive and the other is negative.

Misconceptions are views or opinions that are incorrect because they are based on faulty thinking or misunderstanding. Misconceptions are unavoidable stages in the learning process (Moru & Mathunya 2022). According to Makonye (2012), misconceptions are the underlying wrong beliefs and principles in one's mind that causes a series of errors.

The dictionary.com defines an error as a deviation from accuracy or correctness that is a mistake. An error is an inaccuracy (Luneta and Makonye 2010).

MacMillan (2002) defines learning as the process of gaining knowledge and experience, for example, by studying. Learning is the process by which individuals acquire new knowledge, skills, behaviors or values, not just memorizing or remembering facts but understanding new concepts, developing skills and changing the way we think, feel or behave (Pathak 2024)

According to Grix (2004), teaching is the process of imparting knowledge, skills and attitudes to others. It is a process that facilitates learning. Teaching is the process of sharing knowledge, skills or information with others to help them learn and understand new things which involves a teacher who guides learners through lessons, activities and discussions.

1.10 CHAPTER SUMMARY

This chapter highlighted the background of the study, statement of the problem, research objectives and significance of the study, delimitations, limitations and definition of key terms. Directed numbers are a critical concept in mathematics and real world applications. Learners struggle with addition and subtraction of directed numbers leading to errors and misconceptions. Existing research has gaps in understanding errors and misconceptions in directed number operations. This study aims to investigate the errors and misconceptions made by learners in

addition and subtraction of directed numbers. The study's findings will inform teaching strategies, interventions and learning resources.

CHAPTER 2

2.1 INTRODUCTION

This chapter seeks to review existing literature on errors and misconceptions made by learners when adding and subtracting directed numbers. The first part of this chapter looks at the theoretical frameworks which underpin this study. In this case, the underlying theory is the Constructivist. The second part is concerned with review of related literature.

2.2 THEORETICAL FRAMEWORK

The researcher selected Constructivist theory as the theoretical framework underpinning this study.

2.2.1 CONSTRUCTIVIST TEACHING

Constructivism is "an approach to learning that holds that people actively construct or make their own knowledge and that reality is determined by the experiences of the learner" (Elliott, Kratochwill, Littlefield, & Travers 2000). Mascolo and Fischer (2005) defines constructivism as a learning theory which holds that knowledge is best gained through a process of reflection and active construction in the mind. Constructivism is a theory of learning or meaning-making that enables people to create their own new understanding based on interplay between what they already know and believe and the concepts and knowledge with which they interact (Resnick & Glaser, 2016). A celebrated idea in education, constructivism has been around for a long time.

With constructivist math instruction, students can go beyond rote memorization, gain context that helps them understand the material, and take charge of their education as active learners rather than passive observers. Constructivism's central idea is that human knowledge is constructed, and learners build new knowledge upon the foundation of previous learning. This view of learning sharply contrasts with one in which learning is the passive transmission of information from one individual to another, a view in which reception, not construction is the

key. With constructivism, it is clear that learners are not just the uncritical consumers of knowledge but active makers of knowledge and critical users of knowledge. Contrary to the teacher-centered instructions, constructivism advocates student-centered learning in which the teacher serves as a facilitator of the learning process. The teacher is there to help learners become active participants in their learning and to make meaningful connections between prior knowledge, new knowledge and the learning processes (Tangney, 2014). Constructivist education is effective and benefits students' abilities for the rest of their lives.

Cognitive constructivism, social constructivism, and radical constructivism are the three schools of constructivist education. In a constructivist view, meaning is constructed as children interact in meaningful ways with the world around them. Therefore, cognitive constructivism attaches importance to whole activities as opposed to isolated skill exercises, authentic activities which are inherently interesting and meaningful to the student and real activities that result in something other than a grade on a test. Cognitive constructivism, developed by a young Swiss psychologist Jean Piaget, views learning as an active process where knowledge is constructed by each individual through reflection, exploration, experimentation, problem solving and critical thinking. In this view the teacher's role is to guide students through the problem-solving process, while allowing them to use their own mental capacities to find solutions. Social constructivism, a theory developed by Soviet psychologist Lev Vygotsky, is the view that learning occurs through social interaction and the help of others, often in a group (Lohman & Hurst, 2023). Social constructivism posits that the understanding an individual develops is shaped through social interaction. Radical constructivism is a theory of teaching and learning that encourages students to construct knowledge and build on a conceptual understanding of mathematical knowledge. This theory encompasses the philosophy that knowledge is subjective and that every

student constructs their own knowledge based on personal experiences. Although these schools of thought differ, they are all based on the following fundamental ideas.

Knowledge is constructed. Using our prior knowledge, we build new knowledge by drawing comparisons and making connections. In math this means we use foundational concepts to build the next one (e.g., we learn to add by counting and multiply by adding repeatedly). To learn, students must be part of the process (e.g., problem-solving, productive struggle, debate, etc.) in order to understand new content. Students can receive new information passively (e.g., being lectured to), but this pales in comparison to understanding it through direct, active participation. By active participation in problem solving and critical thinking, students are constructing their own knowledge by testing ideas and approaches based on their prior knowledge and experience, applying them to a new situation and integrating the new knowledge gained with pre-existing intellectual constructs because they find the learning activity is relevant and engaging We learn best by doing something together and interacting with it socially. Our social behaviors and habits influence how we think about what we learn. Knowledge is personal. Subjectivity of experience dictates that we each will differ in how we learn and comprehend a given concept. Another core principle is reality and the mind. Received knowledge is both socially tempered and subjective in nature as it is exclusively a process of the mind. We must agree that knowledge molds our perception of reality. As we learn more, we update our mental models of the world, thus changing how we see and interpret reality.

Teachers who take actions into consideration not only engage students with the content but tend to achieve higher academic results as well. This process takes time, though for significant learning, we need to revisit ideas, ponder them, try them out, play with them and use them. It also takes motivation. Teachers, who create interpersonal connections with students, elicit their

background knowledge, and tailor lessons to their interests are more successful constructivist teachers.

Through differentiated activities, such as group projects, games, and work with the teacher, teachers give their pupils learning opportunities. To illustrate a novel approach to problem-solving, a teacher might also incorporate a brief mini-lesson. To help students learn, think critically, and embrace constructive difficulty, the instructor lets them explore, experiment, and experience controlled failure in the classroom. The teacher should never just provide the solution and assume that the students have grasped the material. Rather, educators honor the variety of viewpoints, approaches, and inquiries that emerge from problem-solving.

