BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE

AND MATHEMATICS EDUCATION DEPARTMENT

ERRORS AND MISCONCEPTIONS DISPLAYED BY LEARNERS IN TRANSFORMATIONAL GEOMETRY AT ORDINARY LEVEL.

BY

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Declaration

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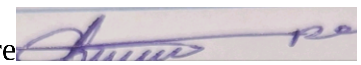
I, Chikwape Knowledge, Registration Number B231620B, declare that this dissertation titled: “*Errors and misconceptions displayed by learners in transformational geometry at ordinary level”* and affirm that it has not been submitted to any other university in support of other similar qualification. I authorize the Bindura University of Science Education to use this dissertation to their institutions or individuals for purposes of scholarly. I declare that all the sources I have used or quoted have been indicated with complete references.

Signature…… Date……….

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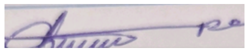
SIGNED BY  **DATE……**

DEDICATION

This research is dedicated to my wife Merylene Mulungisi as without her sacrifice in financial obligations, prayers and support I could never have reached this level. Completing this programme was going to be difficult without this support through all circumstances to reach the stars. l am so thankful to my brother Komborerai Chikwape and my mother Kenita Dokosi you are the reason l have so much courage to move on when the road gets tough. To my dear friends and colleagues I am thankful for all the encouragement you have given me.

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ABBREVIATIONS AND ACRONYMS

ZIMSEC	ZIMBABWE EXAMINATION COUNCIL
T. G	TRANSFORMATIONAL GEOMETRY
O. LEVEL	ORDINARY LEVEL
HBSCED	BACHELORS SCIENCE EDUCATION DEGRE

Abstract

This study investigates the errors and misconceptions exhibited by Ordinary Level learners in transformational geometry at Chionde Secondary School in Muzarabani District, Zimbabwe. Using an action research approach, the study engaged 50 Form Four learners of mixed abilities through a combination of pre and post-tests, written tasks, questionnaires, and semi-structured interviews with mathematics teachers. Sampling was conducted using both purposive and stratified random techniques. All learners selected were actively studying mathematics, ensuring relevance to the research focus. Stratified random sampling was employed to select 15 learners for in-depth interviews, categorized into three performance strata: high (7 learners), average (5 learners), and low (3 learners). This stratification enabled the researcher to capture a range of learner experiences and cognitive challenges. Additionally, two mathematics teachers were purposively selected for interviews based on their experience teaching transformational geometry. The research aimed to identify the nature and causes of learners' errors and misconceptions, and to propose intervention strategies for improving teaching and learning practices. Findings revealed a range of non-systematic errors such as misreading questions and omitting critical information and systematic errors rooted in conceptual misunderstandings, including incorrect plotting of coordinates and misapplication of transformation strategies. The study attributes these challenges to inadequate foundational knowledge, ineffective teaching approaches, and limited visualization skills. Guided by the van Hiele model of geometric reasoning, the research recommends targeted instructional strategies, increased use of visual aids, and hands-on activities to enhance learners' conceptual understanding and performance in transformational geometry.

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CHAPTER ONE: RESEARCH PROBLEM &CONTEX0T

1.1 Introduction

Transformational geometry is a fundamental component of secondary school mathematics, yet a number of learners continue to struggle with its concepts due to persistent errors and misconceptions. These challenges often affect their ability to fully grasp the geometric meaning behind transformations such as translation, rotation, reflection, and enlargement. This chapter introduces the study by providing the background and rationale for investigating these learning difficulties. It presents the statement of the problem, outlines the research objectives and questions, and highlights the significance of the study in improving mathematics instruction. The chapter also defines the study's assumptions, delimitations, and limitations, while offering key definitions to guide the reader. Through this structure, the chapter sets the foundation for a focused exploration of learner misconceptions in transformational geometry.

1.2 Background of the study

In the past years, there have been some considerable numbers of studies about the misconception of various topics in mathematics such as Algebra, Functions and Geometry just to mention the few Fujii (2020). However, there is a dearth of studies on errors and misconceptions in transformational geometry particularly in Zimbabwe (Mukamba & Makamure, 2020). The main purpose of mathematics lessons is for learners to gain problem solving skills, reasoning and providing relationships and communication which gives the researcher the quest to identify some of the things that would prevent learners from acquiring those skills. This study explores learners' errors and misconceptions in learning transformational geometry.

Transformational geometry, also known as geometric transformations, is a branch of geometry that studies the ways in which geometric shapes change when subjected to various transformations (Mitteroecker & Schaefer, 2022). Transformational geometry is a mathematical topic that involves alterations of objects either in position or shape. According to Kekana (2016), transformational geometry consists of movement such as rotation, translations and reflection with dilutions, shrinking, and enlargement. For instance, an object can be moved from point X to point Y, this indicates that the object changed its position. Transformational geometry is a topic that is included in the Zimbabwe Ordinary level Mathematics curriculum. It is considered important because it holds a practical application in the education curriculum and in real life situations (Mukamba & Makamure, 2020). Learners are normally involved in hands-on activities with tangible objects and solve problems using prior knowledge. Practical engagement with physical model enhances learners'motivation and fosters active participation, making abstract transformation concepts more accessible. For instance, teachers provide learners with different types of materials like toothpicks, straws and boxes, and challenge them to construct various shapes such as cubes, pyramids or prisms. This activity helps learners visualise the faces, edges and vertices of three-dimensional shapes.

However, learners at all levels of schooling have challenges with learning geometry (Ibili et al., (2020). For instance, Luneta (2014) posits that learner's basic knowledge of geometry is inadequate. This is evidence that learners often lack deep conceptual understanding in mathematics. According to Wardat et al., (2023), learners without a solid grasp of the underlying concepts, may misinterpret the effects of different transformations. For example, confusing rotations with reflection or dilations with translations can lead to errors in problem solving.

Despite the popularity and importance of transformational geometry in education, ZIMSEC reports (2004-2018) also show that among other components of mathematics the topic has the least pass rate at O level in Zimbabwe. The reports indicate that most learners do not perform well in rotation and reflection. Acosta et al., (2023) discovered that most learners taking mathematics worldwide, at the basic secondary level, were found to have difficulties in understanding transformational geometry concepts. The misconceptions and errors in geometry may be caused by information communicated at different levels of reasoning between the teacher and the learner, which becomes a major cause of the problem (Luneta 2015). Thus, there is a possibility of errors occurring because learners have difficulties in understanding the instructional approaches used by the teacher. For instance, differentiating types of transformational geometry since some have almost similar properties like translation and rotation. Research needs to be done to determine the nature of learners' difficulties with learning geometry and proper possible solutions. An important aspect of this research direction would be the identification of learners' mistakes (Panoura & Gagatsis, 2009).

Luneta (2013) asserts that learners' errors in geometry are because of the procedural way that mathematics is taught. Errors are a result of learners failing to build procedures from conceptual knowledge (Luneta & Makonye, 2010). Transformations can be composed, meaning that applying one transformation followed by other results in a single combined transformation. Hence without a solid conceptual understanding of how transformations compose, learners may incorrectly apply multiple transformations or fail to recognize the net effect of a series of transformations. According to Marek et al., (2011), this results in the rote learning of mathematics where mathematics topics are learnt without connecting them to the underlying semantic information. This may be a source of misconception and the implication here is that learners' errors in geometry are a result of both their learning and the way learners were taught. Errors and misconceptions displayed are proof that learners lack deep conceptional understanding in mathematics.

Shannon and Eu (2013) discovered that learners have difficulties in learning aspects that are linked to transformational geometry. Acosta et al., (2023) argue that teachers have challenges in teaching transformational geometry in a way that stimulates curiosity and motivation among learners. These challenges may breed negative attitudes towards the topic which may culminate into phobia. Mukamba & Makamure, (2020) concur that where learners had challenges in transformational geometry generated from the teaching approaches employed and teaching facilities, the knowledge of teachers was found wanting. In addition, Luneta (2013), identifies teaching habits and ineffective teaching approaches such as persistent teacher-centered approaches as some of the sources of learners' misconceptions and errors. Furthermore, wrong questioning techniques, such as incomplete, ambiguous, or unnecessarily difficult questions can cause learners to make mistakes.

1.3 Statement of the problem

The researcher observed from learners' work in transformational geometry that there were recurring errors and misconceptions, particularly in rotational transformation. Although learners appeared to grasp the algebraic procedures of translation and rotation, their understanding of the geometric meaning behind these concepts remains superficial and fragmented. Mukamba and Makamure (2020) noted that Ordinary Level learners often exhibit such misconceptions due to a lack of conceptual understanding. Since mathematics plays a vital role in national development, effective teaching is essential for equipping learners with real-world problem-solving skills. Transformational geometry, in particular, requires learners to apply specific rules and procedures for each type of transformation (Maharan et al., 2022). Furthermore, misconceptions may stem from challenges in comprehending the instructional approaches used by teachers. To address these issues, this study applies the van Hiele model of geometric thinking to diagnose and interpret learners' errors and misconceptions. According to van Hiele, as cited in Lutfin and Jupri (2020), a learner’s ability to understand geometry is closely linked to their experiences and progression through defined cognitive levels. Therefore, this study seeks to identify the types and causes of learners' errors and misconceptions in transformational geometry and recommend strategies to improve understanding and instruction in the topic.

1.4 Research Objectives

The objectives of this research are:

1. To identify errors and misconceptions displayed by learners when dealing with Transformational geometry at Ordinary level.
2. To identify the possible causes of the errors and misconceptions displayed.
3. To suggest strategies that can be employed to address leaners’ errors and misconceptions

1.5 Research Questions

The research sought to answer the following questions:

1. What errors and misconceptions characterize Ordinary level learners when solving transformational geometry?
2. What are the causes of the errors and misconceptions in learning transformational geometry?
3. How can these errors be addressed for effective learning of transformational geometry?

1.6 Significance of the Study

This research aims to enhance the teaching and learning of transformational geometry by identifying learner-related and teacher-related causes of misconceptions and errors. By exploring these challenges, the research aims to contribute to mathematics pedagogy discourse, ultimately informing strategies to improve learner understanding and performance in transformational geometry. The findings may also give valuable insights for educators, policymakers, and curriculum developers, enabling them to develope more effective instructional approaches and materials that address learners' conceptual gaps in transformational geometry. By addressing errors and misconceptions, this study seeks to promote deeper conceptual understanding and improved problem-solving skills among learners, which are essential for success in mathematics and real-world applications.

1.7 Assumptions of the Study

In this research, it is assumed that teacher-learner ratios are normal in all schools. Panoura & Gagatsis (2009) believe that the teacher’s effectiveness when dealing with a large group is different from when dealing with a small group. It is therefore assumed that the ratio used in this research of one teacher to thirty-five learners at Ordinary Level is generally the same. In addition, Form Four are assumed to have all the relevant knowledge necessaryfor the study. It is assumed that the participants in study will provide accurate answers in the questionnaires and interviews. All the respondents will honestly answer the pre-test and post-test to be administered.

1.8 Delimitation of the study

This study is delimited to one secondary school in Muzarabani District, where the researcher is based. It focuses specifically on Ordinary Level learners and their experiences with transformational geometry. The research explores the types of errors and misconceptions learners’ display, the underlying causes of these errors, and possible strategies to address them.

Conceptually, the study is confined to key components of transformational geometry, namely translation, rotation, reflection, and enlargement, with particular emphasis on rotation and reflection, where most misconceptions were observed. Other geometry topics outside of transformational geometry are not considered. The study will utilize the van Hiele model of geometric thinking to analyse and interpret learners’ levels of understanding. By concentrating on this specific population, content area, and theoretical framework, the study aims to provide focused and in-depth insights into the challenges learners face in mastering transformational geometry.

