BINDURA UNIVERSITY OF SCIENCE EDUCATION

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Real and Imaginary Cats

By

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(HBScED-PHY)

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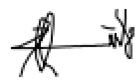
Supervisor: Mr. M. Magama

DECLARATION

I, **Morgan Zimba**, registration number **B212907B** declare that this project is my own original work in partial fulfillment of the requirements for the Bachelor of Science Education Honours Degree in Physics (BScED. Phy) and has not been submitted to any university or other tertiary institution for the award of certification.



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Firstly, I would like to give thanks to the Lord Almighty God for blessing me with the knowledge and wisdom and also for leading me all the way. Secondly, Mr.M. Magama, my supervisor is greatly appreciated for guidance as well as knowledge I gained from him. The research proposal would have not been possible without the help from Mr.M. Magama.

DEDICATION

I dedicate this study to my family for their unwavering support.

APPROVAL

This research project is submitted to the Faculty of Science Education, Department of Mathematics and Science at the Bindura University of Science Education (BUSE) in partial fulfillment of the requirements for the Bachelor of Science Education Honours Degree in Physics (BScEd. Phy).

Name: Morgan Zimba (B212907B) **Signature:** **Date:** 18/11/2022

Supervisor

I, **M. Magama,** declare that I have supervised this research project and now hereby satisfy that it can be submitted to the Faculty of Science Education, of the BINDURA UNIVERSITY OF SCIENCE EDUCATION.

Signature: **Date:** 20/11/2022

Abstract

We construct Schrödinger real and imaginary cat states by superposing the coherent states $|\alpha\rangle$ and $|\alpha^*\rangle$ and study the photon statistics of the superpositions. The photon distribution function (PDF) exhibits an oscillatory character. The physical phenomenon responsible for the resulting PDF is the quantum interference between the superposing components.

Chapter 1: Introduction

1.2 Introduction

In the context of quantum optics, coherent states were introduced by Glauber (1963) as the eigenstates of the annihilation operator, their eigenvalues covering the entire complex plane. The Heisenberg uncertainty relation is minimized in coherent states hence their description as "the most classical quantum states". This makes coherent states the closest representation of classical light. A quantum signal often consists of photons since this enables the use of optical communication fibers, which are very efficient.

1.3 Background of the study

Different finite and infinite sums of coherent states for a single mode quantum harmonic oscillator have been used to construct various non-classical states of light. There is currently much interest in the properties and generation of Schrödinger cat states. A Schrödinger cat is originally a thought experiment where a cat is put in a box with a radioactive substance that can decay in an hour with probability ½ and a bottle of poison that breaks if the radioactive substance decays. In this experiment after an hour, the cat state is in a quantum superposition of the two different classical states: alive and dead: $\frac{1}{\sqrt{2}}(|\text{alive}\rangle + |\text{dead}\rangle)$ (Griffiths and Schroeter, 2018). A Schrödinger cat is any system in a quantum superposition of two different classical states.

The Schrödinger cat states are of particular interest in the area of quantum information theory in which it is studied how quantum mechanics can form a basis of communication and information processing. This has shown promising results such as teleportation S. Perkowita (2011) quantum key distribution D. Bouwmecster,A. Erket, A Zeilinger (2000) quantum computation and high energy Physics J Hidary (2019). The cat states are also important in fundamental research. As a result, the properties and the generation of Schrödinger cats continue to receive much attention.

These quantum superpositions of the finite or infinite number of coherent states exhibit several non-classical characteristics emanating from the quantum interference between summands. Dodonov, Malkin and Man'ko (1974) introduced the concept of even and odd coherent states resulting from the superpositions:$|\text{cat} \pm\rangle = N_{\pm}(|\alpha\rangle \pm |-\alpha\rangle)$. The even and odd Schrödinger cat states satisfy the equation: $\hat{a}^2|\text{cat} \pm\rangle = \alpha^2|\text{cat} \pm\rangle$. Furthermore they exhibit such remarkable non-classical characteristics as oscillations in the photon number distribution. The superpositions $|\text{cat} \pm\rangle = N_{\pm}(|\alpha\rangle \pm |-\alpha^*\rangle)$ which result in the so-called charged Schrödinger cats, have also been considered (Schleich, Dowling, Horowicz and Varro, 1990). These also exhibit a non-Poissonian photon distribution function.

