Bindura University of Science Education

ANALYZING THE ERRORS MADE BY STUDENTS WHEN SOLVING QUADRATIC EQUATIONS USING FACTORIZATION METHOD AT ORDINARY LEVEL: A CASE STUDY OF SETSHANKE HIGH SCHOOL IN NKAYI DISTRICT.

BY

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## DEDICATION

This project is dedicated to my beloved family especially my husband, Daniel Chibwana, who navigated the challenges of my studies alongside me, sacrificing countless moments for this dream. Your belief in me was the anchor that kept me steady. I dedicate this work to pursuit of knowledge and to those who strive to make a difference in Mathematics education. May it contribute in a meaningful way.

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ABSTRACT

This study aimed to analyse the errors made by form three students when solving quadratic equations using the factorisation method at Setshanke High School in Nkayi District, Matebeleland North. The goal was to uncover the nature and causes of these errors. A total of 70 form 3 students participated in the study. Purposive sampling was employed in this research study to select participants for interviews. The sample consisted of five learners (3 females and 2 males) and was selected from all the learners who wrote the test based on their performance. Data was collected using the test and interviews. The data collection process was guided by predefined research questions and ethical considerations were prioritized. A qualitative approach was adopted for this research. Descriptive analysis techniques were utilized to categorize the types of errors made by the students and to examine the data.

The study's findings revealed that the most common errors made by students, were conceptual errors and technical errors. Recommendations were given for practice and further study.

Contents

# CHAPTER 1

# RESEARCH PROBLEM AND SETTING

## 1.1 Introduction

This chapter provides an overview of the research context, including the background and issues that led to the identification of the research problem. It outlines the purpose, objectives, and research questions guiding the study, as well as the hypothesis and significance of the research. The chapter also clarifies the scope and a boundary of the study, acknowledges potential limitations, and defines key terms. Lastly, it concludes with a summary of the chapter's main points.

## 1.2 Background of the Study

Quadratic equations form part of the fundamental topics in the mathematics curriculum in secondary schools which provide a basis for advanced mathematical concepts and real-life applications. However, studies have shown that learners often struggle with solving quadratic equations and make numerous errors.

O'Connor & Norton (2022) are of the view that learners of a secondary school in Australia struggled with manipulating equations and applying the null factor law correctly. Joshi (2019) discovered weak problem-solving skills among learners of a secondary school in Asia when dealing with quadratic equations using factorization method. Similarly, Ly & Takuya (2023) found conceptual errors among learners in Japan that could affect their future mathematical learning.

The challenges with quadratic equations extend beyond specific regions and are prevalent in broader African contexts as well. Thomas & Mahmud (2021) observed that learners of a certain secondary school in Malaysia made errors due to a lack of understanding of basic concepts and learning styles and many students do not master the factoring skills necessary for quadratic expressions because they usually use a scientific calculator to get the answers to the questions involving factorization of quadratic equations. Additionally, Rahman (2019) noted that female learners in Malaysia commonly made comprehension, transformation, and carelessness errors, while male learners had a higher frequency of transformation errors. According to Kim How et al (2020), learners in Malaysia perceived quadratic equations as difficult to learn and struggled to find appropriate learning materials.

Errors made by learners in quadratic equations have also been identified in specific schools in Zimbabwe. A study conducted by Tendere & Mutambara (2020) to form three learners from a certain high school in Zimbabwe revealed that the learners had difficulties in solving symbolic quadratic equations by the factorization method. The same authors identified the following types of errors: conceptual, procedural and technical. Tendere & Mutambara (2020) conducted a research and found that learners had difficulties in solving symbolic quadratic equations using the factorization method and many misconceptions were exposed. These errors significantly impact learners' understanding of mathematics as a whole, as quadratic equations are connected to other mathematical concepts.

In light of these challenges, this study seeks to analyze learners' errors in solving quadratic equations using factorization and aims to assess their understanding using Bloom's Taxonomy of objectives. Understanding the specific errors and challenges faced by these learners will enable the development of targeted instructional strategies and interventions to enhance their understanding and performance.

### 1.3 Purpose of the study

The purpose of this study is to analyze the errors made by form three learners at Setshanke High School in Nkayi District when solving quadratic equations using the factorization method.

The aim is to understand the common errors inherent in these learners in order to develop effective strategies and interventions to improve their understanding and performance in mathematics. By conducting a detailed analysis of the errors, the study intends to provide valuable insights for both teachers and policymakers in designing targeted instructional approaches that address the specific needs of these learners.

## 1.4 Statement of the problem

The problem addressed in this study is the high occurrence of errors made by Form Three learners at Setshanke High School in Nkayi District when solving quadratic equations using the factorization method. This issue is negatively affecting learners' overall performance in mathematics and hindering their ability to progress to more advanced mathematical topics. While previous studies have examined the occurrence of errors in solving quadratic equations, there is a gap in the research regarding the use of technology, the specific types of errors made by these learners, and the underlying causes. Therefore, the purpose of this study is to identify the specific types of errors made by the learners, analyze the underlying causes, and develop effective strategies and interventions that can enhance their understanding and proficiency in solving quadratic equations using the factorization method. Research has shown that learners often struggle with quadratic equations due to misconceptions, poor algebraic manipulation skills, and inadequate understanding of core concepts (Ndlovu & Mhlanga, 2018). These difficulties are more prevalent in rural schools where teaching resources are scarce and learner support is limited.

According to Mtetwa and Garira (2014), Zimbabwean learners frequently display procedural errors (e.g., incorrect factorisation or use of the quadratic formula) and conceptual errors (e.g., misinterpreting the meaning of roots or the nature of solutions). Such errors are compounded by the abstract nature of algebra, especially for learners transitioning from arithmetic operations. Chigonga and Mutambara (2020) emphasize the role of technology in reducing mathematical errors. Their study showed that digital tools such as GeoGebra and graphing calculators can significantly improve learners' conceptual understanding of quadratic functions. However, limited access to such tools in rural schools like Setshanke hinders implementation.

## 1.5 Research Objectives

The research intended to meet the following objectives:

1.4.1 To establish errors made by form three learners when solving quadratic equations.

1.4.2 To assess the fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors.

## 1.6 Research questions

The research questions mentioned below are to be used as a guide to this study:

1.5.1 What are the errors displayed by form three learners when solving quadratic equations?

1.5.2 What is the fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors?

## 1.7 Significance of the Study

A detailed investigation of the errors is an essential criterion for any additional attempt to advance the quality of Mathematics education and the levels of learners' achievement. Aggregating these issues, it can be added that the research findings will help teachers, textbook writers, curriculum planners, and other stakeholders to gain a profound understanding of how errors in solving quadratic equations using factorization can be identified and thoughtfully engaged. A critical grasp of these errors underpinned in solving quadratic equations will provide a new horizon to struggling learners and equip them with essential concepts valuable to be applied in solving complex real word Mathematics problems.

This research will assist learners to identify and address their common errors in solving quadratic equations, leading to improved problem-solving skills and academic performance in mathematics. By understanding learners' common errors, teachers can tailor their teaching methodologies accordingly, providing targeted support and guidance. This benefits learners by enabling them to develop a deeper understanding of quadratic equations and strengthen their overall mathematical abilities.

This research will also inform the teachers about the typical errors made by form three learners in solving quadratic equations using factorization, empowering them to develop targeted instructional strategies. It enables them to identify and address errors early, promoting effective learning and preventing the accumulation of errors. The research contributes to professional development by providing empirical evidence for improving teaching practices and curriculum design.

The analysis will assist school administrators in identifying areas where the curriculum or teaching approaches need revision to better meet the needs of learners. It will also provide a basis for decision-making in resource allocation, such as providing additional support or professional development for teachers. The research will contribute to the overall improvement of mathematics education in the school.

The research also informs educational policies and curriculum development by providing evidence of the common errors made by learners in solving quadratic equations using factorization. It also enables the development of strategies to address these errors at the national level, such as revising textbooks or providing professional development for teachers. The analysis will improve the quality of mathematics education in the country.

This investigation will add to the body of research on learners' errors in solving mathematical problems, particularly quadratic equations. A basis for further exploration of the cognitive and pedagogical factors that contribute to these errors will also be provided. It will inform the development of effective instructional interventions and assessment tools.

## 1.8 Delimitation of the study

The study was carried out at Setshanke High School. The school is located in Nkayi District of Matebeleland North Province in Zimbabwe. The scope of the research is to analyze form three learner's errors in solving quadratic equations using factorization at Setshanke High School. The time frame of this research is from January to June 2025. Of all the concepts in mathematics, the researcher chose quadratic equations because they are important in mathematics as they form the basis for higher-level mathematical concepts and serve as a foundation for understanding and solving complex mathematical problems. The researcher chose to conduct the research at school because of the school's proximity to the researcher. Setshanke High School is close to the researcher's home and place of work, making it convenient to access. Additionally, the school has a reputation for excellence, making it an attractive option for researchers looking to conduct high-quality research.

## 1.9 Limitations

One limitation of this study lies in its sample size and sampling method. The study was conducted with a relatively small group of students of only one school, Setshanke High School which may not fully represent the population of students who struggle with quadratic equations. Additionally, the participants were recruited through purposive sampling just after all the population of 70 learners write the test which could have introduced bias into the sample. As such, the findings of this study may not be generalizable to all students who experience difficulties with quadratic equations. Future research could aim to increase the sample size and conduct the study in multiple schools to increase the generalizability of the findings. Additionally, future research could explore the effectiveness of different teaching strategies in reducing learners' errors in solving quadratic equations using factorization.

## 1.10 Definition of key terms

### Errors

Makonye (2014) cited in Mudavanhu et al. (2023) defines an error as a slip, mistake, or deviation from accuracy.

**Quadratic Equations**

Tendere & Mutambara (2020) view a quadratic equation as an equation of the second degree and the standard form of a quadratic equations is given by $ax^2 + bx + c = 0$ where a, b, c are constants and $x$ is the unknown.

**Factorization**

Factorization, also known as factoring, refers to the process of breaking down an object into a product of other objects, known as factors. These factors, when multiplied together, yield the original number or entity.