1.9 Limitation of the study

The research could have been conducted in the whole district, but it will be restricted to only one school in Muzarabani District where the researcher works, therefore the findings may not be generalized to the rest of the district. Another limitation that may be faced in this study is that some learners who are selected to be part of the sample may withdraw at the time of administering the test. This non-response by some participants may lead to loss of information and underestimation of some aspects of errors and misconception displayed by learners on transformational geometry.

1.10 Definition of terms

Transformational geometry

Transformational geometry is a branch of geometry that studies the ways in which geometric shapes change when subjected to various transformations, such as translation, rotation, reflection, and enlargement Martin (2012). According to Mitra, Pauly, Wand & Ceylan (2013, September) transformational geometry is the study of geometric transformations, including the properties and relationships between shapes and their images under these transformations, which can include changes in position, orientation, direction, or size.

1.10.1 Errors

Errors are "a serious inhibitor to learning as mathematics builds on itself," according to Schnepper and McCoy (2006). Harper (2010) defines an error as a “deviation from accuracy or correctness” while Setiawan (2020) defines errors as “mistakes learners make when solving problems that may be caused by carelessness, misinterpretation of symbols or text; lack relevant experience or knowledge related to that mathematical topic, learning objective, concept; lack of awareness or inability to check the answer given; or the results of misconceptions”.

1.10.2 Misconceptions

Luneta (2015) defines a misunderstanding as an incorrect conclusion derived from flawed reasoning or erroneous or inaccurate facts. Chew & Cerbin, (2021), defines misconceptions as ‘conceptual or reasoning difficulties that hinder learners’ mastery of any discipline’. According to Miedemmaet al, (2022) a misconception could be the result of ‘a misapplication of a rule, an over- or under-genelisation, or an alternative conception of the situation’. Swan (2001, p. 154) views misconceptions as ‘natural stage of conceptual development’.

1.11 Summary

This study explores learners' errors and misconceptions in transformational geometry among Ordinary Level learners in Zimbabwe. The research aims to identify the types of errors and misconceptions learners’ display, the causes of these errors, and potential strategies to address them. The study is grounded in the van Hiele levels of geometric understanding, which suggests that learners' ability to comprehend geometry topics is influenced by their learning experiences. The research highlights the significance of conceptual understanding in transformational geometry and the challenges learners face due to inadequate teaching approaches, lack of hands-on activities, and poor prior knowledge. The study's findings could contribute to improving the teaching and learning of transformational geometry and mathematics pedagogy in general. The research is limited to two schools in Muzarabani District and assumes normal teacher-learner ratios and accurate participant responses. Overall, the study seeks to enhance the understanding of learners' difficulties in transformational geometry and inform strategies to improve their performance.

2 CHAPTER TWO: LITERATURE REVIEW

[redacted]Introduction

This chapter explores the challenges learners face in understanding transformational geometry, a fundamental branch of mathematics that deals with the properties of geometric shapes under transformations such as translations, rotations, reflections, and dilations. Despite its importance, learners often struggle with transformational geometry, exhibiting errors and misconceptions that hinder their comprehension and problem-solving abilities. This review aims to synthesize and critically analyse existing research on errors and misconceptions in transformational geometry education, with a focus on identifying common errors, understanding their root causes, and exploring strategies for mitigation. By examining key studies, theoretical frameworks, and factors contributing to errors and misconceptions, this review seeks to inform the development of effective instructional practices and interventions to improve learner outcomes in transformational geometry.

2.1 Rationale for the inclusion of transformational geometry in the school curriculum

Geometrical transformations were introduced as a curriculum element in many nations because learners in traditional Euclidean geometry had trouble writing proofs and solving geometrical problems. Evboumwan (2013) argues that several groups and individuals came from various countries in support of the inclusion of transformational geometry in the curriculum. Sevgi and Erduran (2020) view the basic facts about transformational geometry which includes translation, reflection and rotation as supposed to be the major objective due to some of its functions.

According to Tursynkulova et al., (2023) the study of geometry contributes to helping learners develop the skills of visualization, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof. Transformational geometry encourages learners to investigate geometry ideas through an informal and intuitive approach. It leads learners to the exploration of abstract mathematical concepts congruency, symmetry, similarity and parallelism that enriches learner's geometrical experience, imagination and thought (Baah-Duobaet al., (2020)

2.2 Some related research on concepts

Kurts (2010) investigated "learner misconceptions and errors in two-dimensional transformation geometry. The subject of the study included 126 bachelor third year learners; data was collected from seven exam questions. The findings of the analysis revealed that these learners did not understand how to apply rotational transformation. The common mistakes observed showed that the learners seemed to know the algebraic meaning of translation and of rotation, but they did not seem to understand the geometric meaning of these concepts.

Evboumwan (2013) conducted research on the difficulties faced by 8th grade learners in the learning of transformation geometry, using van Hiele levels of learning with 90 learners using written test and interview. Findings from the study revealed that learners had difficulties in identifying and naming transformation of rotation, finding the centre, angle of rotation and locating the exact image of a rotated figure after rotation.

Miedema and Fletcher (2022) conducted research on misconception in geometry and suggested solutions for 8th grade learners by focusing on the mistakes learners made in past mid-term examinations. A descriptive methodology and learners' interviews were used in the study to analyse and interpret the results which revealed several misconceptions related to lack of background knowledge, reasoning and basic operation mistakes.

Paudel (2014) conducted a qualitative investigation of difficulties in learning trigonometry at secondary level. Class observations and interviews were used for data collection. The study concludes that there was not sufficient material for learning geometry and due to lack of interactive classroom learners faced challenges in learning trigonometry.

2.3 Misconceptions and associated errors

One important context in which teachers need to play a vital role in trying to improve learner performance in mathematics, particularly geometry, is identifying and addressing learners' errors and misconceptions. According to Panaoura & Gagatsis (2009), errors were seen for the first time in a positive way in the work of Piaget, whereby they allowed the tracing of the reasoning mechanisms adopted by learners. This means that making errors in computation, for example, is not completely bad because it becomes an important part of the learning process if these errors are dealt with diagnostically.

Errors and misconceptions are related even though they are different in many aspects. An error is a mistake, slip, blunder or inaccuracy (Luneta & Makonye 2010). A basic premise in differentiating between an error and a misconception is that errors are easily detected in learners' work such as written text or speech, while misconceptions are often hidden from mere observation without scrutiny. If errors and misconceptions were to be put on a continuum, one would have non-systematic errors on one end, and the more serious systematic errors, which are deeply rooted in misconceptions on the opposite end (Makonye, 2011). The aim of this study was therefore to discover errors and misconceptions in the study of geomatry.

2.4 Classification of errors

Various authors have put forward different ways in which errors can be classified. Scholars generall classify mathematical errors under two broad categories, namely, non-systematic and systematic errors.

2.5 Non-systematic errors

Non-systematic errors are normally defined as slips and might exist due to learners' carelessness, misreading information or forgetting some piece of information, unintentionally. Normally, learners easily correct such errors by themselves because there are no fundamental and faulty conceptual structures associated with them. According to Hensenet al., (2020) teachers are warned that errors in the learning of mathematics are not simply the absence of correct answers or the result of unfortunate accidents. They are the consequences of definite processes whose nature must be discovered. Even if these errors were simple slip ups, they should not be taken lightly because they can demoralise learners and impede their performance when committed, thus becoming a serious inhibitor to learning as mathematics builds on previous understanding (Schnepper & McCoy, 2013).

2.6 Systematic errors

Gad (2022) indicated that systematic errors mean consistent application of faulty methods, algorithms or rules. The cause of systematic errors may relate to learners' procedural knowledge, conceptual knowledge, or links between these two types of knowledge (Mohd, 2002). Examples of systematic errors have been suggested, such as errors due to deficiencies in mastery of prerequisite skills, facts, and concepts and errors due to the application of irrelevant rules or strategies (Edodwatte, 2011).

2.7 The theoretical framework

Piaget (1971) puts forward the view that children’s geometrical understanding develops with age and that for children to create ideas about shapes they need physical interaction with objects. Butterworth (2022) asserts that children’s representation of space is constructed through the progressive organization of the child’s motor and internalized actions.

Problems experienced by learners when learning geometry have been investigated in various studies, many of which used the van Hiele theory to identify learners' misconceptions (Luneta, 2015). This is a useful framework for extracting, measuring, understanding and addressing learners' difficulties with school geometry. In the context of this study, the van Hiele theory played an important role in that it helped diagnose some of the causes of the misconceptions displayed by the learners in that questions were set to cater for particular levels of development of learners' reasoning in geometry. The van Hiele theory was used to address the misconception identified. Understanding these levels enables teachers to identify the general directions of learners’ learning and the level at which they are operating (FengSeret al., 2022). The first three levels involve the development of procedural fluency in geometry, whilst the last two display the development of conceptual understanding (FengSer et al., 2022). Van Hiele’s theory is divided into two parts: the first part is the hierarchical sequence of the levels, which shows that each level must be fully developed by the learner before proceeding to the next level. The second part is the development of intuition in learners and the phases of learning that influence geometric learning (Fitriyani et al., 2020). Van Hiele’s levels provide teachers with a framework within which to conduct geometric activities by designing them with the assumptions of a particular level in mind and they can ask questions that are below or above a particular level (FengSer et al., 2022. The levels are also a good predictor of learners' current and future performance in geometry. According to Arnal-Bailera and Manero (2024), Van Hiele model of mathematical reasoning has become a proven descriptor of the progress of learners' reasoning in geometry and is a valid framework for the design of teaching sequences in school geometry. Van Hiele's (1986) theory of geometry with its focus on geometrical reasoning has been linked to Piaget' five stages of child development and the role they play in learning geometry (Kusuma et al., 2021). Van Hiele's levels of geometrical thought were the guiding principles for studying exiting Grade 12 learners' knowledge of transformation geometry and for determining the level at which the average learner in the sample operated (Kekana, 2016). Significantly, such information can inform tertiary and vocational institutions as to the instructional support in transformation geometry that learners need, as they engage with fundamental mathematics and physics courses.

According to Amidu and Nyarko (2019) van Hiele theory is a theory of geometric thought which helps teachers organize instruction for the effective teaching of geometry. This theory consists of two parts, namely, "levels of thinking" and "phases of learning" (Arnal-Bailera & Manero, 2024). The levels of thinking are levels of understanding through which learners' progress as a result of instruction organized into five phases of learning geometry. According to Mawarsari and Nuriana (2023), Van Hiele categorized these levels as visual, descriptive, abstract/relational, and formal deduction. At the first level, learners identify forms and figures using actual examples. Learners in the second level recognize shapes based on their attributes, such as a rhombus, which has four equal sides. At the third level, learners can identify relationships between classes of figures (for example, that a square is a special form of rectangle) and can discover properties of classes of figures by simple logical deduction. At the fourth level, learners can produce a short sequence of statements to logically justify a conclusion and can understand that deduction is the method of establishing geometric transiformation. According to the van Hiele model it is not possible for learners to bypass a level.

2.8 Causes of errors

Aktas & Ünlü (2017) view that the causes of errors in transformational geometry can be attributed to several factors, including learners' lack of understanding of geometric concepts such as translation, rotation, and reflection. Misconceptions about the properties of shapes under transformation also contribute to errors, as learners often struggle to apply these concepts correctly Özerem (2012). Deficiencies in mastery of prerequisite skills, facts, and concepts, as well as the application of irrelevant rules or strategies, can lead to systematic errors. Additionally, non-systematic errors, such as slips or carelessness, can impede learner performance. Research suggests that learners' procedural knowledge, conceptual knowledge, or links between these two types of knowledge can also contribute to systematic errors. Furthermore, learners' inability to visualize geometric changes and apply transformational rules correctly can lead to errors Aktas, & Ünlü, (2017). Overall, these causes of errors highlight the need for effective instructional practices and interventions to address learners' difficulties in transformational geometry.