1.4 Statement of the problem

Schrödinger cat states are fundamental in the area of quantum information theory in which quantum mechanics can form a basis of communication and information processing. This has shown promising results such as quantum teleportation, quantum key distribution and quantum computation.

1.5 Significance of the study

The knowledge generated by the study of Schrödinger cat states will add to the existing body of knowledge and take us closer to realizing technical breakthroughs in quantum communication systems and quantum computation. The research also provides a glimpse of the types of problems being tackled by physicists working at the cutting edge of the subject.

The research also resonates with the Government policy of stimulating and supporting the study of Science, Technology and Mathematics (STEM) subjects.

1.6 Objectives of the study

The objectives of the project were to:

(a) construct real coherent states (RCS) and imaginary coherent states (ICS); and
(b) to study the photon statistics of the RCS and ICS.

1.7 Scope

The project was limited to the construction of Schrödinger real and imaginary cat states. The methods of generating and detecting these cat states were outside the scope of the study.

1.8 Summary

The outline of the project report is:

Chapter 1: (this chapter) Presents the main objectives and outlines of the study.
Chapter 2: Provides the reader with the necessary background knowledge to understand Schrodinger cat states.
Chapter 3: Presents the construction of the Schrödinger cat states and the determination of the photon distribution function.
Chapter 4: Presents and discusses the results.
Chapter 5: Concludes on the work presented in chapter 3 and discusses future work in field.

Chapter 2: Theoretical background

2.1 Introduction

This chapter presents a review of some basic properties of simple harmonic oscillator states and coherent states within our necessity. The material covered in this chapter can be found in most standard quantum mechanics texts (Griffiths and Schroeter, 2018).

2.2 Quantum simple harmonic oscillator states

The harmonic oscillator is a well-studied system in both classical and quantum physics. In classical physics, the motion is governed by Hooke's law, whereas in quantum physics, the wave function is calculated by solving the Schrödinger wave equation for the simple harmonic oscillator potential. The possible solutions to this equation correspond to the possible energy eigenvalues. The possible solutions are given by:

$$\psi_n(x) = |n\rangle \qquad f_n(t) = exp\left(-\frac{iE(n)t}{\hbar}\right) \qquad E(n) = \left(n + \frac{1}{2}\right)\hbar\omega$$

(2.1)

where $n \in \mathbb{N}$. This gives us that the general solution to the time-dependent Schrödinger equation as

$$\Psi(x,t) = \sum_{n=0}^{\infty} C_n f_n(t)|n\rangle = \sum_{n=0}^{\infty} C_n exp\left(-\frac{iE(n)t}{\hbar}\right)|n\rangle$$

(2.2)

with$C_n \in \mathbb{C}$. The state $|n\rangle \in L^2$ represents the solution to the time-independent Schrödinger equation with energy eigenvalues$E(n)$. These states are also called number states because the index n in $|n\rangle$ represents the number of photons in the system.

The description of the harmonic oscillator is most elegantly achieved in terms of creation and annihilation operators. This is an example of the so-called factorisation method, a method for solving second-order differential equations by purely algebraic means. For a simpler representation of the creation and annihilation operators, we choose $k = 1 = m$. The creation and annihilation operators are defined in the following definition.

Definition 1: The creation operator $\hat{a}^\dagger$ and annihilation operator $\hat{a}$ are defined by

$$\hat{a}^\dagger = \frac{\hat{x} - i\hat{p}}{\sqrt{2\hbar}} \qquad \hat{a} = \frac{\hat{x} + i\hat{p}}{\sqrt{2\hbar}}$$

(2.3)

where $\hat{a}^\dagger$ is the Hermitian conjugate of $\hat{a}$. The number states are defined as the eigenfunctions of $\hat{a}^\dagger\hat{a}$ in the following way

$$\hat{a}^\dagger\hat{a}|n\rangle = n|n\rangle$$

(2.4)

The creation and annihilation operators respectively raise and lower the energy of a number state taking us up and down in the space of simple harmonic oscillator energy eigenstates:

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle \qquad \hat{a}|n\rangle = \sqrt{n}|n-1\rangle \qquad \hat{a}|0\rangle = 0$$

(2.5)

The ket $|0\rangle$ is called the vacuum state.

