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ERRORS AND MISCONCEPTIONS DISPLAYED BY PUPILS IN THE LEARNING OF TRANSFORMATION

BY

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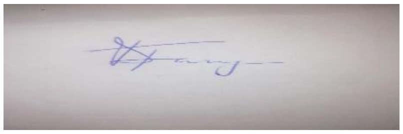
DECLARATION

I Dangiri Tinashe, declare that *Errors and Misconceptions held by students in the learning of transformation geometry at one school in zone B in Insiza District* is my own work and that all the sources that I have used or quoted in the study have been indicated and acknowledged by complete references.



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This research work was undertaken under supervision of Dr Y. Mudavanhu

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APPROVAL FORM

The undersigned certify that they have supervised , read and recommend to Bindura university of science education for acceptance a dissertation entitled Errors and Misconceptions held by pupils in the learning of transformation geometry. Submitted by Dangiri Tinashe (B225540B) in partial fulfilment of the requirements of Honours Bachelors Degree in Science Education mathematics.

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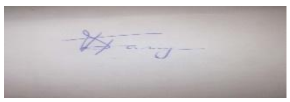
Signature  Date …10…/……06…./…2024………

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DEDICATION

This study is dedicated to my lovely wife Dorothy Chaderopa who encouraged me through all circumstances to reach beyond the stars I am so thankful. To my son Ln D Dangiri you are the reason I have so much courage to move on when the road gets tough. To my dear friends and colleagues I am thankful for all the encouragement you have given me.

ABREVIATIONS/ACCRONYMS

TV- Translation Vector

IM- Image point

HBSCED- Honours Bachelors Science Education Degree

ABSTRACT

This study investigated errors and misconceptions held by O level students in the learning of transformation geometry in Zone B in Insiza District. Students often struggle to grasp the concept of transformation leading to persistent errors and misconceptions. Through a mixed methods approach, questionnaire, interviews and document analysis, the researcher collected data to identify common mistakes and misunderstandings that students hold in transformation topics translation, rotation and reflection. Data was analysed using descriptive statistics and thematic analysis to reveal errors and misconceptions. The findings revealed four primary errors and misconceptions among students namely; students often treat translation vector as matrices leading to incorrect calculations and applications, students frequently interchanged x and y coordinates when plotting points resulting in incorrect graphing and transformation, students commonly mistook the line X = 0 as Y = 0, error of describing reflection as rotation. Teacher participants identified four strategies that can be used to address errors and misconceptions namely: use of traditional teaching methods, use of technology and digital tools during teaching and learning, group work and peer instruction, and hands on activities and manipulatives. The study recommends using visual media in teaching and learning transformation geometry; using various problems of transformation, identifying the correct way of dealing with each problem; using colorful images to help students to recognize the differences between the original objects and image points; and that the government should increase grants given to schools so that the schools can access Wi-Fi and integrate ICT in teaching and learning processes.

The results of this study were expected to provide important insights to educators, policymakers, and local stakeholders seeking to enhance mathematics learning outcomes.

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CHAPTER 1

INTRODUCTION TO THE STUDY

1.1 INTRODUCTION

The background of the problem and sub-problems of the study will be discussed. Research objectives, hypothesis and benefits of the study to the students and teaching fraternity as a whole shall be looked into by the researcher. Assumptions of the study, delimitation and limitation as well as definitions of some key terms will be pointed out. At the end of the chapter, the researcher will give a brief summary to complete the section one of this study.

1.2 BACKGROUND OF THE STUDY

The researcher as an examiner has noticed that pupils are not attempting transformation questions O level examinations. Among those few who attempt the question, they just got marks for plotting the given shapes. This may be caused by pupils' misconception of transformation topic. Only few learners attempt the questions in the external examinations got full marks.

Drew(1987) proposed that misconceptions are widespread and are common among students of all ages and backgrounds, even in subjects like mathematics where there are clear and established principles. They are rooted in cultural or social beliefs or experiences, making them particularly resistant to change. She continues saying misconceptions can be powerful learning tools. When teachers identify and address misconceptions they can use them as a starting point for deeper learning and understanding.

Baldacchino(2014) found that students often confuse transformations with coordinate system, leading to errors in performing transformation. This have significant implications for the teaching and learning of transformation topic. By identifying the misconceptions, teachers can create learning experiences that explicitly address the differences between coordinate system and transformations, which may help students avoid errors and develop a deeper understanding of the concept.

Tsunoda (2013) found that students have difficulties in distinguishing between transformation rules and concrete actions leading to errors in performing transformations. They found that many students tend to see transformation as concrete actions or procedures rather than mathematical operations. For example some students might confuse the rule “rotate clockwise90^0” with the concrete operation of turning the paper clockwise90^0. Brian (1996) added that students often rely on intuition and guesswork when performing transformation of shapes rather than applying a systematic approach.

Mason (1992) discovered that many students struggle to understand the relationship between the original shape and image shape when performing transformations especially rotations and reflections. For example when students asked to perform rotation or reflection of a shape, some students would draw a new shape

During the descriptive questions of the above isometric, pupils are describing rotations of 180 degrees as reflection. They find it difficult to distinguish the two. It is also difficult for them to locate center of rotation using geometric way. There are also missing information in descriptive questions, for example when students are asked to describe a certain transformation he or she did not use specific words like translation, rotation, direction of rotation and reflection Kaiser (2008)

The researcher upon seeing the above grey areas held by students and how delicate the topic in their present and further studies at advanced level is motivated to carry out a study to clear the misconceptions held by the students in the learning of transformation. The idea of helping all pupils in Zimbabwe on the topic is one which arouse interest in the researcher to carry out the study. Transformation is one of the most important topic when building a framework at O level for the further study at other levels in mathematics. Once misconceptions exist at such an early stage of the curriculum, then the whole course at secondary level may tend to be a disaster, hence the researcher felt it worthwhile to carry out this research and help the pupils, teachers and other stakeholders involved.

1.3 PURPOSE OF THE STUDY

The purpose of this study is to investigate the nature of misconceptions displayed by students in the learning of transformation topic and how best they can be addressed and minimized in the classroom situation.

1.4 STATEMENT OF THE PROBLEM

Transformation is an important concept in geometry, yet many students struggle to understand and apply the various types of transformation such as reflections, rotations and translations. Studies have identified common misconceptions that can hinder students learning of this topic. Despite the importance of a clear understanding of transformation, there is limited research on effective methods of identifying and addressing these misconceptions in the classroom. The results of this study will contribute to the existing literature on students' misconception in mathematics and will provide insights into effective strategies for teaching transformation. The findings may inform the development of instructional materials and professional development programs for teachers and ultimately support students learning and understandings of this important mathematical concept.

1.5 RESEARCH QUESTIONS

1.4.1 MAIN RESEARCH QUESTION

What are the errors and misconceptions held by students when learning about transformation?

1.4.2 SUB – RESEARCH QUESTIONS

- What are the sources and nature of errors and misconceptions displayed by students in transformation geometry?
- What are the prerequisite knowledge required to learn transformation?

- What are instructional strategies that can be used to address errors and misconceptions in transformation?

1.5 THE SIGNIFICANCE OF THE STUDY

The significance of this study lies in its potential to provide important insights into how students' misconceptions about transformation can be identified and addressed in the classroom. By gaining a better understanding of the nature and prevalence of these misconceptions, educators can develop more effective instructional strategies and resources to help students overcome them and achieve deeper understanding of the concept.

Furthermore, the findings of this study can inform future research on the teaching and learning of transformations, and may contribute to broader discussions about how best to promote mathematical understanding and reasoning among students.

Additionally, this study has potential implications for educational policy and practice. By identifying specific misconceptions that are common among students, educational policymakers and administrators can make informed decisions about how to design curricula and professional development programs to support teachers in addressing these misconceptions.

Moreover, this research contributes to the growing body of literature on students' misconceptions in mathematics and the importance of addressing them in the teaching and learning process. By highlighting the challenges and opportunities associated with misconceptions in the learning of transformation, this study can help broaden the conversation about how best to support students' mathematical development and understanding.

1.6 ASSUMPTIONS OF THE STUDY

The study makes the following assumptions:

- Misconceptions about transformations are common among students, and are influenced by a variety of factors, including prior knowledge, language, and prior experiences.

- There are instructional strategies and resources that can be effective in addressing misconceptions about transformations.
- The identification and addressing of misconceptions early in the instructional process can lead to improved understanding of transformation.
- Students' understanding of transformations can be measured and evaluated through various assessment methods, such as quizzes, problem sets, and interviews.

1.7 DELIMITATIONS OF THE STUDY

The study is limited to one secondary school. It is also limited to pupils who are following the main stream educational programs (special students are not included). The study also focus on students misconceptions in learning of transformation in mathematics and does not examine other related concepts such as symmetry or congruence. This study also relies on self-reported data from students and teachers, which may be affected by factors such as social desirability bias or recall error. In addition, this study will primarily focus on students misconceptions about specific transformation such as reflection, rotation and translation but may not address other related concept such as enlargement, shear and stretch.

1.8 LIMITATIONS OF THE STUDY

The researcher will face a problem of a sample size. The sample would be pupils at one secondary school who are not a true representation of the total population in Zimbabwe. The identification of misconceptions may be influenced by the specific instructional strategies or resources that are used in the classroom and may not be reflect students' misconceptions in other learning environments or contexts. The study may also be limited by the researchers' own biases and perspectives which may influence the interpretation of data or the conclusions that are drawn.

1.9 DEFINITIONS OF TERMS

The researcher is going to define the following terms contextually. Finally, the researcher will give his own judgment on those definitions.

- Misconceptions
- Errors
- Transformation
- Prerequisite knowledge

In the field of education, a misconception is defined as a commonly held but mistaken belief or idea about a particular subject or topic. In math, misconceptions often arise from misunderstandings of core concepts, such as place value, number operations, fractions and proportional reasoning. Misconceptions can lead to errors in computation, difficulty in problem solving and incorrect solutions to word problems.

Drew (2005:150) postulated that misconceptions is the misapplication of a rule, 'an overgeneralization of the situation.' Pines (1985) view misconception as a display 0f an already acquired system of concepts and algorithms that have been wrongly applied. In other words misconceptions is a misapplication of a rule or overgeneralization of a principle.

Channon and Mcleish Smith, (2011) defines transformation as 'mathematical operations that change the position, shape or size of a figure.' The image of a shape is the figure which results after transforming. Transformation is a marked change in form, nature or appearance. It is the process of changing the position, size and structure of a shape, line or a point by translation, reflection, rotation, enlargement, shear and stretch.

