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ANALYSIS OF STUDENTS' ERRORS IN SOLVING QUADRATIC EQUATIONS USING FACTORISATION

BY

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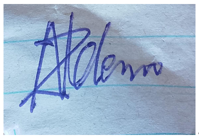………………………………..

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DEDICATION

This research is dedicated to my beloved sons Stanely Junior Maminya, Jephthah Maminya and Joseph Darrell Kendrick Nyakudya.

ABBREVIATIONS AND ACRONYMS:

ZIMSEC: Zimbabwe School Examination Council

ABSTRACT

When students make mistakes, it helps teachers understand where they are getting stuck and how to best support them. This study looked at analysis of students' errors when solving quadratic equations using factorisation. It focused on 75 Form 3 students and analyzed their test answers and interview responses. I found that students make different types of mistakes, like procedural, technical, and conceptual errors, which reveal their level of understanding. The study identified five key areas of knowledge that students need to master to solve quadratic equations: working with directed numbers, basic algebra, factorization, linear equations, and problem-solving skills. The findings suggest that targeted teaching and practice can help fill knowledge gaps and improve math skills. Further research with a larger group of students could provide even more insights.

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CHAPTER 1

Problem and Its setting

1.0 Introduction

This chapter unravels the complexities of the background of the study, shedding light on the pressing concerns that necessitated our investigation, and looks into the purpose of the study, illuminating the research problem and main research question that drive our inquiry. Our research questions and objectives are carefully crafted to guide our exploration, while also exploring the hypothesis and significance of our study, revealing the key stakeholders who stand to benefit and how our findings can impact them. Furthermore, the chapter establishes clear delimitations, defining the scope of our study and acknowledging potential limitations, provides definitions of key terms, and outlines the organization of our study, ultimately summarizing the chapter and offering a concise overview of our research foundation.

1.1 Background of the Study

Quadratic equations are fundamental in mathematics curriculum in secondary schools and provide a basis for advanced mathematical concepts and real-life applications. Previous research has investigated errors and misconceptions in various mathematical topics and these studies have explored areas such as Algebra (Mulungye, 2016; fractions Alghazo and Alghazo, 2017; Low et al., 2020), differentiation (Chikwanha et al., 2022), linear ordinary differential equations (Msomi & Bansilal, 2022), and derivatives of trigonometric functions (Siyepu, 2015). However, studies have shown that learners often struggle with solving quadratic equations and make numerous errors (Vilakazi, 2021; Baybayon and Lapinid, 2024).

O'Connor and Norton (2022) found that learners in a certain secondary school in Australia struggled with manipulating equations and applying the null factor law correctly. Joshi (2019) discovered weak problem-solving skills among learners in a certain secondary school in Asia when dealing with quadratic equations using factorisation. Similarly, Ly and Takuya (2023) found conceptual errors among learners in Japan that could affect their future mathematical learning.

The challenges with quadratic equations extend beyond specific regions and are prevalent in broader African contexts as well. Thomas and Mahmud (2021) discovered that secondary school students in Malaysia struggled with quadratic equations due to a shaky grasp of fundamental concepts and learning styles. Moreover, many students fail to master factoring skills, a crucial step in solving quadratic expressions, as they rely heavily on scientific calculators to arrive at answers. This over-reliance on technology hinders their ability to develop a deeper understanding of mathematical concepts. Additionally, Rahman (2019) noted that female learners in Malaysia commonly made comprehension, transformation, and carelessness errors, while male learners had a higher frequency of transformation errors. Kim How et al (2020) reported that learners in Malaysia perceived quadratic equations as difficult to learn and struggled to find appropriate learning materials.

Errors made by learners in quadratic equations have also been identified in specific schools in Zimbabwe. A revealing study by Tendere and Mutambara (2020) looked into the math struggles of three high school learners in Zimbabwe, uncovering their difficulties in solving symbolic quadratic equations using the factorization method. The researchers identified a trio of error types: conceptual, procedural, and technical. These errors not only highlighted the learners' struggles but also shed light on the areas where their math education needed reinforcement. Tendere and Mutambara (2020) also found that learners had difficulties in solving symbolic quadratic equations by the factorisation method such that many misconceptions were exposed. These errors significantly impact learners' understanding of mathematics as a whole, as quadratic equations are connected to other mathematical concepts.

In light of these challenges, this study seeks to analyse learners' errors in solving quadratic equations using factorisation and aims to assess their understanding using Bloom's Taxonomy of objectives. Understanding the specific errors faced by these learners will

In light of these challenges, this study seeks to analyse learners' errors in solving quadratic equations using factorisation and aims to assess their understanding using Bloom's Taxonomy of objectives. Understanding the specific errors and the fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors will enable the development of targeted instructional strategies and interventions to enhance their understanding and performance.

1.2 Purpose of the study

The purpose of this study is to analyze the errors made by form 3 learners in a certain High school in Rushinga District when solving quadratic equations using the factorisation method. The aim is to understand the common errors faced by these learners and the fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors in order to develop effective strategies and interventions to improve their understanding and performance in mathematics. By conducting a detailed analysis of the errors, the study intends to provide valuable insights for both teachers and policymakers in designing targeted instructional approaches that address the specific needs of these learners.

1.3 Statement of the problem

The problem addressed in this study is the occurrence of errors made by form 3 learners at a certain school in Rushinga District when solving quadratic equations using factorisation. This issue is negatively impacting the learners' overall performance in mathematics and hindering their ability to progress to more advanced mathematical topics. While previous studies have examined the occurrence of errors in solving quadratic equations, there is a gap in the research regarding the fundamental concepts needed when solving quadratic equations using factorization and specific types of errors made by these learners. Therefore, the purpose of this study is to identify the specific types of errors made by the learners and the fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors.

1.4 Main research question

What are the specific types of errors made by third form students when solving quadratic equations using factorisation?

1.5 Research Objectives

The research intended to meet the following objectives;

1.5.1 To establish errors made by form 3 learners when solving quadratic equations using factorization.

1.5.2 To assess the fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors.

1.6 Research questions

The research questions mentioned below are to be used as a guide to this study:

1.6.1 What are the errors exhibited by form 3 learners when solving quadratic equations using factorization?

1.6.2 What is the fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors?

1.7 Significance of the Study

A number of stakeholders are expected to benefit immensely from this research. These stakeholders include the learners, teachers, school administrator, policy makers and researchers.

Learners

This research helps learners identify and address their common errors in solving quadratic equations, leading to improved problem-solving skills and academic performance in mathematics. By understanding learners' common errors, teachers can tailor their teaching methodologies accordingly, providing targeted support and guidance. This benefits learners by enabling them to develop a deeper understanding of quadratic equations and strengthen their overall mathematical abilities.

Teachers

This research informs teachers about the typical errors made by form 3 learners in solving quadratic equations using factorisation, empowering them to develop targeted instructional strategies. Enables them to identify and address errors early on, promoting effective learning and preventing the accumulation of errors. Contributes to professional development by providing empirical evidence for improving teaching practices and curriculum design.

School Administrator

The school sdministrator assists in identifying areas where the curriculum or teaching approaches need revision to better meet the needs of learners. Provides a basis for decision-making in resource allocation, such as providing additional support or training for teachers. Contributes to the overall improvement of mathematics education in the school.

Policymakers

Informs educational policies and curriculum development by providing evidence of the common errors made by learners in solving quadratic equations using factorisation. Enables the development of strategies to address these errors at the national level, such as revising textbooks or providing professional development for teachers. Contributes to improving the quality of mathematics education in the country.

Researchers

Adds to the body of research on learners' errors in solving mathematical problems, particularly quadratic equations. Provides a basis for further exploration of the cognitive and pedagogical factors that contribute to these errors. Informs the development of effective instructional interventions and assessment tools.

1.9 Delimitation of the study

The study was carried out at a certain secondary school. The School is located in Rushinga District in Mashonaland Central Province. The scope of the research is to analyse the learner's errors in solving quadratic equations using factorisation to form 3 learners in Rushinga High School. The time frame of this research is from January to June 2024. Of all the concepts in mathematics, the researcher chose quadratic equations because they are important in mathematics as they form the basis for higher-level mathematical concepts and serve as a foundation for understanding and solving complex mathematical problems. The researcher choose to conduct research at the school because of the school's proximity to the researcher. Rushinga High School is close to the researcher's home and place of work, making it convenient to access. Additionally, the school has a reputation for excellence, making it an attractive option for researchers looking to conduct high-quality research.

1.10 Limitations

One limitation of this study lies in its sample size and sampling method. The study was conducted with a relatively small group of students of only one school, which may not fully represent the population of students who struggle with quadratic equations. Additionally, the participants were recruited through purposive sampling just after all the population of 75 learners wrote the test which could have introduced bias into the sample. As such, the findings of this study may not be generalizable to all students who experience difficulties with quadratic equations. Future research would benefit from including a larger and more representative sample to ensure the generalizability of the results.

1.11 Definition of key terms

Errors

Makonye (2014) cited in Mudavanhu et al. (2023) defines an error as a slip, mistake, or deviation from accuracy.

Quadratic Equations

According to Tendere and Mutambara (2020), quadratic equation is an equation of the second degree, takes on a distinct form - $ax^2 + bx + c = 0$, where a, b, and c represent constants, and x stands as the unknown variable waiting to be unraveled.

Factorisation

Factorization is the art of dissecting a mathematical entity into its fundamental building blocks, known as factors, which, when combined through multiplication, magically reassemble into the original whole.

1.12 Organization of the Study

This study is organized into five chapters.

Chapter 1: It provides an introduction to the study, including the background, purpose, problem statement, research main question, research objectives, research questions, hypotheses/asssumptions, significance, delimitations, limitations, definition of terms, organisation of the study and summary.

Chapter 2: It reviews relevant literature on errors in solving quadratic equations. Theoretical framework. The common errors when solving quadratic equations and the fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors.