Lemma 1: The time-independent wave function of the normalized vacuum $|0\rangle$ is given by

$$|0\rangle = (\pi\hbar)^{-1/4} e^{-x^2/2\hbar} \tag{2.6}$$

The vacuum evolves in time as

$$\Psi(x,t) = exp\left(-\frac{iE(0)t}{\hbar}\right)\Big|0\rangle = exp\left(-\frac{it}{2}\right)|0\rangle \tag{2.7}$$

How the number state $|n\rangle$ can be written in terms of the creation operator $\hat{a}^\dagger$ and the vacuum $|0\rangle$ is presented in Lemma 2.

Lemma 2: Let $n \in \mathbb{N}$ and let $|0\rangle$ be the vacuum as defined in Lemma 1. The number state $|n\rangle$ is expressed in terms of the creation operator $\hat{a}^\dagger$ and the vacuum in the following way

$$|n\rangle = \frac{\left(\hat{a}^\dagger\right)^n}{\sqrt{n!}}|0\rangle \tag{2.8}$$

The Hamiltonian $\hat{H}$ has a very useful expression in terms of the creation and annihilation operators.

Lemma 3: The Hamiltonian operator $\hat{H}$ corresponding to the harmonic oscillator is given by

$$\hat{H} = \hbar\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) \tag{2.9}$$

It gives the total energy of a quantum state in a harmonic oscillator.

With this expression for $\hat{H}$ we see that for the solutions of the time-independent Schrödinger equation (the number states $|n\rangle$ it holds that

$$\hat{a}^\dagger\hat{a}|n\rangle = n|n\rangle \tag{2.10}$$

where $\hat{a}^\dagger\hat{a} \equiv \hat{N}$ is the number operator.

We can express the observables *x* and *p* in terms of $\hat{a}$ and $\hat{a}^\dagger$. This is presented in the following Lemma.

Lemma 4: The observables $\hat{x}$ and $\hat{p}$ are expressed in terms of $\hat{a}$ and $\hat{a}^{\dagger}$ as

$$\hat{x} = \sqrt{\frac{\hbar}{2}}\ \left(\hat{a}^{\dagger} + \hat{a}\right) \qquad \hat{p} = i\sqrt{\frac{\hbar}{2}}\left(\hat{a}^{\dagger} - \hat{a}\right) \tag{2.11}$$

It further holds that

$$\left[\hat{a}, \hat{a}^{\dagger}\right] = \hat{\mathbb{I}} \tag{2.12}$$

where $\hat{\mathbb{I}}$ is the identify operator.

2.3 Coherent states

An important concept which emerges from the study of the quantum harmonic oscillator problem is the notion of coherent states, introduced by Schrödinger (1935) as wavepackets whose dynamics resemble that of a classical particle in a quadratic potential. These states are useful in many branches of physics [Klauder and Skagerstam, 1985;]. We first give the definition of coherent states. Then we state some important properties.

Definition 2: Let $\alpha \in \mathbb{C}$ and $|0\rangle$ be the vacuum such that $\hat{a}|n\rangle = 0$ and $\langle 0|0\rangle = 1$. The coherent state corresponding to the value α is represented by $|\alpha\rangle$ and is defined by

$$|\alpha\rangle = D(\alpha)|0\rangle \qquad D(\alpha) = e^{\alpha\hat{a}^{\dagger} - \alpha^{*}\hat{a}} \tag{2.13}$$

$D(\alpha)$ is called the displacement operator.

A coherent state has some important properties. These are stated in the following lemma.

Lemma 5: Let $\alpha \in \mathbb{C}$ and $|0\rangle$ be the vacuum. Then,

$$|\alpha\rangle = exp\left(-\frac{|\alpha|^2}{2}\right) exp\alpha\hat{a}^\dagger = exp\left(-\frac{|\alpha|^2}{2}\right) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

(2.14)

where $|\alpha|^2$ is the average number of photons in the state and $|n\rangle$ the Fock states. A coherent state is therefore a superposition of number states. It also holds that

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \qquad \langle\alpha|\alpha\rangle = 1$$

(2.15)

This means that a coherent state is normalized and is an eigenstate of the annihilation operator.