Prerequisite knowledge is defined as the knowledge and skills that are necessary for students to understand and apply the concept of transformation. Lesh(2007) proposed that prerequisite knowledge is the prior knowledge and skills that students need in order to learn new concepts or solve problems.

1.10 CHAPTER SUMMARY

This chapter focused on the background of the study, stating problems and sub-problems with how significance the study will be, limitations and delimitations as well as defining the key terms contextually. The next chapter will look into the literature review of the study.

CHAPTER 2
REVIEW OF RELATED LITERATURE

2.4 INTRODUCTION

This chapter focuses on what has been said by different authors concerning the misconceptions in learning transformation. This chapter is composed conceptual framework of the study, transformation misconceptions, common source of errors and misconceptions, ways of addressing and identifying the misconceptions, instructional strategies that promote deep understanding of the concepts as well as different ways of minimizing the misconceptions. At the end of the chapter the conclusion will be given.

2.5 CONCEPTUAL FRAMEWORK OF THE STUDY

The conceptual framework for this study is based on the core concepts which are misconceptions, instructional strategies, teacher knowledge and student learning.

Poor instruction factors.

Poor assessment and feedback.

Poor student understanding of concepts

Lack of pre-requisite knowledge

Misconceptions

Errors

Figure 2.1 Conceptual framework diagram

2.5.1 Misconceptions:

Misconceptions are conceptual misunderstandings that arise from faulty mental models or incomplete knowledge. Misconceptions are conceptual misunderstandings that arise when students have incomplete or incorrect mental models of a concept. In the case of transformation, common misconceptions may include: Confusing translations and reflections. Students may confuse the terms "translation" and "reflection," and may incorrectly apply one term when the other is intended.

Incorrect understanding of rotations. Students may have difficulty in understanding the concept of rotation, particularly the concept of orientation and direction of rotation. According to Fraser and Roberton (1999), who conducted research on students' understanding of transformation in secondary mathematics identified several key misconceptions about rotation including the "spinning around" misconception where students may mistakenly believe that objects rotate around a fixed point, rather than understanding that the object itself rotates. The "center of rotation" misconception in which students may believe that there is a fixed point of rotation, rather than understanding that an object can be rotate around any point.

The "original orientation" misconception. Students may have difficulty understanding that an object can be rotated in any direction, including counterclockwise or clockwise. They may mistakenly believe that there is only one "correct" direction of rotation. The "angular measure" misconception where students may have difficulty understanding the concept of angles and degree measurements, which are important for understanding rotation in mathematics.

According to Fraser and Roberton, addressing these misconceptions requires clear and consistent instruction, as well as opportunities for students to apply their understanding of rotation in practical contexts.

2.5.2 Instructional Practices:

Instructional practices are the methods and strategies used by teachers to promote students' understanding of transformation. The research of Roth et al. (2003) provides insight into effective instructional practices for addressing transformation misconceptions. According to their findings, the following practices can be effective:

2.5.2.1 Model-Based Instruction

Roth et al. found that providing students with hands-on models and manipulative can help them develop more accurate mental models of transformations. For example, using a ruler and protractor to demonstrate rotation can help students understand the concept of angles and degree measurements.

2.5.2.2 Scaffold Instruction

Scaffold instruction refers to a gradual and supportive approach to teaching that provides students with guidance and support as they learn new concepts. Roth et al. found that using scaffold instruction can help students overcome misconceptions and develop a more complete understanding of transformation. Some specific examples of scaffold instruction include, start with familiar examples. Roth et al. suggest starting with familiar examples of transformations, such as reflecting a shape in a mirror or rotating a bike wheel, before moving on to more abstract examples.

The researchers also recommend providing students with worked examples of transformation. Roth et al. suggest that worked examples can be used to model the steps involved in solving a problem and to provide students with a mental framework for understanding the concepts involved. Some strategies for using worked examples effectively include, providing step-by-step explanations. Roth et al. recommend providing students with detailed explanations of each step involved in solving the problem. This helps students to understand the reasoning behind the solution and to develop their own problem-solving skills. The researchers also suggest using highlighting or other visual cues to emphasize key concepts Highlighting key concepts can be an effective instructional strategy for helping students to identify and focus on the most important aspects of the problem. They also suggested specific ways of highlighting key concepts including:

2.5.2.3 Color coding

Roth et al. suggest using color coding to highlight key elements of the problem, such as labeling different transformations with different colors or using color to highlight the steps of a problem-solving process.

2.5.2.4 Bolding or underlining

Using bolding or underlining can also be an effective way to draw students' attention to key terms or ideas in the problem.

2.5.3 Student Learning

Student learning refers to the cognitive processes and outcomes of students' engagement with the material, including their understanding of transformation and the persistence of their misconceptions. According to a study conducted by Lam and Hui (2013), there are several factors that influence student learning of transformation, including, Prior Knowledge. Lam and Hui found that students' prior knowledge and experiences with transformation can have a significant impact on their understanding of the concept. For example, students with more experience with physical transformations (e.g., rotating a bike wheel) may have an easier time understanding mathematical transformations.

Another factor is Intrinsic Motivation. Lam and Hui (2013) found that students who are intrinsically motivated to learn transformation were more likely to persist in their learning and to develop a deeper understanding of the concept. They suggested several strategies that promoting intrinsic motivation in students including making connections. Lam and Hui suggest that helping students to make connections between transformation and their everyday experiences can help to foster intrinsic motivation. Making connections refers to the process of linking new information or concepts to students' existing knowledge and experiences. Lam and Hui (2013) discovered that when students were able to make connections between transformation and their everyday experiences, they were more likely to become intrinsically motivated to learn. Facilitators can make connections between transformation and everyday experiences by giving real-world examples of transformation, such as transformations in art, architecture, or nature, can help students to see the relevance of the concept in their everyday lives.

Problem-based learning is also an educational approach that involves presenting students with real-world problems or scenarios and asking them to develop solutions. Lam and Hui (2013) postulated that using problem-based learning in mathematics instruction can help students to make connections between transformation and their everyday experiences. For example, Lam and Hui describe a study in which students were asked to design and build a model of a bridge using mathematical transformations, such as translation and rotation. By working through real-world problems, students were able to see the practical applications of transformation and develop a deeper understanding of the concept.

2.5.4 Teacher Knowledge

Teacher knowledge refers to the knowledge, beliefs, and dispositions that teachers bring to their instructional practices, including their understanding of students' misconceptions and their understanding of the mathematics they are teaching. : Research by Moon (2015) suggests that teacher knowledge plays a critical role in students' understanding of transformation. He content that teachers who have a deep understanding of transformation themselves are better able to explain the concept to students and to identify and address students' misconceptions. Moon found that teachers' content knowledge, or their knowledge of the mathematics itself, is an important factor in their ability to teach transformation effectively. In the context of transformation, content knowledge would include an understanding of the mathematical principles underlying transformation (for example, the properties of reflections, rotations, and translations), as well as familiarity with the terminology and notations used in the field.

Moon (2015) found that teachers with strong content knowledge of transformation were better able to explain the concepts to students, provide clear examples, and address students' misconceptions. Teachers with weak content knowledge, on the other hand, may struggle to convey the concept to students or may inadvertently reinforce students' misconceptions.

2.6 ERRORS AND MISCONCEPTIONS IN TRANSFORMATION

Misconceptions can be stable and widespread among students. They can be strongly held as resistant to change. Rather than being momentary conjectures that are quickly discarded, misconceptions consistently appear before and after instructions in substantial numbers of students and adults in a wide variety of subjects matter domains and are often actively defended (Clement, 1983).

According to Hill (1996) students shifted the lines while applying reflections and rotations in transformation of shapes. They shift the reflection line and poorly reflect the shapes resulted in producing the wrong image. This misconception mainly resulted from pupils who is incapable of using tracing paper properly and counts the squares on the paper incorrectly.

Linchevski (2010) contends that pupils make mistakes in descriptive questions. They interchange rotation of 180 degrees about the origin with reflection in the line $y = x$. They describe rotation of 180 degrees about the origin as reflection. This misconception emanated from having insufficient knowledge about the topic and forgetting some of the details.

2.6.1 Wrong application of translation vector

Kurtulus (2010) postulated that pupils wrongly apply the concept of translation vector. Most students subtract translation vector from the original points when calculating the image points. Some of them treat the translation vector as matrix. This will result in wrong positioning of the image shape. $(OP - TV = IM)$. This misconception derived from mislearned measures and insufficient concentration by pupils. Pupils calculate wrong image points there by failing to translate the shape or line.

2.6.2 Wrong vocabulary (Transformation ≡ Translation)

Hornsby, McGinnis and Hal on www.googlebookspreview.com suggest that pupils use wrong vocabulary when answering the questions of transformation for example they used the word transformation instead of translation. Many pupils thinks that these two words mean the same and they can be interchanged. The researcher argues that teachers should emphasize formal mathematics terms more during teaching and learning so that pupils will know the proper vocabulary to be used.

2.6.3 Wrong use of protractor

Students have misconception on the use of protractor in rotation of shapes (Baykul 2002). They wrongly position the center of protractor when they are measuring the angles during rotation of shapes. Pupils also usually use the edges of a protractor as the radius or diameter instead of that marked line. So the teacher should always urge the students that they should place their protractor at the correct position when measuring the direction of rotation.

2.6.4 Link between column vector and object matrix

Other misconception which had been postulated by Kufakowadya and Nyamakura (2O1O) in Mathematics today is most students fail to understand the link between the column vector and the object matrix. The column vector must be added to each column of the matrix separately to effect the transformation. But most pupils write as $IP = matrix\ of\ original\ shape + TV$. This is wrong because such matrices do not add and translation vector is not a matrix.

2.6.5 Misapply the concept of direction of rotation

They also misapply the concept of direction of rotation. Most pupils rotate in the wrong direction that is anti-clockwise instead of clockwise. This result in the wrong positioning of the image. This misconception arises from having insufficient knowledge about the direction.

2.7 SOURCES OF MISCONCEPTIONS AND ERRORS IN TRANSFORMATION GEOMETRY

2.7.1 Misunderstanding of coordinate systems

Radford (2003) suggested that many students struggle to understand how coordinate systems work and how they are used to describe transformations. Some students may think that the x-axis and y-axis always represent horizontal and vertical directions, respectively, and may not realize that these directions can change in different coordinate systems.

2.7.2 Student's over-reliance on geometric intuition

Schoenfeld (2007) found that students often rely on their visual intuition or conceptual understanding of transformation, rather than mathematical reasoning, to solve problems. For example, students may attempt to draw a transformation by sight, rather than by calculating the changes in the coordinates of the points. This can lead to errors and misconceptions, especially when dealing with more complex transformations or when students are working with different coordinate systems.