## 1.11 Summary

In this chapter, background of the research was discussed, to try and put the research problem in a well thought out context. Also, in the same chapter the purpose of the study, statement of the problem and main research question have been outlined followed by the research objectives and questions and assumptions of the research study. The significance of the study, the limitations and delimitations of the study were highlighted. In addition to this an attempt to provide definition of terms used in the study was done. The following chapter is projected to provide a review of related literature and a theoretical framework regarding the errors underpinned in solving quadratic equations using factorization.

# CHAPTER 2

# LITERATURE REVIEW

## 2.0 Introduction

This chapter aims to provide a comprehensive review of the related literature, theoretical framework, and research methodology employed in this study. This chapter sets the stage for the exploration of the research questions that will be addressed throughout the course of this research which are: What are the errors displayed by form three learners when solving quadratic equations? What is the fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors?

## 2.1 Theoretical framework

The theoretical framework for analyzing learners' errors in solving quadratic equations using factorization can be developed by incorporating Bloom's Taxonomy of objectives. Bloom's Taxonomy (1956) provides a hierarchical classification of learning objectives, focusing on the cognitive domain, which is relevant for understanding and categorizing the types of errors made by learners (Loveless, 2023).

The cognitive domain encompasses six levels: knowledge, comprehension, application, analysis, synthesis, and evaluation. According to Ahmed (2021), errors at the knowledge level may include forgetting formulas or misconceptions, while errors at the comprehension level may involve misinterpreting the problem or struggling to grasp the underlying concepts (Starr et al., 2019). Loveless (2023) postulates that errors at the application level can arise when learners have difficulties in transferring their knowledge to new situations or executing the factorization process accurately. According to studies conducted by Bloom (1956), on the analysis-level errors involve misidentifying factors or misunderstanding relationships between equation components. On the synthesis-level errors may stem from difficulty in integrating various elements of knowledge and problem-solving strategies (Ahmed, 2021). Loveless (2023) argues that on the evaluation-level errors indicate challenges in critically assessing one's own problem-solving process and justifying solution choices.

By considering these cognitive levels, the theoretical framework can provide a systematic approach to identifying and analyzing the specific types of errors made by learners in solving quadratic equations using factorization. This framework can guide the design of interventions or instructional strategies aimed at addressing these errors and improving learners' problem-solving skills in this domain.

## 2.2 Errors made by learners when solving quadratic equations using the factorization

### 2.2.1 Procedural errors

Mutambara & Bansilal (2021) are of the view that procedural errors are errors which arise as a result of the learner's use of an inappropriate approach in solving a problem or carrying out a procedure incorrectly. Tendere & Mutambara (2020) believe that procedural errors in solving quadratic equations refer to mistakes made in following the correct steps or procedures to find the solution. Tendere & Mutambara (2020) put forward that learners fail to carry out the important working steps in order to solve the given equation due to the lack of prior knowledge of quadratic concepts such as directed numbers while the common error done by the learners is procedural error when using factorization. Johari and Shahrill (2020) classified similar errors as simple mathematical errors of expansion due to carelessness. Incorrectly factoring the quadratic equation is a procedural error. Factoring is a common method to solve quadratic equations, but if the factoring is done inaccurately, it will lead to wrong solutions. For instance, if the quadratic equation $(x + 2)(x + 5) = 0$, is mistakenly factoring it as $(x + 2)(x - 5) = 0$ will give different solutions.

### 2.2.2 Technical / foundational errors

Msomi & Bansilal (2022) define foundational errors as those, which occur because of poor background knowledge such as when a learner incorrectly applies or misinterprets a previously learnt procedure or concept that is being applied in the new concept incorrectly. The learners also experience most of the technical errors due lack of prerequisite knowledge of basic concepts such as directed numbers, operation with negative numbers, substitution in a formula as well as simplifying given expression. For instance, learners were able to write down the correct quadratic formula, but encountered lack of algebraic manipulation. Tendere & Mutambara (2020) are of the view that a learner's existing schemas of a given mathematical concept significantly interferes with the construction of new knowledge, if it is not well mastered.

### 2.2.3 Computational errors

Computational errors in solving quadratic equations involve mistakes made during numerical calculations. This is also in line with Mbewe & Nkhata (2019)'s study who noted that learners difficulties in solving quadratic equations is a result of computational problems, problems with the negative sign as well as the square root. In this research examples include, calculation mistakes in finding the square root: Quadratic equations often involve taking the square root of a number. Errors in calculating the square root can lead to incorrect solutions. Incorrectly distributing signs: Mistakes in distributing signs, like mixing up positive and negative signs in calculations, can cause computational errors. For example, if a learner incorrectly distributes a negative sign $(-)$ when simplifying an equation may alter the value which leads to incorrect solutions (Mudavanhu et al., 2023).

### 2.2.4 Conceptual errors

Mutambara & Bansilal (2021) defines conceptual errors as errors that are a result of deeply held misunderstandings where learners do not have a complete understanding of the new concept. In this research, there are conceptual errors that learners make while applying the factorization method. Mutambara et al, (2019) argues that another error is misinterpreting the zero-product property. Baybayon & Lapinid (2024) argue that the zero-product property states that if the product of two factors is zero, then at least one of the factors must be zero. Some learners mistakenly assume only one factor to be zero. For example, consider the following equation $x^2 - 9 = 0$. If we factorize it as $(x + 3)(x - 3) = 0$, learners might conclude that$x + 3 = 0$. Instead, we need to set each factor equal to zero and solve separately: $x + 3 = 0$ gives$x = -3$, and $x - 3 = 0$ gives $x = 3$ as the correct solutions.

This research analyzes these errors to identify common pitfalls and inform targeted interventions to improve student understanding and problem-solving skills.

## 2.3 The fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors

According to Ly & Takuya (2023) learners require a solid foundation in basic algebraic concepts to successfully solve quadratic equations using factorization with minimum errors. To address the challenges that students face, it is recommended that teachers adopt a constructive approach to learner's errors in solving mathematical problems. This includes recapping vital concepts and skills before teaching quadratic equations, such as directed numbers, factorization, expansion of brackets, and substitution (Tendere & Mutambara, 2020). By doing so, teachers can help learners build a strong foundation in algebra and improve their problem-solving skills, ultimately leading to greater success in solving quadratic equations.

O'Connor and Norton (2022) put forward that understanding variables, constants, and algebraic expressions, as well as familiarity with order of operations and properties of operations such as commutative, associative, and distributive properties will assist in the eradication of the errors inherent in learners when solving quadratic equations using factorization. Additionally, learners need to have a strong grasp of factoring skills which include the ability to factor simple expressions |for example, $x^2 + 5x + 6$| and understanding of factoring techniques such as the greatest common factor and the difference of squares (Joshi, 2019).

Learners also require knowledge of quadratic equation concepts, including understanding quadratic equation forms |for example, $ax^2 + bx + c = 0$| and solutions and their representations |for example, $x = 2$ $or$ $x = -3$|. Furthermore, learners need to possess number sense and properties, including familiarity with integer operations and properties |for example, distributive property| and understanding of negative numbers and their behavior in algebraic expressions (Lester, 2020). According to Polya (1957) in Kim How et al, (2020) problem-solving strategies, such as approaching problems systematically and logically, and familiarity with checking solutions and verifying answers, are also essential. Finally, learners require mathematical notation and terminology knowledge, including understanding mathematical symbols and notation |for example, =, +, -, ×, ÷| and familiarity with mathematical terminology |for example, factor, coefficient and solution| (Rahman, 2019). Thomas and Mahmud (2021) are of the view that basic mathematical concepts, such as understanding place value and decimal arithmetic, and familiarity with fractions and equivalent ratios, are also necessary.

Vilakazi (2021) experiences as a mathematics teacher observed that every quadratic expression can be factored by using any single approach, once having been considered in its general form $ax^2 + bx + c$. Vilakazi (2021) termed this way of looking at quadratics the "fundamental considerations when factoring quadratic expressions”. It is fundamental in a sense that it can be applied to work in factoring all types of quadratic expressions.

## 2.4 Related literature

Over the years, numerous studies have done on the analysis of students' errors in solving quadratic equations shedding light on the misconceptions and challenges that students face in mastering the concept.

Baybayon and Lapinid (2024) carried out a research entitled learners’ common errors in quadratic equations: Towards improved mathematics performance. Baybayon and Lapinid (2024) employed descriptive qualitative research. The participants of that study comprised forty-nine Grade nine learners enrolled in a regular public high school in the Philippines. Permission to conduct the study was granted by the school principal and informed consent was obtained from both learners and their guardians. All forty-six (46) learners in a heterogeneous classroom agreed to participate in the study. They were instructed by the teacher-researcher to do the online tasks as homework in the learning package by watching online instructional videos and self-reading of instructional materials in mathematics learning websites on the following lessons: definition of quadratic equations and its examples; and solving quadratic equations by extracting the square roots, factoring, completing the square, and quadratic formula. Learners were provided with questions to serve as their guide as they go through the online instructional materials. The tiered worksheets served as the primary research instrument in this study for the aforementioned lessons.

In each of the five lessons, the researchers created three worksheets corresponding to learners’ mastery levels: beginning mastery, approaching mastery, and high level of mastery. A total of 15 worksheets were analyzed throughout the duration of the study. Learners were then given worksheets to answer as a formative assessment on the following day in the face-to-face class.

Errors were identified and classified using content analysis by systematically analyzing the learners' solutions and answers in their respective worksheets, and another face-to-face class instruction was conducted to discuss the committed errors in each lesson to underscore the important concepts, further enhance learner's learning, and to help learners correct their errors and misconceptions. Individual learners were interviewed to further validate the interpretation for committing the mistakes.

Baybayon and Lapinid (2024) study's results were as follows: Common errors learners committed were not following directions, mishandling signs, difficulty in recognizing a quadratic equation, inability to distinguish between solving a quadratic equation and simplifying an algebraic expression. Learners also committed errors like failure to express quadratic equations in standard form, disregarding the negative roots, computational errors in basic algebraic conventions in simplifying radical and rational expressions, factoring, performing special products, and in completing the square. Learners were interviewed to validate the error analysis. Immediate feedback through a whole class discussion was conducted the following day to discuss the errors and mistakes committed by learners in order to address these and help them learn the necessary concepts and skills in quadratic equations.