2.9 Strategies

Identification and Addressing of Errors

1. Diagnostic Assessments: Regular diagnostic assessments should be conducted to identify common errors and misconceptions exhibited by learners in transformational geometry. These assessments help uncover both systemic and non-systemic errors, such as misreading instructions or applying incorrect transformation rules (Mbusi, 2016).

2. Error Analysis: Teachers should analyse learners' errors to determine their root causes—whether stemming from gaps in conceptual understanding, procedural knowledge, or spatial reasoning. This analysis enables targeted instructional responses (Mbusi & Luneta, 2023).

Instructional Strategies

1. Intuitive Exploration: Encourage learners to explore geometric concepts through informal, intuitive approaches. This fosters deeper engagement and supports movement through Van Hiele's early phases of geometric thought (Armah et al., 2017; Meng & Sam, 2013).

2. Use of Visual Aids: Incorporate graph paper, manipulatives, and digital micro-worlds to help learners visualize and manipulate geometric transformations. These tools enhance spatial reasoning and support active learning (Mbusi & Luneta, 2023).

3. Step-by-Step Demonstrations: Provide clear, scaffolded instructions for mastering transformations such as rotations, reflections, and translations. This supports procedural fluency while reinforcing conceptual links (Guven, 2012).

Developmental Progression

1. Van Hiele's Levels of Geometric Thought: Instruction should be informed by Van Hiele's theory, guiding learners through levels of visualization, analysis, abstraction, and formal deduction. This progression is essential for building robust geometric understanding (Alex & Mammen, 2016).

2. Gradual Release of Responsibility: As learners advance through Van Hiele's levels, gradually shift responsibility from teacher-led instruction to learner autonomy. This empowers learners to take ownership of their learning and apply concepts independently (Mbusi & Luneta, 2023).

Addressing Misconceptions

1. Targeted Interventions: Design interventions to correct specific misconceptions, such as the belief that rotation involves sliding a shape before turning it. These errors often stem from misapplied procedural rules or inadequate spatial visualization (Bansilas & Naidoo, 2012).

2. Emphasis on Conceptual Understanding: Prioritize conceptual understanding over rote procedures. Learners should grasp the invariant properties of transformations and the relationships between pre-image and image, rather than merely following steps (Luneta, 2015).

2.10 CONCLUSION

The chapter's objective was to review previous studies on errors and misconceptions learners face in transformational geometry and identify the research gap. A rationale for the inclusion of transformation geometry in the Zimbabwean Ordinary level Mathematics curriculum has been given. Overall, the research on errors and misconceptions in geometrical transformations aims to enhance our understanding of how learners learn and conceptualise geometric concepts, as well as inform the development of more effective instructional practices and interventions.

3 CHAPTER THREE: RESEARCH METHODOLOGY

3.1 Introduction

This chapter outlines the methodology used in the study. It describes the research paradigm that guided the investigation, the research design, and the setting in which the study was conducted. The chapter also presents the target population, sampling procedures, and data collection methods employed in gathering information on learners' errors and misconceptions in transformational geometry. Furthermore, it highlights the instruments used in the study, the procedures for analyzing the collected data, and the ethical considerations that were observed. By presenting these aspects, the chapter establishes the framework upon which the validity and reliability of the study are built.

3.2 Research paradigm

The research is pinned on the interpretivist paradigm as it aims at understanding the errors and misconceptions from the learners' perspective (Ary, Jacobs & Razavieh 2002). The research paradigm aims to contribute to a comprehensive exploration of errors and misconceptions in transformational geometry and the development of effective pedagogical strategies and educational intervention (Makinde et al., 2023). According to Muzari et al., (2022) an interpretivist paradigm seeks to understand the situation from the perspective of the participants. The interpretivism paradigm is an approach to social science that asserts that understanding the beliefs, motivation and reasoning of individuals in a social situation is essential to decoding the meaning of the data that can be collected around a phenomenon (Mcleod, 2024). The study is interpretive since it aims at understanding the misconceptions displayed by learners and trying to get the answers from the views of the learners. The research used face to face interactions and detailed written work. The aims were to understand the lives and experiences of individuals and concernedly identify reasons for why they act the way they do.

3.3 Research design

A mixed methods approach was used to collect data from this study. According to Bruce et al., (2023) the mixed-methods approach combines quantitative and qualitative data to investigate errors and misconceptions in transformational geometry. This methodology involves using both numerical data and narratives to provide a comprehensive understanding of the research problem. The researcher’s topic adopted the mixed methods approach, combining the scientific (quantitative) and the humanistic qualitative approaches to research. The exercises and tests form the quantitative data collection instruments whilst interviews form the qualitative data collection instrument. Questionnaires collected both quantitative and qualitative data. The mixed-method approach is chosen for its complementarity, where quantitative and qualitative methods support and elaborate on each other, enhancing the understanding of errors and misconceptions in transformational geometry (Gravada et al., 2023). This facilitates a deeper understanding of learners' mathematical difficulties and aids in identifying patterns of errors and misconceptions in geometry learning.

3.4 Description of the Setting

The investigation was done at Chionde Secondary School in Muzarabani District, Mashonaland Centeral Province. In the District, there are Mission schools, Government schools and Private schools but the research was carried out at a Council school with a total of 7 teachers, 3 females and 4 males.The school is of mixed ability grouping and has one stream per form. The school has learners of the Junior Level (form 1 and form 2) and the Ordinary Level (form 3 and form 4).

3.5 Population

The term population refers to the total group of participants involved in the research. In this study, the population consists of Ordinary Level learners at Chionde Secondary School, specifically those in Form Three and Form Four who are studying Mathematics. A sample of fifty Form Four learners, of mixed abilities, was selected to participate directly in the study and provide the research data.

3.6 Sampling.

Purposive sampling was used to select a relevant group for investigating the phenomenon. Only form four learners who are taking Mathematics were chosen. A group of 50 learners was taught and given tests and exercises, from which the errors and misconceptions displayed, were identified. For interviews, stratified random sampling was employed to select participants in the ratio 7:5:3. The same 15 participants, along with five Mathematics teachers, were interviewed, while all 50 learners completed the questionnaires.

In agreement, Haleen et al. (2022) posit that a sample drawn from the target population can provide insights that allow for generalisations about the larger group. Therefore, a sample of 15 participants was selected for interviews based on their performance in the tests and exercises: 7 high performers, 5 average performers, and 3 low performers. A slightly greater number of low-performing learners was included to gain a deeper understanding of the causes behind their errors and misconceptions. The study also seeks to capture learners' perspectives on possible intervention strategies to prevent or reduce such errors. Since it is not practical to interview all learners, the logic of the study rests on the assumption of sampling, as supported by Pandey and Pandey (2021).

3.7 Research instruments

Takona, (2002, p. 73) defines research instruments as "essential tools to measure such variables as opinion, concepts, attitude, composition and so on", and suggests questionnaires and interviews as effective tools when conducting research in education. Research instruments can be tests, surveys, scales, questionnaires, or even checklists which help the researcher to collect measure and analyse data. In this research, tests and tasks, interviews and questionnaires will be used to collect data.

3.7.1 Tests and tasks

Short tests and exercises will be used to collect data from all the participants. Exercises will be given after each teaching unit to identify errors per unit. Two tests based on errors identified from teachable units will be administered to come up with misconceptions related to the errors. Some items used by Soon (2013) in his research will be adopted to collect data.

3.7.2 Questionnaires

The questionnaires will be` self-administered to a group of 15 participants chosen based on stratified and simple random sampling techniques. Participants will be categorized into better performance, average performance and poor performance groups. The seven learners in the poor performance stratum, five from the average stratum and three from the better stratum will also be interviewed.

Questionnaires are documents which ask the same questions to all individuals collecting qualitative data, quantitative data or both in which research participants write their responses. The questions given may be open-ended, closed or mixed. In this research, the learners will be given questionnaires with closed and open-ended questions to get information on the errors and misconceptions.

Advantages of Questionnaires

A questionnaire is an important data collection tool which is simple, quick, straightforward, uncomplicated, and effective in gathering data as well as coding the responses or answers (Shonhiwa, 2018). This enables collecting much information from a huge population at a lower cost within a short time frame.

Questionnaires can still give reliable and valid results despite being carried out, distributed and collected by another person and not the researcher himself. In this research, the researcher will distribute questionnaires for participants to complete.

Disadivantages of Questionnaires

Interpreting a questionnaire may be difficult for participants thus; results tend to be limited or not answering the questions clearly. Shonhiwa (2018) argues that the research participants making their own assumptions and decisions as to which is crucial and thus, important information or questions may be missed or may be unsuitably answered.

It is also difficult to tell whether the respondent is being truthful. Participants may change behavior and responses as they know that their behavior is being monitored or recorded thus limiting data collection. Shonhiwa (2018) further asserts that close ended questionnaires restrict the respondents to answers provided.

3.7.3 Interviews

Learners will be interviewed so as to identify their misconceptions and the causes of the errors that they displayed. Interviews help the researcher to collect relevant qualitative data from the participants' view.

In-depth interviews are flexible as they can be carried out in a variety of situations such as home, workplace or outdoors (Schlendler & Cooper, 2009). Platoon (2011) adds that the researcher can judge the quality of the response of the subjects, evaluating whether a question has been fully understood and answered and gives room for rephrasing and probing to get more information.

3.8 Rationale for Method Selection

In-class tests provide objective, first-hand evidence of learners' errors and misconceptions. They allow the researcher to assess specific problem-solving processes and measure understanding in a controlled setting. This method aligns with the study's aim to analyse cognitive difficulties and mistake patterns, particularly useful when identifying Van Hiele levels of understanding. Interviews offer rich, qualitative insights into learners' thinking, allowing the researcher to probe deeper into the causes of errors observed in tests. This method supports the identification of both conceptual misunderstandings and affective factors (e.g., language barriers, confidence), which may not be visible in written work alone. Questionnaires enable the collection of broad data from a larger sample. They are effective in identifying trends, beliefs, and contextual factors (e.g., access to resources, teaching methods) that may contribute to learning challenges. Their structured nature supports efficient analysis and comparison across participants.

3.9 Data collection

Data for the study was collected using tests, exercises, interviews, and questionnaires. The tests are designed based on the van Hiele model of geometric thinking, specifically using the framework developed by Soon (1989) to investigate learners' levels of understanding in transformational geometry. Soon's levels provide a structured way to assess how learners comprehend geometric concepts, from basic recognition to logical deduction. These levels are summarized in Table 1, which outlines the characteristics associated with each developmental stage, as adapted by Mbusi (2017).

Level	Characteristic Description
0 – Visual	The learner recognizes shapes based on appearance, not properties
1 – Descriptive/Analytic	The learner identifies properties of shapes but does not understand relationships between them.
2 – Abstract/Relational	The learner understands relationships between properties and can follow logical arguments.

Table 3-1 Performance indictor

These levels will guide the analysis of learner responses to identify their current stage of geometric reasoning and the types of errors and misconceptions they exhibit.

3.10 Collecting Data from Interviews

After administering tests and exercises to all fifty Form Four learners to analyse their errors and misconceptions in transformational geometry, the researcher conducted interviews with a subset of fifteen learners (7 high performers, 5 average performers, and 3 low performers). These participants were purposefully selected based on specific criteria, such as their performance on the tasks and the types of errors they exhibited in concepts like rotation, reflection, translation, and enlargement.