2.7.3 Limited understanding of relationships between variables

As suggested by Marshall and Brown (2014), some students may have difficulty understanding that different variables can be related in different ways in transformation problems. For instance, students may assume that the x-axis always represents time, or that certain transformations (such as rotation) always preserve distances between points. This can lead to errors and misconceptions, particularly in more complex problems. Marshall and Brown recommend that teachers explicitly discuss relationships between variables and provide opportunities for students to explore these relationships in different contexts.

2.7.4 Misunderstanding of invariant properties of shapes.

Invariant properties are properties that do not change under a transformation. According to a study by Bianchini and Cossentino (2017), many students struggle to understand which properties of shapes are invariant (such as area, perimeter, or angle measure) and which properties change (such as position or orientation). This can lead to errors and misconceptions, particularly when students are asked to compare the properties of shapes before and after a transformation.

2.7.5 Lack of understanding of spatial relationships

Battista (2007) suggested that students may struggle to visualize and understand the relationships between shapes and figures in transformation geometry, leading to errors in calculations and applications. This can be due to a lack of experience with spatial reasoning, difficulty with visualizing objects in space, or inadequate instruction on spatial relationships.

2.7.6 Insufficient practice with transformations.

Students may not have sufficient opportunities to practice transformations, leading to a lack of fluency and accuracy in applying transformation geometry concepts (Ashcraft, 2002). This can be due to a lack of time devoted to practice, inadequate resources or materials, or insufficient feedback and support.

2.7.7 Difficulty with abstract thinking.

Transformation geometry requires students to think abstractly and visualize transformations in their mind, which can be challenging for some students (Bianchini, 2017). This can be due to a lack of experience with abstract thinking, difficulty with visualizing abstract concepts, or inadequate instruction on abstract thinking strategies.

2.8 WAYS OF IDENTIFYING AND ADDRESSING ERRORS AND MISCONCEPTIONS IN TRANSFORMATION GEOMETRY

2.8.1 Formative assessment

Identifying and addressing misconceptions and errors in transformation geometry requires a combination of assessment, observation, and instructional strategies. Teachers can identify errors and misconceptions through formative assessments for example quizzes, classwork, observations (Black & Wiliam, 2009) Formative assessments involves use of frequent, low-stakes assessments to monitor student understanding and identify areas where students may be struggling or holding misconceptions.

2.8.2 Student self-reflection

Student self-reflection is also a powerful tool for identifying and addressing errors and misconceptions in transformation geometry (Boud & Garrick, 2007). Self-reflection involves students examining their own thought processes, understanding, and learning strategies, and using this awareness to adjust their approach to learning. To promote student self-reflection, facilitators should encourage students to keep a math journal where they record their thoughts, ideas, and reflections on their learning. This can help them identify areas where they need more practice or review. They also provide students with rubrics that outline the criteria for assessing their understanding of transformation geometry concepts. This can help them evaluate their own work and identify areas for improvement. By incorporating these self-reflection strategies, students can develop a greater awareness of their own learning, identify areas for improvement, and take ownership of their learning process. This can lead to increased motivation, confidence, and success in transformation geometry and other math subjects.

2.8.3 Targeted feedback

Effective strategies for addressing errors and misconceptions include targeted feedback and scaffolding (Hativa, 2013). Scaffolding is the way of offering temporary support and guidance to help students build understanding and correct errors. By so doing students will able to construct meaningful thoughts when solving the given problem. Targeted feedback is a specific and focused type of feedback that aims to improve student understanding and address specific misconceptions or errors. It focuses on a specific aspect of a student performance or understanding rather than providing general comments. It is provided while the student is still engaged with the material, allowing them to make adjustments and apply the feedback immediately.

2.8.4 Using multiple representations visual aids

Teachers can also use various instructional strategies to prevent errors and misconceptions, such as using multiple representations and visual aids (Lesh, 2000). The facilitator should incorporate visual instruction by using visual aids such as graphs, charts, and diagrams to help students understand and visualize transformation geometry. Teachers should provide hands on activities such as puzzles, games and manipulatives to help students explore and understand transformation geometry. These instructional strategies can be combined and adapted to meet the diverse needs of students and promote a deeper understanding of transformation geometry concepts (Sarama & Clements, 2009).

2.9 INSTRUCTIONAL STRATEGIES:

Published researchers suggests that instructional methods and materials can impact students' understanding and error patterns in transformation geometry. Traditional teaching methods and textbooks may reinforce procedural errors and misconceptions (Kilpatrick et al., 2001). Traditional textbooks and workbooks often present transformation geometry concepts in a procedural and formulaic way, which can lead to a focus on memorization rather than deep understanding. This can result in students making errors such as, applying formulas incorrectly, Forgetting to consider spatial relationships and confusing similar concepts for example rotations and reflections.

Technology and digital tools can support visual learning and reduce errors (Sarama & Clements, 2009). Using visual aids such as graphs, charts, and diagrams, and manipulative like puzzles and games, can help students develop a deeper understanding of transformation geometry concepts. However, if these materials are not used effectively, students may make errors such as misinterpreting visual representations and failing to connect manipulative to abstract concepts. Digital tools and software can enhance student understanding of transformation geometry concepts, but if not used appropriately, can also lead to errors such as overreliance on technology, leading to lack of understanding of underlying concepts

Hands-on activities and manipulatives can improve conceptual understanding and reduce errors (Battista, 2007). Student-centered instruction, such as project based learning, inquiry-based learning and problem-solving can promote deeper understanding, but if not implemented effectively, can lead to errors such as students struggling to connect concepts to procedures. Students should be engaged in activities that promote self-construction of knowledge and ideas. The role of the facilitator I to provide a conducive environment that allows pupils to learn by discovering.

By recognizing how instructional methods and materials can influence student understanding, teachers can adapt their instruction to address common errors and misconceptions, promoting a deeper and more accurate understanding of transformation geometry concepts.

2.10 SOME SUGGESTED WAYS OF MINIMISING ERRORS AND MISCONCEPTIONS

There are several ways of minimizing misconceptions. It is noted that students prefer to rely on visual prototype rather than a verbal definitions when classifying and identifying shapes (Linchevski 2010). When a pupil hold both verbal and a visual prototype for a given geometric concepts, the student often recalls upon the visual prototype rather than or in spite of the verbal definitions.

If the proper terminology is used in context verbally when introducing new terms in transformation, students become familiar with the written solutions and this holds them to develop and learn the necessary vocabulary. The teacher can also insists on their using it as much as possible and associates with newly presented topics.

Students have a lot of misconceptions while they are using protractor in rotation. So the teacher should always warn the students that they should always place it in correct position when they measure the direction of rotation. The instruments and their features in mathematics and transformation should be introduced properly. Proper use, sufficient practice and formative assessment guarantees their familiarity with the instruments and enables them to use them correctly (Baykul, 2002).

Another challenge with some students is transformation comprehension. The teacher should analyse students' geometric mistakes and help them to improve their transformation knowledge. The teacher should always make sure that he will use the right word at the right time and place as pupils will usually repeat the wrong things he or she acquired during the lesson.

Misconceptions arise frequently if learners by pass or skip a level from the model (Erkus, 2005). A teacher should get students to explain how they came to their answer or rules so that he or she can analyse the problem, extent the ideas and the new concept. By this way the teacher can understand the reason behind misconceptions and they can be corrected by challenging and contrasting it with the right conception.

Table 2.1 Transformation misconception and the ways to minimize them.

Misconceptions	Reasons	Solutions
Shifting of lines while applying rotation and reflection	Students cannot use the tracing paper properly and counts the square on the paper incorrectly	The use of tracing paper should be shown in detail by using computer based teaching.
The students did mistakes while enlarging objects and wrong use of coordinates.	They did not understood the process of enlarging.	Computer based exercise can be practised so that students can get better knowledge on the enlargement topic. Describe the positions and movements more detail. Two and three dimensional objects should be used to make students able to visualize the images.

2.11 RESEARCH GAP

Previous studies have largely focused on identifying and describing students' misconceptions but there is little research on how to help students overcome these misconceptions and achieve a deeper understanding of the concept. There is also limited research on how students' prior knowledge and experiences influence their understanding of transformations and how these factors can be taken into account when designing instructional strategies. This study will also unpacks the long term impact of addressing misconceptions on students understanding of transformation and their overall development. However, the current study aims to address the specific instructional strategies and resources that are most effective in addressing students' misconception about transformation.

2.12 CHAPTER SUMMARY

The chapter focused on the misconceptions students have on different scenarios of transformation which are common in students. It also looked at factors contributed to the existence of these misconceptions and how to minimize them. The next chapter will look mainly into the research designs, instruments which are going to be used, data collection procedures and data analysis plan of the study.

CHAPTER 3
RESEARCH METHODOLOGY

3.4 INTRODUCTION

This study explores the misconceptions in the learning of transformation, it address the misconceptions in the translation, reflection and rotation of shapes and points. This chapter will focus mainly on the research design, data collection instruments, data collection procedures and data analysis plan of the study as well as ethical considerations.

3.5 RESEARCH DESIGN

According to LeCompte and Schensul (2010) the formal plan of action for a project is called a research design. It shows how the researcher is going to gather and analyse his or her data. The research design is the arrangements of conditions for collection and analysis of data in a manner that aims to combine relevance to the research purpose with economy in procedure. It defines the study type, research questions and hypothesis, variables and data collection methods. Some examples of research designs include descriptive, correlational and experimental.

3.5.1 RESEARCH PARADIGM

The researcher would use a pragmatic research paradigm for this study because it allows for the strength of both qualitative and quantitative to be combined. By using a mixed method approach, the researcher is going to gain insights into both the prevalence and nature of students' misconceptions, as well as the instructional strategies and resources that are most effective in addressing misconceptions. In particular, the quantitative data for example survey response provide a broad overview of the prevalence and nature of students misconceptions, while the qualitative data from questionnaire will provide deeper insights into students thinking and learning processes. By collecting and analysing data from both strands simultaneously, the researcher will be able to develop a richer and more nuanced understanding of the research questions. For example the survey data may reveal that a particular misconception is widespread among students while the interviews may provide insights into why this misconception is so prevalent or how it can be effectively addressed in the classroom. The researcher can integrate these findings to develop a more comprehensive understanding of the issues and identify effective instructional strategies.

The multiple data collection methods that will be used in this study will allow a triangulation of the data which is a key strength of mixed research methods. By gathering data from different sources, the researcher will be able to validate and complement each other's findings, increasing the reliability and validity of the study.