Chapter 3: It describes the research methodology, including the research design, sample selection, data collection instruments, data collection methods and data analysis procedures.

Chapter 4: It presents the findings of the study and discusses the implications for practice and policy.

Chapter 5: It provides a summary of the study, conclusions, and recommendations for future research.

Summary

This chapter explores the research background to provide context for the problem. It outlines the study's purpose, problem statement, and main research question, followed by the research objectives, questions, and hypotheses. The significance, limitations, and delimitations of the study are also highlighted. Additionally, key terms used in the study are defined, and the organization of the study is presented. The next chapter is going to look on literature review.

CHAPTER 2

LITERATURE REVIEW

2.0 Introduction

This chapter aims to provide a comprehensive review of the related literature and theoretical framework. This chapter sets the stage for the exploration of the research questions that will be addressed throughout the course of this research which are: What are the errors exhibited by form 3 learners when solving quadratic equations using factorization? What is the fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors?

2.1 Theoretical framework

The theoretical framework for analyzing learners' errors in solving quadratic equations using factorization can be developed by incorporating Bloom's Taxonomy of objectives. Bloom's Taxonomy (1956) provides a hierarchical classification of learning objectives, focusing on the cognitive domain, which is relevant for understanding and categorizing the types of errors made by learners (Loveless, 2023).

The cognitive domain encompasses six levels: knowledge, comprehension, application, analysis, synthesis, and evaluation. Errors at the knowledge level may include forgetting formulas or misconceptions (Ahmed, 2021), while errors at the comprehension level may involve misinterpreting the problem or struggling to grasp the underlying concepts (Starr et al., 2019). Errors at the application level can arise when learners have difficulty transferring their knowledge to new situations or executing the factorization process accurately (Loveless, 2023). Analysis-level errors involve misidentifying factors or misunderstanding relationships between equation components (Bloom, 1956). Synthesis-level errors may stem from difficulty in integrating various elements of knowledge and problem-solving strategies (Ahmed, 2021). Evaluation-level errors indicate challenges in critically assessing one's own problem-solving process and justifying solution choices (Loveless, 2023).

By considering these cognitive levels, the theoretical framework can provide a systematic approach to identifying and analyzing the specific types of errors made by learners in solving quadratic equations using factorization. This framework can guide the design of interventions or instructional strategies aimed at addressing these errors and improving learners' problem-solving skills in this domain.

2.2 Errors made by learners when solving quadratic equations using the factorisation

2.2.1 Procedural errors

According to Mutambara and Bansilal (2021), procedural errors occur when learners use the wrong approach or make mistakes while following a procedure. Tendere and Mutambara (2020) specifically identify procedural errors in solving quadratic equations as mistakes made while following steps or procedures to find the solution. These errors often arise from a lack of prior knowledge of quadratic concepts, such as directed numbers (Tendere and Mutambara, 2020). Johari and Shahrill (2020) categorize similar errors as simple mathematical errors due to carelessness. For example, incorrectly factoring a quadratic equation, like mistaking (x + 2)(x + 5) = 0 for (x - 2)(x - 5) = 0, leads to wrong solutions and is a classic procedural error (Tendere and Mutambara, 2020).

2.2.2 Technical / foundational errors

According to Msomi and Bansilal (2022), foundational errors occur when learners lack a solid understanding of fundamental concepts, leading to misapplication or misinterpretation of previously learned procedures. This knowledge gap often results in technical errors, particularly in areas like directed numbers, negative number operations, formula substitution, and expression simplification (Msomi and Bansilal, 2022). For instance, learners may correctly recall the quadratic formula but struggle with algebraic manipulation, highlighting the importance of mastering prior knowledge (Tendere and Mutambara, 2020). As Tendere and Mutambara (2020) note, a learner's existing understanding of a mathematical concept significantly influences their ability to construct new knowledge, emphasizing the need for a strong foundation.

2.2.3 Computational errors

Computational errors in solving quadratic equations occur when learners make mistakes during numerical calculations, consistent with Mbewe and Nkhata (2019)'s findings. These errors can arise from struggles with negative signs and square roots. For example, calculation mistakes when finding the square root, such as mistaking the square root of 4 for 5 instead of 2, can lead to incorrect solutions. Additionally, incorrectly distributing signs, like confusing positive and negative signs, can cause errors (Mudavanhu et al., 2023). For instance, mistakenly distributing a negative sign when simplifying an equation can alter the value and result in incorrect solution s.

2.2.4 Conceptual errors

According to Mutambara and Bansilal (2021), conceptual errors occur when learners have a deep-seated misunderstanding of a new concept. In the context of factorization, learners may commit conceptual errors, such as misinterpreting the zero-product property (Mutambara et al, 2019). Baybayon and Lapinid (2024) clarify that this property states that if the product of two factors is zero, at least one of the factors must be zero, not necessarily just one factor. For instance, in the equation $x^2 - 9 = 0$, factoring leads to $(x + 3)(x - 3) = 0$, but learners may incorrectly conclude that only $x + 3 = 0$, overlooking the need to set each factor equal to zero and solve separately, resulting in correct solutions $x = -3$ and $x = 3$. This research examines various errors in solving quadratic equations using factorization, including procedural, technical/foundational, computational, and conceptual errors, to identify common pitfalls and inform targeted interventions for improved student understanding and problem-solving skills.

2.3 The fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors

Learners require a solid foundation in basic algebraic concepts to successfully solve quadratic equations using factorization with minimum errors (Ly and Takuya, 2023). To address the challenges that students face, it is recommended that teachers adopt a constructive approach to learner errors in solving mathematical problems. This includes recapping vital concepts and skills before teaching quadratic equations, such as directed numbers, factorization, expansion of brackets, and substitution (Tendere and Mutambara, 2020). By doing so, teachers can help learners build a strong foundation in algebra and improve their problem-solving skills, ultimately leading to greater success in solving quadratic equations.

O'Connor and Norton (2022) say that this includes understanding variables, constants, and algebraic expressions, as well as familiarity with order of operations and properties of operations such as commutative, associative, and distributive properties. Additionally, learners need to have a strong grasp of factoring skills, including the ability to factor simple expressions (for example, x^2 + 5x + 6) and understanding of factoring techniques such as greatest common factor and difference of squares (Joshi, 2019). Learners also require knowledge of quadratic equation concepts, including understanding quadratic equation forms (for example, ax^2 + bx + c = 0) and solutions and their representations (for example, x = -1, x = 3). Furthermore, learners need to possess number sense and properties, including familiarity with integer operations and properties (for example, distributive property) and understanding of negative numbers and their behavior in algebraic expressions (Lester, 2020). According to Polya (1957) in Kim How et al. (2020) says that problem-solving strategies, such as approaching problems systematically and logically, and familiarity with checking solutions and verifying answers, are also essential. Finally, learners require mathematical notation and terminology knowledge, including understanding mathematical symbols and notation (for example, =, +, -, ×, /) and familiarity with mathematical terminology (for example, factor, coefficient, solution) (Rahman, 2019). Basic mathematical concepts, such as understanding place value and decimal arithmetic, and familiarity with fractions and equivalent ratios, are also necessary (Thomas and Mahmud, 2021).

Vilakazi (2021), a mathematics teacher, observed that all quadratic expressions can be factored using a single approach, starting with the general form $ax^2 + bx + c$. Vilakazi (2021) called this approach the "fundamental considerations when factoring quadratic expressions," as it can be applied to factor all types of quadratic expressions. When factoring quadratic expressions, we consider them in the form $ax^2 + bx + c$ and follow these fundamental considerations:

- Factoring trinomial quadratic expressions with $a \neq 1$: We use the grouping method, as seen in examples like $4x^2 - 20x + 21 = (2x - 3)(2x - 7)$ (Vilakazi, 2021).
- Factoring trinomial quadratic expressions with $a = 1$: We also use the grouping method, as shown in examples like $x^2 - 2x - 15 = (x - 5)(x + 3)$ (Vilakazi, 2021).
- Factoring binomial quadratic expressions with $c = 0$: We consider the expression as having a zero term and apply the grouping method, as in $3x^2 - 5x = x(3x - 5)$ (Vilakazi, 2021).
- Factoring binomial quadratic expressions with $b = 0$: We consider the expression as having a zero term and apply the grouping method, as in $x^2 - 9 = (x - 3)(x + 3)$ (Vilakazi, 2021).

By following these fundamental considerations, we can factor various types of quadratic expressions.

This research analyzes students' errors in solving quadratic equations using factorization, which requires a solid foundation in basic algebraic concepts to minimize errors. Effective factorization involves understanding fundamental considerations and applying techniques like grouping and factoring simple expressions. Learners need to possess knowledge of quadratic equation concepts, factoring skills, number sense, and problem-solving strategies, as well as familiarity with mathematical notation and terminology. By addressing these challenges, teachers can help learners build a strong foundation in algebra and improve their problem-solving skills, leading to greater success in solving quadratic equations. This research aims to identify and analyze common student errors, providing insights for targeted interventions and improved understanding.

2.4 Related literature

Over the years, numerous studies have done on the analysis of students' errors in solving quadratic equations shedding light on the misconceptions and challenges that students face in mastering the concept.

Baybayon and Lapinid (2024) conducted a study on common errors in quadratic equations among Grade 9 learners in the Philippines. The descriptive qualitative research involved 46 learners who completed online tasks and tiered worksheets on quadratic equations. The researchers analyzed the worksheets and identified errors, which were then discussed in class to address misconceptions. Individual interviews validated the error analysis. The study found common errors, including:

- Not following directions
- Mishandling signs
- Difficulty recognizing quadratic equations
- Inability to distinguish between solving and simplifying
- Failure to express quadratic equations in standard form
- Disregarding negative roots
- Computational errors in algebraic conventions

For instance, some learners mistakenly identified an equation as quadratic due to the exponent 2, without considering the distribution of terms (Baybayon and Lapinid, 2024). Others made minor errors in manipulating equations, such as using the Addition Property of Equality incorrectly (Baybayon and Lapinid, 2024). The study highlights the need for targeted instruction to address these common errors and improve mathematics performance.