The theory of constructivism (Meyer, 2009) is apparently supportive to relational understanding because it informs that in the learning process, for example, in the learning of addition and subtraction with negative numbers, giving ready-made rules will not be helpful, rather a learner must be provided with opportunities to be an active participant in the construction of his or her own understanding about addition and subtraction of directed numbers. The theory of constructivism holds that learning occurs as a result of the interaction of a child's existing ideas and new ideas, that is, new ideas are interpreted, organized, and understood in light of the child's own current knowledge. If learners are not given that opportunity to recreate ideas, it will mean for them merely using rules and formulae to fulfill the ritual of getting correct answers.

2.3 LITERATURE REVIEW

In this section the researcher will delve into the opinions of other researchers who have carried out prior studies on errors and misconceptions made by learners in addition and subtraction of directed numbers at form 1 level in secondary schools. By examining various studies, this review aims to identify common challenges faced by learners, the underlying causes of these difficulties

and potential instructional strategies to address them. This will assist in the better understanding of the errors and misconceptions and the best strategies to teach the topic and enhance student learning outcomes.

2.3.1 Errors made by learners and their sources

An error is a deviation from accuracy or correctness; a mistake. Mistakes are not consciously made, are usually once off and made by a few. Generally an error is something a person has done which is reflected to be wrong. A mathematical error is a mistake in mathematical calculation.

Misconceptions are views or opinions that are incorrect because they are based on faulty thinking or misunderstanding. A misconception happens when a student believes a false concept as a true concept. Misconceptions unlike errors are consciously made and are made by many, repeatedly.

Makonye and Fakunde found that 83.3% of students make logical mistakes when adding and subtracting directed numbers, 16.7% make procedural errors, 67% make strategic errors, and 28.6% make misconceptions. When learning to cope with directed numbers, the sources of the errors appeared to be a lack of reference to mediating objects like number lines or other actual contextual situations. As the first number concepts they were exposed to in school, learners appeared to be fixated on positive numbers and addition operation frames. Negative numbers and the subtraction procedure involving negative integers were difficult for them to handle. Poor English ability, which is the language used to teach and understand mathematics, appeared to be another obstacle.

Mathematical errors are significant in practice because they can be used to enhance teaching and learning. Fiori and Zuccherri (2005) found that certain error patterns depend on different ethnic-

cultural situations, teaching methods and algorithm used. Researchers analyzed several types of mathematical errors and categorized them as careless error, random error and systematic error. The Department for Education and Employment (1999) cited by Mukunthan (2021) published a diagnosing student's error as computational error/ careless mistake, misconceptions, wrong operation, over-generalization, under-generalization and random response.

In the study by Khalid and Embong (2020), students were given -6-(-2) and students gave their answer as 4. Makonye and Fakude (2016) reported that the students were asked to solve -7-(3) and the students gave their answer as 4. In both studies, the students ignored the negative sign involved. In a study by Dr Judah Makonye (2014), mathematical errors are important in the classroom because they are the contribution of learners to the learning process and teachers should welcome them instead as seeing them as undesirable. There are different types of errors that learners make when dealing with mathematics and these include systematic errors, constant errors, reading errors, comprehension errors, transformation errors, encoding errors and careless errors.

Some errors discovered in detail are:

1. Procedural errors

   These are mistakes made during the execution of mathematical procedures and these include confusing negative and positive signs, failure to consider the direction of the numbers and incorrectly applying the rules for adding and subtracting directed numbers. Kiat (2005) defines procedural errors as errors that arise while a learner carry out a procedure having grasped the main concepts behind the problem task. Causes of such errors are lack of practice or reinforcement, carelessness or attention deficits and inadequate instruction or guidance. According to Cresswell (201 2), poor understanding and misapplication of rules is one of the

challenges. In the case of integers or directed numbers, teachers prefer to provide students with rules to be memorized and then drill them with enough practice to make them stick to the rules and this might lead to poor understanding and misapplication of rules since students will get confused with so many rules. For example, 4 + (-2), they may get 6 arguing that 2 added to 4 is 6, yet there is a negative number which makes the operation to be subtraction instead of addition. According to Khalid (2018)'s findings students make errors or misconceptions when dealing with directed numbers and the errors were categorized into five categories namely carelessness, poor basic knowledge, inability to assimilate concepts of integers, rule mix-up due to rote learning such as memorizing rules that lead to mix-up and again surface understanding. Basic operations seem simple, yet according to Alsina and Nelsen (2006), the students tend to get confused and struggle when they are asked to solve mathematical problems. Lim (2011) adds that many students seem to depend on rote memorization in learning Mathematics and the teachers teach students using rules and procedures in order to get correct answers, hence neglect their conceptual understanding. Many teachers teach students for the sake of passing examinations instead of emphasizing on the understanding of the concepts. The researcher also viewed that many mathematics teaching in many schools in Malaysia can still be characterized by teacher- centeredness which lead to learners facing challenges in dealing with directed numbers

2. Conceptual errors

These are mistakes made due to poor problem solving strategies or inadequate planning. Ndemo (2023) noted that conceptual errors are similar to what Achterberg, Koch and Martin (2005) refers to as problem-solving related errors. These include inappropriate operation that is using addition instead of subtraction or vice versa, and incorrect ordering that is performing operations

out of order. Causes of such errors include lack of problem-solving experience and limited mathematical vocabulary. According to Kilpatrick (2008), a major challenge in students' learning is poor command of the English Language which is predominantly used during instruction. Brown, Cady and Taylor (2009) noted that mathematics has its terminology, which is different from English. If the language is not transparent, then the objective of learning is invisible. Students could not construct mathematical meaning in the realm of directed numbers if the medium of learning is not transparent. Mathematics is acquired through language-based activities, which involves listening, reading and discussions (Essien, 2018). The students who learn in another language are challenged by concepts than those who are instructed in their native language. According to Msimanga and Lelliott (2014), learning in a second language impose dual risk to the students who are learning the second language, which they are not even good at and using it to acquire the academic content. A number of studies were conducted on the importance of the use of native language in content delivery and acquisition (Essien, 2018; Msimanga & Lelliott, 2014; Taylor & Coetzee, 2013). In several other African countries, such as Eswatine, English is considered as a language of instruction.

3. Logical errors

These are mistakes in reasoning, thinking or decision-making processes which include sign errors (misunderstanding positive and negative signs) and operational errors (confusing addition and subtraction).Causes of such errors include inadequate conceptual understanding and insufficient practice with logical reasoning. Baroody and Dowker (2013) state that if learners do not participate by answering questions, posing questions, doing questions, discussion with others and using their creativity to solve directed number problems, they remain holding various mathematical errors and misconceptions. In the same way, teaching strategies that maximize

learners' participation in mathematics classes help learners to learn and resolve their errors and misconceptions. According to Geary (2011), mathematics is found affected by intelligence, working memory and processing speed. Murayama, Pekrun, Lichtenfeld and Vom Hofe (2013) found out that intelligence is strongly related to the achievement in starting stage but motivation and use of cognitive strategies predicted the growth of achievement. Some learners face the challenge that they are not intelligent enough to grasp the concepts.