3.5.2 MIXED METHOD APPROACH

A mixed method approach is a research methodology that combines both qualitative and quantitative methods to collect and analyze data. This approach allows researchers to triangulate their findings, increasing the validity and reliability of the results. It involves collecting both quantitative data for example surveys, experiments and qualitative data is collected from interviews and observation. It also analyze both types of data separately and together and Integrating the findings to form a comprehensive understanding of the research topic

The mixed method approach offers several advantages as combining quantitative and qualitative data provides a more complete picture of the research topic. Cross-validating findings from multiple methods increases confidence in the results. Mixed methods can increase the validity of findings by confirming or challenging results from different methods.

3.5.3 RESEARCH STRATEGY

Based on my research questions and objectives, the researcher is going to apply sequential explanatory strategy which involves conducting a quantitative survey to identify trends and patterns in students' errors and misconceptions in transformation geometry followed by qualitative interviews to gain deeper insights and explain the findings from the survey.

3.6 POPULATION AND SAMPLING PROCEDURES

3.6.1 POPULATION

A population is the entire group of individuals or items from which a sample may be selected for statistical measurements. The population of the respondents in the study is made up of all two hundred and seventy mathematics ordinary level students and fifteen mathematics teachers of schools in Insiza South Zone B Cluster. The researcher because of the populous number of respondents, resolved to draw up a sample of the students.

3.6.2 SAMPLING PROCEDURES

A sample is a representative of a population that is examined to gain statistical information about the whole of the population. Due to the limited time, the researcher is going to carry out the study at one high school in the cluster with five mathematics teachers and seventy mathematics ordinary level students. All 5 teachers are going to be included in the study because of their small number. The researcher is going to use systematic sampling method to select an appropriate sample of students. The sample is going to have ten boys and ten girls. Every nth individual is selected from the population where n is a fixed number.

3.7 RESEARCH INSTRUMENTS

Research instruments are those techniques used in the gathering of data. They are not necessarily the act of collecting data but the procedures involved, which are established and not merely haphazard. Research instruments involve a spectrum of methods and of the most commonly used may be interviews, observation, testing, questionnaire, document analysis. The researcher after comparing the research instruments listed above deemed that the appropriate research instruments to be used in this study for collecting the data would be testing, interviews and questionnaire.

3.7.1 QUESTIONNAIRE

Questionnaires allow for large amounts of data to be collected quickly and efficiently, which is important in a study with a large population. The use of questionnaires allows students to respond anonymously, which may encourage more honest and candidate responses. This is particularly important when collecting data on misconceptions, which students may be reluctant to admit to in a face-to-face interview. They can also be designed to include items that are reliable and valid measures of students' understanding and misconceptions. For example, the use of multiple-choice items with distractors based on known misconceptions can help identify common misconceptions in the population. Questionnaires can be used to compare the prevalence of misconceptions across different groups of students (e.g., by gender, grade level, or school type), which can provide valuable insights into the nature of these misconceptions.

3.7.2 TESTS

Grindal (2006) defines test as an assessment or evaluation used to measure knowledge, skills or performance. It can be used to evaluate understanding or comprehension. This gives an impression that tests solicit understanding in behaviour after learning a certain concept. The instruments are basically research tools used in gathering data.

Tests are a research’s foundation in which a researcher is able to build in preparation for implementing solutions. Best and Khans (1923) proposed that tests are used to diagnose strengths and weakness hence the teacher has a view on how and where to concentrate his or her attention. In this study the researcher is using tests to solicit the actual misconception that pupils have in the learning of transformation and develop some alternate solution to counter them. Tests are also cheap to administer as compared to other gathering instruments. This will help the researcher to minimize the costs and serve time. Comparison can be made against a baseline test to measure how far a concept has been grasped.

However, tests do not take into account feelings and attitudes of pupils being measured. The researcher cannot identify some of the underlying intrinsic factors in an individual. They can also fail to give an exact picture of what the researcher to get for example copying may affect the objectivity of the test. The researcher may give an unbiased conclusion of the group.

3.8 DATA COLLECTION, PRESENTATION AND ANALYSIS PROCEDURES

Two self-administered questionnaires are used to collect the categorical data set. Open and closed ended questions are going to be operationalized and statistical software is utilized to manipulate the data such as geogebra, spread sheet and originPro. The variable that transformation topic is hard is taken to be dependent variable and is operationalized from the data collected.Explanatory variables used are instructional strategies, background information on the topic, prerequisite knowledge required. Open and closed ended questions will be administered and charts drawn to present the data which is discussed in terms of meaning. The survey data will be summarised using descriptive statistics such as tables, mean, mode, median, variance, standard deviations and ranges. Finally graphs such as bar graph, pie charts will be modelled to show the relationship between the comparable data.

3.8.1 ANALYSIS OF QUANTITATIVE DATA

Analysis of quantitative data involves using statistical methods and techniques to identify patterns and trends and relationships within numerical data. In this study the researcher will check for missing values, outliers and errors to ensure that the data is in a suitable format for analysis. The researcher is going to use descriptive statistics that involves calculation of measure of central tendencies include mean, mode, median, quartiles and variability of data to understand the data distribution. The researcher will also use plots and charts such as pie charts, histograms to visualize the data and identify patterns. In addition, inferential statistics will also applied by using statistical tests, t-tests to draw conclusion about the population.

3.8.2 ANALYSING OF THE QUALITATIVE DATA

Analysing of qualitative data involves identifying patterns, themes and meanings within non numerical data such as text, images, or observations. In this study, the researcher will read, watch and listen to the data multiple times to understand the content and context, assigning labels to relevant segments of data to identify initial themes and patterns. The researcher will also use tables, figures and diagrams to visualize themes and patterns. Finally making inferences, identifying implications and draw conclusions based on the findings.

3.9 VALIDITY AND RELIABILITY

Validity and reliability are two important concepts in this study that help to ensure quality and accuracy of the findings. Validity refers to whether the research measures what it claims to measure. The research questions and interview guides of this study has a potential to measure what they claim to measure, which is student misconceptions and errors in transformation geometry and how teachers can best support students in overcoming these challenges. The study is based on theoretical concepts of transformation geometry and student learning and the interviews questions are designed to tap into these concepts. The research covers a range of aspects of student errors and misconceptions and in transformation geometry including, prerequisite knowledge, sources of errors and identifying and addressing these misconceptions. The research is intended to predict how teachers can best support students in overcoming errors and misconceptions in transformation geometry and the findings can be used for instructional purposes.

Reliability refers to the consistency and dependability of the research findings. This ensures that the study produce same results when repeated. The research use a consistent interview guide and data collection method which would likely produce consistent results if repeated.

This study also uses a systematic and transparent approach to data analysis which helps to ensure consistency within the findings.

3.10 ETHICAL CONIDERATIONS

The conducting of research requires not only expertise and diligence, but also honesty and integrity. This is done to recognize and protect the rights of human subjects. To render the study ethical, the right to self-determined anonymity, confidentiality and informed consent were observed. Subjects consent is obtained before they completed the questionnaires and interview questions. Burns and Grove (1993) defined informed consent as the prospective subject's agreement to participate voluntarily in a study which is reached after assimilation of essential information about the study. Pupils will be informed of their rights voluntarily consent or decline to participate and to withdraw participation and to withdraw at any time without penalty.

Participants will be informed about the purpose of the study, the procedures that would be used to collect the data and assured that there are no potential risks or costs involved.

Anonymity and confidentiality will be maintained throughout the study. Burns and Grove (1993) defined anonymity as when subjects cannot be linked even by the researcher with his or her individual response. In this study anonymity will be assured by not disclosing student's name on the questionnaire and interview scripts and research reports and detaching the written consent from the questionnaire.

3.11 CHAPTER SUMMARY

The chapter has described the research design as a conceptual framework within which research is done. It comprises of the stages and procedures for collection, measurement and analysis of data, decisions regarding what, where, when and how much, by what means a research designs were taken. The chapter has regarded the research design as an arrangements of condition for collection and analysis of data in a way which seeks to combine relevance to the research purpose. The chapter has explained the descriptive survey designs, the targeted population and sampling procedures employed. Attempts were made to justify the use of data collection methods. The three data collection instruments were examined. The next chapter covers data presentation and analysis.

CHAPTER 4

DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.0 INTRODUCTION

This chapter focuses on the presentation of data sourced from tests and questionnaires. The analysis and interpretation of data would be done following the sequences of research questions. Subsequently the chapter would conclude with the summary of the chapter. In this study qualitative and quantitative method of collecting data were used. The data gathering tools were designed in such a way that they gave an insight into the participants understanding of transformation geometry.

The focus of this study is exploring students' errors and misconceptions in the learning of transformation geometry at Ordinary level. Four research questions were used to unpack the research problem. The research questions were:

- What are the errors and misconceptions held by students when learning about transformation?
- What are the sources of errors and misconceptions in transformation geometry?
- What are the prerequisite knowledge required to learn transformation?
- What instructional strategies to address errors and misconceptions in transformation?

The researcher is going to present and analyse data basing on these three sub research questions and were possible the researcher will present the results especially of tests scripts analysis. To explore student errors and misconceptions of transformation topic, a question by question analysis of test questions was done. The researcher will also use the response from the questionnaires to also explore the source of errors and misconceptions and also identify the best ways of addressing and identifying the errors and misconceptions in transformation geometry.

4.1 BIOGRAPHY OF THE PARTICIPANTS

4.1.1 Biography of teacheM1, M2 M1, M2 and M3 are the three male teachers participating in this project. M1 holds a Postgraduate Diploma in Education, with 8 years of teaching experience.M2 has a Bachelor of Education in Mathematics and 5 years of teaching experience. M3 holds a Postgraduate Diploma in Education and has been teaching for 7 years. F1 and F2 are the two female teachers participating in the project. F1 has a Bachelor of Science in Mathematics and a Postgraduate Diploma in Education, with 9 years of teaching experience.F2 holds a

Postgraduate Diploma in Education and has been teaching for 4 years. All teachers have a strong background in mathematics education and are committed to helping students overcome errors and misconceptions in transformation topics.

4.1.2 Biography of learners

The 20 Ordinary Level students participating in this project are a representative sample of secondary school learners. The group consists of 10 boys and 10 girls, all between 15 and 17 years old. They are currently studying mathematics and are engaged in the learning process of transformation topics. Their diverse backgrounds and abilities will provide valuable insights into the errors and misconceptions that arise during this learning process.