Baybayon and Lapinid (2024) gave some recommendations and these include the strategies that specifically address these prerequisite skills and concepts to improve learners' performance in mathematics. These strategies could include explicit instruction, guided practice, and frequent review of these concepts. Additionally, teachers can provide clear and detailed instructions to ensure that learners understand the requirements of each problem and the necessary steps to solve it. Double-checking answers and practicing careful attention to detail should also be emphasized to reduce careless mistakes. Overall, it is recommended to implement targeted interventions and instructional strategies to address learners' weaknesses in prerequisite skills and concepts in order to improve their performance in mathematics.

Tendere and Mutambara (2020) conducted a study on errors and misconceptions in solving quadratic equations. The objectives of the study were to identify the types of errors and misconceptions exhibited by form three learners in Zimbabwe when using factorization and the quadratic formula, as well as to determine interventions to address these issues.

The population for the study of Tendere and Mutambara (2020) consisted of three mathematics teachers and ninety-six learners, with a sample size of thirty learners selected using purposive and stratified sampling methods. Data was collected using a test and follow-up interviews. The test assessed learners' knowledge of quadratic equations, with questions categorized into solving equations through factorization, using the quadratic formula, and involving word problems. The researchers ensured the reliability and validity of the test items by including diverse question types and drawing from past exam papers. Follow-up interviews provided insights into learners' thinking processes and allowed for the identification of misconceptions. The study revealed that most errors were conceptual and technical in nature, indicating a lack of prerequisite knowledge and difficulties with algebraic manipulation.

Tendere and Mutambara (2020) recommended that teachers address learner errors by reviewing essential concepts and skills before teaching quadratic equations, focusing on one point at a time and promoting critical reasoning. They also suggested further research to utilize identified misconceptions and errors in assessments and to explore the development of learner errors and misconceptions in algebra.

The study conducted by O'Connor and Norton (2022) focused on exploring the challenges faced by learners in understanding quadratic equations and examining the curriculum structure and implementation related to this topic. The study employed an exploratory methodological approach to understand the reasons behind learners' difficulties in solving quadratic equations (Creswell, 2012). The researchers chose the topic of quadratics because it builds upon the algebraic conventions studied in earlier grades. Data collection occurred in two phases: a written test followed by diagnostic interviews to gain a nuanced understanding of learners' written responses. The analysis of learners' written responses involved the generation of categories explaining errors, using an emergent design approach rather than predetermined categories. The data analysis followed an inductive approach, moving from specific analysis of individual learner errors to broader comparisons between error patterns, categories of errors, and other categories.

O'Connor and Norton (2022) developed interview questions based on the written test results to understand learners' thought processes and conceptual understanding of quadratics, solution techniques, and solution meanings. The interview data supported the written test findings. The researchers also analyzed curriculum documents to understand the context. According to O'Connor and Norton (2022), the study revealed two significant issues in the teaching and learning process:

1. Learners showed limited initial understanding of quadratic equations, indicating a lack of conceptualization and schema construction.

2. Learners lacked proficiency in basic algebraic conventions, which should have been mastered earlier, suggesting a gap in prior learning or pedagogy.

These findings highlight the need for a stronger focus on conceptual understanding and foundational algebra skills to support learners' development in mathematics.

O'Connor and Norton (2022) recommend incorporating remediation practices and allocating sufficient time for revision to enhance learner retention. The limited time for remediation may be due to classroom discourse, but the curriculum structure is also a likely contributor. This supports previous concerns about time constraints resulting from an overcrowded curriculum (Donnelly & Wiltshire, 2014; Snider, 2004). The study highlights flaws in a curriculum design that briefly covers mathematics topics, assuming learners will master them in subsequent years without reinforcement.

2.5 Literature gap

The literature gap in this research refers to a specific area that has not been adequately addressed or explored in previous studies. This gap consists of three main aspects. Firstly, there is a need for a comprehensive understanding of the specific errors and challenges that Form 3 learners face when solving quadratic equations using factorization. Secondly, there is a lack of understanding of the fundamental knowledge and skills required to address these errors and improve learners' problem-solving skills in this area. Thirdly, previous research did not look the different cognitive levels of understanding among learners, such as knowledge, comprehension, application, analysis, synthesis, and evaluation, which are crucial in understanding the challenges learners face when solving quadratic equations.

While previous studies have investigated learner errors and misconceptions in mathematics, they have not specifically focused on identifying and analyzing the types of errors made by Form 3 learners when solving quadratic equations using factorization. Additionally, there is a lack of research on determining the essential knowledge and skills necessary to support learners in overcoming these errors and developing proficiency in solving quadratic equations.

By addressing this literature gap, the research aims to contribute meaningfully to the field of mathematics education. The study seeks to provide valuable insights into the challenges faced by Form 3 learners in solving quadratic equations using factorization, develop targeted instructional strategies and interventions to support learners in overcoming these challenges, and enhance mathematical literacy and problem-solving skills for learners. To achieve this, the research will conduct a comprehensive analysis of learner errors and misconceptions, the fundamental knowledge and skills required to address these issues and different cognitive level of knowledge of learners. By filling this literature gap, the study aims to provide a deeper understanding of the challenges faced by Form 3 learners in solving quadratic equations using factorization and contribute to the existing body of research in mathematics education.

Summary

This chapter served as a solid foundation for the research, presenting an overview of the literature, theoretical framework, methodology of related literature and literature gap. It sets the stage for the subsequent exploration of the research questions, which focus on common errors in solving quadratic equations using factorization and the fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors. Thus chapter provides a roadmap for the remainder of the study and creates a framework for understanding and addressing the complexities of quadratic equation solving. The next chapter is about methodology.

Chapter 3

Research Methodology

3.0 Introduction

This chapter provides an overview of the research methodology. In this chapter, the research design for the study on using problem-based learning in the teaching of quadratic equations will be discussed. It will outline the instruments used to collect data, describe the target population and sample, discuss the sampling technique employed, and provide an outline of the procedures for collecting data. Additionally, the chapter will cover the data presentation, analysis procedures and summary.

3.1 Research design

A qualitative approach wad adopted for this research. According to Tenny et al. (2022), a qualitative approach is a research method that delves deeper into real-world issues, seeking to understand participants' experiences, perspectives, and behaviors. This approach provides rich insights into complex problems, offering a nuanced understanding of the research context. Qualitative research methodology has been effectively utilized by various researchers (Chikwanha et al., 2022; Johari and Shahrill, 2020; Msomi and Bansilal, 2022; Siyepu, 2015). This strategy allows for an in-depth exploration on analysising student's errors in solving quadratic equations using factorisation (Mudavanhu et al., 2023).

3.2 Instruments used to collect data

To identify errors in solving quadratic equations using factorization, a two-pronged approach was employed: a written test and follow-up interviews. These research instruments, as defined by Collins (2021), are tools used to collect, measure, and analyze data. The test assessed learners' problem-solving skills, while the interviews provided insight into their thought processes and understanding, enabling a comprehensive understanding of their errors.

3.2.1 Test

As Adom et al. (2020) explain, tests are systematic tools used to assess a specific aspect of behavior by presenting a set of questions or tasks. This standardized approach enables researchers to measure and evaluate a particular skill or knowledge area, providing valuable insights into learners' abilities and understanding. The quadratic equations knowledge test was conducted among all the students in the third form. The test assessed their understanding of fundamental properties related to quadratic equations. The test items were categorized into two groups: solving equations through factorization and solving word problems. The test was created using ZIMSEC O-Level past examination papers. These test items were selected based on their adherence to ZIMSEC standards. This is supported by Mudavanhu et al. (2023) who say that selecting questions from past examination papers ensures content validity and reliability in distinguishing learners with varying abilities. Additionally, a grid specification method was employed to ensure that the test items covered the first four levels of Bloom's taxonomy, namely remembering, understanding, applying and analyzing (Johari and Shahrill, 2020). The test aimed to identify areas where errors occurred, such as reading and comprehending the questions, understanding their structure, translating them, and solving them correctly. To ensure the reliability of the test items, a diverse group of students was selected for testing. Sufficient time was provided to the students to complete the test. Additionally, the test was designed to adequately cover the content and assessment domains.

3.2.2 Interview

To gain a deeper understanding of learners' thought processes, the researcher conducted follow-up interviews, allowing learners to share their thoughts and explain their actions. As defined by George (2022), interviews are a qualitative research method involving questions to collect data, typically between two or more people, including an interviewer. These interviews helped identify contradictions, deficiencies, or errors in learners' understanding. Additional questions were posed based on the initial test solutions, aiming to uncover learners' conceptualizations, interpretations, and thinking processes (Tendere and Mutambara, 2020). Selective interviews with students provided valuable insights into the reasons behind specific error types and learning gaps, shedding light on areas requiring targeted support.

3.3 Population

This study focused on a specific group of 75 Form 3 learners, which represents the target population. According to Bhandari (2020), a population refers to the entire group that you want to draw conclusions about in research. It's important to note that a population can comprise various elements, not just people. It could be a group of objects, events, organizations, countries, species, or anything else that is being studied.

3.4 Sample

A diverse sample of 5 learners (3 females and 2 males) was strategically selected from the test-takers based on their performance. As emphasized by Sriram (2024), including both females and males in the sample enhances the study's validity, ensuring more representative and generalizable findings that can be applied to the larger population. According to Bhandari (2020), a sample is a subset of the population, chosen to provide insights and data that can be used to make inferences about the population's characteristics. Selecting a sample allows for a more manageable group to collect and analyze data while maintaining sufficient representation, making it a crucial step in research (Bhandari, 2020).