2.3.1 Orthogonality relations

Employing lemma 5 for two coherent states we can write

$$\langle\alpha|\beta\rangle = exp\left(-\frac{|\alpha|^2}{2} - \frac{|\beta|^2}{2}\right) \sum_{m,n=0}^{\infty} \frac{(\alpha^*)^n \beta^m}{\sqrt{m!\,n!}} \langle n|m\rangle$$

(2.16)

and due to the orthonormality of the n-states we get

$$\langle\alpha|\beta\rangle = exp\left(-\frac{|\alpha|^2}{2} - \frac{|\beta|^2}{2} + \alpha^*\beta\right)$$

(2.17)

It follows its modulus (sometimes called the state overlap) is

$$|\langle\alpha|\beta\rangle|^2 = exp\left(-|\alpha|^2 - |\beta|^2 + \alpha^*\beta + \alpha\beta^*\right) = e^{-|\alpha-\beta|^2}$$

(2.18)

Hence coherent states are approximately orthogonal only in the limit of large separation of the two eigenvalues, $|\alpha - \beta| \longrightarrow \infty$.

2.3.2 Photon distribution function

The number distribution for the coherent states is

$$P(n) = |\langle n|\alpha\rangle|^2 = e^{-|\alpha|^2} \frac{\left(|\alpha|^2\right)^n}{n!}$$

$$(2.19)$$

This is a Poisson distribution whose mean and variance are equal to $|\alpha|^2$.

2.4 Summary

The algebraic solution to the quantum simple harmonic oscillator problem was presented and the observables were cast in terms of creation and annihilation operators. The coherent states were defined by the action of the displacement operator on the vacuum state. They were also presented as a superposition of number states or eigenstates of the annihilation operator. Coherent states are approximately orthogonal only in the limit of large separation of the two eigenvalues. The photon distribution function in the coherent state is the Poisson distribution, which is a smooth function with a single maximum near the mean photon number.

Chapter 3: Schrödinger cat states

3.1 Introduction

In optics the quantum description of a classical state of light is the coherent state. The quantum superpositions of the finite or infinite number of coherent states have various non-classical characteristics emerging due to the quantum interference of the summands.

3.2 Schrödinger real and imaginary cat states

Consider the superposition of two macroscopically distinguished coherent states

$$|\text{cat}\pm\rangle = N_{\pm}(|\alpha\rangle \pm |\alpha^*\rangle) \qquad \alpha \equiv |\alpha|e^{i\varphi} \tag{3.1}$$

where $N_{\pm}$ is the normalization constant and * is the complex conjugation operator. Using the expansion given in Eqn. (2.14)

$$|\alpha\rangle = exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

we construct the real and imaginary cat states. From Eqns. (3.1) we have

$$\alpha^n = |\alpha|^n e^{in\varphi} = |\alpha|^n(cosn\varphi + isinn\varphi)$$
$$(\alpha^*)^n = |\alpha|^n e^{-in\varphi} = |\alpha|^n(cosn\varphi - isinn\varphi) \quad (3.2)$$

For the case $|\text{cat}+\rangle$ we have:

$$|\text{cat}+\rangle = N_+(|\alpha\rangle + |\alpha^*\rangle)$$

$$= N_+ exp\left(-\frac{|\alpha|^2}{2}\right)\left[\sum_{n=0}^{\infty}\frac{|\alpha|^n(cosn\varphi + isinn\varphi)}{\sqrt{n!}}|n\rangle + \sum_{n=0}^{\infty}\frac{|\alpha|^n(cosn\varphi - isinn\varphi)}{\sqrt{n!}}|n\rangle\right]$$

Therefore, the expansion for real cat states (or RCS) is given by

$$|\text{cat}+\rangle = 2N_+ exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty}\frac{|\alpha|^n}{\sqrt{n!}}cos(n\varphi)|n\rangle \qquad 0 \le \varphi < 2\pi \quad (3.3)$$

where $|n\rangle$ is the vector of the Fock space.

Following the same procedure as above, it easy to derive the corresponding expansion for imaginary cat states (or ICS) which is given by

$$|\text{cat}-\rangle = 2iN_- exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=1}^{\infty}\frac{|\alpha|^n}{\sqrt{n!}}sin(n\varphi)|n\rangle \qquad 0 \le \varphi < 2\pi \quad (3.4)$$

where $|n\rangle$ is the vector of the Fock space.