4.2 What is the performance of students in transformation?

The results of the test are tabulated as shown in the table 1:1 below. The test was marked out of twenty-five and it consist of two questions that includes drawing and descriptive questions. The researcher used grading system used at O level to create class intervals.

Table 4.1 SHOWING TEST RESULTS (n=20)

% mark	0$\leq x < 35$	35$\leq x < 50$	50$\leq x < 60$	60$\leq x < 70$	70$\leq x < 100$
Frequency	9	7	0	3	1

Modal class = $0 \leq x < 35$

Mean = 34.5%

Median class = 35$\leq x < 50$

Percentage pass rate = 20%

Standard deviation (s) = 21.65

Variance = 445.44

From the above table and calculations, the performance of pupils is below standard. Most pupils scored below the average mark of the test. Their percentage pass rate is very low 20%, only four out of twenty attained marks greater than 50%. So far the data shows that pupils studied were weak in Mathematics as evidenced by the majority who scored less than 50% in the test administered.

The mean score of 34.5% means students in this school are not good in mathematics. The study went further to explore errors and misconceptions displayed by the students and pre-requisite knowledge required by the students to do well in the learning of transformation.

4.3 What are the sources and nature of errors and misconceptions held by students in the learning of transformation?

4.3.1 Misconception of considering y=0 as y-axes and x=0 as x-axes

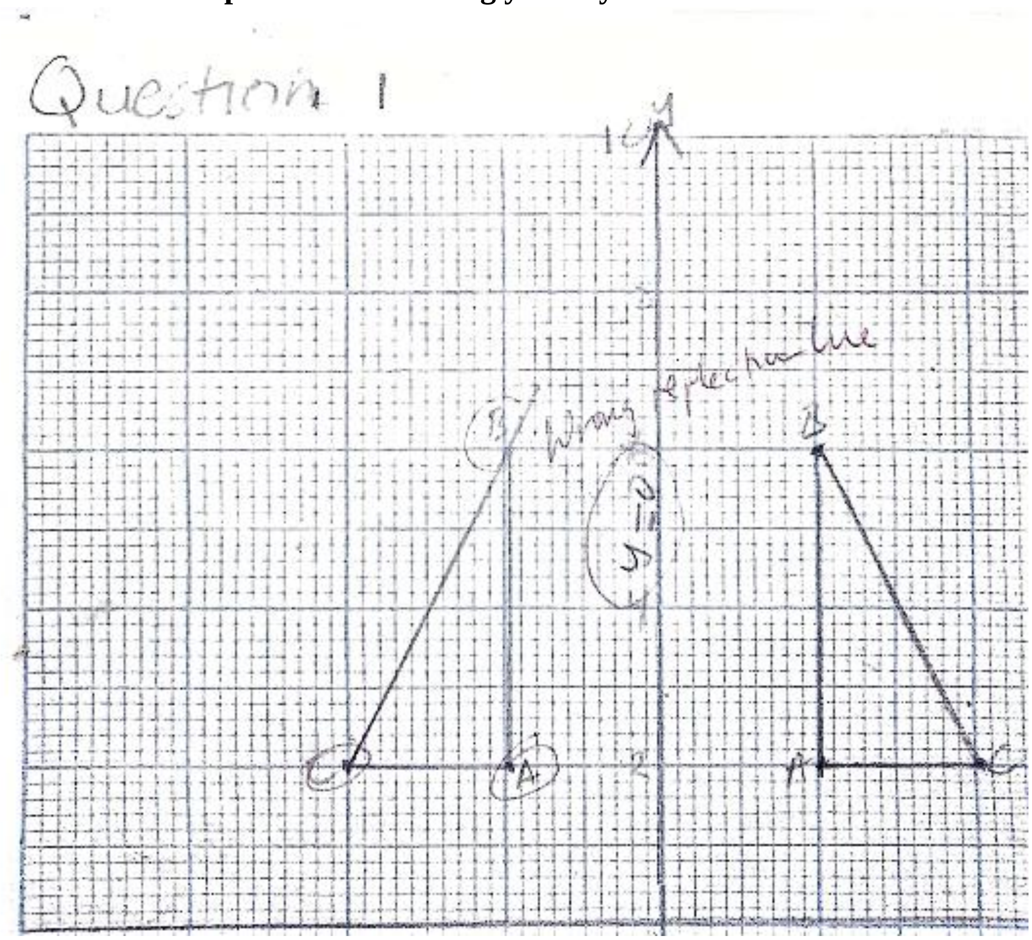

Figure 4.1 Error in identifying x = 0 as y =0 (y axes)

As shown on figure 4.1 the student has an error of treating y axes as y=0. A number of students has got this misconceptions. In an interview one learner explained that the vertical line labelled y axis is passing through zero so it is the same as y=0. The researcher explained to the student that horizontal lines represents different values of y and that all vertical lines represent different values of x. The student admitted the error.

4.3.2 Error of treating TV as an object matrix

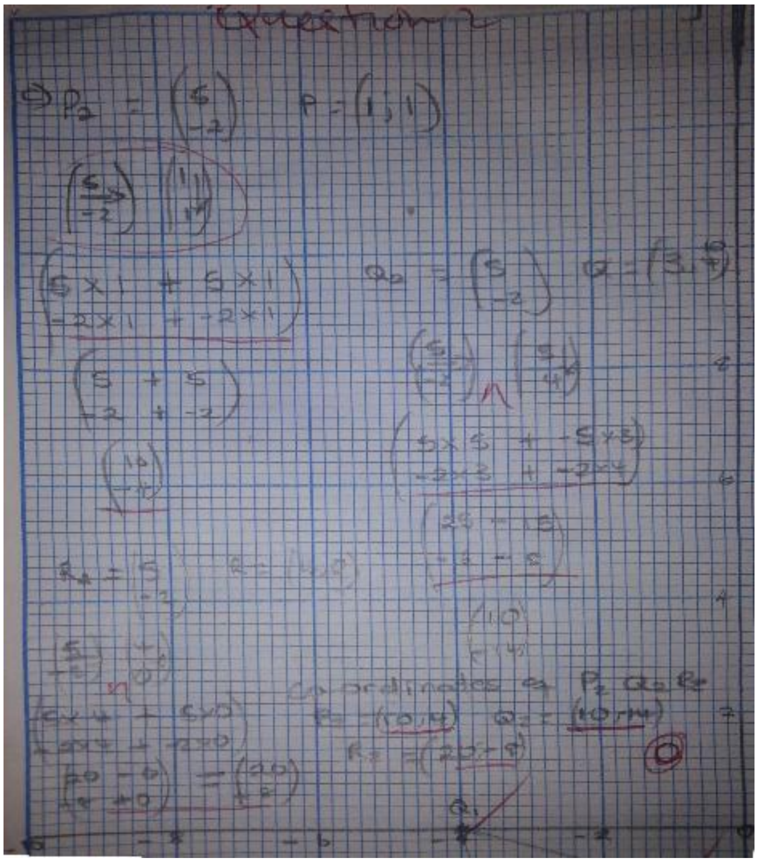

Figure 4.2 Error of treating TV as Object matrix

Most learners failed to calculate coordinates of the image points using TV as shown on figure 4.2. Students who got the question wrong were treating TV as an object matrix instead of a vector. During the interview, they suggest that the TV is a

$$2 \text{ by } 1 \text{ matrix } \ (5 \ -2\ )$$

This was an error and misconception because TV is a column vector. Further, the students were asked to explain and went on to state that

$$(5 \ -2\ )(1\ 3\ 4\ 1\ 4\ 0\ )$$

are two matrix that can be multiplied which is wrong. The acceptable solution is to calculate image points by adding TV on the original points of the triangle (image points = original points + TV). That is

$$P_2 \ = \ (5\,;-2\,) + \ (1;\ 1\,),$$

$$Q_2 \ = \ (5\,;-2\,) + \ (3;\ 4\,),$$

$$R_2 \ = \ (5\,;-2\,) + \ (4;\ 0\,)$$

then express the answers in coordinate form. Students seem to have forgotten the difference between a vector and a matrix.

4.3.3 Interchanging the x-co-ordinate and the y-co-ordinate when plotting

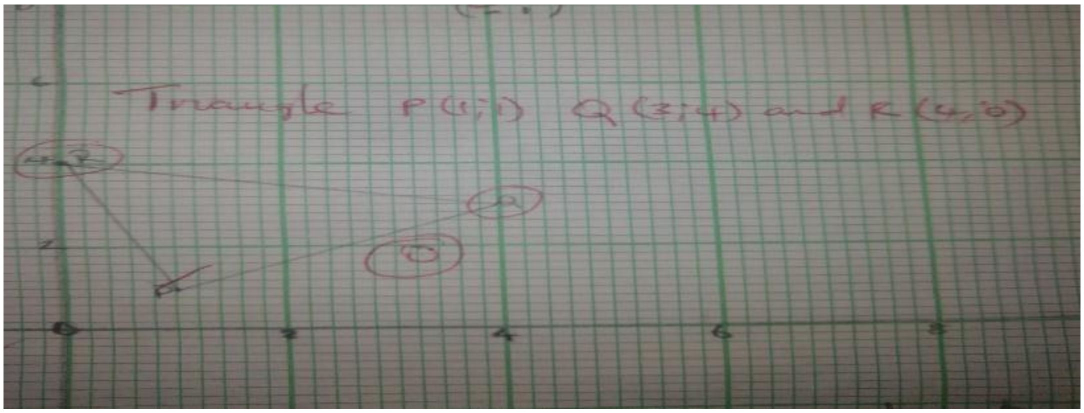

Figure 4.3 Error of interchanging x and y coordinate when plotting

Figure 4.3 above shows student's script who has an error of interchanging coordinates when plotting on the Cartesian plane. Only few students displayed this error. They were plotting x coordinate on the y-axes and y coordinate on the x-axes resulting in obtaining wrong shape. During the interview the students who got the question wrong, explained that they were forgetting where to place x and y values in the coordinate system (x ; y). The researcher explained the concept by emphasizing that the first coordinate represent the x value and the second coordinate is the y value.