3.5 Sampling technique to be used in the study

This research employed a combination of census and purposive sampling techniques. According to Smith (2023), a sampling technique is a deliberate and systematic approach used to select a representative subset of individuals, objects, or events from a larger population. By strategically choosing participants, researchers can ensure a more accurate representation of the population, enhancing the study's validity and reliability.

3.5.1 Census sampling technique

The researcher administered the test to all 75 learners (the entire population) and this is an example of census sampling, where data is collected from the entire population rather than a sample. Sriram (2024) defines a cencus sampling method as an attempt to gather information from each and every person of interest. By testing all individuals, the researcher aimed to gather information about the errors made by the entire group of form three learners when solving quadratic equations using factorisation. This approach allows for a comprehensive understanding of the errors and patterns within the population, providing a more accurate representation of their performance and enabling meaningful analysis and conclusions about the errors in solving quadratic equations using factorisation.

3.5.2 Purposive sampling technique

This study utilized purposive sampling to select participants for in-depth interviews. As defined by Nikolopoulou (2022), purposive sampling involves deliberately selecting individuals with specific characteristics that align with the research goals. In this case, the researcher administered a test on quadratic equation factorization to the entire population of 75 Form Three learners. After reviewing the test results, five students (3 females and 2 males) were chosen for interviews based on their varying performance levels and error types. By intentionally selecting students with diverse error patterns, the researcher aimed to gain a deeper understanding of the common mistakes made by Form Three learners when solving quadratic equations using factorization. This targeted approach enabled the researcher to gather rich insights and contribute to the study's findings and conclusions.

3.6 Procedures for collecting data

Data was collected using the test and interviews. The data collection process was guided by predefined research questions and ethical considerations were prioritized.

3.6.1 Procedures of collecting data from the test

In this research, the researcher conducted a test to collect data on the errors made by all 75 form three learners when solving quadratic equations using factorisation. The researcher began by designing a test specifically tailored to evaluate their understanding and proficiency in this area. The test consisted of various questions covering different types of quadratic equations and factorisation techniques. Next, the researcher administered the test to all 75 learners, ensuring that each student received a copy of the test and clear instructions for completing it. Throughout the test administration, the researcher monitored the students to ensure compliance with instructions and maintain test integrity. After the students finished the test, the researcher collected their answer sheets, ensuring that all papers were accounted for without any missing data. With the collected test papers from all 75 learners, the researcher compiled the data by organizing the responses and noting the errors made by each student. This data formed the basis for the researcher's subsequent analysis, where the researcher categorized the types of errors, identifies mistakes, and examined patterns or misconceptions exhibited by the learners.

3.6.2 Procedures of collecting data from the interviews

After administering the test to all 75 form three learners to analyze their errors in solving quadratic equations using factorisation, the researcher conducted interviews with a subset of 5 learners (3 females and 2 males). These participants were purposefully selected based on specific criteria such as their performance on the test or the types of errors they made. Mudavanhu et al. (2023) say that the interviews were designed to get deeper into their experiences and gather detailed insights regarding their thought processes, understanding of concepts, and challenges faced during quadratic equation solving using factorisation. The interviews took place in a supportive environment, and the researcher actively listened to thelearners' responses, recording their answers either through detailed notes and audio recordings.

3.7 Data presentation procedures

In the research study analyzing students' errors in solving quadratic equations using factorization through a qualitative approach, the data presentation pro cedures focus on presenting the qualitative data concisely and insightfully, while also incorporating visual elements and learners' workings. To begin, the qualitative data obtained from interviews can be organized and categorized using thematic analysis. Recurring themes, patterns, and subthemes related to the errors made by students can be identified and presented systematically (Mudavanhu et al., 2023). Supporting quotes or excerpts from the interviews can be included alongside these themes, providing evidence and enhancing the understanding of the errors. Additionally, visual representations such as diagrams or flowcharts can be utilized to present learners' workings and illustrate the problem-solving process. These visuals can help readers visualize the steps taken by learners and better comprehend the specific errors made. Furthermore, narrative description plays a crucial role in presenting the qualitative data. Descriptive narratives can provide a detailed account of the errors, including the context, learners' experiences, and challenges encountered during the problem-solving process. These narratives can be presented coherently, engaging readers and immersing them in the learners' perspectives (Tendere and Mutambara, 2020). Direct quotes from the interviews can be integrated into the narratives, adding authenticity and offering a firsthand glimpse into the thought processes of the learners.

3.8 Data Analysis Procedures

The researcher employed descriptive analysis techniques to scrutinize the data, categorizing student errors and utilizing a coding and tally system to quantify specific error occurrences and opinions. In-depth content analysis was conducted to pinpoint areas of difficulty, address research questions, and identify errors and misconceptions. Visual representations of students' written work were generated, providing comprehensive and detailed data. Interviews were transcribed and analyzed alongside written work, offering a holistic understanding. Focusing specifically on quadratic equation factorization errors, the study employed audio recordings, transcription, and coding to explore student challenges in differentiation questions. This methodology aligns with similar studies analyzing diverse data sources, including test scripts and interviews (Tendere and Mutambara, 2020; Chikwanha et al., 2022; Mudavanhu et al., 2023).

Summary

This chapter looked into the research methodology that underpinned the investigation into students' errors in solving quadratic equations using factorization. The research design, data collection instruments, target population, sample, and sampling technique were all carefully considered and explained. The procedures for collecting data were also outlined in detail. Furthermore, this chapter shed light on the data presentation and analysis procedures, setting the stage for the next chapter, which will dive into the data presentation, interpretation, and analysis, uncovering valuable insights and findings.

Chapter 4

Data presentation, analysis and discussion

4.1 Introduction

This chapter provides an overview of data presentation, analysis and discussion. The purpose of this chapter is to address the research questions: 1. What are the errors exhibited by form 3 learners when solving quadratic equations using factorisation? 2. What is the fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors? This chapter is divided into six sections: introduction, methodology, results, discussion, conclusion, and limitations.

4.2 Methodology

This research used a qualitative approach to investigate how students solve quadratic equations using factorisation, giving a detailed understanding of their problem-solving methods. By using this approach, I gathered detailed information on the mistakes students made when trying to solve quadratic equations using factorisation. Many researchers have successfully used this approach (Chikwanha et al., 2022; Johari & Shahrill, 2020; Msomi & Bansilal, 2022; Siyepu, 2015).

The study involved 75 Form 3 students from a certain High School in Rushinga District. I created a 25-minute test using past exam papers from ZIMSEC, which ensured the test was fair and accurate in assessing students' abilities. The test had five questions on quadratic equations using factorisation, designed to test students' remembering, understanding, applying, and analyzing skills. I gave the test and analyzed the results to identify students' mistakes. I also interviewed five students to understand their thought processes when solving quadratic equations using factorisation. The interviews helped me understand what students need to know to solve these equations successfully.

4.3 Results

4.3.1. What are the errors exhibited by form 3 learners when solving quadratic equations using factorization?

4.3.1.1 Students' response to the first question.

In the first question, a significant disparity in understanding was revealed. While 23 learners demonstrated a solid grasp of the concept, successfully solving the question, the remaining 52 learners struggled, exhibiting a range of conceptual and procedural errors. The question that revealed this divide is presented below, highlighting the specific challenges that tripped up many learners.

1. Solve the equation x^2-5x+6=0

Procedural error for question 1

The analysis revealed that 22 students, similar to Student D exhibited the same errors that is procedural errors when solving the first question.

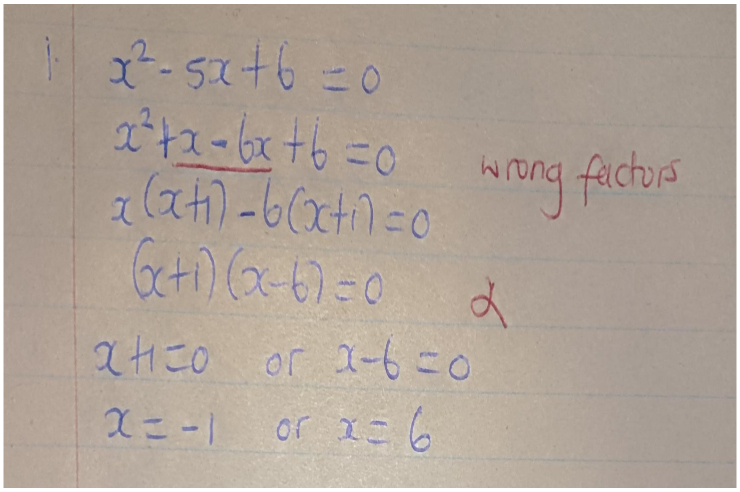


Figure 1: Student D's Problem-Solving Snapshot - Question 1 Response

The learner was knowing how to solve the equation but using wrong factors. He found the factors of the sum and ignored the product. The learner was displaying some procedural errors. The correct factors were -2x and -3x. According to Bloom's Taxonomy of objectives under the cognitive domain, this error falls under the "Comprehension" level (Level 2). The learner struggles to understand the underlying concepts and relationships, despite being familiar with the procedures. A Conversation Unfolds with student D.

Researcher: How did you go about factoring that expression?

Student D: ummmm I found 2 terms that when I add them I get -5x which are -6x and 1x

Researcher: What was your next step?

Student D: Then I factorise by grouping.

The conversation with student D shows that the error of Student D is the procedural error of focusing on finding terms that add up to -5x (the coefficient of the linear term) rather than finding factors that multiply to the constant term (+6). This shows a misunderstanding of the factoring concept, as factoring involves finding numbers whose product is the constant term, not finding terms whose sum is the coefficient of the linear term. In this case, Student D should have been looking for factors whose product is $6x^2$, such as -2x and -3x, rather than only focusing on the sum of -6x and 1x.

Conceptual error for question 1

Thirty learners exhibited conceptual errors.