Difficulty in recognizing a quadratic equation was one of the errors identified. Some of the learners showed difficulty in discerning whether a given equation is a quadratic equation based on its degree. This is in relation to the question which prompts the learners to determine whether a particular mathematical equation is an example of a quadratic equation or not. For example, $4x(x^2-5x) +6x=6$, seven (15%) learners immediately said that the given equation is a quadratic equation since they saw the exponent 2. They did not realize that they still have to distribute 4x to all the terms contained inside the parenthesis which further result to a cubic equation. Also, inability to express the quadratic equation in its standard form was reviewed. There were also learners (3 or 7% of them) who committed minor errors in terms of manipulating equations specifically the use of Addition Property of Equality. For example $3x-2x^2=7$

Baybayon and Lapinid (2024) gave some recommendations and these include the strategies that specifically address these prerequisite skills and concepts to improve learners' performance in mathematics. These strategies could include explicit instruction, guided practice, and frequent review of these concepts. Additionally, teachers can provide clear and detailed instructions to ensure that learners understand the requirements of each problem and the necessary steps to solve it. Double-checking answers and practicing careful attention to detail should also be emphasized to reduce careless mistakes. Overall, it is recommended to implement targeted interventions and instructional strategies to address learners' weaknesses in prerequisite skills and concepts in order to improve their performance in mathematics.

Tendere & Mutambara (2020) conducted a study on errors and misconceptions in solving quadratic equations. The objectives of the study were to identify the types of errors and misconceptions displayed by form three learners in Zimbabwe when using factorization and the quadratic formula, as well as to determine interventions to address these issues.

The population for the study of Tendere & Mutambara (2020) consisted of three mathematics teachers and ninety-six learners, with a sample size of thirty learners selected using purposive and stratified sampling methods. Data was collected using a test and follow-up interviews. The test assessed learners' knowledge of quadratic equations, with questions categorized into solving equations through factorization, using the quadratic formula, and involving word problems. The researchers ensured the reliability and validity of the test items by including diverse question types and drawing from past exam papers. Follow-up interviews provided insights into learners' thinking processes and allowed for the identification of misconceptions.

The study revealed that most errors were conceptual and technical in nature, indicating a lack of prerequisite knowledge and difficulties with algebraic manipulation.

Tendere & Mutambara (2020) recommended that teachers address learner errors by reviewing essential concepts and skills before teaching quadratic equations, focusing on one point at a time and promoting critical reasoning. They also suggested further research to utilize identified misconceptions and errors in assessments and to explore the development of learner errors and misconceptions in algebra.

The study conducted by O'Connor & Norton (2022) focused on exploring the challenges faced by learners in understanding quadratic equations and examining the curriculum structure and implementation related to this topic. The study employed an exploratory methodological approach to understand the reasons behind learners' difficulties in solving quadratic equations (Creswell, 2012). The researchers chose the topic of quadratic equations because it builds upon the algebraic conventions studied in earlier grades. Data collection occurred in two phases: a written test followed by diagnostic interviews to gain a nuanced understanding of learners' written responses. The analysis of learners' written responses involved the generation of categories explaining errors, using an emergent design approach rather than predetermined categories. The data analysis followed an inductive approach, moving from specific analysis of individual learner errors to broader comparisons between error patterns, categories of errors, and other categories.

O'Connor & Norton (2022) constructed interview questions based on the results observed in the written test. These questions aimed to ascertain learners' reasoning behind the strategies employed and assess their conceptual understanding of quadratics, solving techniques, and the meanings of solutions. The interview data helped validate the findings from the analysis of the written scripts. A critique of curriculum documents was also considered to provide a contextual background for interpreting the results. Studies conducted by O'Connor & Norton (2022), revealed that many learners demonstrated limited proficiency in foundational algebra, indicating potential failures at two levels in the teaching and learning process. Firstly, learners displayed key conceptual errors related to quadratic equations, suggesting limited initial conceptualization and schema construction in this topic. The data did not provide information on whether this was due to pedagogy that lacked emphasis on conceptualization or a lack of time for appropriate pedagogy. Secondly, learners lacked understanding and fluency with basic algebraic conventions that should have been mastered in prior years.

O'Connor & Norton (2022) study recommends the integration of remediation practices and the allocation of more time for remediation and revision to improve retention. The lack of time may be attributed to classroom discourse, but curriculum structure is likely a contributing factor. The findings support previous concerns about time constraints due to a crowded curriculum (Donnelly & Wiltshire, 2014; Snider, 2004). The study suggests flaws in a curriculum design that covers mathematics topics infrequently, assuming mastery in subsequent years.

## 2.5 Literature gap

While the literature review provides valuable insights into the common errors made by learners when solving quadratic equations and the potential impact of these errors, there are several research gaps that can be addressed in future studies. These research gaps include contextual factors, effective instructional strategies, individual differences, longitudinal studies and learner perspectives.

The literature review does not extensively explore the influence of contextual factors on learners' errors in solving quadratic equations. Factors such as cultural background, educational system, and instructional approaches may play a role in the occurrence and persistence of errors. Future research could investigate how these contextual factors influence learners' understanding and performance in quadratic equation solving. Although the review mentions the importance of providing clear explanations and conducting class discussions, it does not delve into specific effective instructional strategies that can address learners' errors. Further research could explore and evaluate different instructional approaches, such as problem-based learning, technology integration, or manipulative use, to determine their effectiveness in addressing specific types of errors.

The review does not extensively discuss the role of individual differences, such as learners' prior knowledge, cognitive abilities, or learning styles, in their errors when solving quadratic equations. Investigating how individual differences contribute to the occurrence and persistence of errors can help tailor instructional interventions to meet the specific needs of different learners. Most of the studies mentioned in the literature review focus on cross-sectional data or short-term interventions. Longitudinal studies that follow learners over an extended period can provide insights into the long-term effects of errors in quadratic equation solving and their impact on subsequent mathematical learning. Such studies can also shed light on the effectiveness of interventions in addressing errors over time.

The literature review primarily focuses on the perspective of researchers and educators. Including the perspectives of learners themselves can provide valuable insights into their experiences, challenges, and strategies for overcoming errors in quadratic equation solving. Future research can employ qualitative methods, such as interviews or think-aloud protocols, to capture learners' perspectives. Addressing these research gaps can contribute to a deeper understanding of the nature of errors in solving quadratic equations and inform the development of effective instructional strategies to improve learners' mathematical proficiency in this area.

### 2.6 Summary

This chapter served as a solid foundation for the research, presenting an overview of the literature, theoretical framework, methodology of related literature and literature gap. It sets the stage for the subsequent exploration of the research questions, which focus on common errors in solving quadratic equations, their impact on overall learner performance, and interventions to address these errors. The chapter provides a roadmap for the remainder of the study and creates a framework for understanding and addressing the complexities of quadratic equation solving. The next chapter will explore on the research methodology.

# CHAPTER 3

# RESEARCH METHODOLOGY

## 3.1 Introduction

This chapter provides an overview of the research methodology. In this chapter, the research design for the study on using problem-based learning in the teaching of quadratic equations will be discussed. It will outline the instruments used to collect data, describe the target population and sample, discuss the sampling technique employed, and provide an outline of the procedures for collecting data. Additionally, the chapter will cover the data presentation, analysis procedures and summary.

## 3.2 Research design

A qualitative approach was adopted for this research. Tenny et al. (2022) define qualitative approach as a type of research that explores and provides deeper insights into real-world problems and it gathers participants' experiences, perceptions, and behavior. Qualitative research methodology has been effectively utilized by various researchers (Chikwanha et al., 2022; Johari and Shahrill, 2020; Msomi and Bansilal, 2022; Siyepu, 2015). This strategy allows for an in-depth exploration on analyzing student's errors in solving quadratic equations using factorization (Mudavanhu et al., 2023). It focuses on understanding the "how" and "why" on errors made by students when solving quadratic equations by factorization, providing rich, in-depth insights into student experiences, and perceptions (Moser & Korstjens, 2018). This is crucial for understanding individual interpretations on the analysing of student's errors when solving quadratic equations by factorization (Moser & Korstjens, 2018).

## 3.3 Population

The research targeted seventy(70) form three learners. Bhandari (2020) defines a population as the entire group that you want to draw conclusions about in a research. A population does not always refer to people. It can mean a group containing elements of anything you want to study, such as objects, events, organizations, countries, species, organisms and so forth.

### 3.4 Sample

The sample consisted of five learners. A sample of five learners (3 females and 2 males) was selected from all the learners who wrote the test based on their performance. Sriram (2024) states that having a diverse sample comprising both females and males enhances the validity of the study by ensuring a more representative and generalizable set of findings that can be applied to the larger population. Bhandari (2020) defines a sample as the specific group that you will collect data from and the size of the sample is always less than the total population. It is chosen from a population to provide insights, information, or data that can be used to make inferences about the characteristics of the population. Selecting a sample from the population allows for a more manageable group to collect and analyze data while still maintaining sufficient representation (Bhandari, 2020).

### 3.5 Sampling technique to be used in the study

In this study purposive sampling was used. Smith (2023) defines sampling technique as a systematic approach or strategy employed to select a subset of individuals, objects, or events from a larger population.

#### 3.5.1 Purposive sampling technique

Purposive sampling was employed in this research study to select participants for interviews. Nikolopoulou (2022) defines purposive sampling as a group of non-probability sampling techniques in which units are selected because they have characteristics that you need in your sample. The researcher began by administering a test on solving quadratic equations using factorization to the entire population of form three learners, which consisted of 70 students. Following the completion of the test, the researcher carefully reviewed the test results and identified five students who exhibited varying levels of performance and different types of errors in solving quadratic equations to interview them.

These five students (3 females and 2 males) were then chosen to participate in the interviews. The purpose of purposive sampling in this study was to intentionally select individuals who could provide valuable insights into the specific topic of interest: analyzing students' errors in solving quadratic equations using factorization. By focusing on a subset of students who demonstrated different patterns of errors, the researcher aimed to gain a comprehensive understanding of the common mistakes made by form three learners, which would contribute to the overall findings and conclusions of the research.

**3.6 Instruments used to collect data**

A test and follow up interview were the instruments used to identify errors made by learners in solving quadratic equations using factorization. According to Collins (2021) a research instrument refers to any tool that is used by a scientist to obtain, measure, and analyze data.