According to Mudavanhu et al. (2023), interviews serve to explore learners' deeper thinking and provide insight into their understanding of mathematical concepts. In this study, the interviews were designed to gain a deeper understanding of learners' thought processes, conceptual understanding, and the challenges they encountered while solving transformational geometry problems.

The interviews were conducted in a quiet, supportive environment to encourage open discussion. The researcher actively listened to the learners' responses and documented their input through detailed note-taking and audio recordings to ensure accuracy and allow for thorough analysis during the data interpretation phase.

3.11 Data Presentation Procedures

In this research study analyzing learners' errors and misconceptions in solving problems related to transformational geometry through a qualitative approach, the data presentation procedures aim to present findings clearly and insightfully. Qualitative data obtained from interviews will be organized and analysed using thematic analysis. Recurring themes, patterns, and sub-themes related to specific errors in transformations such as rotation, reflection, translation, and enlargement will be systematically identified and presented (Mudavanhu et al., 2023). Supporting quotes or excerpts from learner and teacher interviews will be included under each theme to provide evidence and deepen understanding.

In addition, visual representations such as learners' actual workings, diagrams, and flowcharts will be incorporated to illustrate their thought processes and problem-solving approaches. These visuals will help readers trace the steps taken by learners and highlight exactly where conceptual or procedural errors occurred.

3.12 Data analysis

Data analysis is the process of systematically applying statistical and/or logical techniques to describe and illustrate, condense and recap, and evaluate data. Indeed, researchers generally analyse for patterns in observations through the entire data collection phase (Li & Zhang, 2021). Johnson and Christensen (2024) define it as the process of coding, categorizing and clustering information. While Brown (2014) regards data analysis as a process of inspecting, cleansing, transforming, and modelling data with the goal of discovering useful information, informing conclusions, and supporting decision-making.

Data, when initially obtained, must be processed or organized for analysis (Nelson, 2014). Brown (2014) states that once processed and organized the data may be incomplete, contain duplicates, or contain errors. Common tasks include record matching, identifying inaccuracy of data, and overall quality of existing data, deduplication, and column segmentation.

In this research, the data is organized to reduce duplication and match errors and misconceptions. This ensures the error has been recorded once though with alternatives. Content analysis of scripts will be done together with responses from the questionnaire and interview results. This is to be done to determine common difficulty patterns and errors that should be used to formulate misconceptions related to each transformation. The data is analysed inductively to generate theory.

3.13 Ethical considerations

Ethical considerations in research are a set of principles that guide research design and practice. These principles include voluntary participation, informed consent, anonymity, confidentiality, minimizing potential harm, and transparent communication of results. According to Bhandari (2021), research ethics are essential for maintaining scientific integrity, respecting human rights and dignity, and fostering collaboration between science and society. Bohannon (2016) also emphasizes that ethical principles ensure that research participation is voluntary, informed, and safe, while protecting individuals and communities from harm or unethical conduct.

In this study, written consent will be obtained from all participants, and they will be fully informed about the purpose and procedures of the research. They will be assured of confidentiality, anonymity, voluntary participation, safety, and inclusivity. To protect participant identities, shadow names (pseudonyms) will be used in all data presentation and reporting. For instance, learners will be referred to as "Learner A," "Learner W1," up to "Learner W50" for interview participants, while teachers will be referred to as "Teacher X1" and "Teacher X7".

Furthermore, a follow-up intervention will be conducted to ensure that all participants, including those in the pilot group, benefit from the research outcomes. This approach aligns with ethical standards aimed at maximizing benefit and minimizing harm.

3.14 CONCLUSION

The chapter focused on research paradigm, research design and data collection methods. It also focused on sampling techniques and data analysis procedures. The following chapter will dive in to the presentation, analysis, and discussions of the findings of the study focusing on learners' errors and misconceptions in transformational geometry.

4 CHAPTER FOUR: DATA PRESENTATION, ANALYSIS, AND DISCUSSION

4.1 Introduction

This chapter presents, analyses, and discusses the findings of the study focused on learners' errors and misconceptions in transformational geometry. A total of fifty (50) learners participated in the research. Data were collected using a test, interviews, and questionnaires. The results are presented using descriptive statistics, tables, and charts, followed by a qualitative discussion. Errors are categorized into non-systematic and systematic errors. Analysis of learners' responses and interviews revealed numerous misconceptions that hinder understanding of transformational geometry concepts. This chapter concludes with a summary of the key findings.

4.2 Demographics Descriptive of respondents

The demographic characteristics of learner respondents are presented in Table 4.2 below

Table 4-1 Demographic Descriptive of respondents

Variable	**Categories**	**Frequency**	**Percentage (%)**
Age	15 years	17	34%
	16 years	22	44%
	17 years and above	11	22%
Gender	Male	18	36%
	Female	32	64%

The demographic information shows that learner respondents are evenly distributed across a number of categories. Regarding age distribution, 34% (17 learners) are of the ages of 15; 44% (22 learners) are of the ages of 16, which is the normal for secondary school learners; and the remaining 22 (11 learners) are 17 years of age and older, which may include final-year learners or those with repeated grades. In terms of gender, there is a slight gender imbalance favouring females, with females making up 64% of the learner body (32 learners) and males making up 36 (18 learners).. Overall, the data shows a typical secondary school environment, with a higher percentage of learners in their final year of study, slight female gender dominance, and the majority of learners in the expected age range of 16 years.

4.2.1 Demographic Information of Teachers

The demographic information was based on their gender. To establish the gender of Teachers, they were asked to indicate it. Their response is presented below.

Table 4-2 Distribution of Teachers by gender

Gender	Total	Percentage
Males	4	57.14%
Females	3	42.85%

The gender distribution of teachers 57.14% male and 42.85% female though based on a small sample, highlights the importance of diversity in mathematics instruction. In relation to the Van Hiele model, varied facilitator perspectives can enrich how learners progress through levels of geometric understanding, especially by offering different scaffolding approaches and fostering inclusive, learner-centered environments.

4.3 Results

What errors and misconceptions characterize Ordinary level learners when solving transformational geometry tasks?

The results show that several learners exhibited both significant non-systematic and systematic (conceptual) errors in solving problems related to transformational geometry. Non-systematic errors included reversing the axes, misreading questions, omission of key information, and incomplete descriptions of transformations. These errors appeared to be due to carelessness or a lack of attention to detail.

On the other hand, systematic (conceptual) errors reflected deeper misunderstandings. These included errors in basic operations, incorrect plotting of coordinates, misapplication of transformation rules, and misconceptions about diagonal reflections. For example, some learners confused reflection over the line $y = x$ with reflection over the x-axis or y-axis. Others applied rotation without using the correct angle or center of rotation.

4.4 Type of Non-Systematic Errors displayed

4.4.1 Reversing the Axes

In Question 1, learners were asked: Using a scale of 2cm to represent 1unit on both axes, draw the x-axes and y-axes for $-3 \leq x \leq 5$ and $-3 \leq y \leq 4$. Triangle ABC has vertices at A (2; 1), B (3; 1) and C (3; 3). *Draw and label clearly triangle ABC.*

The analysis revealed that seven learners displayed errors in drawing the Cartesian plane. Among these, three learners reversed the x- and y-axes, two used inconsistent scales, and another two did not adhere to the specified range. These errors suggest a lack of understanding of the basic structure of the coordinate system. Interview data further indicated that these learners had a weak foundational understanding of number lines and coordinate systems, highlighting gaps in prerequisite knowledge necessary for accurate graph construction.

In line with the van Hiele model, these learners appear to be functioning at Level 0 (Visualization) or Level 1 (Analysis), where they recognize geometric figures but struggle to understand their properties and relationships. Their inability to correctly represent the Cartesian plane suggests they have not yet progressed to Level 2 (Informal Deduction), where understanding of the structure and logic of geometric systems becomes evident.

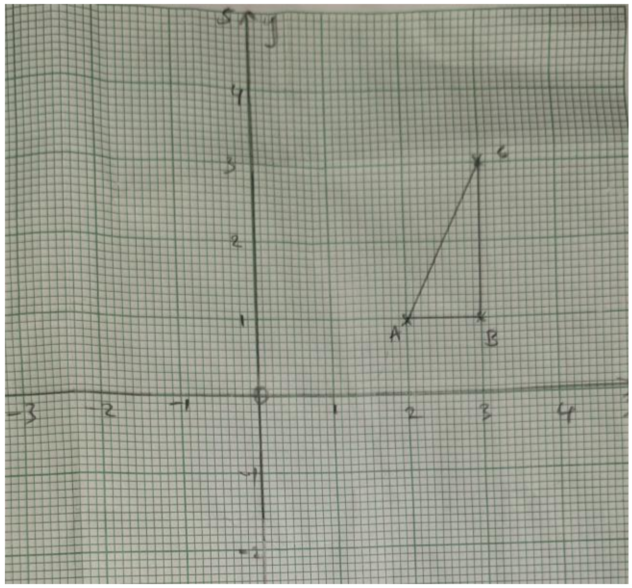

Figure 4-1 Response for learner W22

For instance, Figure 4.1: Response from Learner W22 suggests a possible misunderstanding of coordinate orientation in Question 1. While the diagram is unclear—particularly around the placement of the value 5 on the x-axis—the plotting may reflect a reversal of the x- and y-axes. This kind of error indicates a conceptual gap in understanding how ordered pairs relate to the Cartesian plane, and reinforces the need for instruction that supports learners in progressing through Van Hiele's geometric thinking levels. To confirm this interpretation, a diagnostic interview was conducted.

Teacher: Let's talk about how you plotted the point (3, 5) in Question 1. Can you walk me through what you did?

Learner W22: Sure. I looked at the first number, 3, and went up three units. Then I looked at the second number, 5, and went across five units.

Teacher: So you went up first, then across?

Learner W22: Yes. I thought the first number goes up and the second goes sideways.

Teacher: Interesting. Can you show me where the x-axis and y-axis are on your diagram?

Learner W22: Umm… I think the x-axis is the one going up, and the y-axis is the one going across.

Teacher: Okay, thanks for explaining. Just to clarify, in standard coordinate geometry, the x-axis runs horizontally and the y-axis runs vertically. So when we plot (3, 5), we go 3 units across on the x-axis, then 5 units up on the y-axis.

Learner W22: Oh! I didn't know that. I thought it was the other way around.

Teacher: That's a really common mistake, and it's great that you explained your thinking. We'll work on that together.

This interview confirms that Learner W22 reversed the axes conceptually, interpreting the x-axis as vertical and the y-axis as horizontal. This places the learner at Van Hiele Level 1 (Visualization), where orientation and relational properties are not yet internalized. Instruction should now focus on explicit axis labeling and orientation activities, kinesthetic plotting (e.g., walking out coordinates on a grid) and scaffolded tasks that reinforce the meaning of (x, y) as horizontal then vertical movement.

4.4.2 Misreading the Questions

In Question 2, learners were asked: ΔPQR *has vertices P(2, 3), Q(4, 5), and R(6, 2). Draw*

$\Delta P_2Q_2R_2$*, the image of* ΔPQR *under a reflection in the x-axis. Label the vertices of* $\Delta P_2Q_2R_2$ *clearly.*

The analysis shows that learners experienced difficulties in carefully interpreting transformation questions, particularly in identifying the object and its image. Many struggled to differentiate between the original and transformed shapes. For example, learners misinterpreted the instruction "Draw $\Delta P_2Q_2R_2$, the image of ΔPQR under a reflection in the x-axis" as being different from "ΔPQR is mapped onto $\Delta P_2Q_2R_2$ by a reflection in the x-axis. Draw and label clearly $\Delta P_2Q_2R_2$." This confusion led to incorrect constructions or, in some cases, failure to draw the required triangle altogether.