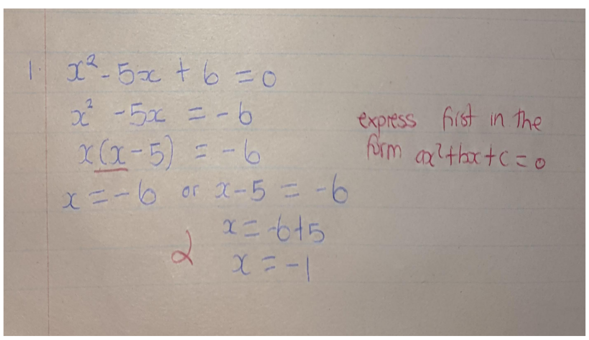


Figure 2: student A response to question 1

The learner was struggling with a fundamental concept in algebra, specifically when solving quadratic equations. She did not understand that one side of the equation needs to be set equal to zero, which is a crucial step in the problem-solving process of quadratic equation. This misunderstanding indicates a conceptual error, meaning that the learner has a gap in their understanding of the underlying mathematical concepts. According to Bloom's Taxonomy of objectives under the cognitive domain, this error falls under the "knowledge" level (Level 1), which represents the most basic level of understanding, where learners are still developing their knowledge and comprehension of key concepts. Addressing this conceptual error is crucial to build a strong foundation in algebra and progress to higher levels of understanding, application, and problem-solving skills. A conversation unfolds with student A.

Researcher: What kind of equation is this?

Student A: I don't know.

Researcher: How did you go about factoring that expression?

Student A: I grouped the numbers and factorise the left side.

Researcher: What was your next step?

Student: I equate the factor to -6

This conversation between the researcher and Student A reveals several errors and misconceptions in the learner's understanding of algebra. Student A is unable to identify the type of equation, demonstrating a knowledge gap, and subsequently uses an incorrect problem-solving approach by grouping numbers and factoring the left side. Furthermore, they equate the factor to -6, showcasing a conceptual error by failing to set the factored expression equal to zero, a crucial step in solving quadratic equations.

4.3.1.2 Students' response to the second question.

Twenty learners wrote the second question correctly. The other fifty-five learners displayed conceptual, technical and procedural errors. Question 2 is exhibited below

2. Solve the equation $x^2+5x=24$

Technical error for question 2

Nine learners exhibited technical errors just like student F did.

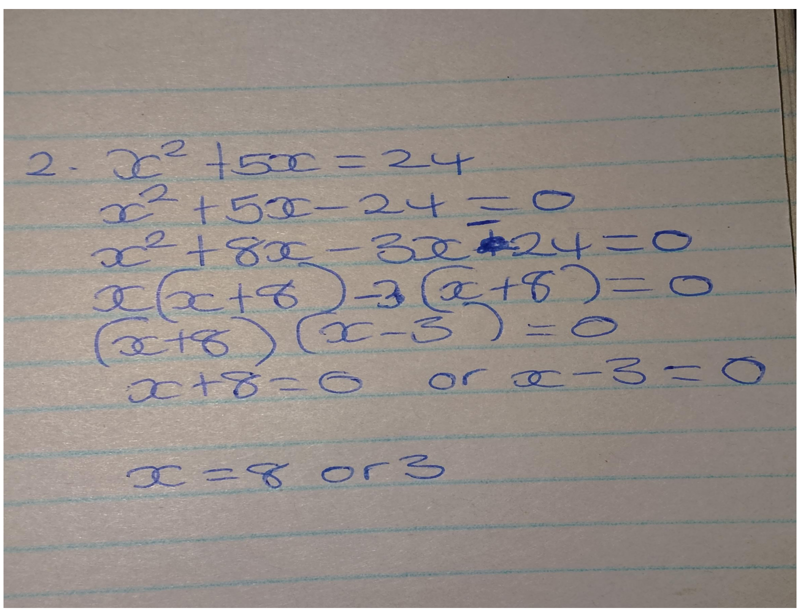


Figure 3: Student F's Problem-Solving Snapshot - Question 2 Response

The learner's technical error resulted in an incorrect solution, obtaining x = 8 or x = 3 instead of the correct solutions x = -8 or x = 3. This type of error is categorized as a "technical error indicating a mistake in the execution of a mathematical procedure or algorithm. In this case, the learner likely made a mistake when applying the rules of algebra, such as incorrectly handling the sign of the terms or making a calculation mistake. According to Bloom's Taxonomy of objectives under the cognitive domain, this error falls under "Application" (Level 3). This level involves the ability to apply learned concepts, procedures, and algorithms to solve problems and complete tasks. The learner has demonstrated a technical error in applying the rules of algebra, specifically in handling the sign of terms or making a calculation mistake, which is a characteristic of errors at the Application level. A conversation unfolds with student F.

Researcher: What kind of equation is this?

Student F: Quadratic equation

Researcher: What does it mean when an equation is quadratic?

Student F: A quadratic equation has a highest power 2 and has 2 solutions.

Researcher: How did you solve this?

Student F: I arrange the equation equal to 0 and then factorise.

Researcher: How did you get 8?

Student F: ummmm I made a mistake I wrote 8 instead of -8.

This conversation indicates that Student F is making a technical error, specifically a calculation error or a mistake in executing a mathematical procedure. Student F demonstrates an understanding of the concept of quadratic equations and the correct procedure for solving them (factoring), but makes a mistake in the calculation, writing 8 instead of -8. This type of error is characterized by a mistake in the execution of a mathematical procedure or algorithm, rather than a misunderstanding of the underlying concept.

Conceptual errors for question 2

Twenty-seven learners exhibited conceptual errors.

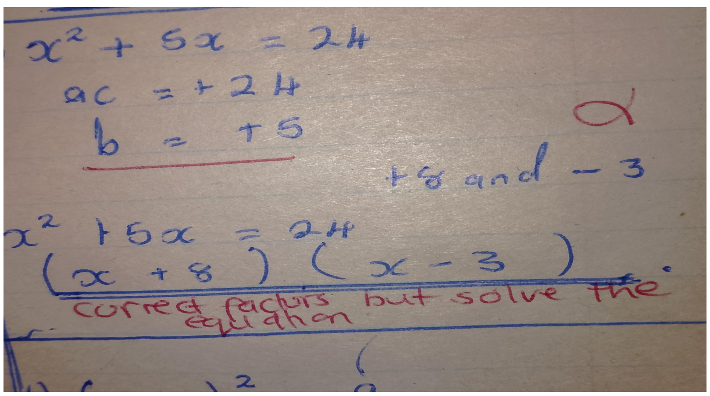


Figure 4: Student G's Problem-Solving Snapshot - Question 2 Response

The learner demonstrates a procedural understanding of factorization, correctly factoring the expression as (x + 8)(x - 3), but displays conceptual errors and a limited understanding of solving equations and algebraic concepts, failing to recognize the need to equate the expression to zero, instead solely relying on factorization, revealing a gap in her understanding of the fundamental principles of equation solving. According to Bloom's Taxonomy of objectives under the cognitive domain, this learner's errors fall under the "Understanding" level (Level 2), indicating a lack of comprehension of the underlying concepts and principles of equation solving, and a need to develop a deeper understanding of algebraic concepts and problem-solving strategies to progress to the "Application" level (Level 3) where they can apply their knowledge.

Procedural errors in question 2

Nineteen learners demonstrated a familiarity with the procedures for solving quadratic equations when tackling question 2, specifically the equation $x^2 + 5x = 24$. However, their responses revealed a crucial mistake - they consistently confused the signs when transposing the constant term (+24) to the other side of the equation, indicating a lapse in attention to detail and a need to reinforce the importance of precise calculations in algebraic manipulations.

4.3.1.3 Students' response to the third question.

Fifteen learners wrote the third question correctly. The other sixty learners exhibited conceptual errors. Question 3 is exhibited below.

3. A garden has an area of 33 square meters. Its length is 5 meters more than certain number x and width is 3 meters less than a certain number x. What are the possible values of x?

Conceptual errors for question 3.

The analysis revealed that most students exhibited conceptual errors, demonstrating a fundamental misunderstanding of quadratic functions. Specifically, sixty students struggled to translate word problems into algebraic equations, failing to grasp the underlying mathematical concepts. As a result, they formulated incorrect quadratic equations, highlighting a gap in their understanding of how to represent real-world scenarios mathematically.

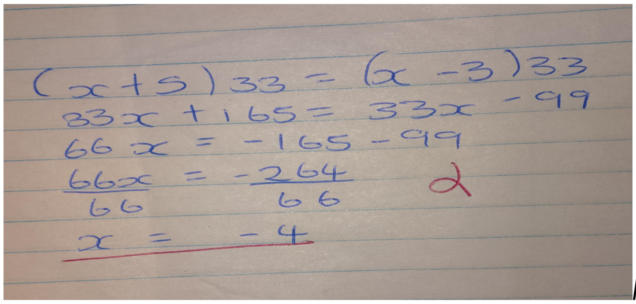


Figure 5: Student M's Problem-Solving Snapshot - Question 3 Response

Student M exhibited conceptual error because she failed to make a quadratic equation. It suggests that students need targeted support to develop a deeper understanding of quadratic functions and how to apply them to solve problems, rather than just memorizing procedures. According to Bloom's Taxonomy of objectives under the cognitive domain, this type of error indicates that students are struggling to reach the "Application" level (Level 3), where they can apply their knowledge of quadratic functions to solve problems and complete tasks, revealing a need for support to move beyond mere comprehension and develop the ability to apply mathematical concepts in practical situations.

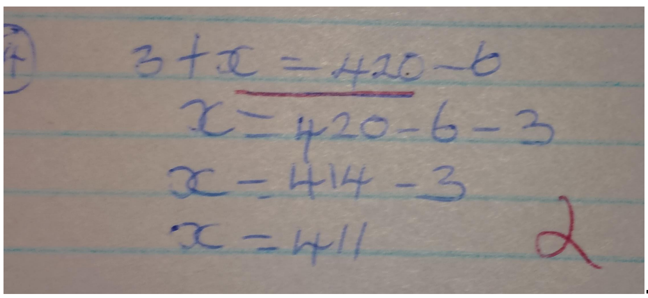
4.3.1.4 Students' response to the fourth question.