4.3.4 Wrong description of reflection as rotation

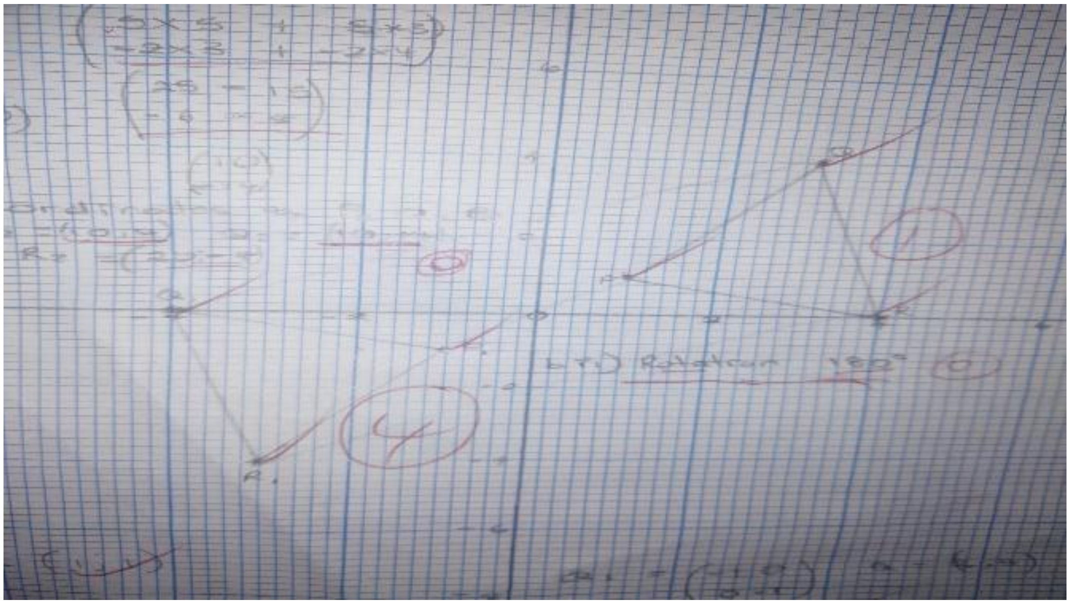

Figure 4.4 Wrong description of reflection as rotation

During analysis of the marked tests scripts, the researcher identifies that some students were describing reflection as rotation 180^0 and others were describing it as translation. Students displayed error of describing rotation as translation and vice versa. During interviews students admitted that the concepts are similar and confusing. Translation, reflection and rotation are all isometric, the original shape and the image have the same area, dimension and size. Students should make careful comparison and identify the differences especially in descriptive questions.

Table 4.2 Summary of Errors and Misconceptions made by pupils in a test

Concept	Question number	Script with wrong solutions	Misconception	Error
Plotting and drawing triangle	1(i), 2a	5	Interchanging x and y coordinate	Wrong shape
Reflection of shapes	1(ii), 1(iii),	10	Treating y=0 as y axes and x=0 as x axes	Wrong position of shapes
Reflection using matrix	2b	15	Adding the matrix	Wrong image coordinates
Description of the image formed	1(iv)	18	Describing rotation as reflection in y= -x	Wrong description

			Wrong visualization of shapes	Fail to identify centre of rotation
Description of reflection	2(bii)	14	Reflection is only done on vertical and horizontal lines	Fail to identify reflection line Wrong description
Calculation of coordinates of image given TV	2c(i)	13	Treating TV as matrix Wrong use of the formula Subtracting TV on the original shape	Wrong coordinates of the image Writing the answer in column form

The table above summaries the errors and misconceptions displayed by pupils in different questions leading to wrong answers. A number of pupils failed to answer question 2b which was demanding pupils to describe rotation. Pupils were describing it as reflection. Description questions are very challenging to students. Teachers should take time assisting pupils in descriptive questions. The concept of reflection was also challenging to the students. They were treating y=0 as y axis and x=0 as x axes. This result in using wrong reflection lines result in wrong positioning of shape. Pupils were also treating TV as a matrix which is wrong. They were also subtracting TV from the original points leading to wrong image points.

The results that the researcher got from the survey indicate that 12 out of 20 students questioned have a misconception that reflection changes the size and shape of an object. Students may misinterpret diagrams or illustrations, thinking that a change in orientation or position implies a change in size or shape. 15 students also said rotation of 90 degrees will also result in the same shape. 6 students also indicate that translation is the same as rotation. This shows that pupils have misconceptions towards transformation geometry.

4.4 What are the sources of errors and misconceptions in the learning of transformation geometry?

Of the 20 students asked, 50% of them suggest that confusion between similar concepts is always the major source of errors and misconceptions in transformation geometry for example reflection on the vertical axis and translation. Rotation and reflection are all isometric, they maintain the same size, area and dimension of the original shape. The difference is that reflection is the flipping of a point or figure over line of reflection while rotation is the turning of a figure about a fixed point called center of rotation.

18 students support that lack of practice and reinforcement always contributed to existence of errors and misconceptions in the topic. Teachers should give pupils more exercises and tests so that they become familiar with different concepts of transformation.

75% of the respondents suggest that sometimes lack of connection to real word application is a source of errors and misconceptions. Teachers should give real world examples that shows the application of transformation for example the image formed in the mirror, which is an example of application of reflection.

12 students agree that sometimes the source of errors and misconceptions in transformation geometry is inadequate understanding of coordinate geometry.

Table 4.3 Results of respondents on sources of errors and misconceptions

	Respondents			
Source of errors and misconceptions	**Never**	**Rare**	**Sometimes**	**Always**
Confusion between similar concepts			**3**	**2**
Difficult with visualizing and representing transformation			**1**	**4**
Inadequate understanding of coordinate geometry		**1**	**3**	**1**
Language barriers	**1**		**3**	**1**
Lack of practice and reinforcement			**1**	**4**
Difficult in abstract thinking		**1**	**2**	**2**
Lack of connection to real world application			**3**	**2**

Of the five teachers questioned, 80% of teachers outlined that lack of practice and reinforcements (45) and difficult with visualization and representing transformation (42) are always the major source of errors and misconceptions in transformation. 60% of the teachers said sometimes confusion between similar concepts, inadequate understanding of coordinate geometry, lack of connection with real world application and language barriers lead to errors and misconceptions in transformation. This is in line with the researcher's assumption that lack of pre-requisite knowledge can cause the existence of errors and misconceptions in transformation geometry. Teachers and students have the same views on the sources of errors and misconceptions in transformation, however some of the students were blaming teachers for lacking enough content on the topic. Some teachers fail to explain the concept in detail, usually they teach the topic for the purpose of answering paper one with structured questions not for the optional questions in paper two.

4.5 Pre-requisite knowledge for learning Transformation.

The document analysis of the responses on test scripts and questionnaires allowed the researcher to determine the pre-requisite knowledge required by students in order to develop the full scope of understanding. The data gathered helped the researcher to identify the student's weaknesses about transformation and infer the prerequisite knowledge required by students. Some4 participants were lacking the concept of coordinate system. Students should do the topic of straight lines first where they plot and draw straight lines graphs. This helps them to understand the concept of coordinate system. Some students were lacking the concepts of matrices. They were not able to multiply the given matrix with the original points to get image points.

4.6 Identifying and addressing errors and misconceptions in transformation geometry.

Among 20 pupils questioned, 50% said they address their errors and misconceptions through online resources or tutorials and working with study groups. 75% of the respondents seek for teacher or peer help and 60% sometimes review notes and textbook. Teachers should make sure that the is enough textbook for use by pupils. Pupils should have access to free Wi-Fi and allowed to bring their cell phones to school so that they search information on internet.

Table 4.4 ***Teachers' view about the effective instructional method and materials in teaching transformation geometry***

	Respondents			
Instructional method	**Never**	**rare**	**Sometimes**	**Always**
Traditional method			**4**	**1**
Technology and digital tools			**3**	**2**

Hands on activities and manipulative		**1**	**4**	
Group work and peer instruction				**5**
Individualized instruction and feedback				**5**

The table above shows information about the respondents on effective instructional method and materials in teaching transformation that assist in identifying and addressing errors and misconceptions. 100% of the teachers' support that group work, peer instruction and individualize instruction and feedback always assist in identifying and addressing errors and misconceptions held by pupils in the topic of transformation. 80% of the respondents suggest that traditional teaching methods and hands on activities and manipulative sometimes helps in addressing errors and misconceptions in pupils towards transformation geometry. This goes in line with the researcher that lecture method helps the teacher to explain and demonstrate key concepts of the topic as well as geometrical software will also assist pupils to visualize the shape as it moves from one point to another.

From the graph above 100% of the teachers agreed that they use formative assessment and peer instruction to identify and address errors and misconceptions held by students in transformation. 80% of the respondents use quizzes and tests and interviewing. 100% 0f them also agreed that observation do not apply in identifying errors and misconceptions held by learners. This support researcher's suggestion that formative assessment helps to identify and addressing misconceptions held by learners in transformation topic. By giving pupils more exercise, the facilitator will eventually identify the gaps in pupils and assist them through individual feedback.

4.7 DISCUSSION

The findings of this study align with the existing literature on errors and misconceptions in transformation topics. Radford (2003) and Battista (2007) reported that students struggle with conceptual understanding and visualization of transformation concepts, which is consistent with the results of this study. Specifically, students in this study exhibited difficulties with understanding the underlying mathematical concepts and applying transformations to real-world scenarios. Students were also facing some challenges in descriptive questions for example describing reflection as rotation. Educators should asses students more on descriptive questions so that they become aware of the differences between the types of transformation. The findings also shows that pupils exhibited error of treating TV as an object matrix resulting in obtaining wrong image points. Facilitators should explain to the students the difference between a matrix and vector. A number of students were struggling to do the correct operations. The study also revealed that pupils consider the y axis as y=0 instead of the x axis. Students needs proper coaching on drawing the reflections line. Teacher should make sure that pupils have understood the concept of drawing straight line graphs before introducing the topic of transformation.

Another findings of this study on errors and misconceptions is interchanging the x and y coordinate when plotting. Some students were reversing points during plotting on the graph. Facilitators should emphasize the concept of placing numbers in coordinate system. The first value represent the x coordinate followed by y coordinate.

Similarly, Brown (2014) and Hativa (2013) found that students' difficulties with transformation topics are often rooted in a lack of understanding of prerequisite concepts, such as coordinate geometry and functions. This study supports this notion, highlighting the importance of a strong foundation in these concepts for successful learning of transformations.

However, Lesh (2000) emphasized the role of conceptual understanding in mathematical problem-solving, which is also reflected in this study's findings. Students who struggled with transformation concepts often lacked a deep understanding of the underlying mathematics.

In terms of instructional methods, the findings of this study suggest that group work and peer instruction, hands on activities and manipulative are effective in reducing errors and misconceptions in transformation topics. This is consistent with Lam and Hui (2013), who found that a balanced approach to instruction, including both explicit teaching and student centered activities is beneficial for student learning. Additionally, Moon (2015) emphasized the importance of using multiple representations such as graphs, tables and equations to help students develop a deeper understanding of mathematical concepts. The study’s findings suggest that the use of technology, such as graphing software can enhance student understanding of transformation topic. This is in consistent with Moon (2015) who found that technology can facilitate deeper understanding and improve student engagement. However, some students may rely too heavily on technology and fail to develop a conceptual understanding of the underlying mathematics.