2.4 Suggested teaching methods and solutions to the problem

Using models to teach directed numbers may aid students in developing conceptual and relational understanding (Ali, Haider, Munir, Khan & Ahmed, 2013). The use of number lines as a visual aid to aid pupils in comprehending the idea of directed numbers is supported by Van de Walle (2013). By graphically depicting movement in both directions, number lines can help make the addition and subtraction of positive and negative integers more understandable.

Based on the National Council of Teachers of Mathematics (NCTM), directed numbers must be connected to actual circumstances, including variations in temperature or financial settings (credit versus debit). With this method, abstract ideas become more relatable to students' experiences and tangible.

According to Kullberg (2010), some learners rush to use calculators, which can incline them to instrumental comprehension of directed numbers rather than relational understanding. Calculators can still be used wisely in explanatory contexts, but their usage should be postponed until students acquire a relational grasp of directed numbers.

Use of code-switching between the learners' mother tongue and English must be encouraged to overcome English Language barrier in the teaching and learning of directed numbers. Mathunya

(2022) agreed to the use of indigenous language as a strategy to minimize misconceptions in directed numbers.

In the opinion of Johnson and Johnson (1999), teachers should encourage students to learn collaboratively by assigning them to solve directed number problems in pairs or small groups. By encouraging conversation and giving students a platform to express their ideas, this exchange assists in eliminating misunderstandings. Interactive practice requires the use of educational tools and online platforms (Hattie 2009). To accommodate varied learning styles, technology can provide differentiated instruction and instant feedback.

2.5 RESEARCH GAPS

Although a variety of teaching techniques are proposed, not enough empirical research has been done to assess how well they work in varied classroom environments. To determine which approaches produce the best results for various learner demographics, more thorough research is required. The ways in which individual characteristics, such as learning impairments or giftedness, affect the comprehension of directed numbers require further investigation. There is much research being done on how to modify teaching methods to accommodate different demands.

2.6 CONCLUSION

The chapter has examined the typical mistakes and misunderstandings that students have while attempting to add and subtract directed numbers. It is evident from an analysis of procedural, conceptual, and logical errors that these difficulties can have a substantial effect on students' comprehension of mathematics. It was determined that a number of factors, such as the complexity of mathematical language and educational approaches, were the causes of the misconceptions. These observations emphasize how important it is for teachers to understand the

particular difficulties that students face so they may modify their instruction accordingly. As mentioned above, other authors suggested using visual aids, encouraging conceptual conversations, and applying real-world contexts to address these concepts. By using these strategies, teachers can assist students in developing a deeper comprehension of directed numbers, which will ultimately improve their general mathematical abilities.

CHAPTER 3

3.1 INTRODUCTION

This chapter outlines the research methodology employed to investigate errors and misconceptions made by learners in addition and subtraction of directed numbers. The study adopted a mixed-methods approach, combining quantitative and qualitative data collection and analysis methods. The chapter discusses the research design, including the selection of participants, sampling strategy and data collection instruments. It then describes the procedures for data collection and explains the data analysis procedures. It addresses issues of validity, reliability and ethical considerations in the research.

3.2 RESEARCH DESIGN

According to Kumar (2005), research design is a procedural plan that is adopted by the researcher so as to be able to answer questions validly, objectively, accurately and economically. Research design is a format and theoretical structure under which the study was carried out. This study is grounded in a constructivist paradigm, which posits that learners construct knowledge through their experiences and interactions with the world. This perspective emphasizes the importance of understanding students' thought processes and the meanings they attach to mathematical concepts. By employing a mixed methods approach, the researcher seeks to capture both statistical trends in student errors and the nuanced insights into the students' reasoning and understanding. The mixed-methods design combines quantitative and qualitative research methods to provide a richer understanding of the research problem. The quantitative component involves the written test to assess students' understanding of directed numbers, while the qualitative component involves teachers and students' interviews as well as the questionnaires to gain insights into the students thought processes and misconceptions.

A qualitative approach according to Thomas (2011) is a systematic subjective approach used to describe life experiences and to give them meaning. According to Makore-Rukuni (2001), qualitative research is a process of systematic inquiry into the meanings which people employ to make sense of their experiences and guide their actions. The goal of the qualitative method is to have a comprehensive understanding of human behavior. It looks into the how and why of making decisions. With a qualitative technique, the researcher can discover more about a phenomenon's depth, richness, and complexity. Reality is subjective, yet it is holistic. Since the research was conducted in a genuine social situation rather than one that was fabricated or artificial, the qualitative research approach was more suitable in this case. Human behavior according to Vaughn, Bos and Schumm (2007) is seen as a contextual bound that is, there are social, historical, political and cultural antecedence that explain human behavior. The qualitative approach is a multi-method approach to capture the diversity and complexity of ideas and situations. Bordens and Abbort (2007) say that on data collection procedures the research relies heavily on field work method, in which both the researcher and the participants interact to gather the lived world experiences of participants. The sample used in this study was randomly selected from the target population and was meant to be a representative of the population. Barood and Dowker (2013) suggested that the quality of a sample should have the same characteristics as the population it represents so that the results of the information would be valid. Of the challenges in answering tasks on addition of directed numbers with no preconceived hypothesis to be tested, this approach was more preferable than the others because one would be able to describe systematic observation on the study (McMillan & Schumacher, 2014). After that, errors on written tasks on directed numbers were noted. Basing on those errors, in-depth interviews were held with the learners to further elicit and probe learners' thinking behind those errors (Mears,

2012). As this research was seeking to find the errors and misconceptions in addition of integers, in-depth interviews were viewed to be helpful to gain better insight into the categories of errors and misconceptions that the learners exhibited, and find out the causes thereof.

3.2.1 Population

Haralambos and Holborn (2008) describe a population as a group of elements, individuals, objects or events that conform to the specific criterion to which the researcher intends to generate his or her study. It refers to all people or items with the characteristic one wish to understand. There is rarely enough time and resources to gather information from everyone in a population, thus the goal becomes finding a representative sample or subject of that population. The target population refers to members, events or objects to which the researcher would generalize the study. The population of this research comprised secondary school teachers and learners who are currently learning about directed numbers at Seke 2 High School.