Two learners wrote the fourth question correctly. The other seventy-three learners exhibited conceptual and procedural errors. Question 4 is exhibited below:

4. The father is three times older than his daughter. If we multiply their ages from six years ago, we get 420. Can you figure out how old they are today?

Conceptual errors for question 4

All the learners wrote the fourth question incorrectly. They exhibited conceptual errors. According to Bloom's Taxonomy of objectives under the cognitive domain, the type of errors indicate that all students are struggling to reach the analysis level (level 4) because they are having difficulty breaking down complex information, identifying patterns, and making connections to solve problems or understand concepts. All learners failed to set the correct algebraic presentation and equation.



. *Figure 6: Student F's Problem-Solving Snapshot - Question 4 Response*

Student F exhibited conceptual errors in their solution because they misinterpreted the "three times younger" relationship, setting up an incorrect equation (3+x), and failed to understand the "six years ago" clause, subtracting 6 from the product of their ages (420-6) instead of considering the ages at that time. Additionally, they incorrectly applied algebraic operations, subtracting 3 from 420-6 instead of dividing the result by 3. These errors demonstrate a lack of understanding of the problem's context, algebraic representations, and mathematical relationships, leading to an incorrect solution, and highlighting the need for guidance on correctly interpreting the problem and setting up and solving algebraic equations accurately.

4.3.1.5 Students' response to the fifth question

All students wrong the fifth question incorrectly. They exhibited conceptual errors. According to Bloom's Taxonomy of objectives under cognitive domain, the learners failed to have knowledge on general form of quadratic equation and to analyze the question. All 75 learners failed to understand the demand of the question. The question 5 is exhibited below

5. The solutions x = -1 and x = 3 are given. Can you find the quadratic equation that matches these solutions?

Conceptal errors on question

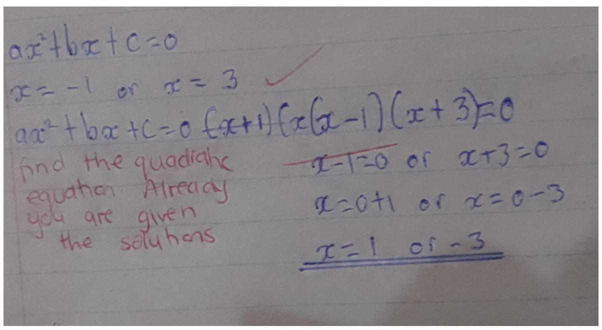


Figure 7: Student Z2's Problem-Solving Snapshot - Question 5 Response

Student Z2 failed to understand the demand of the question, leading to conceptual errors. Instead of formulating the quadratic equation using the given roots (x = -1 and x = 3), Student Z2 attempted to find the roots again, demonstrating a lack of understanding of the question's requirements. This indicates a deficiency in the analysis level of Bloom's Taxonomy, where students are expected to break down complex information, identify patterns, and make connections. By not recognizing the need to formulate the equation, Student Z2 showed a lack of analytical skills, highlighting the need for targeted instruction to develop this critical cognitive domain. The conversation unfolds with student Z2.

Researcher: Read the question please

Student Z2: He reads the question fluently

Researcher: What is the demand of the question?

Student Z2: To solve the equation

Researcher: What is the general form of quadratic equation

Student Z2: ummmmm I don't know.

Researcher: How did you solve this?

Student Z2: I found the solutions of the quadratic equation

The conversation with student Z2 indicated that student Z2 is unable to write a quadratic equation given the roots reveals errors in both Knowledge (Level 1) and Analysis (Level 4) under Bloom's Taxonomy of objectives in the cognitive domain. The student's lack of understanding of the question's demand indicates a deficiency in Knowledge, as they failed to recall the concept of forming a quadratic equation from its roots. Furthermore, their inability to apply this knowledge to formulate the equation demonstrates an error in Analysis, where they struggled to break down the information, identify patterns, and make connections to solve the problem. This highlights the need for targeted instruction to address both knowledge gaps and analytical skills.

4.3.2 Fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors.

Through a comprehensive analysis of student responses on assessment test scripts and follow-up interviews, researchers identified five essential categories of pre-requisite knowledge crucial for learners to attain a comprehensive understanding of quadratic equations using factorisation. These categories include: directed numbers, basic algebra, factorisation, linear equations and problem solving skills.

The data showed that a solid understanding of directed numbers is essential, as seen in the following two excerpts, which highlight the importance of this concept as a prerequisite for further learning.

Researcher: Hello, what challenges do you face when solving quadratic equations using factorisation?

Learner: directed numbers

Researcher: What are directed numbers

Learner: Directed numbers? I'm not sure what that is. I think it might have something to do with negative numbers.

Researcher: What about positive numbers?

Learner: I'm quite not sure.

This conversation indicates that the learner has a partial understanding of directed numbers, associating them primarily with negative numbers but not having a clear grasp of the concept as a whole, including positive numbers. This lack of comprehensive knowledge suggests a need for further explanation and instruction on directed numbers before proceeding to more advanced topics like solving quadratic equations that involve directed numbers.

Secondly, the data revealed that a strong foundation in basic algebra is crucial, as evident in the following two excerpts, which highlight the importance of this concept as a prerequisite for further learning.

Researcher: What topics should be covered before learning quadratic equations using factorization, and how can consistent practice help?

Learner: Algebra and equations.

Researcher: Can you describe the concept of basic algebra to me?

Learner: Basic algebra? I'm a bit fuzzy on that. I think it has something to do with solving for x.

Researcher: That's a good start! How about expanding and simplifying algebraic expressions?

Learner: Honestly, I'm not really sure how to do that.

This conversation indicates that the learner has a limited understanding of basic algebra, grasping the general idea of solving for a variable but struggling with more complex concepts like expression manipulation. This knowledge gap suggests a need for additional instruction and practice in basic algebra before moving on to more advanced topics like quadratic equations.

Thirdly, the data revealed that a strong foundation in factorisation is crucial, as evident in the following two excerpts, which highlight the importance of this concept as a prerequisite for further learning.

Researcher: What prior knowledge is essential for solving quadratic equations using factorization?

Learner: Factorisation

Researcher: Can you explain the process of factorising a quadratic expression?

Learner: Factorising? I'm not entirely sure... I think it involves finding two numbers that multiply to give the last term, and add up to give the coefficient of the middle term.

Researcher: That's close! How about factorising expressions with negative coefficients, like x^2-5x+6?

Learner: Hmm... I'm really not sure. I always get confused when the signs are different.

This conversation indicates that the learner has a partial understanding of factorisation, grasping the general idea of finding two numbers that multiply and add up to certain values, but struggling with more complex scenarios involving negative coefficients. This knowledge gap suggests a need for additional instruction and practice in factorisation before moving on to more advanced topics like solving quadratic equations.

Fourthly, the data revealed that a strong foundation in linear is crucial, as evident in the following two excerpts, which highlight the importance of this concept as a prerequisite for further learning.

Researcher: What prerequisite skills necessary to solve quadratic equations.

Learner: Linear equations

Researcher: Are you able to solve linear equations

Learner: I've worked with linear equations in my math class and feel comfortable solving them.

Researcher: Excellent! Solving linear equations is a fundamental skill required to solve quadratic equations. Can you demonstrate how you would solve a simple linear equation like 2x + 3 = 7?

Learner: Sure, I'd subtract 3 from both sides to get 2x = 4, then divide both sides by 2 to solve for x, which equals 2.

Researcher: solve (x+3)(x-2)=0

Learner: I use linear equations x+3=0   x=-3. x-2=0 x=2.

Researcher: That's good.

The learner possesses a solid understanding of linear equations, demonstrated by their ability to solve simple linear equations, such as 2x + 3 = 7, with ease and confidence. They have a strong foundation in this area, which is evident in their ability to apply linear equation skills to solve quadratic equations through factorization, as seen in their solution to the equation (x+3)(x-2)=0. The learner successfully identifies the two linear equations, x+3=0 and x-2=0, and solves them to find the values of x, (-3 and 2 respectively). This showcases their ability to build upon their linear equation knowledge to tackle more complex quadratic equations, highlighting their readiness for further learning and development in mathematics.

Fifthly, the data revealed that a strong foundation in problem-solving skills is crucial, as evident in the following two excerpts, which highlight the importance of this concept as a prerequisite for further learning.

Researcher: Can you walk me through your thought process when approaching a complex math problem?

Learner: Problem-solving? I'm not entirely sure... I usually try to find a formula or equation that fits the situation, but sometimes I get stuck.

Researcher: That's a good start! How about when the problem involves multiple steps or unknowns, like solving a system of equations?

Learner: Hmm... I'm really not sure. I often get overwhelmed and don't know where to begin.

This conversation indicates that the learner has a partial understanding of problem-solving skills, grasping the general idea of applying formulas and equations, but struggling with more complex scenarios involving multiple steps or unknowns. This knowledge gap suggests a need for additional instruction and practice in problem-solving skills before moving on to more advanced topics like quadratic equations.

4.4 Discussion

The purpose of this study was to investigate the errors exhibited by form three learners when solving quadratic equations using factorization, as well as the fundamental knowledge needed for this process. The research questions that guided this study were:

1. What are the errors exhibited by form 3 learners when solving quadratic equations using factorisation?

2. What is the fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors?

4.4.1 Errors exhibited by Form Three Learners when Solving Quadratic Equations using Factorization

The findings of this study indicate that form three learners exhibited several common errors when attempting to solve quadratic equations using factorization. These errors included: procedural, conceptual and technical errors.