Addressing and identifying errors and misconceptions in transformation topics is crucial for student learning. Lesh (2000) emphasized the importance of understanding students thinking and conceptual framework to identify and address errors and misconceptions. This study‘s findings support this notion, highlighting the need for educators to understand students’ prior knowledge and conceptual understanding before introducing new concepts. Black and William (2009) noted that formative assessment is essential for identifying and addressing errors and misconceptions. This study’s findings align with this, suggesting that regular, targeted assessment can help educators identify areas where students require additional support.

While the findings of this study are consistent with the literature, there are some differences in the types of errors and misconceptions exhibited by students. For instance, Radford (2003) reported that students primarily struggled with graphing transformations, whereas this study found that students also struggled with understanding the mathematical concepts.

More so, the major source of errors and misconceptions as highlighted by students and teachers in the questionnaire was lack of practice and reinforcement. Teachers should give pupils lot of exercise and tests for practice and offer individual feedback to minimize the existence of errors and misconceptions in transformation geometry.

The other source of errors posed by pupils is confusion between similar concept and lack of connection to real world applications. The isometric concept are similar to each other result in students become confused. Teachers and students suggested the use of technology and digital tools in the teaching and learning of transformation to minimize errors.

Overall, the findings of this study contribute to the existing body of knowledge on errors and misconceptions in transformation topics by providing new insights into the types of errors and misconceptions exhibited by students and the factors that contribute to these difficulties. The study supports the importance of conceptual understanding, visualization, and application of transformation concepts, as well as a strong foundation in prerequisite concepts.

4.8 CHAPTER SUMMARY

This chapter has focused on the presentation, analysis and interpretation of data collected using tests and questionnaires. Data was collected from the selected sample of 20 O level students and 5 teachers at one school in zone B in Insiza District. Tables and charts were used to present and analyse the data collected. A detailed discussion of the errors and misconceptions that existed was presented. The data collected reflected that pupils have various errors and misconceptions towards the topic of transformation that includes describing rotation as reflection, treating TV as matrix, considering y=0 as y axis, interchanging y and x coordinate when plotting. The study also identifies effective instructional method that can be used to identify and address the errors and misconceptions. The next chapter will draw conclusion from the findings and then make recommendation of the research.

CHAPTER 5
SUMMARY, CONCLUSION AND RECOMMENDATIOS

5.4 INTRODUCTION

This chapter focuses on the major findings related to the research topic. The chapter also provides the summary and conclusions that are derived from the research findings, thereby making inferences from the review of related literature. Recommendations shall be outlined to enhance the significance of the study.

5.5 SUMMARY

The purpose of this study was to find out errors and misconceptions held by students in the learning of transformation topic by ordinary level students at one school in zone B in Insiza district. The aim of the study was to identify errors and misconceptions held by students in the learning of transformation, finding their source and ways of addressing them.

In this study the mixed method approach method was used because of its merits of collecting both qualitative and quantitative data. The tools used by the researcher to gather information were tests and questionnaire and their advantage and disadvantages were highlighted. Responses were tabulated, frequencies, totals and the measure of central tendencies were calculated to analyse the data. Bar graph, pie chart and frequency polygon were drawn to interpret the data obtained from tests and questionnaires.

5.6 CONCLUSIONS

5.6.1 Sources of errors and misconceptions in transformation

The study found out that students displayed confusion between translation and rotation, difficulties with reflection and symmetry, and struggled to apply transformation concepts to real-world problems. The study concluded that the sources of errors and misconceptions in transformation were confusion between similar concepts, lack of practice and reinforcement, inadequate understanding of coordinate system and lack of connection to real world application.

1. Confusion between similar concepts

Students struggle to distinguish between translation, rotation and reflection leading to errors in transformation application. This is because all the three types of transformation are isometric.

That is the reason many students describe rotation as reflection. Facilitators should use visual aids and technology to enhance students understanding of transformation.

2. Lack of practice and reinforcement

Insufficient practice and reinforcement of transformation concept resulted in students' inability to apply them correctly. Transformation questions are optional questions in O level mathematics paper two, therefore learners do not take it serious and practice it during revisions. So teachers do not teach the topic and discourage pupils to select the questions during the examination. Teachers should encourage students to ask questions and explore transformation concepts in a supportive learning environment.

3. Inadequate understanding of coordinate system

Students' weakness in coordinate geometry hinder them in grasping transformation concepts. It was clearly reflected in the written test that those pupils who were struggling in plotting points, drawing straight lines on the coordinate plane were facing challenges in answering the questions. Teachers should make sure that pupils have clearly understood the coordinate system before introducing transformation topic.

5.6.2 Prerequisite knowledge required by students in transformation

The study concluded that the following prerequisite knowledge is required by learners before they are introduced to the topic transformation: construction, matrices, vectors, coordinates system, straight line graphs and functions. Errors and misconceptions were identified in areas like coordinate geometry, drawing of reflection lines and matrices operation. Teachers emphasized the importance of prior knowledge in algebra, matrices and construction for successful learning of transformation topic. The findings suggest that students' difficulties in learning transformation topics stem from inadequate prerequisite knowledge. Teacher's perceptions highlighted the crucial role of prior knowledge in facilitating students understanding of transformations. The study supports the notion that prerequisite knowledge is a critical factor in the learning of mathematics.

5.6.3 Instructional strategies to address errors and misconceptions in transformation

This study investigated instructional strategies to address errors and misconceptions in transformation topic among ordinary level students. The findings suggest that a combination of direct instruction, formative assessment, practice and feedback, group work and peer instruction and technology and digital tools effectively address student's errors and misconceptions.

5.7 RECOMMENDATIONS

As recommendation towards addressing the misconceptions identified in this study, teachers should provide students with opportunities to experience hands on manipulative that will give them plenty of practice to manipulate in order to get visual perspectives of how shapes move as they are translated, rotated and reflected. This should be achieved before any rules are learned or emphasized. Thereafter students should work on step by step instructions on how to relate the transformed shapes with the properties of both objects and images so that they can discover and master the rules themselves.

Another recommendation can be given to the teachers and students who took the course. When teaching the subject of transformation which covers translation, reflection, rotation, enlargement, shear and stretch, concepts should be well explained. The information given should be comprehensive and descriptive. In order to improve the knowledge of students while teaching the subject, it is better to give concrete examples first then it is appropriate to give the abstract examples later. When teaching the subject, the facilitator must ensure that pay attention to both the relationship and the differences between the concepts. If the concept is expressed clearly, then the probability of students' errors and misconceptions will decreases.

In addition, teachers should integrate ICT in all their teaching and learning processes. The use of technology and computers in the teaching of transformation improves the understanding of concepts and pupils will be in a position to see how shapes, lines and points are shifted from one position to the other. To achieve this strategy, teachers should allow pupils to bring ICT gadgets in class such as smart phones so that they can use them during the lessons.

More so, teachers should encourage collaborative learning activities such as group projects and peer to peer teaching, to foster a deeper understanding of transformation and promote problem solving skills.

To the government, a call is presented to provide free wifi, laptops and projectors to the schools so that teachers and students will integrate ICT in their lessons effectively. This will assist learners to use the heritage based curriculum where learners will have to do examples by themselves and only to inquire to teachers on the tougher ones. The government through ZIMSEC, should make transformation questions compulsory so that teachers and students are taken to ask without either part persuading.

Future research should continue to explore student learning and misconceptions in mathematics, investigating innovative teaching approaches and interventions to address common errors and improve student understanding. By prioritizing research in mathematics education, we can work towards ensuring that all students develop a deep and lasting understanding of mathematical concepts, preparing them for success in an increasingly complex and technological world.

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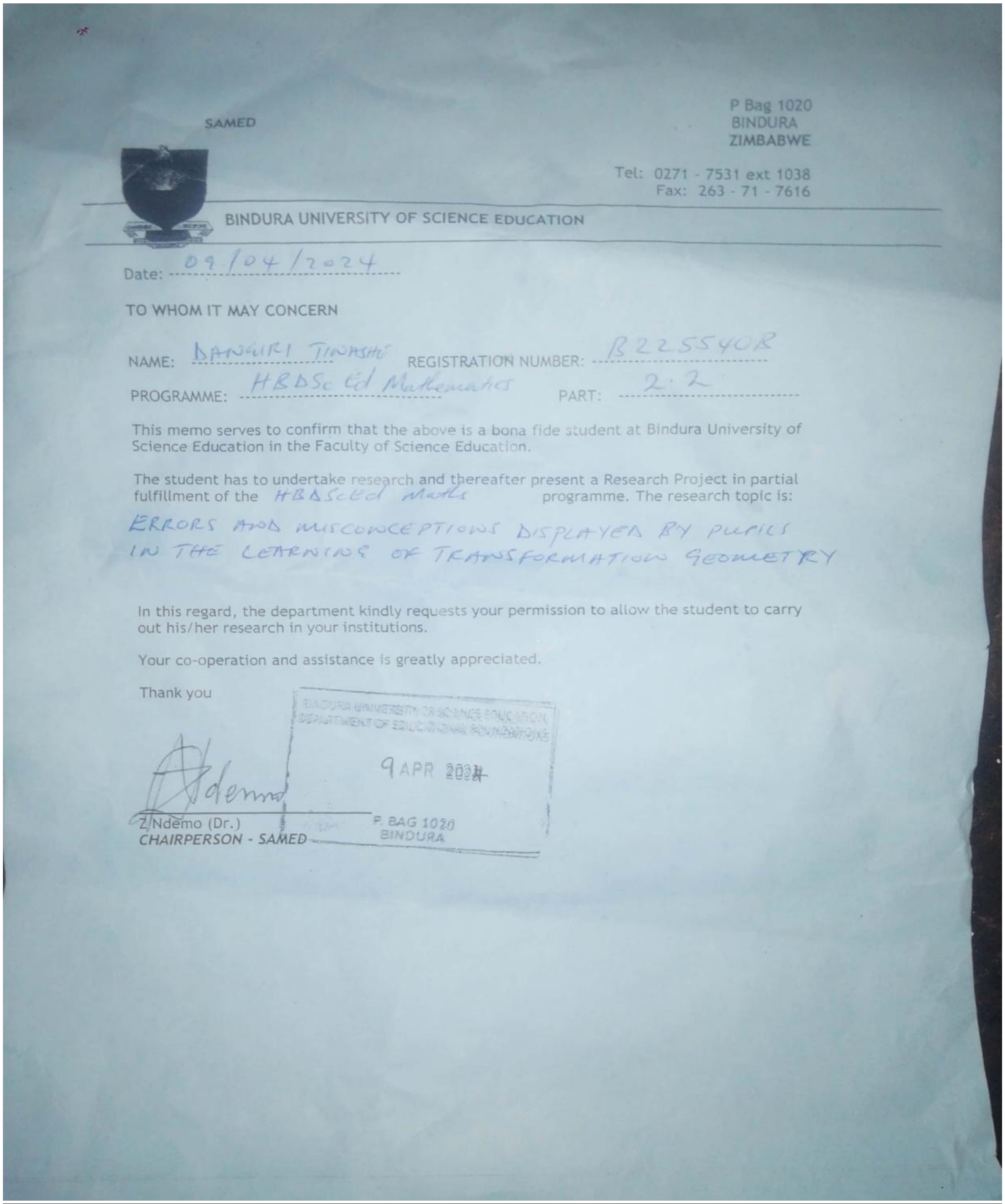
***APPENDIX A**: Request to carry out a research*