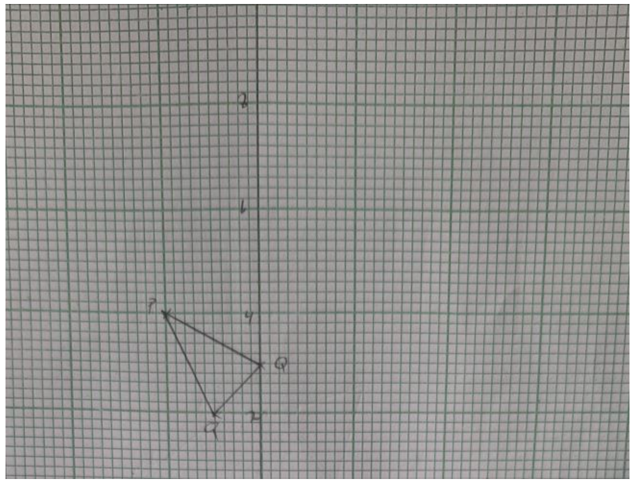

Fig 4-1 written response for learner W18

Figure 4.4: Written Response from Learner W18's written response illustrates a fundamental challenge in transformational geometry: the failure to distinguish between the pre-image (original triangle) and the image (reflected triangle). In this case, the learner omitted the original triangle entirely, suggesting that they may have confused the transformation instruction or misunderstood the task requirements.

Upon reviewing the diagram, it was evident that points P, Q, and R were not correctly plotted. The triangle shown did not match the coordinates of the original triangle provided in the question. This error indicates that the learner may not have grasped the foundational concept of plotting points on a Cartesian plane a prerequisite for performing accurate transformations.

Interview take place between a teacher and Learner W18

Teacher: I noticed your diagram only shows one triangle. Can you explain which triangle you drew?

Learner W18: I thought I was supposed to draw the reflected triangle. I didn't know I had to show the original one too.

Teacher: Did you plot the points P, Q, and R from the question before doing the reflection?

Learner W18: I tried, but I wasn't sure which was which. The instructions were in English, and I got confused with the words like "pre-image" and "image."

Teacher: That's okay. Those terms can be tricky. Did you understand what reflection means in this context?

Learner W18: I think it means flipping the shape, but I wasn't sure how to do it. I just guessed where the triangle should go.

Teacher: Thanks for explaining. It sounds like you're still learning how to read and interpret transformation instructions. We'll work on that together.

Learner W18's response and interview place her at Van Hiele Level 1 (Visualization) not Level 2 (Analysis), as previously stated. At Level 1, learners recognize shapes and transformations based on appearance but struggle to apply formal procedures or distinguish between relational concepts like pre-image and image. Her confusion with mathematical language (e.g., "pre-image," "reflection") and inability to correctly plot points P, Q, and R suggest that she has not yet developed the spatial structuring or procedural fluency required for Level 2 (Informal Deduction), where learners begin to understand relationships between figures and transformation rules.

4.4.3 Incomplete Transformation Descriptions

In Question 4, learners were asked:

"ΔABC has vertices A(2, 3), B(4, 5), and C(6, 2). After a rotation, the image is ΔA'B'C' with vertices A'(-3, 2), B'(-5, 4), and C'(-2, 6). Describe the rotation fully."

The analysis showed that many learners failed to provide comprehensive descriptions of transformations, particularly with rotations. For instance, when asked to describe a rotation, some learners mentioned only the angle and direction, but omitted the center of rotation. This indicates a misconception about the importance of fully specifying all elements of a transformation. Many learners assumed that providing the diagram alone was sufficient and did not recognize the need for clear verbal or written explanations. This highlights a gap in their understanding of mathematical communication, especially regarding the precision required in describing transformations.

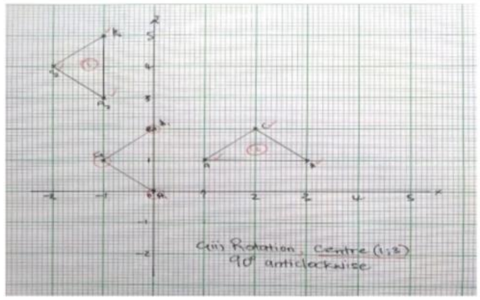

Fig 4-2 Learner W10's response

Analysis shows that Learner W10 identified the point (1, 3) as the center of rotation for a transformation involving point B and its image B′. However, the diagram and written response do not show the method used to arrive at this point such as halving the segment BB′ or constructing perpendicular bisectors. Without explicit evidence, it is speculative to conclude that the learner used midpoint reasoning.

To clarify the learner’s thinking, a follow-up interview was conducted. The transcript below reveals the learner’s strategy and misconceptions.

Teacher: I noticed you chose (1, 3) as the center of rotation. Can you explain how you found that point?

Learner W10: I looked at point B and B′, and I thought the center should be halfway between them. So I just found the midpoint.

Teacher: Did you check if that point works for other pairs of points, like A and A′ or C and C′?

Learner W10: No, I just used B and B′. I thought if it works for one pair, it should be okay.

Teacher: That’s an interesting idea. Did you consider using perpendicular bisectors to find the center?

Learner W10: I’ve heard of that, but I wasn’t sure how to do it. I thought midpoint was enough.

Teacher: I see. So you were estimating based on one pair of points. That’s a good start, but sometimes the center of rotation isn’t exactly halfway it has to satisfy all point-image pairs.

Learner W10: Oh… I didn't think of that. I guess that's why my answer was wrong.

Van Hiele Interpretation

Learner W10's reasoning places them at Level 2 (Informal Deduction) of the Van Hiele model. They recognize relationships between figures—such as the idea of "halfway"—but lack the formal reasoning needed to verify the center using multiple geometric principles. Their reliance on midpoint estimation without testing other pairs or constructing perpendicular bisectors shows they are not yet at Level 3 (Deduction), where learners can follow and construct logical arguments.

4.5 Type of Systematic (Conceptual) Errors displayed by learners

4.5.1 Errors with Basic Operations

In Question 5, learners were instructed: *"Using a scale of 2 cm to represent 1 unit on both axes, draw the x- and y-axes for $-3 \leq x \leq 5$ and $-3 \leq y \leq 4$. Triangle ABC has vertices at A(2, 1), B(3, 1), and C(3, 3). Triangle $A_1B_1C_1$ is the image of ABC under a translation.* $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$ *(i) Find the coordinates of A_1, B_1, and C_1."*

As noted in the analysis, several learners' demonstrated poor mastery of basic arithmetic operations, particularly involving directed numbers. This foundational weakness significantly impacted their ability to perform transformations accurately. For example, 30 out of 50 learners (60%) made errors when adding or subtracting directed numbers while calculating new coordinates during translation tasks and 8 learners (16%) showed confusion with the order of operations involving negative numbers when applying rotation formulas. These errors highlight the need for remedial support in basic arithmetic before introducing more advanced tran

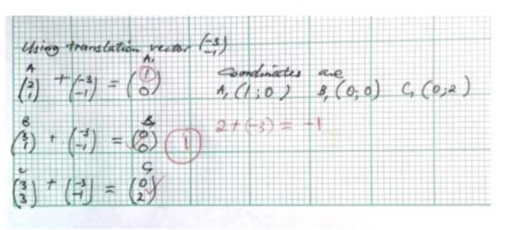

Fig 4-3 Work for learner W5

Figure 4.8: Work of Learner W5 reveals that the learner was unable to correctly add directed numbers, leading to incorrect coordinates. This indicates cognitive difficulty in performing fundamental operations necessary for transformation tasks. The following interview excerpt further illustrates the issue:

Researcher: Can you briefly explain how you have obtained the vertices of the triangle?

Learner W5: Oh yes, I add -3 to 2.

Researcher: How did you add?

Learner W5: I followed the rule which says when one digit has a negative sign, you subtract.

Researcher: Why don't you use a number line?

This exchange demonstrates that Learner W5 lacks a conceptual understanding of directed numbers, relying instead on incomplete rules without visual or number line support.

According to the van Hiele model, this learner is likely at Level 0 (Visualization) or struggling to transition to Level 1 (Analysis). At Level 0, learners recognize shapes based on appearance but do not understand the properties or operations associated with them. Without secure number sense and the ability to reason with coordinates and transformations, learners cannot progress to higher levels of geometric thinking. Strengthening numerical foundations is therefore essential to support their movement through van Hiele's levels.

4.5.2 Incorrect Plotting of Coordinates

In Question 6, learners were asked: *"Plot the following points on a coordinate plane: A(2, -2), B(2, -4), C(3, -4). Explain the steps you took to plot each point, ensuring you identify the correct quadrant and order of coordinates (x, y)."*

The analysis showed that many learners struggled with plotting coordinates correctly, often placing points in the wrong quadrants or reversing the order of coordinates, for example, plotting (y, x) instead of the correct (x, y). This indicates confusion regarding coordinate conventions and spatial reasoning, both of which are foundational for success in transformational geometry.

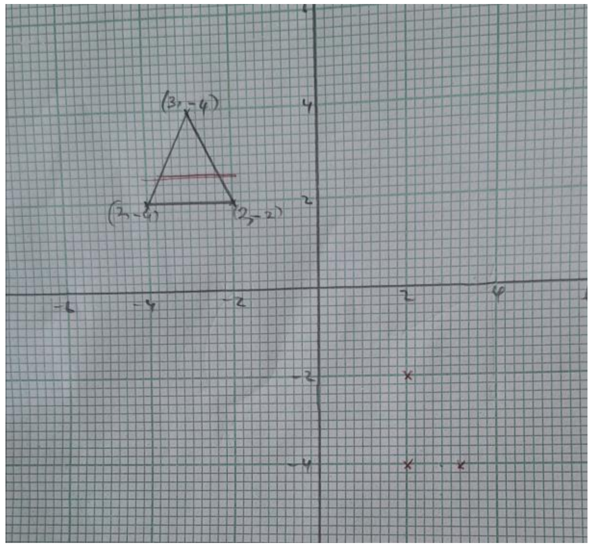

Fig 4-4 Response for learner W7

Figure 4.9: Response from Learner W7 shows that the learner was unable to plot the triangle correctly, revealing conceptual misunderstandings. The following interview excerpt provides further insight:

Researcher: Can you briefly explain how you have drawn the above triangle?

Learner W7: Oh yes, I plotted the points first.

Researcher: How did you plot the points?

Learner W7: I followed the x-axis and the y-axis.

Researcher: What did you do next?

Learner W7: I plotted a point where the lines intersect.

Researcher: Do you know which lines are the x- and y-axis?

Learner W7: I was thinking that the horizontal lines are the y-axis. I was wrong the horizontal lines are the x-axis.

This exchange highlights cognitive confusion around the orientation of the axes and the correct ordering of coordinates. According to the van Hiele model, Learner W7 is likely functioning at Level 0 (Visualization), where learners recognize shapes and figures only by their appearance and lack awareness of the properties and relationships governing them. Misunderstanding coordinate plotting suggests that the learner has not yet transitioned to Level 1 (Analysis), where learners begin to identify component properties like axes, coordinates, and positions in space. Instruction that emphasizes visual reasoning, hands-on graphing practice, and clarification of coordinate rules is needed to support learners at this level and help them advance.

4.5.3 Misapplication of Transformation Strategies

In Question 7, learners were asked: "*Given a transformation matrix and the points A (1,1), B(2,1), and C(3,3), apply the transformation matrix* $\begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix}$ *to these points using matrix multiplication. Show the step-by-step process for obtaining the image triangle* $A_1B_1C_1$*.*"

The results indicate that the misapplication of transformation strategies was a significant issue among learners. Specifically, some learners incorrectly applied transformation matrices—for example, adding matrix elements instead of using matrix multiplication. This error points to a fundamental misunderstanding of matrix operations and their essential role in geometric transformations.