Learners exhibited several procedural errors when solving quadratic equations using factorization. One common error was the inability to correctly identify the appropriate factors of the quadratic expression, a crucial step in the factorization process. Additionally, mistakes were made in the actual factorization process, such as incorrect placement of the factors or sign errors. Furthermore, learners struggled to connect the factorized form to the process of solving the quadratic equation, indicating a lack of understanding of the procedural steps involved.

Conceptual errors were also prevalent, revealing a lack of understanding of the fundamental concepts underlying quadratic equations. Many learners failed to grasp the need to set one side of the equation equal to zero, a basic concept in solving quadratic equations. Moreover, difficulties in translating word problems into the corresponding quadratic equations revealed gaps in understanding how to represent real-world scenarios mathematically. Learners also struggled to analyze complex information, identify patterns, and make connections to solve problems, indicating deficiencies in higher-order cognitive skills.

Technical and calculation errors were also observed, including mistakes in the execution of algebraic procedures and calculations. Learners made errors such as incorrectly handling the signs of terms or making calculation errors, demonstrating a lack of technical proficiency in algebraic manipulations. These errors highlight the need for learners to develop a strong foundation in algebraic procedures and calculations to succeed in solving quadratic equations using factorization.

The findings of this study align with the existing literature, which highlights learners' struggles with the conceptual understanding of factorization and its relationship to solving quadratic equations (Msomi and Bansilal, 2022; Tendere and Mutambara, 2020; Mutambara and Bansilal, 2021). The errors which are consistent with literature review include procedural, technical, and conceptual errors, are consistent with the common challenges reported in previous research. However, this study takes it a step further by revealing that the learners' errors can be attributed to various levels of understanding according to Bloom's Taxonomy of educational objectives in the cognitive domain. Specifically, the errors range from knowledge-level gaps (e.g., recognizing the type of equation) to higher-order analytical skills (e.g., breaking down complex information and making connections), which is a novel contribution to the existing body of research.

4.4.2 Fundamental Knowledge Needed when Solving Quadratic Equations using Factorization

The findings of this study emphasize the critical importance of ensuring that learners have a strong grasp of the prerequisite knowledge necessary for developing a comprehensive understanding of quadratic equations using factorization. The five essential categories identified - directed numbers, basic algebra, factorization, linear equations and problem-solving skills - are all fundamental building blocks that must be firmly in place for learners to successfully navigate the complexities of working with quadratic equations.

The results regarding directed numbers reveal that learners' understanding of positive and negative numbers is often partial, with a primary association with negative numbers. This suggests that more explicit instruction and practice are needed to solidify learners' conceptual grasp of the entire directed number system. Strengthening this foundation will better equip them to work with the positive and negative coefficients inherent in quadratic equations.

Similarly, the limitations observed in learners' basic algebra skills, particularly in manipulating algebraic expressions and solving for variables, highlight the need for additional targeted instruction and practice in these fundamental algebraic concepts. A robust understanding of basic algebra is a prerequisite for successfully applying algebraic methods to the solutions of quadratic equations.

The findings related to factorization skills also point to the need for more comprehensive instruction and practice, as learners struggled with more complex scenarios involving negative coefficients. Mastering the ability to identify the appropriate factors that satisfy the conditions of a quadratic expression is a crucial step in being able to employ factorization as a problem-solving strategy.

The study notes that oearners need linear equations as a prerequisite for solving quadratic equations using factorization because linear equations provide the foundation for understanding equations, introduce algebraic skills, and serve as building blocks for factoring quadratic expressions. Mastering linear equations enables learners to recognize and factor simple quadratic expressions, factor more complex ones by identifying linear factors, and solve quadratic equations by setting each factor equal to zero.

Finally, the study's insights regarding learners' partial understanding of problem-solving skills, particularly in applying formulas and equations to multi-step or unknown-laden scenarios, underscore the importance of developing strong problem-solving abilities. These skills are essential for translating real-world quadratic problems into mathematical representations and then utilizing appropriate strategies to reach viable solutions.

The findings of this study align with existing literature, which highlights that solving quadratic equations using factorization requires specific fundamental knowledge areas, including directed numbers, basic algebra, factorization, and problem-solving skills (Ly and Takuya, 2023; Tendere and Mutambara, 2020). This means that the study's results are consistent with previous research, which has also identified these knowledge areas as essential for successfully solving quadratic equations using factorization. However, the study also reveals that solving linear equations is another crucial fundamental knowledge area needed to solve quadratic equations, emphasizing the importance of a strong foundation in this skill as well.

4.4.3 Implications and Significance of the Findings

The implications and significance of this study are multifaceted. Firstly, the study provides a detailed analysis of the specific errors exhibited by form three learners when solving quadratic equations using factorization, including procedural, conceptual, and technical errors. This taxonomy of errors can help educators better understand the areas of difficulty for students and design targeted interventions.

Secondly, the study highlights the fundamental knowledge areas that are essential for effectively solving quadratic equations using factorization, including directed numbers, basic algebra, factorization skills, and problem-solving abilities. The findings emphasize the need to ensure students have a solid grasp of these prerequisite concepts before attempting more advanced topics.

Thirdly, the results of this study are consistent with previous research, which also identified similar challenges and knowledge gaps among students solving quadratic equations. This reinforces the need to address these common issues in education.

Fourthly, the study goes beyond just identifying errors, by also examining them through the lens of Bloom's Taxonomy. This provides a more nuanced understanding of the cognitive skills required, ranging from basic knowledge to higher-order analytical abilities. This can inform the design of instructional strategies that target different levels of understanding.mathematics

Fifthly, the findings can guide teachers in developing more effective instructional approaches for teaching quadratic equations. This includes focusing on strengthening the fundamental knowledge areas, providing targeted practice, and scaffolding instruction to address the various error types and cognitive levels observed.

Lastly, by exploring the specific errors and prerequisite knowledge needed for solving quadratic equations using factorization, this study adds to the existing body of research in mathematics education. The insights generated can inform future studies and help develop a more comprehensive understanding of the challenges students face in this domain. In summary, this study has important implications for improving mathematics instruction and student learning, particularly in the context of solving quadratic equations.

4.5 Conclusion

The main findings of this study reveal that Form Three learners exhibit various errors when solving quadratic equations using factorization. These errors encompass procedural, technical, and conceptual mistakes. Notably, the study discovers that learners' errors can be attributed to different levels of understanding, ranging from knowledge-level gaps, such as recognizing equation types, to higher-order analytical skills, like breaking down complex information and making connections, as categorized by Bloom's Taxonomy. This discovery offers a nuanced understanding of learners' errors and highlights the need for targeted instruction and practice to address these knowledge gaps and enhance learners' proficiency in mathematics.

The study's findings reveal that solving quadratic equations using factorization requires specific fundamental knowledge areas, including directed numbers, basic algebra, factorization, and problem-solving skills. Additionally, the study highlights the crucial importance of a strong foundation in solving linear equations as a necessary precursor to successfully solving quadratic equations. These findings emphasize the need for learners to develop a comprehensive understanding of these fundamental knowledge areas to navigate the complexities of quadratic equations and their factorization.

These findings address the research questions, identifying the errors exhibited by Form Three learners and the fundamental knowledge necessary for effective solution using factorization, highlighting the need for targeted instruction and practice to address knowledge gaps and enhance learners' proficiency in mathematics.

4.6 Limitations

This study had some limits. We only looked at a small group of students from one school. To make the results more widely applicable, future studies should include more students and multiple schools. Additionally, future research could investigate which teaching methods work best to help students avoid mistakes when solving quadratic equations using factorization.

Summary

This chapter presents the findings from a qualitative data analysis, which identified and classified errors made by Form Three learners when solving quadratic equations using factorization. The data was collected and analyzed using a systematic approach, and the errors were categorized according to Bloom's Taxonomy of objectives under the cognitive domain. The next chapter is going to look at summary, conclusions and recommendations.

CHAPTER 5: SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

This chapter seeks to give a summary of the research, recommendations and conclusions on analysis of students' errors when solving quadratic equations using factorisation.

5.2 Summary

This study is about "analysis of students' errors when solving quadratic equations using factorisation." Chapter 1 introduces the research study on analyzing learner errors in solving quadratic equations using factorization. The background of the study highlights the importance of quadratic equations in the mathematics curriculum and the challenges learners face in solving them. The purpose of the study is to identify and analyze the specific errors made by Form 3 learners in a certain school in Rushinga District when solving quadratic equations using factorization. The research objectives include establishing the errors made by learners, assessing the fundamental knowledge necessary for effective solution, and identifying the underlying causes of errors. The study aims to provide valuable insights for teachers, policymakers, and researchers to develop targeted instructional approaches and interventions to improve learner understanding and performance in mathematics. The chapter also outlines the significance of the study, limitations, delimitations, definition of key terms, and the organization of the study.

Chapter 2 provides a comprehensive review of the literature related to the research study on analyzing learner errors in solving quadratic equations using factorization. The theoretical framework for the study is based on Bloom's Taxonomy of objectives, which provides a hierarchical classification of learning objectives in the cognitive domain. The literature review highlights the common errors made by learners when solving quadratic equations, including procedural, technical/foundational, computational, and conceptual errors. The study also identifies the fundamental knowledge necessary for learners to effectively solve quadratic equations using factorization, including a solid foundation in basic algebraic concepts, factoring skills, number sense, problem-solving strategies, and familiarity with mathematical notation and terminology. The literature review also discusses various studies that have analyzed student errors in solving quadratic equations, shedding light on the misconceptions and challenges that students face in mastering the concept. The studies highlight the importance of targeted interventions and instructional strategies to address learners' weaknesses in prerequisite skills and concepts, such as explicit instruction, guided practice, and frequent review. The chapter concludes by emphasizing the need for a comprehensive approach to address learner errors and improve student understanding and problem-solving skills in mathematics.