3.2.2 Sample

A sample according to Haralambos and Holborn (2008) is a part of a population under study and which has been selected to be a representative of the larger population. It is a small proportion of population chosen for observation, study and analysis. It is therefore a representative of a union for whose characteristics are of interest to the researcher. A sample is all about making a selection from the sampling frame in order to identify the people or issues to be included in the research. White (2005) also defines a sample as the subset of the population which is used to represent the whole population. Sampling is concerned with the selection of individuals from within a statistical population to estimate characteristics of the population.

3.2.2.1 Sampling techniques

The random sampling method was used in the study. The respondents were selected on the basis that they were mathematics teachers and learners who were in the area of the study. Two

mathematics teachers were considered out of the ten teachers in the mathematics department. To select the teachers, the researcher took note of the teachers who were in the session of form 1 learners as the school has hot sitting, and there is morning and afternoon session. The teachers in the session were four altogether including the researcher. The other teacher of the four was on sick leave thus the number ended up at two teachers being selected for the study. As for the learners, the researcher cut small pieces of papers which were written yes and no and they were placed in the hat. 30 of them were written yes and 50 were written no as the researcher used two out of nine classes of mixed ability at the school. Each learner was given chance to pick one paper from the hat with each learner having an equal chance of being selected. The learners who were considered for the study were the ones who picked a yes.

3.3 DATA COLLECTION METHODS

The sample consisted of 30 form 1learners (15 boys and 15 girls) of Seke 2 High School in Chitungwiza. Written tasks on addition and subtraction of directed numbers were given to the learners in the form of a test, and their exercise books were analyzed. The test was marked and errors noted. Learners who performed poorly in the written tasks were interviewed to get an understanding of the errors and misconceptions they exhibited in the tasks. Six learners were individually interviewed (three boys and three girls) in about twenty-five minutes by the researcher. The researcher wanted to understand learners' reasoning within and beyond their written responses on the tasks and wanted to provide the learners with an opportunity to say more than they wrote and also check whether the researcher had interpreted their responses correctly or the learners themselves had understood their own thinking. Questionnaires were given to the learners to respond to on top of the test written. All ethical protocols were strictly followed and adhered to.

The researcher consulted the two teachers on different days briefing them on the purpose of the

visit and seeking their permission. The researcher distributed questionnaires to the two teachers and learners and collected them at the end of the same day of distribution. Follow-up interviews were then conducted with two teachers who had earlier responded to questionnaires. This was a bid to gather information from them as there was room for probing.

3.4 ETHICAL CONSIDERATIONS

Certain ethical values were considered by the researcher in this study. Ethical issues considered include informed consent, anonymity of informants, avoidance of harm, respect of participants' privacy and confidentiality.

3.5 Validity and reliability

The researcher used a variety of data collection techniques, including questionnaires and interviews, to guarantee the validity and reliability of the instruments. Reliability and validity reduce mistakes. According to Middleton (2019) validity refers to how accurately a method measures what it is intended to measure. Reliability basically refers to how trustworthy or dependable the measurement is (Haralambos and Holborn 2008). It clarifies the extent to which a test reliably assesses the things it is supposed to assess. Reliability is about the consistency of a measure whereas validity is about accuracy of a measure. To ensure the validity and reliability of the tools used in this study, the interview questions were given to math teachers at the school.

3.6 RESEARCH INSTRUMENTS

Grey (2009) defines research instruments as tools used to collect data to find solutions to problems under investigation. These are the tools that the researcher used to find information. In this study, the following instruments were used; questionnaires, interviews, and test.

3.6.1 Questionnaires

Questionnaires were used to collect data. According to Grey (2009), a questionnaire is a research tool through which people are asked to respond to some set of questions in a predetermined

order. The researcher used questionnaires for mathematics teachers and learners in this study because:

(i) Respondents were likely to give true information, in a freeway and in privacy. The researcher had assured respondents of confidentiality.

(ii) Collected information would be easily tabled and represented in graphical form.

(iii) The researcher could get a general overview of attitudes and experience of the respondents since they were answering exactly the same questions.

Despite the advantages, questionnaires has its own weaknesses which include;

(i) the participants would need thorough monitoring since some respondents could copy each other, thereby giving inaccurate information on an individual's opinion.

(ii) If the instructions on the questionnaires are difficult to understand, the respondents could be confused and give wrong information.

3.6.2 Interviews

Interviews were done in order to probe learners and teachers of Seke 2 high school on the errors and misconceptions by learners in manipulating directed numbers. According to Mears (2012), in-depth interviews were held with learners to further elicit and probe learners thinking behind those errors identified on written tests on directed numbers which were noted. They were used as a follow-up to the mathematics teachers and learners who gave responses to the questionnaires. Researcher used interviews because;

(i) they created the opportunity for face to face meeting with respondents to get in- depth information relevant to the study.

(ii) They allowed high response rate and the method was flexible because if the questions were not understood the researcher could ask in a different way.

iii) Non-verbal responses could be noted through gestures, emotional changes and facial expressions.

Interviews also have weaknesses which were noted during its conduction;

(i) It took much time to conduct interviews, hence the process was tiresome.

(ii) It was difficult to record responses; hence the researcher could miss some important points.

(iii) In some examples, respondents could be reserved to give responses to certain questions.

3.6.3 Tests

The researcher also designed a test to compliment views given by students and teachers in the questionnaires given and interviews conducted. The test was given to 30learners and contained 8 questions. Addition and subtraction were the operations that were considered in the test.

The test had the following advantages;

(i) First-hand information on the learners' problems when dealing with directed numbers would be contained.

(ii) Tests would also present substantial evidence of pupils' understanding of subject matter.

(iii) The actual performance of individual learner would be identified from the test results.

(iv)The test was easy to administer.

The following weaknesses were noted during the test;

(i) Tests seemed not to be reliable since the results could be affected by illness and any other discomforts during the test like social problems from home.

(ii) Some pupils would not participate whole- heartedly, resulting in the researcher not to obtain true problems that learners face when writing the tests.

(iii) Some learners could also cheat, that is copying their neighbor peer's work

3.7 DATA ANALYSIS

Data analysis is the process of evaluating data using analytical and logical reasoning to examine each component of the data provided. Data was collected and then presented through the use of a table and a pie chart. It was reviewed and then analyzed.

3.8 CHAPTER SUMMARY

This chapter represents the methodology used for data collection. Qualitative approach was done by the researcher. Random sampling technique was used to make sure that the objectives of the study were achieved successfully. Population and sample of the study were also discussed. Confidentiality of respondents was also considered and respondents were respected and protected.

CHAPTER 4

4.1 INTRODUCTION

This chapter presents the findings derived from the research on errors and misconceptions made by form 1 learner in addition and subtraction of directed numbers. The three primary research questions are addressed. Data is analyzed and discussed. Data is presented in the verbatim form and pie chart.