**3.6.1 Test**

According to Adom et al. (2020), tests can be seen as standard procedures used to systematically measure a sample of behaviour by posing a set of questions. The quadratic equations knowledge test was conducted among all the students in the third form. The test assessed their understanding of fundamental properties related to quadratic equations. The test items were categorized into two groups: solving equations through factorization and solving word problems. The test was created using ZIMSEC O-Level past examination papers. These test items were selected based on their adherence to ZIMSEC standards. This is supported by Mudavanhu et al. (2023) who say that selecting questions from past examination papers ensures content validity and reliability in distinguishing learners with varying abilities.

Additionally, a grid specification method was employed to ensure that the test items covered the first four levels of Bloom's taxonomy, namely remembering, understanding, applying and analyzing (Johari & Shahrill, 2020). The test aimed to identify areas where errors occurred, such as reading and comprehending the questions, understanding their structure, translating them, and solving them correctly. To ensure the reliability of the test items, a diverse group of students was selected for testing. Sufficient time was provided to the students to complete the test. More so, the test was designed to adequately cover the content and assessment domains.

### 3.6.2 Interview

The researcher employed follow-up interviews to enable learners to express their opinions and articulate their actions, thereby validating the insights into their thought processes. George (2022) defines an interview as a qualitative method that relies on asking questions in order to collect data and it involves two or more people, one of whom is the interviewer asking the questions. These interviews provided an opportunity to identify any contradictions, deficiencies, or errors among the students.

Additional questions were posed to the learners based on the solutions derived from the initial test. Tendere & Mutambara (2020) are of the view that the primary objective of the follow-up interviews was to gain a deeper understanding of the learners' conceptualizations, interpretations, and thinking processes. Furthermore, the selected students were instrumental in shedding light on the reasons behind specific types of errors committed by the students and identifying any gaps in their learning.

### 3.7 Procedures for collecting data

Data was collected using the test and interviews. The data collection process was guided by predefined research questions and ethical considerations were prioritized.

#### 3.7.1 Procedures of collecting data from the test

In this research, the researcher conducted a test to collect data on the errors made by all seventy (70) form three learners when solving quadratic equations using factorization. The researcher began by designing a test specifically tailored to evaluate their understanding and proficiency in this area. The test consisted of various questions covering different types of quadratic equations and factorization techniques. Next, the researcher administered the test to all seventy learners, ensuring that each student received a copy of the test and clear instructions for completing it. Throughout the test administration, the researcher monitored the students to ensure compliance with instructions and maintain test integrity.

After the students finished the test, the researcher collected their answer sheets, ensuring that all papers were accounted for without any missing data. With the collected test papers from all seventy learners, the researcher compiled the data by organizing the responses and noting the errors made by each student. This data formed the basis for the researcher's subsequent analysis, where the researcher categorized the types of errors, identified mistakes, and examined patterns or misconceptions exhibited by the learners.

#### 3.7.2 Procedures of collecting data from the interviews

After administering the test to all seventy form three learners to analyze their errors in solving quadratic equations using factorization, the researcher conducted interviews with a subset of five learners (3 females and 2 males). These participants were purposefully selected based on specific criteria such as their performance on the test or the types of errors they made.

Mudavanhu et al. (2023) say that the interviews were designed to get deeper into their experiences and gather detailed insights regarding their thought processes, understanding of concepts, and challenges faced during quadratic equation solving using factorization. The interviews took place in a supportive environment, and the researcher actively listened to the learners' responses, recording their answers either through detailed notes and audio recordings.

### 3.8 Data presentation procedures

In the research study analyzing students' errors in solving quadratic equations using factorization through a qualitative approach, the data presentation procedures focus on presenting the qualitative data concisely and insightfully, while also incorporating visual elements and learners' workings. To begin, the qualitative data obtained from interviews can be organized and categorized using thematic analysis. Recurring themes, patterns, and sub-themes related to the errors made by students can be identified and presented systematically (Mudavanhu et al., 2023). Supporting quotes or excerpts from the interviews can be included alongside these themes, providing evidence and enhancing the understanding of the errors.

Additionally, visual representations such as diagrams or flowcharts can be utilized to present learners' workings and illustrate the problem-solving process. These visuals can help readers visualize the steps taken by learners and better comprehend the specific errors made. Furthermore, narrative description plays a crucial role in presenting the qualitative data. Descriptive narratives can provide a detailed account of the errors, including the context, learners' experiences, and challenges encountered during the problem-solving process. These narratives can be presented coherently, engaging readers and immersing them in the learners' perspectives (Tendere & Mutambara, 2020). Direct quotes from the interviews can be integrated into the narratives, adding authenticity and offering a firsthand glimpse into the thought processes of the learners.

### 3.9 Data Analysis Procedures

The researcher utilized descriptive analysis techniques to examine the data, including categorizing the types of errors made by students. A coding and tally system was employed to quantify the occurrence of specific errors and opinions. In-depth content analysis was conducted to identify areas of difficulty and answer the research questions by recognizing errors and misconceptions. The analysis incorporated visual representations of students' written work, generating comprehensive and detailed data. Interviews were transcribed and analyzed alongside the written work, providing a holistic understanding. The study specifically focused on analyzing errors in solving quadratic equations using factorization. Interviews were audio recorded, transcribed, and coded to explore the specific challenges encountered by students in differentiating questions. Similar methodologies have been employed in analyzing various data sources such as test scripts and interviews (Tendere & Mutambara, 2020; Chikwanha et al., 2022; Mudavanhu et al., 2023).

### 3.10 Ethics

There has been a notable surge in academic interest in research ethics in recent years (Armond et al., 2021). Ethics, as defined by Makore-Rukuni (2001), pertains to the philosophical branch that governs human behavior, distinguishing right from wrong actions and assessing the morality of motivations and outcomes. As a researcher, it is imperative to uphold ethical standards, ensuring confidentiality, informed consent, privacy, and minimizing harm to respondents. By adhering to these ethical principles, researchers can conduct studies that are not only methodologically sound but also morally responsible. In this research, the following ethical considerations were taken into account to ensure the integrity of the study.

#### 3.10.1 Confidentiality

According to Bros (2020), confidential information refers to personal details that individuals wish to keep private. As such, researchers have an ethical responsibility to protect respondents' identities and responses. To uphold confidentiality in this study, participants were assigned numerical and alphabetical codes instead of using their actual names on research instruments, thereby ensuring their anonymity and privacy.

### 3.10.2 Informed consent

According to Urombo (2000), informed consent is an ethical imperative that requires researchers to fully disclose the potential risks and benefits of participation, allowing respondents to make an informed decision about their involvement. In this study, informed consent was ensured through the use of an ethical information document, which was provided to all 70 participants and relevant stakeholders. Participants were given the autonomy to choose whether to participate in the study or not, respecting their right to self-determination.

### 3.10.3 Harm to respondent

According to Urombo (2000), researchers have a moral obligation to ensure that participants are not harmed or put in harm's way during the research process. Harm can manifest in various forms, including irritation, anger, negative labeling, and invasion of privacy. To uphold this principle, the researcher took proactive measures to safeguard participants from potential harm. Specifically, the study was designed to prevent exposure to distressing or uncomfortable situations, and participants' responses were kept anonymous to protect their identities and maintain confidentiality.

### 3.11 Summary

In this chapter, an overview of the research methodology utilized in the study on analysis of students errors in solving quadratic equations using factorization. The research design, including the instruments used for data collection, the target population, sample, and sampling technique, was discussed. Additionally, the procedures for data collection were outlined. The chapter also highlighted the data presentation and analysis procedures. Moving forward, the next chapter will look into data presentation, interpretation, and analysis.

# CHAPTER 4

# DATA PRESENTATION, ANALYSIS AND DISCUSSION

## 4.1 Introduction

This chapter provides an overview of data presentation, analysis and discussion. The purpose of this chapter is to address the research questions: What are the errors made by students in solving quadratic equations using factorization? What are the sources of these errors? And what measures can be taken to minimize these errors? This chapter is divided into six sections: introduction, methodology, results, discussion, conclusion, and limitations.

## 4.2 Methodology

This research used a qualitative approach to investigate how students solve quadratic equations using factorization, giving a detailed understanding of their problem-solving methods. By using this approach, I gathered detailed information on the errors students made when solving quadratic equations using factorization. Many researchers have successfully used this approach (Chikwanha et al., 2022; Johari & Shahrill, 2020; Msomi & Bansilal, 2022; Siyepu, 2015).

The study involved seventy (70) form three students from Setshanke High School in Nkayi District. I created a 30-minute test using past exam papers from ZIMSEC, which ensured the test was fair and accurate in assessing students' abilities. The test had four questions on quadratic equations using factorization, designed to test students' remembering, understanding, applying, and analyzing skills. I gave the test and analyzed the results to identify students' errors. I also interviewed five students to understand their thought processes when solving quadratic equations using factorization. The interviews helped me understand what students need to know to solve these equations successfully.

## 4.3 Results

4.3.1 What are the errors displayed by form three learners when solving quadratic equations?

To answer the research questions, twenty-three learners wrote the first question correctly. The other 47 learners displayed conceptual and procedural errors. For example Figure 1 displayed below shows a student who manifested a procedural error.

### 4.3.1 Procedural error for question 1

The analysis revealed that twenty students, similar to Student A exhibited the same errors in the process of solving the first question.

Solve the equation $x^2 - 5x + 6 = 0$.

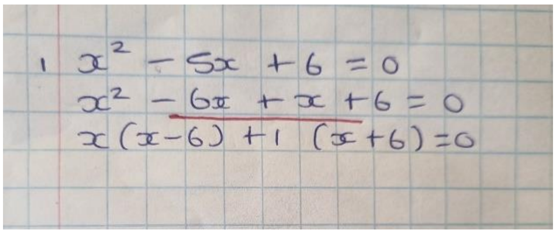


*Figure 1: Written response for student A*

The learner had knowledge on how to solve the equation but using wrong factors. He found the factors of the sum and ignored the product. The learner was displaying some procedural errors. The correct factors were -2x and -3x. According to Bloom's Taxonomy of objectives under the cognitive domain, this error falls under the "Comprehension" level (Level 2). The learner struggles to understand the underlying concepts and relationships, despite being familiar with the procedures. The following interview took place with student A.