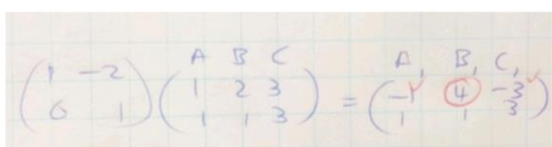

Fig 4-5 Response for learner W12

Figure 4.10: Response from Learner W12 reveals that the learner did not follow the correct procedure. Instead of multiplying the matrix by each coordinate vector, Learner W12 added the values, resulting in inaccurate image points.This reflects a lack of conceptual understanding of transformation matrices, and confusion between scalar operations and matrix operations.

According to the van Hiele model, this suggests that the learner is likely functioning at Level 1 (Analysis) able to identify individual components (like matrices or coordinates) but unable to understand the relationships between them or how they function within a transformation. The correct application of matrix multiplication requires progression to Level 2 (Informal Deduction), where learners can reason logically about the processes and relationships involved in applying transformations.

To support learners in advancing to higher van Hiele levels, targeted instruction, concrete examples, and guided practice in matrix operations are essential. Using visual aids or dynamic geometry software can also help learners see the connection between algebraic procedures and geometric outcomes, thus reinforcing their conceptual understanding.

4.5.4 Misconceptions about Diagonal Reflections

In Test 2, learners were instructed: *"Draw triangle $A_2B_2C_2$, the image of triangle ABC. A (3,1), B(4,1), and C(4,3), under reflection in the line y = x."*

Reflections over diagonal lines, such as y = x or y = -x, posed significant challenges for many learners. A common observation was that learners defaulted to reflecting over horizontal (x-axis) or vertical (y-axis) lines, avoiding diagonal lines altogether. This suggests a difficulty in visualizing symmetry along sloped lines, which require a more abstract understanding of spatial relationships.

Among the learners who displayed conceptual errors, several failed to accurately draw reflections over diagonal lines, primarily because they could not construct or interpret the diagonal lines correctly. Instead, they attempted to reflect the image using incorrect mirror lines, leading to distorted or misplaced images.

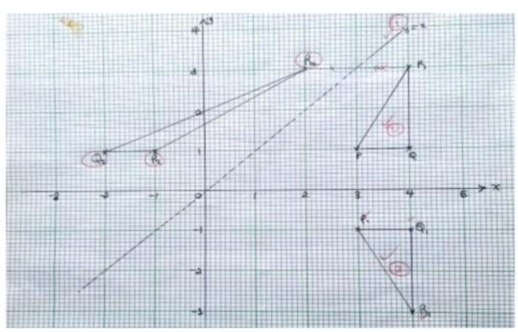

Fig 4-6work for learner W4

Figure 4.11: Work of Learner W4 demonstrates this difficulty. Although the learner was able to draw the line y = x, she failed to reflect triangle ABC correctly across it. During the follow-up interview, Learner W4 acknowledged that diagonal reflections are confusing, stating that she tends to think only in terms of horizontal or vertical reflections.

According to the van Hiele model, this error reflects a learner operating at Level 1 (Analysis), where they can recognize and name properties (like a line of reflection), but may lack relational understanding of how those properties interact in geometric transformations. These learners have not yet reached Level 2 (Informal Deduction), where they would be able to reason logically about transformations, including symmetry across slanted lines like y = x.

Targeted instruction using visual aids, dynamic geometry software, and guided construction exercises is necessary to help learners visualize and understand diagonal reflections, thereby supporting their progression to higher van Hiele levels.

4.6 Interview Findings on the Real-Life Relevance of Geometrical Transformations

Question 1: Is the learning of geometrical transformations important in real life?

The majority of teachers interviewed affirmed that learning geometrical transformations is highly valuable. They emphasized its role in enhancing critical thinking, spatial awareness, and problem-solving skills. Teachers cited real-life applications in fields such as architecture, art, design, and engineering, noting that transformation concepts serve as foundational tools in these professions. Additionally, they highlighted that transformational geometry is a core component of the mathematics curriculum, essential for developing learners' understanding of space, structure, and symmetry (see IB Pros, 2025).

In contrast, learner responses revealed a significant disconnect. Out of 30 learners interviewed, only 21 recognized the relevance of geometrical transformations, while 9 learners expressed skepticism. As illustrated in the pie chart referenced in the report, these learners perceived the concept as abstract and disconnected from everyday life. For instance, Participant W56 stated that transformations "seem to be added just to make mathematics more difficult and to grade learners rather than to teach useful skills."

These contrasting views can be interpreted through the lens of the Van Hiele model of geometric thought. Learners who failed to recognize the real-life relevance of transformations likely operate at Level 0 (Visualization) or Level 1 (Analysis). At these levels, learners tend to see shapes as static images and struggle to connect geometric operations with abstract reasoning or functional applications (Alex & Mammen, 2016; Mbusi & Luneta, 2023). Without progressing to Level 2 (Informal Deduction) where learners begin to understand relationships between figures and apply transformation rules geometry remains disconnected from real-world contexts.

This underscores the need for context-rich and applied instruction in geometry. Teaching transformations through real-life scenarios, design challenges, and interactive tools can help learners move beyond surface-level recognition to deeper conceptual understanding. Such approaches not only foster engagement but also support learners in transitioning through Van Hiele levels toward formal deductive reasoning.

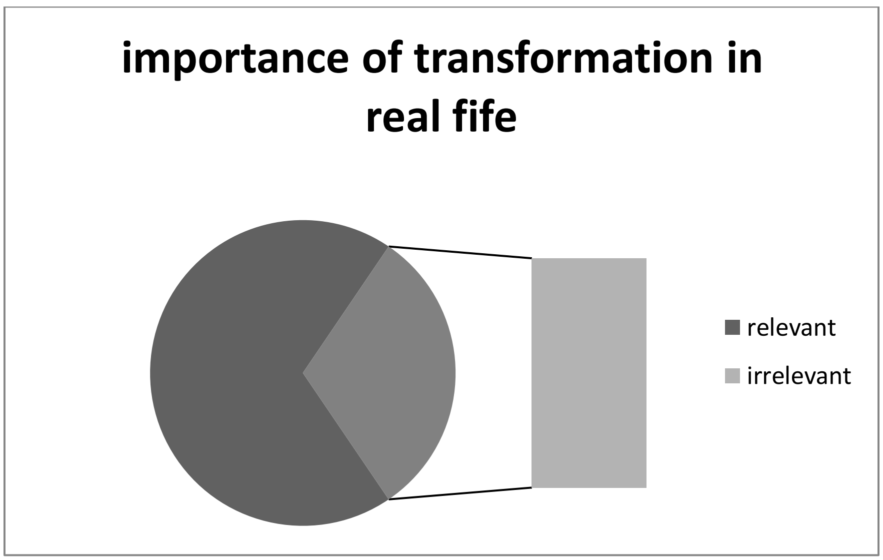

Fig 4-7 importance of transformation in real life

This highlights the importance of teaching geometry in context-rich and applied ways, so that learners can move beyond recognizing shapes and operations to understanding why transformations matter both within and outside the classroom.

Question 2: **What do you think are the reasons or causes of the errors learners make in dealing with transformational geometry?**

Both learners and teachers identified several key causes of errors and misconceptions in transformational geometry. These factors are linked to cognitive challenges, instructional limitations, and resource constraints. The main causes include:

- Carelessness: Some errors stem from simple mistakes such as plotting points in the wrong quadrant, misreading coordinates, or skipping steps in transformations. These are not due to lack of understanding but rather inattention during problem-solving.
- Shortage of reading materials: Many learners lack access to adequate textbooks or revision materials that explain transformational geometry in depth, making it difficult to practice and reinforce learning outside the classroom.
- Limited access to audio-visual aids: The absence of teaching tools such as projectors, computers, or dynamic geometry software reduces opportunities for learners to visualize transformations dynamically. Visualization is a key skill in mastering this topic, and its lack hinders conceptual understanding.
- Lack of assumed knowledge: Transformational geometry requires a strong foundation in coordinate geometry, number lines, and basic arithmetic. Learners who lack this prior knowledge struggle to understand transformations conceptually and apply them accurately.
- Absence of physical manipulatives or real-life examples: Teachers noted that without concrete teaching aids (like mirrors for reflection, tracing paper, or physical grid models), learners find it hard to grasp abstract ideas, especially diagonal reflections or rotations.
- Similarity of transformations: Learners often confuse different types of transformations due to their structural similarities. For instance, they may mix up reflection and rotation, or translation and enlargement, especially when questions are not clearly worded or when diagrams are omitted.

In light of the van Hiele model, many of these causes suggest that learners are stuck at lower levels (Level 0 or Level 1), where they can recognize shapes but struggle to reason about relationships or rules behind transformations. Progression to higher levels requires intentional instructional strategies, resources, and support to build conceptual depth and spatial reasoning.

Question 3: How can these errors be overcomedior addressed?

Based on learners' and teachers' suggestions, several strategies can be implemented to reduce errors and improve learners' understanding of transformational geometry. These include both pedagogical and systemic interventions:

Teaching vector and matrix topics beforehand: It was noted that vectors and matrices should be introduced and fully understood before transformational geometry is taught. These concepts form the foundation for transformations such as translation (using vectors) and rotation or reflection (using matrices).

Integration of technology: Learners suggested using computers and interactive displays during lessons. Tools like GeoGebra, Desmos, and other dynamic geometry software enhance visualization and help explain complex concepts more clearly than static chalkboard drawings. Some teachers struggle to draw or explain transformations legibly, and digital tools can bridge this gap.

Mastery of foundational concepts: Teachers should reinforce basic concepts, such as coordinate systems, number lines, and arithmetic operations (especially with directed numbers), before introducing transformations. Building strong foundational knowledge is essential for higher-level geometric reasoning.

Use of physical teaching aids: Incorporating real-life objects, models, and manipulatives (such as cut-out shapes, mirrors, or tracing paper) helps learners visualize and internalize the effects of different transformations, especially reflections and rotations.

Availability of textbooks and learning materials: There should be adequate learning resources so learners can study independently. Many learners lack access to practice materials at home, limiting their ability to reinforce what is learned in class.

ZIMSEC exam structure adjustments: Learners suggested that transformational geometry should be made a compulsory section in the ZIMSEC Mathematics examination. This would increase attention to the topic by both teachers and learners and ensure thorough preparation.

These suggestions align with the van Hiele model, particularly the need for visualization (Level 0) and analysis (Level 1) before progressing to informal deduction (Level 2). Effective teaching should support learners' gradual movement through these levels using concrete experiences, guided instruction, and technology-enhanced learning.

Question 4: Which transformation or its sub-concept is more challenging than others?

The isometric transformations was not mentioned, most of the learners stated the non-isometric transformations enlargement, shear and stretch. Some stated the two isometrics; reflection and rotation.

Table 4-3 comparison between reflection and rotation

Reflection	Rotation
Finding the mirror line Finding the matrix after a reflection	Finding the centre of rotation Finding the rotation matrix

The above table reveals that learners have problems with finding the matrices for rotation and reflection. Eight of the interviewed learners argued that they crammed and memorized the matrices but because of their similarities, cannot identify the one required. An observation made here is that teachers have to teach learners to derive the matrices from using the unit square method so that learners will not face challenges but get all matrices by themselves.

Table 4-4 Challenges on enlargement, shear and stretch

Enlargement	Shear	Stretch
• Finding the centre of enlargement • Finding the enlargement scale factor (for descriptions) • Multiplication iof matrices	• Finding the scale factor iof shear. • Using matrices to get images	• Differences with enlargement • Finding scale factors and invariant lines for descriptions • Positioning of scale factors in the general matrix

Table 4.2 challenges on enlargement, shear and stretch.