4.2 DATA PRESENTATION, ANALYSIS AND DISCUSSION

Errors and misconceptions made by learners were explained in their types:

4.2.1 Common errors

I. Conceptual errors

Due to their difficulties with positive and negative signs, learners produced inaccurate figures. By applying the rules for adding and subtracting positive and negative numbers incorrectly, errors were made in identifying the result's right sign. On the specified test, an improper change in the result's sign was observed. For instance, given (–1) + (-3), the expected response was -4 but learners could just write 4 leaving the negative sign. Some errors made were just careless mistakes because one could see that the learner understood the concept but made a careless mistake in calculation. Learners also misinterpreted the number line and moved in the wrong direction when performing operations. In this case one could move to the right side of the number line because of the addition sign yet a positive and negative sign produces a negative sign which means one was supposed to move to the left side of the number line.

II. Procedural errors

These errors arise from incorrect application of rules for addition and subtraction of directed numbers. Learners failed to apply rules consistently. Rules were inconsistently applied for

combining like terms leading to erroneous results. Given (4) + (-2), learners could treat both numbers needing to be added, yet the 2 need to be subtracted as a negative and positive sign gives a negative sign resulting in the need for 2 to be subtracted. Instead of giving a 2 as the result, one could give a 6. Thus students forgot to change the sign when adding a negative number.

III. Logical errors

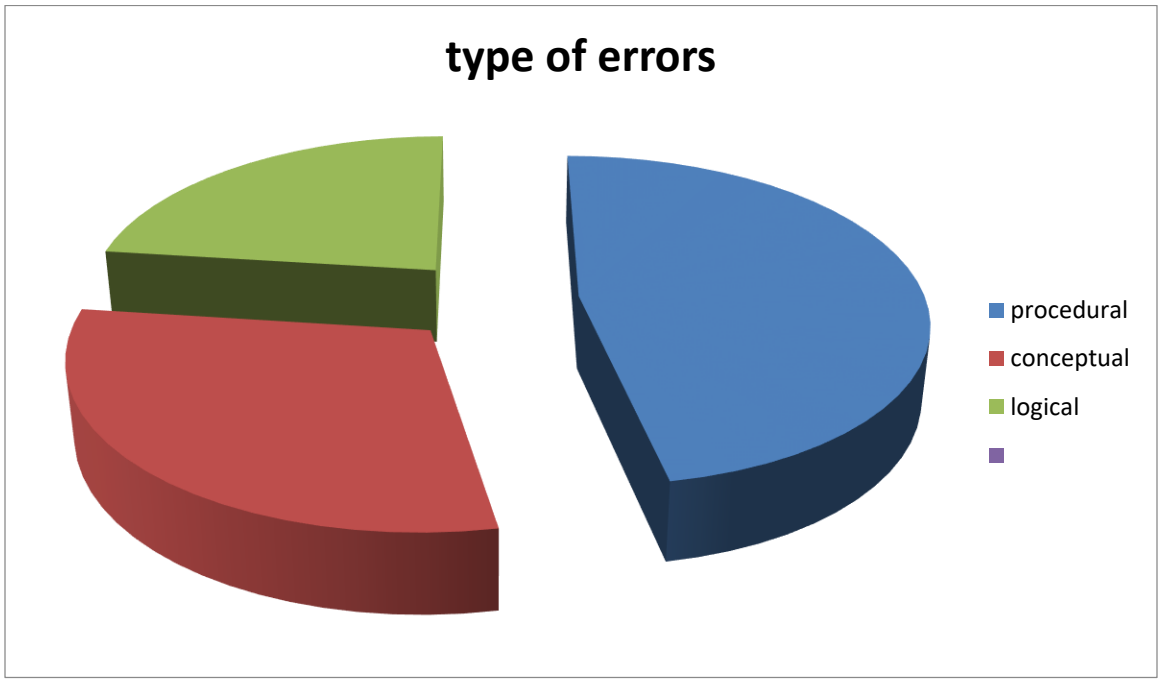
Learners struggled with word problems involving directed numbers. Learners exhibited difficulties in just explaining what needs to be done for example which direction does one move when adding a negative number, is it to the right/left side of the number line. Memory recall was also a challenge. The researcher noted that learners would forget fundamental rules such as two negatives make a positive for example in the given question (4) – (-2), some instead of writing 6, the answer was given as 2.

4.2.2 Misconceptions

The researcher noted from the student interviews that many students did not understand directed numbers as quantities having direction, such as temperature and elevation, but instead saw them as just arbitrary values. Confusion resulted from this misunderstanding in real-world situations. Additionally, students struggled to distinguish between addition and subtraction. Students failed to understand that subtracting a negative integer is the same as adding it. Expressions like4 – (-2) were miscalculated as a result of this mistake.

Data of errors noted was presented on a table below and on a pie chart.

Type of error	**Number of learners who made the error**	**Percentage**
Procedural	14	47%
Conceptual	9	30%
Logical	7	23%



4.3 DISCUSSION

From the data drawn, there is evidence that most errors made were procedural errors when computing directed numbers. All participants were learners in form 1. From the learners' questionnaires, almost three quarters of the learners responded that they were not comfortable with the use of positive and negative numbers and the researcher was inquisitive about this response and had to probe more for the reason in an interview with the learners. The reason from

the interview was that math is difficult and the signs of a number being negative and positive cause confusion. From the interview with the learners, the learners noted that despite mathematical and instructional challenges, learners just feel anxious about making mistakes when working with positive and negative numbers. They don't have confidence in themselves. Peer pressure also led to discomfort of the learners.

Considering specific instruction, learners did not put the blame on the teachers as their response to receiving specific instruction was a yes. However learners feel that the best instructional methods that help them understand directed numbers is more of group work with their peers rather than the demonstration method on the board and one learner is also asked to come and demonstrate to the whole class. Their comment to this was that they are afraid of trying in front of the whole class as they are ashamed that their peers will laugh at them if they fail. The learners' feeling on how teachers can do differently to help learners avoid errors is through individual assistance and more extra work on the operations. Another response was that the rules of addition and subtraction are too much for them as at times they seem to contradict themselves. For example when we subtract we move to the left side of the number line but when one subtract a negative number if you move to the left hand side the answer becomes wrong. When asked what strategies they use to add or subtract directed numbers, others responded that they use their fingers; others said they use a calculator and others responded that they use their head that is mental work or use the number line but a query arose from the use of visual representations like the number line. Learners responded that the number line helps in understanding the topic and the operations of addition and subtraction as it clearly shows how one moves when adding or subtracting whether to the left or right but the challenges comes when the operations involves bigger numbers, the number line fail to accommodate them.