### 4.3.2 An interview with student A

**Researcher:** *How did you go about factoring that expression?*

**Student A:** *ummmm I found 2 terms that when I add them I get $-5x$ which are $-6x$ and $1x$*

**Researcher:** *What was your next step?*

**Student A:** *Then I factorized by grouping.*

The conversation with student A shows that the error of Student A is the procedural error of focusing on finding terms that add up to $-5x$ (the coefficient of the linear term) rather than finding factors that multiply to give the constant term $(+6)$. This shows a misunderstanding of the factoring concept, as factoring involves finding numbers, whose product is the constant term, not finding terms whose sum is the coefficient of the linear term. In this case, Student A should have been looking for factors whose product is$6x^2$, such as $-2x$ and$-3x$, rather than only focusing on the sum of $-6x$ and$1x$.

### 4.3.3 Conceptual error for question 1

Twenty-seven learners exhibited conceptual errors just like student B.

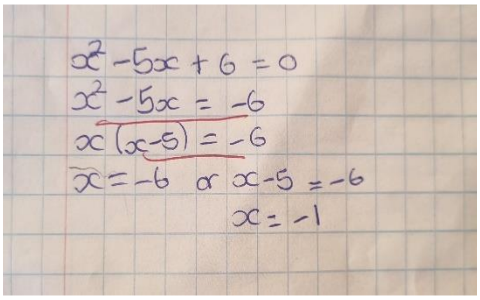


*Figure 2: Written response for student B*

The learner was struggling with a fundamental concept in algebra, specifically when solving quadratic equations. She did not understand that one side of the equation needs to be set equal to zero, which is a crucial step in the problem-solving process of quadratic equation. This misunderstanding indicates a conceptual error, meaning that the learner has a gap in the understanding of the underlying mathematical concepts. According to Bloom's Taxonomy of objectives under the cognitive domain, this error falls under the "knowledge" level (Level 1), which represents the most basic level of understanding, where learners are still developing their knowledge and comprehension of key concepts. Addressing this conceptual error is crucial to build a strong foundation in algebra and progress to higher levels of understanding, application, and problem-solving skills. The following interview took place with student B.

### 4.3.4 An interview with student B

***Researcher:*** *What kind of equation is this?*

***Student B:*** *I don't know.*

***Researcher:*** *How did you go about factoring that expression?*

***Student B:*** *I grouped the numbers and factorize the left side.*

***Researcher:*** *What was your next step?*

***Student B:*** *I equate the factor to* $-6$

This conversation between the researcher and Student B reveals several errors misconceptions in the learner's understanding of algebra. Student B is unable to identify the type of equation, demonstrating a knowledge gap, and subsequently uses an incorrect problem-solving approach by grouping numbers and factoring the left side. Furthermore, they equate the factor to $-6$, showcasing a conceptual error by failing to set the factored expression equal to zero, a crucial step in solving quadratic equations.

**4.3.5 Students' response to the second question.**

Twenty learners wrote the second question correctly. The other fifty learners displayed conceptual, technical and procedural errors. Question 2 is displayed below:

Solve the equation $x^2 + 5x = 24$.

**4.3.6 Technical error for question 2**

Nine learners exhibited technical errors just like student C did.

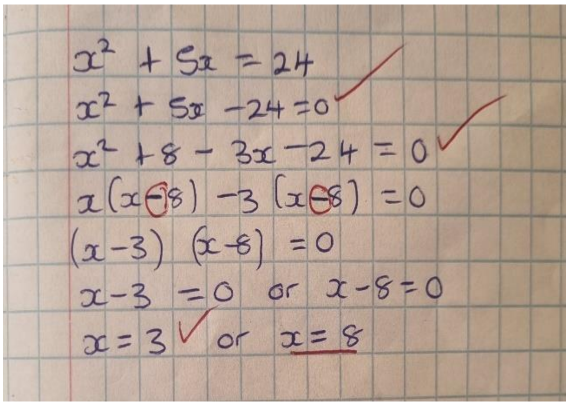


*Figure 3: student C response to question 2*

The learner's technical error resulted in an incorrect solution, obtaining $x = 8$ or $x = 3$ instead of the correct solutions $x = -8$ or $x = 3$. The student also wrote -8 instead of -8x. This type of error is categorized as a "technical error" or "procedural error," indicating a mistake in the execution of a mathematical procedure or algorithm. In this case, the learner likely made a mistake when applying the rules of algebra, such as incorrectly handling the sign of the terms or making a calculation mistake. According to Bloom's Taxonomy of objectives under the cognitive domain, this error falls under "Application" (Level 3). This level involves the ability to apply learned concepts, procedures, and algorithms to solve problems and complete tasks. The learner has demonstrated a technical error in applying the rules of algebra, specifically in handling the sign of terms or making a calculation mistake, which is a characteristic of errors at the Application level. The following interview took place with student C.

**4.3.7 An interview with student C**

***Researcher:*** *What kind of equation is this?*

***Student C:*** *Quadratic equation*

***Researcher:*** *What does it mean when an equation is quadratic?*

***Student C:*** *A quadratic equation has a highest power* $2$ *and has* $2$ *solutions.*

***Researcher:*** *How did you solve this?*

***Student C:*** *I arrange the equation equal to* $0$ *and then factorize.*

***Researcher:*** *How did you get* $8$*?*

***Student C:*** *ummmm I made a mistake I wrote* $8$ *instead of* $-8$.

This conversation indicates that Student C is making a technical error, specifically a calculation error or a mistake in executing a mathematical procedure. Student C demonstrates an understanding of the concept of quadratic equations and the correct procedure for solving them (factoring), but makes a mistake in the calculation, writing 8 instead of -8. This type of error is characterized by a mistake in the execution of a mathematical procedure or algorithm, rather than a misunderstanding of the underlying concept.

#### 4.3.8 Conceptual errors for question 2

Twenty-two learners exhibited conceptual errors.

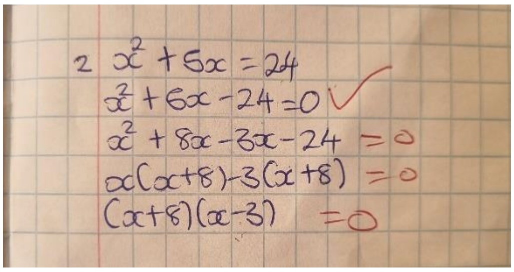


*Figure 4: student D response to question 2*

The learner demonstrates a procedural understanding of factorization, correctly factoring the expression as $(x + 8)(x - 3)$, but displays conceptual errors and a limited understanding of solving equations and algebraic concepts, failing to recognize the need to equate the expression to zero, instead solely relying on factorization, revealing a gap in her understanding of the fundamental principles of equation solving. According to Bloom's Taxonomy of objectives under the cognitive domain, this learner's errors fall under the "Understanding" level (Level 2), indicating a lack of comprehension of the underlying concepts and principles of equation solving, and a need to develop a deeper understanding of algebraic concepts and problem-solving strategies to progress to the "Application" level (Level 3) where they can apply their knowledge.

### 4.3.9 Procedural errors in question 2

Nineteen learners demonstrated a familiarity with the procedures for solving quadratic equations when tackling question 2, specifically the equation $x^2 + 5x = 24$. However, their responses revealed a crucial mistake - they consistently confused the signs when transposing the constant term $(+24)$ to the other side of the equation, indicating a lapse in attention to detail and a need to reinforce the importance of precise calculations in algebraic manipulations. Learners also revealed procedural errors of taking the equation as an algebraic expression even though they were able to carry out factorization as shown below

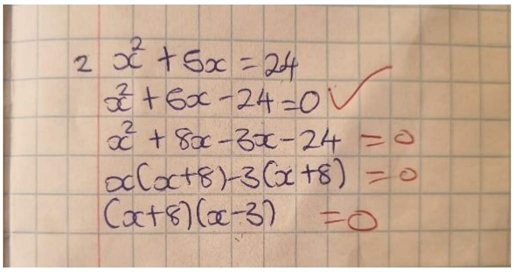


*Figure 5: student D response to question 2*

### 4.3.10 Students' response to the third question.

Fifteen learners wrote the third question correctly. The other fifty-five learners displayed conceptual errors. Question 3 is displayed below.

> A garden has an area of 33 square meters. Its length is 5 meters more than a certain number $x$ and the width is 3 meters less than a certain number $x$. What are the possible values $x$?

### 4.3.11 Conceptual errors for question 3

The analysis revealed that most students exhibited conceptual errors, demonstrating a fundamental misunderstanding of quadratic functions. Specifically, sixty students struggled to translate word problems into algebraic equations, failing to grasp the underlying mathematical concepts. As a result, they formulated incorrect quadratic equations, highlighting a gap in their understanding of how to represent real-world scenarios mathematically.

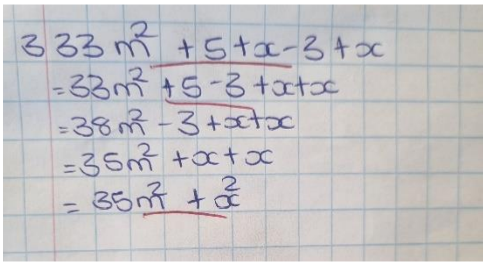

*Figure 6: Student E response to question 3*

Student E displayed conceptual error because she failed to make a quadratic equation. It suggests that students need targeted support to develop a deeper understanding of quadratic functions and how to apply them to solve problems, rather than just memorizing procedures. According to Bloom's Taxonomy of objectives under the cognitive domain, this type of error indicates that students are struggling to reach the "Application" level (Level 3), where they can apply their knowledge of quadratic functions to solve problems and complete tasks, revealing a need for support to move beyond mere comprehension and develop the ability to apply mathematical concepts in practical situations.

**4.3.12 Students' response to the fourth question.**

Two learners wrote the fourth question correctly. The other sixty-eight learners displayed conceptual and procedural errors. Question 4 is displayed below

> The father is three times older than his daughter. If we multiply their ages from six years ago, we get 420. Can you figure out how old they are today?

**4.3.13 Conceptual errors for question 4**

Almost all the learners wrote the fourth question incorrectly. They displayed conceptual errors. According to Bloom's Taxonomy of objectives under the cognitive domain, the type of errors indicate that all students are struggling to reach the analysis level (level 4) because they are having difficulty breaking down complex information, identifying patterns, and making connections to solve problems or understand concepts. Almost all learners failed to set the correct algebraic presentation and equation.