The table 4.2 shows that these three transformations are a great challenge.

4.7 Perceptions of Relevance in Daily Life

Teachers affirmed the relevance of transformational geometry in everyday life, highlighting its role in developing critical thinking, spatial reasoning, and visualization skills. They also noted its practical applications in fields such as architecture, design, engineering, and the arts. These observations align with the view that transformational geometry fosters higher-order thinking skills essential for solving real-world problems.

However, nine learners expressed doubts about the practical usefulness of learning transformations, indicating disconnect between classroom instruction and real-life applications. This may reflect their current Van Hiele level of understanding, particularly those still at Level 0 (Visualization) or Level 1 (Analysis), where learners recognize shapes and transformations but may not yet understand the underlying principles or their real-world relevance.

The study identified two broad categories of errors:

- Non-systematic errors, often resulting from carelessness, poor question interpretation, or language difficulties. These are more likely to occur at lower Van Hiele levels, where learners rely heavily on surface features without deeper reasoning.

- Systematic errors, which stem from conceptual misunderstandings of transformation properties, such as incorrectly identifying the center of rotation or failing to apply vector rules in translations. These errors suggest learners are struggling to transition from Level 1 (Analysis) to Level 2 (Informal Deduction), where they must begin reasoning about relationships and properties logically.

Key areas of difficulty included:

- Incorrect coordinate plotting and quadrant identification.
- Misunderstanding the properties of transformations, such as reflection lines or rotation direction.
- Challenges with mathematical language interpretation, especially for learners who do not speak the language of instruction as their first language.
- Limited ability to visualize transformations, particularly diagonal reflections or composite transformations.

These findings underscore the need to strengthen foundational geometry and arithmetic skills, integrate language support strategies into mathematics instruction and use dynamic geometry tools and interactive visuals to help learners at lower Van Hiele levels progress toward higher stages of understanding. By aligning instructional strategies with the Van Hiele model, teachers can better scaffold learners' development, guiding them from visual recognition to deeper conceptual understanding of geometric transformations.

4.8 CONCLUSION

In the chapter above, the researcher presented, analysed, and discussed the findings of the study focused on learners' errors and misconceptions in transformational geometry. A total of fifty (50) learners participated in the research. Data were collected using a test, interviews, and questionnaires. The results were draw in the format of descriptive statistics followed by qualitative analysis including non systematic errors, systematic errors, analysis of interview questions and a discussion on the perceptions of relevance in daily life

5 CHAPTER FIVE SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

The study investigated learners' errors and misconceptions in transformational geometry during teaching and learning. This chapter summarises the study and the findings and puts forward relevant recommendations.

5.2 Summary of the research

Chapter one focused on defining transformational geometry and its importance in learning mathematics. The research aimed to inquire into the common errors and misconception displayed by learners in transformational geometry, shedding light on their origin and implications. This chapter further disussed the background of the study, research problem, aims and research questions. The background of the study explains the errors and misconceptions done by learners in solving transformational geometry. The study was guided by three questions:

1. What errors and misconceptions characterize Ordinary level learners when solving transformational geometry tasks?
2. What are the causes of the errors and misconceptions in learning transformational geometry?
3. How can these errors be addressed for effective learning of transformational geometry?

Chapter two synthesized and critiqued existing research on errors and misconception in transformational geometry education. Chapter three deals with research design and methodology in conducting the research and focuses on the research paradigm and research instruments used for data collection, sampling techniques and data analysis. The researcher used pre and post test, questionnaires and interviews to collect data in identifying errors and misconceptions. Chapter four focused on analysis of the learner test scripts and interpreting interviews which confirmed that the learners have various misconceptions and associated errors with the topic of transformation. Learners further highlighted the shortage of resources, lack of relevant mathematical pre-requisite knowledge, and inadequate comprehensive examples and practice as some of the major issues compromising teaching and learning of transformation geometry.

5.3 DISCUSSION OF RESULTS

Various researches relate learner's poor performance in geometry to poor pedagogical practices employed in the classroom (Boaler, 2016). According to The Curriculum Development Unit (CDU) cited in Mashingaidze (2012), teachers are held responsible for learners' poor performance in mathematics. Geometrical transformations require a number of skills (assumed knowledge that learners ought to understand before embarking on the concept. This is because early strengths support later progress while, earlier weaknesses compound into greater debility.

This research tried to answer the following questions:

What errors and misconceptions are displayed by Form Four learners at Chionde Secondary School when solving the transformational geometry?

A lot of errors were identified and classified into non-conceptual errors and conceptual errors. The non-conceptual errors are misreading and missing important information. These are unintentional slips or mistakes due to carelessness or errors due to the incorrect performance of steps in mathematical processes. They include the misreading of information by learners or unintentionally leaving out an important piece of information.

The conceptual errors involve basic operations, language, and terminology, misinterpretation of instructions, application of irrelevant rules and strategies, inability to discover properties or changes, and introduction of a system of axes where they do not exist. These errors emanate from inaccuracies inherent in their understanding of mathematical concepts. These errors are common when dealing with multi-step word problems, multi-digit multiplication, or long division.

Learners compute and apply wrong information using the wrong procedure or steps. Most learners could not follow the step-by-step process when answering questions involving multiple sub-concepts. Consequently, the steps were not related to each other, and a single solution that satisfied the conditions was never found.

An error due to misreading which figure is the original and which one is the image was displayed, leading to an incorrect answer. Many learners read "anti-clockwise" as "clockwise," leading to incorrect identification of figures and images. However, this error could also be associated with language problems, as learners mentioned that they often confuse clockwise and anti-clockwise directions.

Learners made errors in writing incorrect ordered pairs by omitting the negative sign, even though the points were correctly labeled or identified on the system of axes. For instance, missing signs led to wrong explanations on questions involving the rotation matrix. This example concurs with Schnepper's (2013) idea of an error involving having the right solution to the wrong question.

Furthermore, learners also made errors in misinterpreting instructions. For example, learners were instructed to draw or locate the angle of rotation on the diagram, but it was interpreted as drawing or writing the size of the angle on the diagram. In another instance, a question required learners to describe two different transformations that would move a figure onto its image, and three-quarters of the learners only mentioned the type of transformation without describing it.

What are the causes of the errors and misconceptions in learning transformational geometry?

The research used questionnaires and interviews to collect information on the causes of errors and misconceptions from teachers and learners. Learners stated that they lack conceptual knowledge in tackling transformation questions. For instance, tests indicated that learners do not understand which concepts are appropriate to solve the problems posed. Interviews with several learners revealed that they are unsure whether they have used the right concepts in problem-solving. Additionally, they could not distinguish between the concepts of rotating and the methods used to simplify tasks. This finding is in accordance with the findings of Kharis et al. (2018) that learners still have difficulty due to a lack of understanding of the concept of the problem.

Moreover, learners make mistakes in writing that lead to wrong solutions. They lacked information in descriptive questions, particularly specific words like transformation and rotation. Wrong wording, such as translation instead of transformation, was also common.

Some of the challenges identified as causes of errors and misconceptions in transformational geometry include carelessness, shortage of reading materials and audio-visual aids for enhancing visualization skills, lack of assumed knowledge, lack of physical objects (real-life application), lack of support from home (no extra tuition outside of school), teacher incapacity, cramming of matrices, inability to identify the properties of changes

Teachers must recognize that errors in the learning of mathematics are not simply the absence of correct answers or the result of unfortunate accidents; they are the consequence of a definite process whose nature must be discovered (Wiens, 2007).

Calculation errors can be generalized as mistakes in addition, subtraction, and multiplication when solving related problems. Carelessness and lack of attention can result in calculation errors, especially in directed numbers, where learners tend to have challenges operating with directed numbers and fail to solve the problem.

Language problems in transformational geometry result in errors and misconceptions during learning. Learners tend to confuse the terminology used in transformation, especially rotation and reflection. For instance, many learners could not understand the meaning of translation and did not make use of appropriate words.

How can these errors be addressed for effective learning of transformational geometry?

The research employed both questionnaires and interviews to gather data on the causes of errors and misconceptions in transformational geometry from teachers and learners. Learners consistently reported a lack of conceptual understanding when tackling transformation questions. For instance, test results revealed that many learners were unable to identify the appropriate concepts required to solve transformation problems. Interviews further indicated that several learners were uncertain whether they had applied the correct concepts, and some struggled to distinguish between transformation types—such as rotation—and unrelated problem-solving strategies.

This finding aligns with the work of Kharis et al. (2018), who observed that learners often experience difficulty due to a lack of understanding of the underlying concepts in transformation geometry.

Additionally, learners made writing errors that led to incorrect solutions. These included vague or incorrect responses to descriptive questions, particularly involving terms like transformation and rotation. A common mistake was the use of the word translation when referring to transformation, indicating confusion in mathematical terminology.

Several challenges were identified as contributing factors to these errors and misconceptions, including ,arelessness and lack of attention, shortage of reading materials and audio-visual aids to support visualization, insufficient prior knowledge of foundational geometry concepts, lack of real-life applications and physical manipulatives, limited support from home, such as absence of extra tuition, teacher incapacity to scaffold transformation concepts effectively, over-reliance on memorization, especially in matrix operations and inability to identify and describe properties of transformations

As Wiens (2007) aptly notes, errors in mathematics should not be viewed merely as the absence of correct answers or unfortunate accidents. Rather, they are the result of a definite cognitive process whose nature must be understood and addressed through targeted instruction.

Calculation errors were also prevalent, particularly in addition, subtraction, and multiplication during transformation tasks. These were often attributed to carelessness and difficulty with directed numbers, where learners struggled to apply correct operations involving positive and negative values.

Finally, language-related challenges emerged as a significant source of misconceptions. Learners frequently confused transformation terminology—especially rotation, reflection, and translation. Many were unfamiliar with the precise meanings of these terms and failed to use them appropriately in their responses, reinforcing the need for language-sensitive instruction in mathematics classrooms.

5.4 Conclusion

This research sought to identify errors and misconceptions displayed by Ordinary Level learners in transformational geometry, examine the causes of these errors, and propose strategies for addressing and preventing them during instruction. Learners exhibited misconceptions in the concept under investigation, likely due to a lack of opportunities to practice visualization skills during their Junior Level education. It is believed that implementing the recommendations outlined in this study can enhance the teaching and learning of geometry, thereby promoting deeper conceptual understanding. Both teachers and learners stand to benefit if educators make deliberate efforts to familiarize themselves with learners' errors and misconceptions, and adopt targeted strategies to address them.This is especially critical in the Zimbabwean context, where error analysis and diagnostic approaches are not formally embedded in teacher training or mathematics education courses. Integrating these practices into professional development and classroom instruction could significantly improve learners' engagement and performance in transformational geometry.

5.5 Recommendations

A major recommendation arising from this research is that teachers should provide learners with opportunities for hands-on manipulation. Such experiences allow learners to practice and develop visual perspectives on how shapes move when they are translated, reflected, or rotated. Learners should be guided through step-by-step instructions that help them relate transformed shapes to the properties of both the original objects and their images. This approach enables learners to discover and internalize transformation rules, fostering deeper conceptual understanding.

This study builds on a broader intervention program that addresses misconceptions through a series of lessons structured around the Van Hiele phases of geometric learning. These phases offer a progressive framework, helping learners advance from one level of geometric thinking to the next. Classroom instruction based on this model supports learners in moving from visual recognition to formal deductive reasoning.