On the questionnaire, learners defined a positive number as numbers that we use in everyday life and a negative number as a number with a negative sign before it or a number to the left side of the number line. Learners gave their thoughts when adding two positive numbers saying that the result is bigger than the two original numbers being added for example 2+3=5 and 5 is greater than both 2 and 3. When you add a positive and negative number, some responded that the numbers cancel each other giving an example of 2 + (-2) = 0 and some responded that the number becomes small. The misconceptions that we have as learners is putting a negative sign where it does not fit or at times leaving the sign where it is needed. Learners found textbooks very useful in understanding directed numbers but the challenge is that the textbooks are few and we have to share the books was the response from the learners. The internet can also be useful but we don't have gadgets that are capable and also data is a challenge even if we have the gadgets, was also another response from another learner.

The teachers involved in the study all were teaching form 1 and 2 with teaching experiences of 15 years and the other one five years but during all those years no specific training was received on teaching of directed numbers as a core concept in mathematics. Of course some wok shops and staff development meetings were done in mathematics department at the school and even with other schools but not specifically on directed numbers. From the teachers' responses, pair work and group work are most effective when teaching directed numbers but time does not allow us to do that and also we lack motivation on our earnings thus we end up just demonstrating to the class and give them an exercise to do. We also use visual aids like the number line and they are effective with small numbers but if used to the number line it becomes a difficulty when it comes to large numbers. We assess learners' understanding on directed numbers by the performance on the given in class exercises or test exercises. To address the errors and

misconceptions made by the learners we give feedback to the learners by their marked books and revise as the whole class. Most errors are found when subtracting a negative number from a negative number for example -2-3 or -5- (-3), learners will not write the negative sign on the solution. The teachers responded that due to the changes in technology at a faster rate as it is, the curriculum should also adapt leading to the use of projectiles and videos so that learners may understand the concept thus also making learning interesting. From my experience, learners have to be caught young earlier than form 1and the concept continue to be emphasized so that it is fully grasped because it is difficult to rub the thought of it can't learnt at primary level and it seem to cause confusion in the mind of the learners, was the response of one of the teachers.

4.4 EXPLANATIONS FOR THE ERRORS AND MISCONCEPTIONS

From the findings from the interviews and questionnaires with the teachers, it was noted that learners in form 1 are at a developmental stage where abstract thinking is still developing making it challenging to grasp the concept of directed numbers. Some of the reasons found explaining the root cause of the errors are in detailed below:

4.4.1 Prior knowledge

Prior knowledge is crucial since it serves as the basis for mathematics and has been identified as a cause of mistakes and misunderstandings. Some students demonstrated that they lacked a solid comprehension of fundamental arithmetic processes, which made it difficult for them to comprehend more complicated ideas like directed numbers. When directed numbers are involved, learners who have not fully learnt addition and subtraction of whole numbers find it difficult. Applying the principles for directed numbers correctly was also hampered by the learners' prior understanding of operations with whole numbers. It is difficult to translate intuitions that were acquired for positive numbers. When it comes to directed numbers, it can be

difficult as at primary school level students are taught that a smaller number cannot subtract a larger number. The ‘it can’t’ thinking is difficult to rub in the mind of the learners.

4.4.2 Instructional methods

Misconceptions may be unintentionally reinforced by certain teaching strategies. The learners' questionnaires revealed rote memory without context since they were unable to provide an explanation for their responses. Learners may make recurrent mistakes if the first directed numbers teaching is inadequate or ignores common misconceptions. Students need to have a firm grasp of how positive and negative numbers are represented and what they mean. According to the teachers' comments, traditional teaching techniques were beneficial, but they prioritized rote repetition over conceptual comprehension. Teachers at the school under investigation were complaining about time because they only had 35 minutes for the lesson and at times 30 minutes as the school has hot sitting, which made it hard to use a range of teaching strategies. There were no staff development sessions or seminars on these mistakes committed by students, according to the instructor questionnaires and interview. Thus, the educators just continue using the methods of education they are familiar with.

Additionally, learners were not contextualized, meaning they were unable to connect directed numbers to real-world situations. Understanding of mathematical terms is further hampered by a lack of proficiency in the language of instruction. The fact that the researcher had to translate some of the interview questions into the student's mother tongue in order for the student to respond to the questions showed that the English language was a barrier. Because the test was constructed with strong guidelines against using a calculator, the researcher also identified over-reliance on them as a contributing factor. According to the students' responses during the interview, a calculator allows them to find the answer more quickly and with less thought.

4.4.3 Cognitive overload

Errors were also caused by cognitive overload throughout the learning process. Learners may become overwhelmed by the sheer volume of rules and exceptions required in directed number operations, which could result in misunderstandings and errors. There is a significant learning curve for students when they are first exposed to directed numbers. A difficult cognitive demand is created when operational regulations and the idea of positivity against negativity must be mastered at the same time.

The students' nervousness related to mathematics was also seen. The most common response to the questionnaire's extra comments was that arithmetic is hard. Because of this, students may be less inclined to interact with the content and absorb it superficially as a result of their high level of math anxiety.

4.4.4 Linear dependency model

Many students perceive addition and subtraction strictly in linear terms, where they mistakenly believe that moving towards greater numbers is addition and moving towards lesser numbers is subtraction. This led to confusion when incorporating negative numbers as they could not visualize these operations on a number line properly. On the test exercise written, 4 + (-2), because of the addition sign one could write the answer as 6 ignoring the negative sign on 2 and likewise on 4 – (-2), one would just write 2 because subtraction move towards a smaller number.

4.5. Effective teaching strategies

The two teachers were interviewed as well as the learners. The finest recommendations for addressing the directed numbers idea and lowering student mistakes and misunderstandings are effective tactics. Several successful methods for instructing students in directed number addition and subtraction were identified:

4.5.1 Conceptual understanding

Teachers should emphasize understanding the concept of directed numbers through tangible examples. Using visual aids and contextual examples, such as number lines (including labeling and directional arrows) or temperature scales, can help students grasp the meaning of positive and negative values that is enhancing conceptual understanding. Teachers should incorporate number lines as a visual tool to help students understand the concept. Number lines can clarify the addition and subtraction of positive and negative integers by visually representing movement in both directions. So conceptual understanding is emphasized by providing learners with concrete representations and models to develop deeper understanding of directed numbers.

4.5.2 Real-life Applications

Teachers should integrate real-life contexts into lessons to provide relevance. Utilizing real scenarios, such as illustrating upward and downward movement for example in elevation can help solidify students' understanding. Teachers should relate directed numbers to real-life situations, such as temperature changes or financial contexts (debt versus credit). This approach makes abstract concepts more tangible and relevant to students' experiences.