SAMED

P Bag 1020
BINDURA
ZIMBABWE

Tel: 0271 - 7531 ext 1038
Fax: 263 - 71 - 7616

BINDURA UNIVERSITY OF SCIENCE EDUCATION

Date: 09/04/2024

TO WHOM IT MAY CONCERN

NAME: DANGIRI TINASHE REGISTRATION NUMBER: B225540R

PROGRAMME: HBDSc Ed Mathematics PART: 2.2

This memo serves to confirm that the above is a bona fide student at Bindura University of Science Education in the Faculty of Science Education.

The student has to undertake research and thereafter present a Research Project in partial fulfillment of the HBDScEd Maths programme. The research topic is:

ERRORS AND MISCONCEPTIONS DISPLAYED BY PUPILS IN THE LEARNING OF TRANSFORMATION GEOMETRY

In this regard, the department kindly requests your permission to allow the student to carry out his/her research in your institutions.

Your co-operation and assistance is greatly appreciated.

Thank you

BINDURA UNIVERSITY OF SCIENCE EDUCATION
DEPARTMENT OF EDUCATIONAL FOUNDATIONS
9 APR 2024
P. BAG 1020
BINDURA

Z Ndemo (Dr.)
CHAIRPERSON - SAMED

APPENDIX B: *PERMISSION TO CARRY OUT A RESEARCH*

Gumbalo secondary school

Box 82

Filabusi

10 May 2024

The Head of Department

Bindura University of Science Education

P Bag 1020

Bindura

Zimbabwe

Dear Sir/Madam

Re: Permission to carry out a research

Dangiri Tinashe , EC number 5845848G is a mathematics teacher at Gumbalo secondary school. He has been granted permission to carry out a research at the school on **Errors and Misconceptions displayed by pupils in the learning of Transformation geometry.**

Yours faithfully

Sidhleni Cloud (Head)

Phone number 0771405155

Signature__________________

Stamp

THE HEAD
GUMBALO SEC SCHOOL
2024 -05- 10
P. O. BOX 82
FILABUSI

Scanned with CamScanner

__APPENDIX C:__ QUESTIONNAIRE FOR STUDENTS

My name is Dangiri Tinashe a student at BUSE. I'am carrying a research on the misconception of pupils in the learning of transformation by O level pupils at Gumbalo secondary school in Insiza District.

The reason for this questionnaire is to gather data for academic purposes. All the information you provide is strictly confidential. Please do not write your name and name of school on this questionnaire.

Your consideration is greatly appreciated.

Section 1: Demographics

1) Grade level: ______________
2) Gender: ______________
3) Age: ______________

Section 2: Misconceptions about Transformations

4. How confident do you feel in your ability to apply transformations to points and shapes on the coordinate plane? (tick where appropriate)

Not confident Very confident

MISCONCEPTIONS	TRUE	FALSE

5. A reflection changes the size and shape of a shape.		
6. A rotation of 90 degrees will always result in the same shape.		
7. A translation is the same as a rotation.		
8. If a shape is reflected over a vertical line, the left and right sides of the shape will switch places.		

Section 3: Sources of Errors and Misconceptions

Which of the following sources of errors and misconceptions do you experience most frequently? (Select all that apply)

Sources of errors and misconceptions experienced	**Never**	rarely	sometimes	always
9. Confusion between similar concepts	**1**	2	3	4

10. Difficult with visualizing and representing transformation				
11. Inadequate understanding of coordinate geometry				
12. Language barriers				
13. Lack of practice and reinforcement				
14. Difficult in abstract thinking				
15. Lack of connection to real world applications				

Section 4: Instructional Methods and Materials.

What instructional methods and materials have been most helpful to you in learning transformation geometry? (Select all that apply)

16. Instructional methods and materials helpful	never	rarely	sometimes	always
	1	2	3	4
17. Traditional teaching methods (e.g., lectures, textbooks)				
18. Technology and digital tools (e.g., GeoGebra, Khan Academy)				
19. Hands-on activities and manipulatives (e.g., geometry software, puzzles)				

20. Group work and peer instruction				
21. Individualized instruction and feedback				

Section 5: Identifying and Addressing Misconceptions.

What strategies do you use to address or correct your misconceptions? (Select all that apply)

Strategies used to address or correct misconceptions	never	rarely	Always	Sometimes
	1	2	3	4
22. Asking teacher or peer for help				
23. Reviewing notes and textbook				
24. Practicing additional problems				
25. Using online resources or tutorials				
26. Working with a study group				
27. Other (please specify)				

What resources or tools would you find most helpful in identifying and addressing misconceptions in transformation geometry? (Select all that apply)

Resources or tools helpful in identifying misconceptions in transformation geometry	helpful	Not helpful
28. Diagnostic assessments or quizzes		
29. Formative feedback from teachers		
30. Peer review and feedback		
31. Graphic organizers or visual aids		
32. Online resources or tutorials		
33. Other (please specify)		

Section 6: Prerequisite knowledge for learning about transformations:

34. Can you describe the coordinate system used in the Cartesian plane?

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……………………………………………………………………………………………………

35. What is the difference between positive and negative numbers in the Cartesian plane?

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36. How do you plot points on the Cartesian plane using ordered pairs?

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APPENDIX D : <u>QUESTIONNAIRE FOR TEACHERS</u>

<u>OPEN ENDED QUESTIONS</u>

Section 7: Demographics

37. Grade level taught: ______________

38. Subject area: ______________

39. Years of teaching experience: ______________

Section 8: Student Errors and Misconceptions

40. What common misconceptions or errors have you observed students making in transformation geometry?

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Section 9: Sources of Errors and Misconceptions

Sources of errors and misconceptions	Never	Rarely	Sometimes	Always
	1	2	3	4
41. Confusion between similar concepts				
42. Difficult with visualizing and representing transformation				
43. Inadequate understanding of coordinate geometry				
44. Language barriers				
45. Lack of practice and reinforcement				
46. Difficult in abstract thinking				
47. Lack of connection to real world applications				

Section 10: Identifying and Addressing Misconceptions:

What instructional methods and materials do you find most effective in teaching transformation geometry? (Select all that apply)

Instructional methods and materials helpful	Never	rarely	sometimes	always
	1	2	3	4
48. Traditional teaching methods (e.g., lectures, textbooks)				
49. Technology and digital tools (e.g., GeoGebra, Khan Academy)				

50. Hands-on activities and manipulatives (e.g., geometry software, puzzles)				
51. Group work and peer instruction				
52. Individualized instruction and feedback				

How do you identify and address misconceptions in transformation geometry in your teaching practice? Select all that apply

Method of identifying and addressing errors and misconceptions	Applicable	Do not apply
	1	2
53. Quizzes and tests		
Projects and presentations		
54. Interviewing		
55. Formative assessments and observations		
56. Peer instruction		
57. Observation		

APPENDIX E : TEST FOR PUPILS

QUESTION 1

On graph paper, draw x-axis and y-axis for -7≤x≤10 and -9≤x≤10 using a scale of 2cm to represent 2units on both axis.

I. Draw and label triangle P with vertices at (2;2) , (2;6) and (4;2). [1]
II. Reflect triangle P onto triangle P_1 in the line $y = 0$. [2]
III. Reflect P_1 onto triangle P_2 in the line $x = 0$. [2]
IV. Describe fully the transformation which maps P onto P_2. [3]
V. What do you notice? [1]
VI. Now reflect triangle P onto P_3 in the line $y = x$. [3]

QUESTION 2 N2014 Q8

Answer the whole of this question on the sheet of graph paper.

Using a scale of 2cm to 2units on each axis, draw axis for values of x from -10 to 10 and for values of y from -6 to 6.

a. Draw and label Triangle PQR with vertices P(1;1) , Q(3;4) , R(4;0). [1]
b. Triangle PQR is mapped onto triangle $P_1 Q_1 R_1$ by a transformation N represented by a matrix $[-1\ 0\ 0\ -1\ ]$.
   i. Draw and label triangle $P_1 Q_1 R_1$. [4]

ii. Describe fully the single transformation represented by matrix N. [3]

c. Triangle PQR is mapped onto triangle $P_2\, Q_2 R_2$ by a Translation T $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$.

i. Calculate the coordinates of the vertices $P_2\, Q_2 R_2$. [3]

ii. Draw and label triangle $P_2\, Q_2 R_2$. [2]

Solutions

Question 1

i. Correct triangle **(1)**

ii. Correct reflection line **(M1).** Triangle with coordinates (2;-2); (2;-6); (4;-2) **(A1)**

iii. Correct reflection line **(M1).**Triangle with coordinates (-2;-2); (-2;-6) (-4;-2) **(A1)**

iv. Rotation, centre (0;0); 180^0 **(B3)** or **(B0SCB1)** for stating that it is rotation.

v. Reflection along the x-axis and then reflect the resultant image along the y axis result in rotation. **(B1)**

Question 2

a. Drawing triangle PQR on the graph. **(B1)**

b.

i. Triangle with coordinates $P_1\ (-1;-1)\ \ Q_1(-3;-4)\ \ R_1(-4;0)$ **(B4)**

ii. Rotation, 180^0; centre of rotation origin. **(B3)**

c. i. Coordinates of $P_2\ (6;-1)\ \ Q_2(8;2)\ \ R_2(9;-2)$. **(B3)**

ii. Drawing triangle $P_2\ Q_2R_2$ on the graph. **(B2)**