Teachers and learners alike would benefit if educators made deliberate efforts to identify and understand learners' errors and misconceptions, and implemented targeted strategies to address them. This is especially important in the Zimbabwean context, where error analysis and diagnostic teaching are not yet systematically embedded in teacher education programs.

Resourcefulness and Curriculum Support

Teachers must be resourceful and avoid relying solely on a single textbook. They should access a variety of texts and online resources, which offer diverse ideas and teaching methods for transformational geometry. Learners should also be encouraged to use online platforms to reinforce their understanding.

In addition, the government should ensure that every learner has access to a mathematics textbook, as emphasized by Shiza and Kariwo (2011). This provision would empower learners to engage with curriculum content independently and seek teacher support only when necessary.

Enhancing Visualization through Technology

Schools should invest in internet facilities to support the development of visualization skills, which are essential for mastering transformational geometry. With internet access, learners can explore interactive graphs, educational videos, and dynamic geometry software, such as GeoGebra. These tools make abstract concepts more tangible and engaging, helping learners understand and apply transformations—such as rotations, reflections, and translations—with greater confidence. By leveraging technology, schools can create immersive learning environments that foster mathematical thinking and prepare learners for success in a digitally driven world.

Instructional Clarity and Conceptual Sequencing

Teachers should ensure that all sub-topics within transformational geometry are thoroughly explained. Instruction should be comprehensive and descriptive, beginning with concrete examples before progressing to abstract representations. Educators must emphasize both the relationships and distinctions between transformation types to reduce misconceptions and enhance understanding.

Moreover, teachers must effectively teach the assumed knowledge that underpins transformational geometry. Foundational topics such as geometrical constructions, coordinate geometry, directed numbers, graphics, vectors, and matrices should be covered prior to introducing transformations.

Teacher Development and Language Support

To improve pedagogical approaches, staff development workshops should be conducted at school, cluster, district, and provincial levels. These workshops should focus on effective strategies for teaching transformational geometry and diagnosing learner misconceptions.

The Curriculum Development Unit (CDU) should provide teachers with manuals and training materials, emphasizing the use of mathematical language and terminology. Learners have reported that language barriers hinder their understanding of transformational geometry, making it essential for teachers to use precise and accessible mathematical jargon.

Learners must be exposed to hands-on activities that develop their visual and spatial reasoning, enabling them to observe and understand changes in geometric properties. As Cohen and Manion (1984, p.106) argue, interacting with individual learners is a vital skill in mixed-ability classrooms. Teachers must design appropriate individual tasks and monitor progress to ensure that each learner receives the support needed to succeed.

INTERVIEW GUIDE FOR TEACHERS

I am Chikwape Knowledge HBScEd Mathematics learner from BUSE.I am conducting a research ion implementation of competence based learning in mathematics. This research is expected to benefit the entire education fraternity and your participation is received with much appreciation.

Duration (in I minutes)	**Interview question**
2	Is the learning of geometrical transformations important in real life
3	Which transformation is a challenge to you? Describe why the transformation is still a challenge
3	In your opinion, what are the causes of errors and misconceptions?
3	How can these errors and misconceptions be overcome?
4	Which transformation or its sub-topic is more challemging than others? Describe why the transformation remains a challenge

QUESTIONNAIRE GUIDE FOR LEARNERS

Title: Understanding Errors and Misconceptions in Transformational Geometry

Introduction: This questionnaire aims to gather information about the difficulties and misconceptions faced by learners in transformational geometry.Your responses will help us to identify common errors and misconceptions, and to develop strategies for improving learning in this area.

Section 1: Background Information

Section A: Background Information

a) What is your age?

15yrs [ ]   16yrs [ ]   17yrs and above [ ]

b) What is your gender?

FEMALE [ ]   MALE [ ]

c) Have you taken any courses in transformational geometry before?

Yes [ ]

No [ ]

d) How iwould you rate your understanding of transformational geometry?

Very good []

Good []

Average []

Below []

Below average []

Section 2: Errors and Misconceptions

a) What difficulties have you experienced in understanding transformational geometry concepts?

Visualizing transformations []

Applying transformations to real-world problems []

Other (please specify) []

b) Have you encountered any common errors or misconceptions in transformational geometry?

Yes []

No []

If you answered "yes" to question 2, please describe the errors or misconceptions you have encountered.

Section 3: Teaching and Learning

a) How do you prefer to learn transformational geometry concepts?

Lecture []

Hands-on activities []

Group work []

Online resources []

Other (please specify) []

b) Have you found any teaching methods or resources to be particularly helpful in iunderstanding itransformational igeometry?

Yes []

No []

c) If you answered "yes" to question 2, please specify the teaching methods or resources that you found helpful.

d) Have you ever received feedback on your work in transformational geometry?

Yes []

No []

e) If you answered "yes" to question 4, how helpful was the feedback in improving your understanding?

Very helpful []

Neutral []

Not helpful []

Section 4: Suggestions for Improvement

a) What strategies or resources do you think would help improve learning in transformational geometry?

More hands-on activities []

Improved visualization tools []

Better teaching methods []

More online resources []

b) Do you have any suggestions for how teachers can better support learners in understanding transformational geometry?

Provide more examples and practice problems []

Use more visual aids []

Encourage group work and discussion []

Provide more feedback on learner work []

Thank you for taking the time to complete this questionnaire. Your feedback is invaluable in helping us to improve learning in transformational geometry

RESEARCH NSTRUMENT

TRANSLATION

Task1. Translation

Source: MOPSE Level 11 volume 2

Task 2 Translation

Using a scale of 2cm to represent 1unit on both axes, draw the x-axes and y-axes for $-3 \leq x \leq 5$ and $-3 \leq y \leq 4$. Triangle ABC has vertices at A (2; 1), B (3; 1) and C (3; 3).

3 Draw and label clearly triangle ABC.

4 Triangle $A_1B_1C_1$ is the image of ABC under a translation

(i) Find the coordinates of A_1, B_1 and C_1

(ii) Draw and label clearly triangle $A_1B_1C_1$

5 Translate triangle ABC by vector onto triangle $A_2B_2C_2$

REFLECTION

Task 1 Reflection

Triangle PQR has vertices at P (3; 1), Q (4; 1) and R (4; 3). Taking a scale of 2cm to represent 1 unit on both axes, draw the x and y axes for $-3 \leq x \leq 5$ and $-3 \leq y \leq 4$

a) Draw and label clearly PQR

b) Draw and label clearly $\Delta P_1Q_1R_1$ the image of ΔPQR under a reflection in the x-axis

c) Triangle ΔPQR is mapped onto $\Delta P_2Q_2R_2$ by a reflection in the line $y = x$

i) Draw the line $y = x$

ii) Draw the $\Delta P_3Q_3R_3$

d) Reflect PQR onto $P_4Q_4R_4$ in the y-axis

Task 2 Reflection

Triangle ABC has vertices A (2; 1), B (4; 1) and C (4; 4). Using a scale of 1cm to represent 1 unit on both axes, draw the x and y-axes for $-4 \leq x \leq 14$ and $-10 \leq y \leq 12$

a) Draw and label triangle ABC.

b) Triangle $A_1B_1C_1$ is a reflection of triangle ABC in the line $y = -2$. Draw and label clearly triangle $A_1B_1C_1$

c) Triangle $A_2B_2C_2$ is the image of ABC under a certain **single** transformation. Describe this transformation completely.

ROTATION

Task 1 Rotation

Taking a scale of 2cm to represent 1 iunit on axes, draw the x and y axes for -45 and -4y4
Triangle A has vertices at (3; 1), (4; 1), and (4; 3)

a) Draw and label clearly triangle A

b) Triangle B is the image of triangle A under a reflection in x = 2. Draw and label clearly triangle B

c) Triangle A is mapped onto triangle C by a rotation centre (0; 0), 90° clockwise.

- State the matrix for this transformation
- Draw and label clearly triangle C

d) Triangle D is the image of triangle B under a rotation centre (0; -1), 90° anticlockwise. Draw and label clearly triangle.

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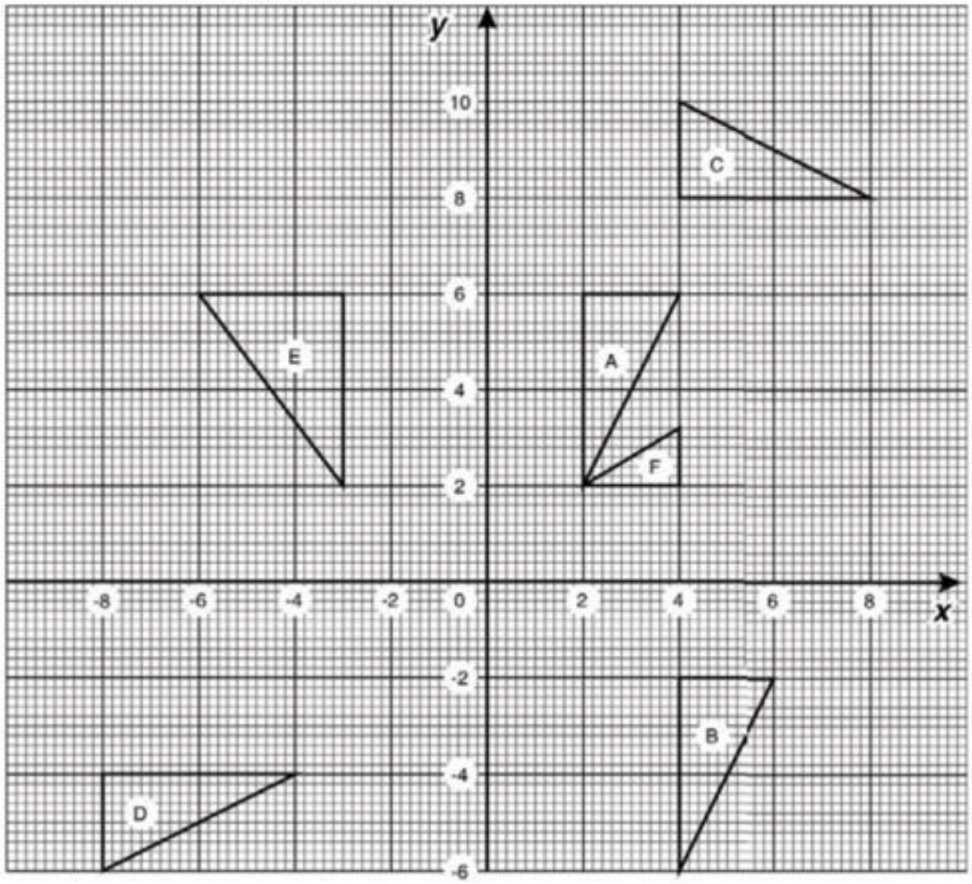

(i) ΔA is mapped onto ΔB by a translation. Write down the column vector for this translation. [1]

(ii) Describe **fully** the **single** transformation which maps ΔA onto ΔC. [3]

(iii) A reflection maps ΔA onto ΔD. Write down the equation of the mirror line. [2]

(iv) ΔD is mapped onto ΔF by a single transformation. Write down the matrix which represents this transformation. [2]

Procedure for approval

To conduct research on errors and misconceptions in transformational geometry, I will begin by preparing a formal request letter detailing the study's objectives, methodology, and ethical considerations. This letter will first be submitted to the Provincial Education Director for Mashonaland Central Province to seek approval. Once granted, I will forward the approved request to the District Schools Inspector in Muzarabani District for district-level clearance. With district approval, I will then approach the Head of School where the research will take place to obtain school-level consent. After securing all necessary permissions, I will proceed to obtain informed consent from participating learners, their guardians, and teachers. The research will be conducted ethically, ensuring confidentiality, voluntary participation, and minimal disruption to teaching and learning activities.

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