The use of tangible examples like counters or chips also helps students to grasp the meaning of positive and negative numbers. These hands on approach can allow students to visualize operations and understand the additive inverse concept.

4.5.3 Collaborative Learning

Teachers should promote collaborative learning by having students work in pairs or small groups to solve problems involving directed numbers. This interaction fosters discussion and allows students to articulate their reasoning, helping to clarify misconceptions. Group discussions and peer-teaching sessions encourage students to articulate their reasoning and confront misconceptions in a supportive environment.

4.5.4 Use of Technology

Technology is currently transforming the planet on a worldwide scale. By using technology, such as interactive math platforms and instructional software, students can be engaged and given immediate feedback, which helps to reinforce sound thinking and offers practice chances without the stress of traditional tests. To accommodate varied learning styles, technology can provide differentiated instruction and instant feedback. Online manipulative, instructional games and digital simulations can improve comprehension and engagement.

Calculators must be used later, even if technological use is encouraged. Before utilizing calculators, students must be encouraged to answer problems by hand. This will help them understand techniques more thoroughly.

4.5.5 Differentiated Instruction

Differentiated instruction that is tailored to each learner's needs can help reduce frustration and foster mastery of directed numbers by providing the right challenges and support. Teachers should be aware that children learn at different paces. Encourage an atmosphere in which students can freely express their mathematical ideas without worrying about rejection or humiliation from their peers. When adding or subtracting directed numbers, students should be encouraged to explain their reasoning as this can assist discover and address misconceptions. Different teaching strategies will make learning engaging and joyful, which will help students develop a good attitude toward mathematics. To accommodate individual variances and meet the various demands of the students, modify your teaching methods, resources, and tempo.

4.5.6 Scaffolding Techniques

It is necessary to use scaffolding techniques, which progressively raise the level of difficulty of assignments using directed numbers. Introduce increasingly complicated scenarios gradually while starting with simpler problems to help pupils develop their comprehension. Introduce

formal rules and processes gradually after starting with informal, intuitive ways that build on learners' prior knowledge, such as employing analogies and real-world contexts. Employ a methodical approach that includes clear instructions and lots of practice opportunities.

4.5.7 Formative assessment (reflection and self-correction)

Facilitators need to teach students to evaluate their work critically, spot possible mistakes, and fix themselves. The development of students' metacognitive abilities is necessary to track their comprehension and problem-solving procedures. Students can develop a deeper understanding and self-awareness of their mistakes by being encouraged to reflect on how they solve problems. By looking at common errors in the addition and subtraction of directed numbers, students can practice error analysis. Recognizing and talking about mistakes strengthens learning and deepens comprehension.

By using continuous assessment techniques, teachers can receive timely feedback that helps them spot and correct misconceptions as they arise.

4.6 CHAPTER SUMMARY

The types of errors and misconceptions that form 1 students make when dealing with directed numbers were described in this chapter. The chapter explored the root causes of these errors and offered useful teaching methods for these ideas. Gaining an understanding of these elements can greatly improve instructional strategies and, in the end, enable a stronger mathematical foundation when working with directed numbers.

CHAPTER 5

5.1 INTRODUCTION

As students advance through mathematical concepts, it is crucial that they comprehend addition and subtraction procedures using directed numbers. In context with the body of research on the teaching and comprehension of directed numbers, this chapter analyzes the results from the preceding chapter and interprets their importance. Along with suggestions for additional research, the consequences for educational practice are examined. The objective is to offer a thorough grasp of how mistakes and misunderstandings can be successfully handled while teaching directed numbers.

5.2 SUMMARY

The purpose of this study was to investigate the challenges faced by form 1 learners in addition and subtraction of directed numbers at Seke 2 high school in Chitungwiza District, Harare province, where the researcher was teaching. Thirty learners from form a form 1 class of mixed abilities and two mathematics teachers, all from Seke 2 high school were taken as subjects in this study. Random sampling was used to select learners to make sure that there was no bias. Qualitative and quantitative methods were used to collect data though questionnaires, interviews observations and a test. Various constraints were met during the research period of the study and these might have had a negative effect to this study. Time was one of the constraints. However, besides the constraints, the research was successfully completed. Errors and misconceptions made by learners were noted and their causes together with suggested effective strategies to teach directed numbers.

5.3 FINDINGS OF THE STUDY

The research showed that, accounting for 47% of all errors found, procedural errors were the most common. The rules for adding and subtracting directed numbers were not well understood

by many students. This result is in line with earlier studies showing that when the learners first face directed numbers, they frequently struggle with procedural fluency. Thirty percent of the mistakes found were conceptual in nature, mostly involving the number line. This supports the claim made by Clements and Sarama (2009) that visual aids are essential for fostering a strong grasp of mathematical ideas. The findings suggest that students are prone to misunderstand directed number operations if they lack a solid conceptual basis.

Deeper mathematical understandings that learners carry a cognitive load give rise to misconceptions. The cognitive load rises as students come across combinations of both positive and negative numbers. Overwhelmed by this intricacy, they frequently turn to rote memorization of procedures, which results in mistakes rather than a deeper comprehension of the underlying ideas. Strong challenges may result from past experiences with whole numbers. In order to accommodate the peculiarities of directed numbers, students occasionally use rules that are only applicable to natural numbers, which frequently results in knowledge gaps. Misconceptions may be unintentionally reinforced by certain teaching strategies.

Effective techniques, including the use of concrete models, improved visual aids, interactive activities, and targeted feedback, were proposed to address these mistakes and misconceptions.

5.4 CONCLUSION

The common mistakes and misunderstandings that students make when working with addition and subtraction of directed numbers have been uncovered in this chapter. Finding these problems lays the groundwork for creating focused educational interventions meant to raise students' understanding as well as achievement in this crucial area of mathematics. Teachers can promote a better understanding of directed numbers and ultimately pave the road for success in more complex mathematical concepts by confronting these misconceptions straight away.

5.5 RECOMMENDATIONS

- Curriculum that emphasizes conceptual knowledge of directed numbers before procedural fluency is necessary in light of the aforementioned conclusions. For students to find the concepts more accessible and realistic, curricula should include real-world applications and visual representations. Students will be better prepared for more complex mathematical topics if there is a significant emphasis on fundamental concepts.
- The goal of professional development programs should be to give educators the tools they need to teach directed numbers. This involves instruction in error analysis methods and the use of visual aids. Encouraging teachers to reflect their own misconceptions can also enhance their ability to identify and address similar issues in their students.
- Educational tools and software that provide interactive experiences with directed numbers must be incorporated.
- It is necessary to include educational resources and software that offer interactive experiences with directed numbers. Longitudinal studies should be a part of future research to monitor students' progress in understanding directed numbers. These kinds of studies could help guide teaching strategies and shed light on how early misconceptions impact later mathematics performance.
- To assess how technology affects directed number learning, more research is required. Examining the ways in which digital technologies might improve conceptual understanding and deliver instant feedback may open up new possibilities for successful teaching.
- Encourage teachers to engage in reflective practices where they examine their own understanding of directed numbers. This can help them identify potential biases in their teaching and better support their students.