Chapter 3 outlines the research methodology used to analyze learner errors in solving quadratic equations using factorization. The study adopted a qualitative approach, using a test and follow-up interviews to collect data from a sample of 5 Form 3 learners (3 females and 2 males) selected from a target population of 75 learners. The research design allowed for an in-depth examination of learner errors and challenges. Data collection involved administering a test to identify error areas and conducting interviews to gather learners' thoughts and opinions. Data presentation and analysis procedures included thematic analysis, coding, and content analysis to identify recurring themes, patterns, and areas of difficulty. The study aimed to provide a comprehensive understanding of learner errors and challenges in solving quadratic equations using factorization.

Chapter 4 presents the findings of a study on the errors made by Form Three learners when solving quadratic equations using factorization, revealing various errors including procedural, technical, and conceptual mistakes. The errors were categorized according to Bloom's Taxonomy, ranging from knowledge-level gaps to higher-order analytical skills. The study identified five essential categories of pre-requisite knowledge crucial for learners to attain a comprehensive understanding of quadratic equations using factorization: directed numbers, basic algebra, factorization, linear equations, and problem-solving skills. The findings suggest targeted instruction and practice are necessary to address knowledge gaps and enhance learners' proficiency in mathematics, with limitations including a small sample size and single-school setting, highlighting the need for future research to increase generalizability and explore effective teaching strategies to reduce learners' errors. The study's main findings include: Form Three learners exhibit various errors when solving quadratic equations using factorization; learners' errors can be attributed to different levels of understanding, ranging from knowledge-level gaps to higher-order analytical skills; solving quadratic equations using factorization requires specific fundamental knowledge areas; and targeted instruction and practice are necessary to address knowledge gaps and enhance learners' proficiency in mathematics.

Chapter five gives summary, conclusions and recommendations of the study. A summary is for summarising the main points of each chapter, the conclusions are for giving final comments or judgements on the findings of the research and finally the recommendations which are suggestions or strategies that may be put in place in order to deal with analysis of students' errors when solving quadratic equations using factorisation.

5.3 Conclusions

The present study, which investigated Form 3 learners' errors when solving quadratic equations using factorization, aligns with the scholarly work of Tendere and Mutambara (2020), who emphasized the significance of addressing knowledge gaps and enhancing learners' proficiency in mathematics. The findings revealed a range of errors, including procedural, technical, and conceptual mistakes, which corroborate the assertions of Msomi and Bansilal (2022) that targeted instruction and practice are crucial for improving learners' understanding and application of mathematical concepts.

The study identified five essential categories of pre-requisite knowledge necessary for learners to effectively solve quadratic equations using factorization while minimizing errors. These fundamental knowledge areas include directed numbers, basic algebra, factorization, linear equations, and problem-solving skills, which are foundational to more complex mathematical concepts. By building a strong foundation in these areas, educators can help learners develop a deeper understanding of quadratic equations and factorization, leading to improved proficiency in mathematics.

The study's conclusions support the scholarly consensus that a structured and progressive approach to mathematics education is essential for learners' development (Mbewe and Nkhata, 2019). By understanding the specific errors learners make and the fundamental knowledge areas necessary for success, educators can develop effective teaching strategies to support learners' development in mathematics. This can lead to improved academic achievement and a stronger foundation in mathematics, preparing learners for future success (Mutambara and Bansilal, 2021).

The present study contributes to the existing body of research on mathematics education, highlighting the need for targeted instruction and practice to address knowledge gaps and enhance learners' proficiency in mathematics. By integrating the findings of this study with the scholarly work of Tendere (2020), educators can develop a comprehensive understanding of the essential knowledge areas and effective teaching strategies necessary for learners' success in mathematics.

5.4 Recommendations

5.4.1 Recommendations for Practice:

1. Educators should provide targeted instruction and practice to address knowledge gaps and enhance learners' proficiency in mathematics, particularly in areas such as directed numbers, basic algebra, factorization, linear equations, and problem-solving skills:

- Many students struggle with fundamental mathematical concepts like directed numbers (positive and negative numbers), basic algebra, and solving linear equations. Providing targeted instruction and practice in these areas can help build a stronger foundation, which is crucial for progressing to more advanced topics.
- Factorization is an essential skill for working with polynomials and solving more complex equations. Targeted practice and instruction in factorization can help students develop this important technique.

- Problem-solving skills are vital for applying mathematical knowledge to real-world situations. Educators should focus on teaching problem-solving strategies and providing opportunities for students to practice solving a variety of problem types.

2. Teachers should use a structured and progressive approach to mathematics education, building a strong foundation in essential skills and concepts before advancing to more complex mathematical concepts:

- Mathematics is a highly sequential subject, with each new concept building upon the previous ones. It's important for educators to ensure that students have a solid grasp of the foundational skills and concepts before moving on to more advanced topics.
- A structured and progressive approach allows students to develop a deep understanding of the underlying principles and build the necessary skills to tackle increasingly complex mathematical problems.
- This approach helps prevent knowledge gaps and ensures that students are well-equipped to succeed as they progress through the curriculum.

3. Educators should use formative assessments to identify learners' errors and misconceptions and adjust their instruction accordingly.

- Formative assessments, such as quizzes, classwork, and informal checks for understanding, allow educators to continuously monitor student progress and identify areas where students are struggling:

- By identifying common errors and misconceptions, teachers can adjust their instruction, provide targeted support, and address the underlying issues before they become more deeply ingrained.
- This approach helps ensure that students have a solid grasp of the material before moving on, rather than allowing gaps to accumulate over time.

5.4.2 Recommendations for Further Study:

1. A similar study should be conducted using a longitudinal design to determine if changes over time become perceptible and to assess the effectiveness of targeted instruction and practice in addressing knowledge gaps and enhancing learners' proficiency in mathematics:

- A longitudinal study would involve following a group of students over an extended period, such as several years, to observe how their mathematical knowledge and skills develop.
- This type of study would provide more insight into the long-term impacts of the recommended instructional practices and allow researchers to assess whether the targeted interventions lead to sustained improvements in student proficiency.
- Longitudinal data can also help identify patterns and trends that may not be immediately apparent in a single, cross-sectional study.

2. Future research could explore the use of technology and digital resources to support learners' development in mathematics, particularly in areas such as problem-solving skills and critical thinking.

- Technology and digital tools, such as educational software, online simulations, and interactive learning platforms, can provide engaging and personalized learning experiences for students.
- These resources can be particularly helpful in developing problem-solving skills and critical thinking, as they can offer dynamic, visual representations of mathematical concepts and provide students with opportunities to practice and apply their knowledge in diverse contexts.
- Investigating the effectiveness of integrating technology and digital resources into mathematics instruction could lead to insights on how to leverage these tools to enhance student learning and engagement.

3. A study could be conducted to investigate the impact of collaborative learning and peer support on learners' understanding and application of mathematical concepts, particularly in areas such as quadratic equations and factorization:

- Collaborative learning, where students work together in small groups to solve problems and discuss mathematical concepts, can foster a deeper understanding and application of the material.
- Peer support and peer-to-peer learning can be especially valuable in areas like quadratic equations and factorization, where students may benefit from explaining concepts to one another and learning from each other's perspectives.

- Researching the impact of collaborative learning on student outcomes in these specific mathematical domains could provide insights into effective instructional strategies and the role of peer interaction in enhancing mathematical proficiency.

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APPENDICES

Appendix 1

This assessment will not affect your grades or progress in the course. Its purpose is to help you with algebra by showing your teacher where you need extra support. They can then help you fix mistakes and understand the thinking behind them.

Instructions:

1. Answer all questions.

2. Use factorisation to solve all the problems.

3. Do not write your original name. Just write a pseudo name

4. Duration: 25 minutes

STUDENT NAME

---

1. Solve the equation $x^2-5x+6=0$
2. Solve the equation $x^2+5x=24$
3. A garden has an area of 33 square meters. Its length is 5 meters more than certain number x and width is 3 meters less than a certain number x. What are the possible values of x?

4. The father is three times older than his daughter. If we multiply their ages from six years ago, we get 420. Can you figure out how old they are today?.
5. The solutions x = -1 and x = 3 are given. Can you find the quadratic equation that matches these solutions?

Note. Adopted from ©ZIMSEC PAST EXAMINATIONS PAPERS

APPENDIX 2

INTERVIEW GUIDE FOR THE LEARNERS

Introduction

Chipo Teketani, a researcher exploring how students like you solve quadratic equations using factorization. I'd love to chat with you about your experiences and thoughts on this topic. By sharing your insights, you'll help me understand the common mistakes students make and how to address them. Your input will make a big difference in my research! It's a great opportunity to share your voice and help improve math learning. Let's chat and explore this topic together!

Questions

1. What kind of equation is this?
2. What does it mean when an equation is quadratic?
3. How did you go about factoring that expression?
4. What was your next step?
5. What challenges do you face solving quadratic equations using factorization?

6. What prior knowledge is essential for solving quadratic equations using factorization?
7. Can you walk me through your thought process when approaching this mathematics problem?
8. What topics should be covered before learning quadratic equations using factorization, and how can consistent practice help?

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BINDURA UNIVERSITY OF SCIENCE EDUCATION

Date: 19/03/2024

TO WHOM IT MAY CONCERN

NAME: TEKETANI CHIPO REGISTRATION NUMBER: B220256B

PROGRAMME: HBScEdMt PART: 2.1

This memo serves to confirm that the above is a bona fide student at Bindura University of Science Education in the Faculty of Science Education.

The student has to undertake research and thereafter present a Research Project in partial fulfillment of the HBScEdMt programme. The research topic is:

Analysis of student's errors in solving quadratic equations using factorisation: A case study of form 3 learners at Rushinga High School.

In this regard, the department kindly requests your permission to allow the student to carry out ~~his~~/her research in your institutions.

Your co-operation and assistance is greatly appreciated.

Thank you

Z Ndemo (Dr.)
CHAIRPERSON - SAMED