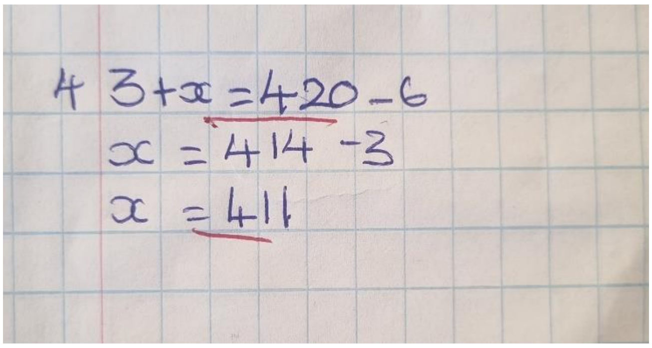


*Figure 7: Student F response to question 4*

Student F displayed conceptual errors in their solution because they misinterpreted the "three times younger" relationship, setting up an incorrect equation $(3 + x)$, and failed to understand the "six years ago" clause, subtracting 6 from the product of their ages $(420 - 6)$ instead of considering the ages at that time. Additionally, they incorrectly applied algebraic operations, subtracting 3 from $420 - 6$ instead of dividing the result by$3$. These errors demonstrate a lack of understanding of the problem's context, algebraic representations, and mathematical relationships, leading to an incorrect solution, and highlighting the need for guidance on correctly interpreting the problem and setting up and solving algebraic equations accurately.

### 4.3.14 Students' response to the fifth question

All students wrote the fifth question incorrectly. They displayed conceptual errors. According to Bloom's Taxonomy of objectives under cognitive domain, the learners failed to have knowledge on general form of quadratic equation and to analyze the question. All 70 learners failed to understand the demand of the question. The question 5 is displayed below:

> The solutions $x = -1$ and $x = 3$ are given. Can you find the quadratic equation that matches these solutions?

### 4.3.15 Conceptual errors for question 5

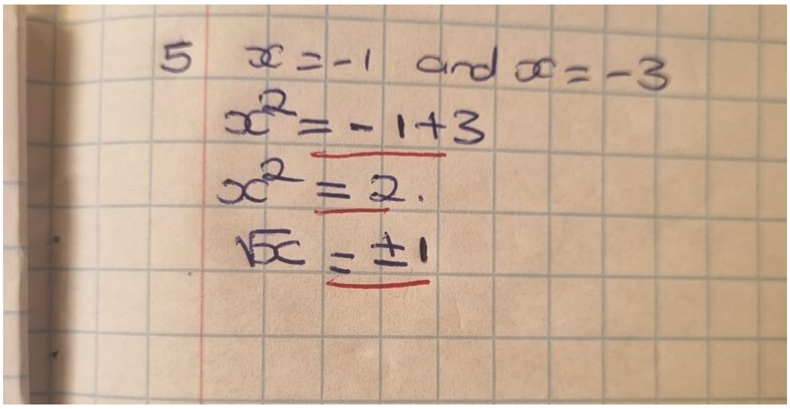


*Figure 8: Student G response to question 5*

Student F failed to understand the demand of the question, leading to conceptual errors. Instead of formulating the quadratic equation using the given roots (x = -1 and x = 3), Student F attempted to find the roots again, demonstrating a lack of understanding of the question's requirements. This indicates a deficiency in the analysis level of Bloom's Taxonomy, where students are expected to break down complex information, identify patterns, and make connections. By not recognizing the need to formulate the equation, Student F showed a lack of analytical skills, highlighting the need for targeted instruction to develop this critical cognitive domain. The following interview took place with student G.

***Researcher:*** *Read the question please*

***Student G:*** *He reads the question fluently*

***Researcher:*** *What is the demand of the question?*

***StudentGF:*** *To solve the equation*

***Researcher:*** *What is the general form of quadratic equation?*

***Student G:*** *ummmm I don't know.*

***Researcher:*** *How did you solve this?*

***Student G:*** *I found the solutions of the quadratic equation*

The conversation with student G indicated that student G is unable to write a quadratic equation given the roots reveals errors in both Knowledge (Level 1) and Analysis (Level 4) under Bloom's Taxonomy of objectives in the cognitive domain. The student's lack of understanding of the question's demand indicates a deficiency in Knowledge, as they failed to recall the concept of forming a quadratic equation from its roots. Furthermore, their inability to apply this knowledge to formulate the equation demonstrates an error in Analysis, where they struggled to break down the information, identify patterns, and make connections to solve the problem. This highlights the need for targeted instruction to address both knowledge gaps and analytical skills.

### 4.4 Fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors.

Through a comprehensive analysis of student responses on assessment test scripts and follow-up interviews, researchers identified five essential categories of pre-requisite knowledge crucial for learners to attain a comprehensive understanding of quadratic equations using factorization. These categories include: directed numbers, basic algebra, factorization, linear equations and problem solving skills.

The data showed that a solid understanding of directed numbers is essential, as seen in the following two excerpts, which highlight the importance of this concept as a prerequisite for further learning.

***Researcher:*** *Hello, what challenges do you face when solving quadratic equations using factorization?*

***Learner:*** *directed numbers*

***Researcher:*** *What are directed numbers?*

***Learner:*** *Directed numbers? I'm not sure what that is. I think it might have something to do with negative numbers.*

***Researcher:*** *What about positive numbers?*

***Learner:*** *I'm quite not sure.*

This conversation indicates that the learner has a partial understanding of directed numbers, associating them primarily with negative numbers but not having a clear grasp of the concept as a whole, including positive numbers. This lack of comprehensive knowledge suggests a need for further explanation and instruction on directed numbers before proceeding to more advanced topics like solving quadratic equations that involve directed numbers.

Secondly, the data revealed that a strong foundation in basic algebra is crucial, as evident in the following two excerpts, which highlight the importance of this concept as a prerequisite for further learning.

***Researcher:*** *What topics should be covered before learning quadratic equations using factorization, and how can consistent practice help?*

***Learner:*** *Algebra and equations.*

***Researcher:*** *Can you describe the concept of basic algebra to me?*

***Learner:*** *Basic algebra? I'm a bit fuzzy on that. I think it has something to do with solving for x.*

***Researcher:*** *That's a good start! How about expanding and simplifying algebraic expressions?*

***Learner:*** *Honestly, I'm not really sure how to do that.*

This conversation indicates that the learner has a limited understanding of basic algebra, grasping the general idea of solving for a variable but struggling with more complex concepts like expression manipulation. This knowledge gap suggests a need for additional instruction and practice in basic algebra before moving on to more advanced topics like quadratic equations.

Thirdly, the data revealed that a strong foundation in factorization is crucial, as evident in the following two excerpts, which highlight the importance of this concept as a prerequisite for further learning.

***Researcher:*** *What prior knowledge is essential for solving quadratic equations using factorization?*

***Learner:*** *Factorization*

*Researcher: Can you explain the process of factorizing a quadratic expression?*

***Learner:*** *Factorizing? I'm not entirely sure... I think it involves finding two numbers that multiply to give the last term, and add up to give the coefficient of the middle term.*

*Researcher:* *That's close! How about factorizing expressions with negative coefficients, like*$x^2 - 5x + 6$*?*

**Learner:** *Hmm... I'm really not sure. I always get confused when the signs are different.*

This conversation indicates that the learner has a partial understanding of factorization, grasping the general idea of finding two numbers that multiply and add up to certain values, but struggling with more complex scenarios involving negative coefficients. This knowledge gap suggests a need for additional instruction and practice in factorization before moving on to more advanced topics like solving quadratic equations.

Fourthly, the data revealed that a strong foundation in linear is crucial, as evident in the following two excerpts, which highlight the importance of this concept as a prerequisite for further learning.

**Researcher:** *What prerequisite skills necessary to solve quadratic equations.*

**Learner:** *Linear equations*

**Researcher:** *Are you able to solve linear equations*

*Learner: I've worked with linear equations in my math class and feel comfortable solving them.*

**Researcher:** *Excellent! Solving linear equations is a fundamental skill required to solve quadratic equations. Can you demonstrate how you would solve a simple linear equation like* $2x + 3 = 7$?

**Learner:** *Sure, I'd subtract 3 from both sides to get* $2x = 4$*, then divide both sides by 2 to solve for x, which equals 2.*

**Researcher:** *solve* $(x + 3)(x - 2) = 0$

**Learner:** *I use linear equations* $x + 3 = 0 \quad x = -3. x - 2 = 0 \ x = 2.$

**Researcher:** *That's good.*

The learner possesses a solid understanding of linear equations, demonstrated by their ability to solve simple linear equations, such as $2x + 3 = 7$, with ease and confidence. They have a strong foundation in this area, which is evident in their ability to apply linear equation skills to solve quadratic equations through factorization, as seen in their solution to the equation $(x + 3)(x - 2) = 0$. The learner successfully identifies the two linear equations, $x + 3 = 0$ and $x - 2 = 0$, and solves them to find the values of $x$, ($-3$ $and$ $2$ respectively). This showcases their ability to build upon their linear equation knowledge to tackle more complex quadratic equations, highlighting their readiness for further learning and development in mathematics.

Fifthly, the data revealed that a strong foundation in problem-solving skills is crucial, as evident in the following two excerpts, which highlight the importance of this concept as a prerequisite for further learning.

***Researcher:*** *Can you walk me through your thought process when approaching a complex math problem?*

***Learner:*** *Problem-solving? I'm not entirely sure... I usually try to find a formula or equation that fits the situation, but sometimes I get stuck.*

***Researcher:*** *That's a good start! How about when the problem involves multiple steps or unknowns, like solving a system of equations?*

***Learner:*** *Hmm... I'm really not sure. I often get overwhelmed and don't know where to begin.*

This conversation indicates that the learner has a partial understanding of problem-solving skills, grasping the general idea of applying formulas and equations, but struggling with more complex scenarios involving multiple steps or unknowns. This knowledge gap suggests a need for additional instruction and practice in problem-solving skills before moving on to more advanced topics like quadratic equations.

**4.5 Discussion**

The purpose of this study was to investigate the errors exhibited by form three learners when solving quadratic equations using factorization, as well as the fundamental knowledge needed for this process. The research questions that guided this study were:

1. What are the errors displayed by form 3 learners when solving quadratic equations using factorization?

2. What is the fundamental knowledge necessary for students to effectively solve quadratic equations using factorization while minimizing errors?