- Initiate research that targets diverse student populations to better understand how cultural and economic factors influence the understanding of directed numbers, informing more inclusive teaching practices.
- Foster collaborative learning environments where students can work together to solve problems involving directed numbers. Group discussions and collaborative problem-solving can enhance understanding and allow students to articulate their thought process.

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RESEARCH INSTRUMENTS

APPENDIX A
INTERVIEW QUESTIONS FOR TEACHERS

1. What are some common misconceptions learners have when learning about addition and subtraction of directed numbers?
2. When a learner makes an error in this area, how do you typically respond to help them understand their mistake?
3. What types of feedback do you provide to learners on their directed number work?
4. Have you observed any specific patterns or trends in learners' misconceptions related to these operations?
5. Do you think learners are aware of their misconceptions? If not, how can teachers help them recognize these gaps in understanding?
6. Have you engaged in any professional development that focuses on misconceptions in mathematics, especially directed numbers? How has it influenced your teaching?
7. How do you stay current with best practices in math education?
8. What methods do you use to teach directed numbers?
9. What strategies do you use to address these misconceptions in your teaching?
10. Which resources do you use to teach directed numbers? Are there any gaps in the resources identified?

APPENDIX B
INTERVIEW QUESTIONS FOR LEARNERS

1. Can you explain what directed numbers are and how they differ from regular numbers?
2. What mistakes do you often make when adding or subtracting directed numbers? Can you give an example?
3. Why do you think negative numbers are difficult to work with?
4. How do calculators affect your understanding of operations with directed numbers?
5. What methods or tools help you understand addition and subtraction of directed numbers better?

APPENDIX C
QUESTIONNAIRE FOR LEARNERS

Answer questions in the spaces provided.

SECTION 1: BACKGROUND INFORMATION

1. What is your current grade level?.....
2. How comfortable do you feel with using positive and negative numbers?

   Comfortable very comfortable not comfortable
3. Have you received any specific instruction on directed numbers? Yes/ No

SECTION 2: CONCEPTUAL UNDERSTANDING

4. How would you define a positive number?

   ……
5. How would you define a negative number?

   ……
6. What do think happens when you add two positive numbers?

   ……
7. What do you think happens when you add a positive number and a negative number?

   ……

   ……
8. What do you think happens when you subtract a positive number from a negative number?

……

……

SECTION 3: PROBLEM SOLVING

9. Solve -3 + 5 =?

   a) What did you think about when solving this?

   ……..

   b) Did you encounter any difficulties? If so, please explain.

   ……

   ……

10. Solve -2-4=

    Did you make any errors? If yes, what were they?

    ……

    ……

11. What strategies do you use to add or subtract directed numbers?

    ……

    ……

12. Have you ever made a mistake while working with directed numbers? If so, can you describe the mistake?

    ……

    ……

SECTION 4: MISCONCEPTIONS

13. What misconceptions do you think learners have about adding and subtracting directed numbers?

……

……

14. Do you think that the visual representation of directed numbers (like a number line) helps in understanding? Give a reason for your answer.

……

……

15. How do you feel about the rules for adding and subtracting directed numbers?

……

……

SECTION 5: INSTRUCTIONAL STRATEGIES

16. What teaching methods have helped you understand directed numbers better?

……..

17. What do you think teachers could do differently to help learners avoid errors with directed numbers?

……

……

18. Are there specific resources (books, websites, videos) that you found helpful in understanding directed numbers?

……

……

SECTION 6: ADDITIONAL COMMENTS

19. Is there anything else you would like to share about your experiences with addition and subtraction of directed numbers?

……

……

……..

APPENDIX D
QUESTIONNAIRE FOR TEACHERS

Answer questions in the spaces provided.

SECTION 1: BACKGROUND INFORMATION

1. What grade level do you teach? .....
2. How many years of experience do you have teaching mathematics? ……
3. Have you received any training specific to teaching directed numbers? Yes/ No

SECTION 2: OBSERVATIONS OF STUDENT UNDERSTANDING

4. In your experience, how comfortable are your learners with directed numbers?

   Comfortable Very comfortable Not comfortable

5. What common errors do you observe learners making when subtracting directed numbers?

   ……

   ……

   ……

6. What common errors do you observe learners making when adding directed numbers?

   ……

   ……

   ……

7. Can you identify specific misconceptions that learners have about directed numbers?

……

……

8. How do students typically respond when encountering addition or subtraction problems involving negative numbers?

……

……

……..

SECTION 3: INSTRUCTIONAL STRATEGIES

9. What teaching methods do you find most effective for helping students understand directed numbers?

……

……

10. Do you use visual aids (like number lines or diagrams) when teaching these concepts? If yes, how effective do you find them?

……

……

11. What specific strategies do you employ to address common errors or misconceptions?

……

……

12. How do you assess learner understanding of addition and subtraction of directed numbers?

……

……

……

SECTION 4: CURRICULUM AND RESOURCES

13. What resources (textbooks, online tools, etc.) do you find most helpful in teaching directed numbers?

……

……

14. Have you noticed any gaps in the curriculum regarding the teaching of addition and subtraction of directed numbers? If so, please explain.

……

……

……

……

SECTION 5: PROFFESSIONAL DEVELOPMENT

15. What additional training or resources would you find beneficial in improving your teaching of teaching of directed numbers?

……

……

16. Have you participated in any workshops or professional development focused on learner misconceptions in mathematics? If yes, what were the key takeaways?

……

……

……

SECTION 6: ADDITIONAL COMMENTS

17. Is there anything else you would like to share about your experiences teaching addition and subtraction of directed numbers?

……

……

……

……

18. Do you have any recommendations for other teachers facing similar challenges?

……

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……

……

APPENDIX E
TEST QUESTIONS

Answer all questions showing clear working where necessary. You can use a number line

1. -3 + 0
2. 5 – 7
3. -2 – 3
4. 4 + (-2)
5. -1 + (-3)
6. 4 – (-2)
7. -5 – (-3)
8. True/ False
    a) Adding a negative number moves you left on a number line
    b) Subtracting a negative number moves you right on a number line