3.3 Photon distribution function

Using expansion (3.3) we can calculate the photon distribution function (PDF) for the real cat states. We begin by calculating the inner product $\langle n|\text{cat}+\rangle$:

$$\langle n|\text{cat}+\rangle = \frac{2N_{+}exp\left(-\frac{|\alpha|^2}{2}\right)|\alpha|^n cosn\varphi}{\sqrt{n!}}$$

We obtain the PDF from

$$P_{+}(n) = |\langle n|\text{cat}+\rangle|^2 = \frac{2N_{+}^2 e^{-|\alpha|^2}|\alpha|^{2n}cos^2 n\varphi}{\sqrt{n!}}$$

Using the trigonometric identity: $cos2\theta = 2cos^2\theta - 1$; the PDF reduces to

$$P_{+}(n) \sim [1 + cos(2n\varphi)]e^{-|\alpha|^2}\frac{\left(|\alpha|^2\right)^n}{n!} \equiv [1 + cos(2n\varphi)]P(n) \tag{3.5}$$

where $P(n)$ is the Poisson distribution function.

Following the same procedure as above we can show that the PDF for imaginary cat states is given by

$$P_{-}(n) \sim [1 - cos(2n\varphi)]e^{-|\alpha|^2}\frac{\left(|\alpha|^2\right)^n}{n!} \equiv [1 + cos(2n\varphi)]P(n) \tag{3.6}$$

Eqns. (3.5) and (3.6) can be recast in a compact form as

$$P_{\pm}(n) \sim [1 \pm cos(2n\varphi)]e^{-|\alpha|^2}\frac{\left(|\alpha|^2\right)^n}{n!} \equiv [1 \pm cos(2n\varphi)]P(n) \tag{3.7}$$

3.4 Summary

RCS and ICS ($|\text{cat}\pm\rangle$) were constructed using the quantum superposition of an infinite number of coherent states. The PDFs of the two cat states were also determined.

Chapter 4: Discussion of results

4.1 Real and imaginary Schrödinger cat states

The superposition of the coherent states $|\alpha\rangle$ and $|\alpha^*\rangle$ yields

$$|\text{cat}+\rangle = 2N_+ exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty}\frac{|\alpha|^n}{\sqrt{n!}}cos(n\varphi)|n\rangle \qquad 0 \le \varphi < 2\pi$$

$$|\text{cat}-\rangle = 2iN_- exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=1}^{\infty}\frac{|\alpha|^n}{\sqrt{n!}}sin(n\varphi)|n\rangle \qquad 0 \le \varphi < 2\pi \tag{4.1}$$

where $|n\rangle$ is the vector of the Fock space and $|\text{cat}\pm\rangle$ are real and imaginary Schrödinger cat states.

4.2 Photon distribution functions

The photon statistics of the Schrödinger cat states, $|\text{cat}\pm\rangle$, are given by

$$P_{\pm}(n) \sim [1 \pm cos(2n\varphi)] e^{-|\alpha|^2} \frac{\left(|\alpha|^2\right)^n}{n!} \equiv [1 \pm cos(2n\varphi)] P(n) \tag{4.2}$$

where $P(n)$, the Poisson distribution function, is modulated by the oscillatory function $[1 \pm cos(2n\varphi)]$. Therefore, the real and imaginary Schrödinger cat states manifest a modulated Poissonian photon distribution function. This is a result of the interference between the coherent states $|\alpha\rangle$ and $|\alpha^*\rangle$. This statistical property depends on both the modulus $|\alpha|$ and the phase difference 2φ between α and α^*.

Oscillations of the PDF are shown graphically in Figs. (4.1) and (4.2) both for RCS and ICS in the representative cases of $|\alpha|^2 = 1.5$ and $|\alpha|^2 = 9.0$, $\varphi = 0.3\pi$ in the Appendix.

For the case $\varphi = 0.5\pi$ we have

$$P_{+}(n) \sim [1 + (-1)^n] P(n) = \begin{cases} 2P(n) & \text{for even } n \\ 0 & \text{for odd } n \end{cases}$$

(4.3)

$$P_{-}(n) \sim [1 - (-1)^n] P(n) = \begin{cases} 2P(n) & \text{for odd } n \\ 0 & \text{for even } n \end{cases}$$

Therefore, for this case we recover the Poisson distribution function for the even values of n for the $|\text{cat}+\rangle$ quantum states and for odd values of n for the $|\text{cat}-\rangle$ quantum states.