### 4.5.1 Errors displayed by Form Three Learners when Solving Quadratic Equations using Factorization

The findings of this study indicate that form three learners exhibited several common errors when attempting to solve quadratic equations using factorization. These errors included: procedural, conceptual and technical errors.

Learners exhibited several procedural errors when solving quadratic equations using factorization. One common error was the inability to correctly identify the appropriate factors of the quadratic expression, a crucial step in the factorization process. Additionally, mistakes were made in the actual factorization process, such as incorrect placement of the factors or sign errors. Furthermore, learners struggled to connect the factorized form to the process of solving the quadratic equation, indicating a lack of understanding of the procedural steps involved.

Conceptual errors were also prevalent, revealing a lack of understanding of the fundamental concepts underlying quadratic equations. Many learners failed to grasp the need to set one side of the equation equal to zero, a basic concept in solving quadratic equations. Moreover, difficulties in translating word problems into the corresponding quadratic equations revealed gaps in understanding how to represent real-world scenarios mathematically. Learners also struggled to analyze complex information, identify patterns, and make connections to solve problems, indicating deficiencies in higher-order cognitive skills.

Technical and calculation errors were also observed, including mistakes in the execution of algebraic procedures and calculations. Learners made errors such as incorrectly handling the signs of terms or making calculation errors, demonstrating a lack of technical proficiency in algebraic manipulations. These errors highlight the need for learners to develop a strong foundation in algebraic procedures and calculations to succeed in solving quadratic equations using factorization.

The findings of this study align with the existing literature, which highlights learners' struggles with the conceptual understanding of factorization and its relationship to solving quadratic equations (Msomi and Bansilal, 2022; Tendere and Mutambara, 2020; Mutambara and Bansilal, 2021). The errors which are consistent with literature review include procedural, technical, and conceptual errors, are consistent with the common challenges reported in previous research. However, this study takes it a step further by revealing that the learners' errors can be attributed to various levels of understanding according to Bloom's Taxonomy of educational objectives in the cognitive domain. Specifically, the errors range from knowledge-level gaps (e.g., recognizing the type of equation) to higher-order analytical skills (e.g., breaking down complex information and making connections), which is a novel contribution to the existing body of research.

### 4.5.2 Fundamental Knowledge needed when Solving Quadratic Equations using Factorization

The findings of this study emphasize the critical importance of ensuring that learners have a strong grasp of the prerequisite knowledge necessary for developing a comprehensive understanding of quadratic equations using factorization. The four essential categories identified - directed numbers, basic algebra, factorization, and problem-solving skills - are all fundamental building blocks that must be firmly in place for learners to successfully navigate the complexities of working with quadratic equations.

The results regarding directed numbers reveal that learners' understanding of positive and negative numbers is often partial, with a primary association with negative numbers. This suggests that more explicit instruction and practice are needed to solidify learners' conceptual grasp of the entire directed number system. Strengthening this foundation will better equip them to work with the positive and negative coefficients inherent in quadratic equations.

Similarly, the limitations observed in learners' basic algebra skills, particularly in manipulating algebraic expressions and solving for variables, highlight the need for additional targeted instruction and practice in these fundamental algebraic concepts. A robust understanding of basic algebra is a prerequisite for successfully applying algebraic methods to the solutions of quadratic equations.

The findings related to factorization skills also point to the need for more comprehensive instruction and practice, as learners struggled with more complex scenarios involving negative coefficients. Mastering the ability to identify the appropriate factors that satisfy the conditions of a quadratic expression is a crucial step in being able to employ factorization as a problem-solving strategy.

Finally, the study's insights regarding learners' partial understanding of problem-solving skills, particularly in applying formulas and equations to multi-step or unknown-laden scenarios, underscore the importance of developing strong problem-solving abilities. These skills are essential for translating real-world quadratic problems into mathematical representations and then utilizing appropriate strategies to reach viable solutions.

The findings of this study align with existing literature, which highlights that solving quadratic equations using factorization requires specific fundamental knowledge areas, including directed numbers, basic algebra, factorization, and problem-solving skills (Ly and Takuya, 2023; Tendere and Mutambara, 2020). This means that the study's results are consistent with previous research, which has also identified these knowledge areas as essential for successfully solving quadratic equations using factorization. However, the study also reveals that solving linear equations is another crucial fundamental knowledge area needed to solve quadratic equations, emphasizing the importance of a strong foundation in this skill as well.

### 4.5.3 Implications and Significance of the Findings

The implications and significance of this study are multifaceted. Firstly, the study provides a detailed analysis of the specific errors exhibited by form three learners when solving quadratic equations using factorization, including procedural, conceptual, and technical errors. This taxonomy of errors can help educators better understand the areas of difficulty for students and design targeted interventions.

Secondly, the study highlights the fundamental knowledge areas that are essential for effectively solving quadratic equations using factorization, including directed numbers, basic algebra, factorization skills, and problem-solving abilities. The findings emphasize the need to ensure students have a solid grasp of these prerequisite concepts before attempting more advanced topics.

Thirdly, the results of this study are consistent with previous research, which also identified similar challenges and knowledge gaps among students solving quadratic equations. This reinforces the need to address these common issues in education.

Fourthly, the study goes beyond just identifying errors, by also examining them through the lens of Bloom's Taxonomy. This provides a more nuanced understanding of the cognitive skills required, ranging from basic knowledge to higher-order analytical abilities. This can inform the design of instructional strategies that target different levels of understanding Mathematics

Fifthly, the findings can guide teachers in developing more effective instructional approaches for teaching quadratic equations. This includes focusing on strengthening the fundamental knowledge areas, providing targeted practice, and scaffolding instruction to address the various error types and cognitive levels observed.

Lastly, by exploring the specific errors and prerequisite knowledge needed for solving quadratic equations using factorization, this study adds to the existing body of research in mathematics education. The insights generated can inform future studies and help develop a more comprehensive understanding of the challenges students face in this domain. In summary, this study has important implications for improving mathematics instruction and student learning, particularly in the context of solving quadratic equations.

### 4.6 Conclusion

The main findings of this study reveal that Form Three learners exhibit various errors when solving quadratic equations using factorization. These errors encompass procedural, technical, and conceptual mistakes. Notably, the study discovers that learners' errors can be attributed to different levels of understanding, ranging from knowledge-level gaps, such as recognizing equation types, to higher-order analytical skills, like breaking down complex information and making connections, as categorized by Bloom's Taxonomy. This discovery offers a nuanced understanding of learners' errors and highlights the need for targeted instruction and practice to address these knowledge gaps and enhance learners' proficiency in mathematics.

The study's findings reveal that solving quadratic equations using factorization requires specific fundamental knowledge areas, including directed numbers, basic algebra, factorization, and problem-solving skills. Additionally, the study highlights the crucial importance of a strong foundation in solving linear equations as a necessary precursor to successfully solving quadratic equations. These findings emphasize the need for learners to develop a comprehensive understanding of these fundamental knowledge areas to navigate the complexities of quadratic equations and their factorization.

These findings address the research questions, identifying the errors displayed by Form Three learners and the fundamental knowledge necessary for effective solution using factorization, highlighting the need for targeted instruction and practice to address knowledge gaps and enhance learners' proficiency in mathematics.

### 4.7 Summary

This chapter presents the findings from a qualitative data analysis, which identified and classified errors made by Form Three learners when solving quadratic equations using factorization. The data was collected and analyzed using a systematic approach, and the errors were categorized according to Bloom's Taxonomy of objectives under the cognitive domain. This discovery offers a nuanced understanding of learners' errors and highlights the need for targeted instruction and practice to address these knowledge gaps and enhance learners' proficiency in mathematics. The study's findings reveal that solving quadratic equations using factorization requires specific fundamental knowledge areas. Additionally, the study highlights the crucial importance of a strong foundation in solving linear equations as a necessary precursor to successfully solving quadratic equations. These findings emphasize the need for learners to develop a comprehensive understanding of these fundamental knowledge areas to navigate the complexities of quadratic equations and their factorization.

These findings address the research questions, identifying the errors displayed by Form Three learners and the fundamental knowledge necessary for effective solution using factorization, highlighting the need for targeted instruction and practice to address knowledge gaps and enhance learners' proficiency in mathematics. The next chapter is going to look at summary, conclusions and recommendations.

# CHAPTER 5: SUMMARY, CONCLUSION AND RECOMMENDATIONS

## 5.1 Introduction

This chapter seeks to give a summary of the research, recommendations and conclusions on analysis of students' errors when solving quadratic equations using factorization.

## 5.2 Summary

This study is about "analysis of students' errors when solving quadratic equations using factorization." Chapter 1 introduces the research study on analyzing learner errors in solving quadratic equations using factorization. The background of the study highlights the importance of quadratic equations in the mathematics curriculum and the challenges learners face in solving them. The purpose of the study is to identify and analyze the specific errors made by Form 3 learners in a certain school in Nkayi District when solving quadratic equations using factorization. The research objectives include establishing the errors made by learners, assessing the fundamental knowledge necessary for effective solution, and identifying the underlying causes of errors. The study aims to provide valuable insights for teachers, policymakers, and researchers to develop targeted instructional approaches and interventions to improve learner understanding and performance in mathematics. The chapter also outlines the significance of the study, limitations, delimitations, definition of key terms, and the organization of the study.

Chapter 2 provides a comprehensive review of the literature related to the research study on analyzing learner errors in solving quadratic equations using factorization. The theoretical framework for the study is based on Bloom's Taxonomy of objectives, which provides a hierarchical classification of learning objectives in the cognitive domain. The literature review highlights the common errors made by learners when solving quadratic equations, including procedural, technical/foundational, computational, and conceptual errors. The study also identifies the fundamental knowledge necessary for learners to effectively solve quadratic equations using factorization, including a solid foundation in basic algebraic concepts, factoring skills, number sense, problem-solving strategies, and familiarity with mathematical notation and terminology. The literature review also discusses various studies that have analyzed student errors in solving quadratic equations, shedding light on the misconceptions and challenges that students face in mastering the concept. The studies highlight the importance of targeted interventions and instructional strategies to address learners' weaknesses in prerequisite skills and concepts, such as explicit instruction, guided practice, and frequent review. The chapter concludes by emphasizing the need for a comprehensive approach to address learner errors and improve student understanding and problem-solving skills in mathematics.