4.3 Summary

The superposition of the coherent states $|\alpha\rangle$ and $|\alpha^*\rangle$ yields Schrödinger real and imaginary cat states $|\text{cat}\pm\rangle$. The PDF was found to be a modulated Poisson distribution function. The oscillatory modulating function arises due to the quantum interference between the superposing coherent states. The Poisson distribution function re-emerges for the even (odd) values of n for the $|\text{cat}+\rangle$ ($|\text{cat}-\rangle$) quantum states in the case $\varphi = 0.5$

Chapter 5: Conclusions and recommendations

5.1 Conclusions

We constructed Schrödinger real and imaginary cat states $|\text{cat}\pm\rangle$ by superposing the coherent states $|\alpha\rangle$ and $|\alpha^*\rangle$. The oscillatory character of the photon distribution function arising from the quantum interference between the superposing components was demonstrated. The objectives of the study as outlined in section 1.5 were met.

5.2 Recommendations

There are numerous other research directions that can be undertaken and these include the construction of other types of Schrödinger cat states, the study of other aspects such as quadrature squeezing, the generation and detection of Schrödinger cat states and the practical applications of Schrödinger cat states. The possibilities are many as areas like quantum information theory are still relatively new areas of physics.

Appendix

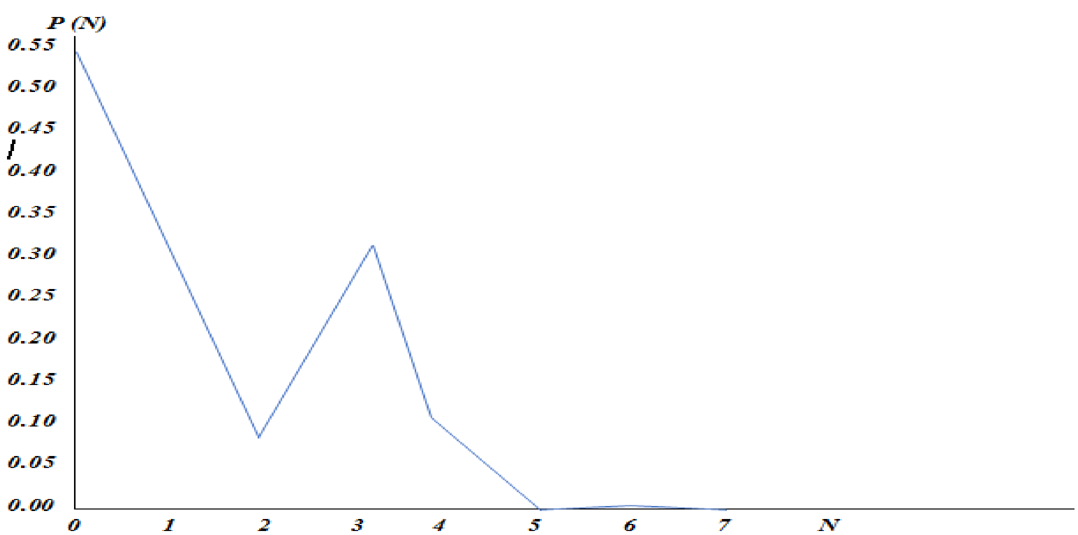

Figure 4.1(a) A plot of PDF for $P_+(n)$ for $|+\rangle$ with $|\alpha|^2 = 1.5$ *and* $\varphi = 0.3\pi$.