Chapter 3 outlines the research methodology used to analyze learner errors in solving quadratic equations using factorization. The study adopted a qualitative approach, using a test and follow-up interviews to collect data from a sample of 5 Form 3 learners (3 females and 2 males) selected from a target population of 70 learners. The research design allowed for an in-depth examination of learner errors and challenges. Data collection involved administering a test to identify error areas and conducting interviews to gather learners' thoughts and opinions. Data presentation and analysis procedures included thematic analysis, coding, and content analysis to identify recurring themes, patterns, and areas of difficulty. The study aimed to provide a comprehensive understanding of learner errors and challenges in solving quadratic equations using factorization.

Chapter 4 presents the findings of a study on the errors made by Form Three learners when solving quadratic equations using factorization, revealing various errors including procedural, technical, and conceptual mistakes. The errors were categorized according to Bloom's Taxonomy, ranging from knowledge-level gaps to higher-order analytical skills. The study identified five essential categories of pre-requisite knowledge crucial for learners to attain a comprehensive understanding of quadratic equations using factorization: directed numbers, basic algebra, factorization, linear equations, and problem-solving skills. The findings suggest targeted instruction and practice are necessary to address knowledge gaps and enhance learners' proficiency in mathematics, with limitations including a small sample size and single-school setting, highlighting the need for future research to increase generalizability and explore effective teaching strategies to reduce learners' errors. The study's main findings include: Form Three learners exhibit various errors when solving quadratic equations using factorization; learners' errors can be attributed to different levels of understanding, ranging from knowledge-level gaps to higher-order analytical skills; solving quadratic equations using factorization requires specific fundamental knowledge areas; and targeted instruction and practice are necessary to address knowledge gaps and enhance learners' proficiency in mathematics. The main findings of this study reveal that Form Three learners exhibit various errors when solving quadratic equations using factorization. These errors encompass procedural, technical, and conceptual mistakes. Notably, the study discovers that learners' errors can be attributed to different levels of understanding, ranging from knowledge-level gaps, such as recognizing equation types, to higher-order analytical skills, like breaking down complex information and making connections, as categorized by Bloom's Taxonomy. This discovery offers a nuanced understanding of learners' errors and highlights the need for targeted instruction and practice to address these knowledge gaps and enhance learners' proficiency in mathematics.

The study's findings reveal that solving quadratic equations using factorization requires specific fundamental knowledge areas, including directed numbers, basic algebra, factorization, and problem-solving skills. Additionally, the study highlights the crucial importance of a strong foundation in solving linear equations as a necessary precursor to successfully solving quadratic equations. These findings emphasize the need for learners to develop a comprehensive understanding of these fundamental knowledge areas to navigate the complexities of quadratic equations and their factorization.

These findings address the research questions, identifying the errors displayed by Form Three learners and the fundamental knowledge necessary for effective solution using factorization, highlighting the need for targeted instruction and practice to address knowledge gaps and enhance learners' proficiency in mathematics.

Chapter five gives summary, conclusions and recommendations of the study. A summary is for summarizing the main points of each chapter, the conclusions are for giving final comments or judgments on the findings of the research and finally the recommendations which are suggestions or strategies that may be put in place in order to deal with analysis of students' errors when solving quadratic equations using factorization.

### 5.3 Conclusions

The present study, which investigated Form 3 learners' errors when solving quadratic equations using factorization, aligns with the scholarly work of Tendere and Mutambara (2020), who emphasized the significance of addressing knowledge gaps and enhancing learners' proficiency in mathematics. The findings revealed a range of errors, including procedural, technical, and conceptual mistakes, which corroborate the assertions of Msomi and Bansilal (2022) that targeted instruction and practice are crucial for improving learners' understanding and application of mathematical concepts.

The study identified five essential categories of pre-requisite knowledge necessary for learners to effectively solve quadratic equations using factorization while minimizing errors. These fundamental knowledge areas include directed numbers, basic algebra, factorization, linear equations, and problem-solving skills, which are foundational to more complex mathematical concepts. By building a strong foundation in these areas, educators can help learners develop a deeper understanding of quadratic equations and factorization, leading to improved proficiency in mathematics.

The study's conclusions support the scholarly consensus that a structured and progressive approach to mathematics education is essential for learners' development (Mbewe and Nkhata, 2019). By understanding the specific errors learners make and the fundamental knowledge areas necessary for success, educators can develop effective teaching strategies to support learners' development in mathematics. This can lead to improved academic achievement and a stronger foundation in mathematics, preparing learners for future success (Mutambara and Bansilal, 2021).

The present study contributes to the existing body of research on mathematics education, highlighting the need for targeted instruction and practice to address knowledge gaps and enhance learners' proficiency in mathematics. By integrating the findings of this study with the scholarly work of Tendere (2020), educators can develop a comprehensive understanding of the essential knowledge areas and effective teaching strategies necessary for learners' success in mathematics.

## 5.4 Recommendations

### 5.4.1 Recommendations for Practice:

1. Educators should provide targeted instruction and practice to address knowledge gaps and enhance learners' proficiency in mathematics, particularly in areas such as directed numbers, basic algebra, factorization, linear equations, and problem-solving skills:

- Many students struggle with fundamental mathematical concepts like directed numbers (positive and negative numbers), basic algebra, and solving linear equations. Providing targeted instruction and practice in these areas can help build a stronger foundation, which is crucial for progressing to more advanced topics.

- Factorization is an essential skill for working with polynomials and solving more complex equations. Targeted practice and instruction in factorization can help students develop this important technique.
- Problem-solving skills are vital for applying mathematical knowledge to real-world situations. Educators should focus on teaching problem-solving strategies and providing opportunities for students to practice solving a variety of problem types.

2. Teachers should use a structured and progressive approach to mathematics education, building a strong foundation in essential skills and concepts before advancing to more complex mathematical concepts:

- Mathematics is a highly sequential subject, with each new concept building upon the previous ones. It's important for educators to ensure that students have a solid grasp of the foundational skills and concepts before moving on to more advanced topics.
- A structured and progressive approach allows students to develop a deep understanding of the underlying principles and build the necessary skills to tackle increasingly complex mathematical problems.
- This approach helps prevent knowledge gaps and ensures that students are well-equipped to succeed as they progress through the curriculum.

3. Educators should use formative assessments to identify learners' errors and misconceptions and adjust their instruction accordingly.

- Formative assessments, such as quizzes, classwork, and informal checks for understanding, allow educators to continuously monitor student progress and identify areas where students are struggling:

- By identifying common errors and misconceptions, teachers can adjust their instruction, provide targeted support, and address the underlying issues before they become more deeply ingrained.
- This approach helps ensure that students have a solid grasp of the material before moving on, rather than allowing gaps to accumulate over time.

**5.4.2 Recommendations for Further Study:**

1. A similar study should be conducted using a longitudinal design to determine if changes over time become perceptible and to assess the effectiveness of targeted instruction and practice in addressing knowledge gaps and enhancing learners' proficiency in mathematics:

- A longitudinal study would involve following a group of students over an extended period, such as several years, to observe how their mathematical knowledge and skills develop.
- This type of study would provide more insight into the long-term impacts of the recommended instructional practices and allow researchers to assess whether the targeted interventions lead to sustained improvements in student proficiency.
- Longitudinal data can also help identify patterns and trends that may not be immediately apparent in a single, cross-sectional study.

2. Future research could explore the use of technology and digital resources to support learners' development in mathematics, particularly in areas such as problem-solving skills and critical thinking.

- Technology and digital tools, such as educational software, online simulations, and interactive learning platforms, can provide engaging and personalized learning experiences for students.
- These resources can be particularly helpful in developing problem-solving skills and critical thinking, as they can offer dynamic, visual representations of mathematical concepts and provide students with opportunities to practice and apply their knowledge in diverse contexts.
- Investigating the effectiveness of integrating technology and digital resources into mathematics instruction could lead to insights on how to leverage these tools to enhance student learning and engagement.

3. A study could be conducted to investigate the impact of collaborative learning and peer support on learners' understanding and application of mathematical concepts, particularly in areas such as quadratic equations and factorization:

- Collaborative learning, where students work together in small groups to solve problems and discuss mathematical concepts, can foster a deeper understanding and application of the material.
- Peer support and peer-to-peer learning can be especially valuable in areas like quadratic equations and factorization, where students may benefit from explaining concepts to one another and learning from each other's perspectives.

- Researching the impact of collaborative learning on student outcomes in these specific mathematical domains could provide insights into effective instructional strategies and the role of peer interaction in enhancing mathematical proficiency.

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## Appendix 1

This assessment will not affect your grades or progress in the course. The main aim is to help you with algebra by showing your teacher areas you need extra support. The teacher will the assist you in fixing errors and understand the thinking behind them.

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**Instructions:**

- *Answer all questions.*
- *Use factorisation to solve all the problems.*
- *Do not write your name but rather use pseudo names.*
- *Duration: 25 minutes.*

**STUDENT NAME**

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1. Solve the equation $x^2 - 5x + 6 = 0$.
2. Solve the equation $x^2 + 5x = 24$.
3. A garden has an area of 33 square meters. Its length is 5 meters more than a certain number $x$ and the width is 3 meters less than a certain number $x$. What are the possible values $x$?
4. The father is three times older than his daughter. If we multiply their ages from six years ago, we get 420. Can you figure out how old they are today?
5. The solutions $x = -1$ and $x = 3$ are given. Can you find the quadratic equation that matches these solutions?

**Note: adopted from ZIMSEC PAST EXAMINATION PAPERS**

**Appendix 2**

## INTERVIEW GUIDE FOR THE LEARNERS

### Introduction

Bondora Rutendo is a Bindura University of Science Education student who is researching about errors learners have in solving quadratic equations using the method of factorization. As learners, I would like to ask you about your experiences and thoughts on this topic. By sharing your insights, you might help me to understand the common mistakes learners make and how to address them. Your input will make a big difference in my research and it is a great opportunity to share your voice and help improve mathematics learning.

### Questions

- What kind of an equation is this?
- What does it mean when an equation is quadratic?
- How did you go about factoring?
- What was your next step?
- What challenges do you face solving quadratic equations using factorisation?
- What prior knowledge is essential for effective solving of quadratic equations using factorisation?
- Can you walk me through your thought process when approaching this mathematics problem?