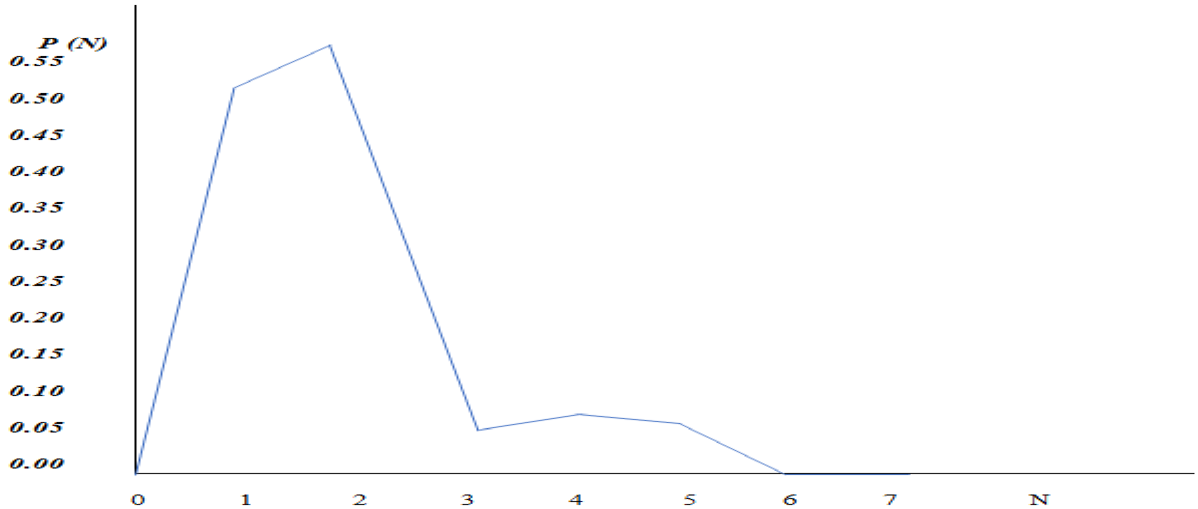

Figure 4.1(b) A plot of PDF for $P_{-}(n)$ for $|-\rangle$ with $|\alpha|^2 = 1.5$ *and* $\varphi = 0.3\pi$

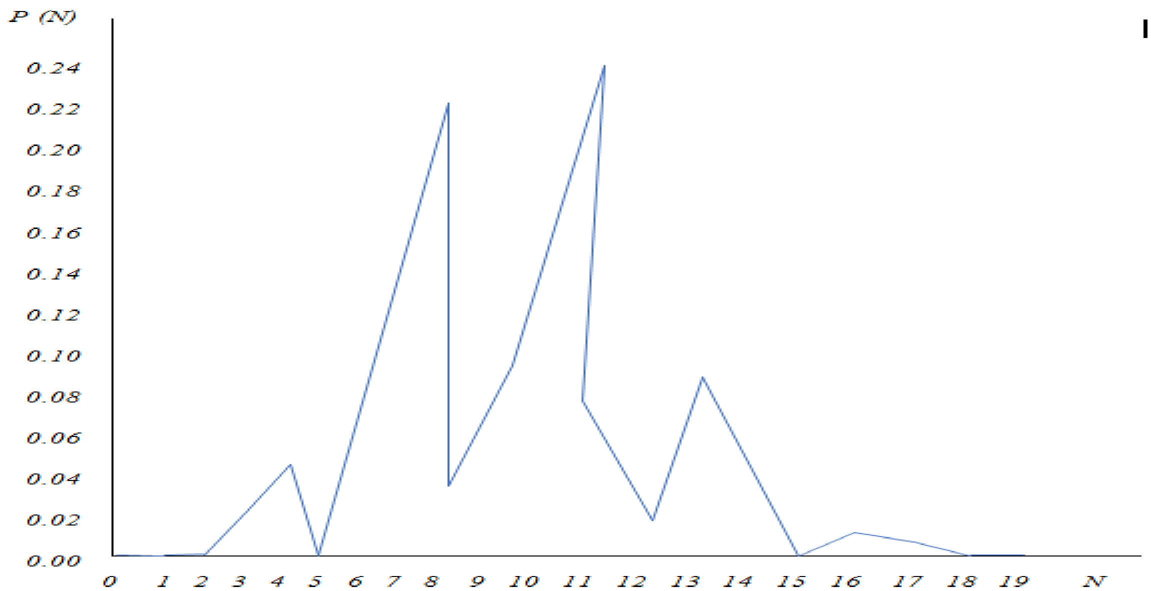

Figure 4.2(a) a plot of PDF for $\mathrm{P}_{+}(n)$ for $|+\rangle$ with $|\alpha|^2 = 9.0$ *and* $\varphi = 0.3\pi$

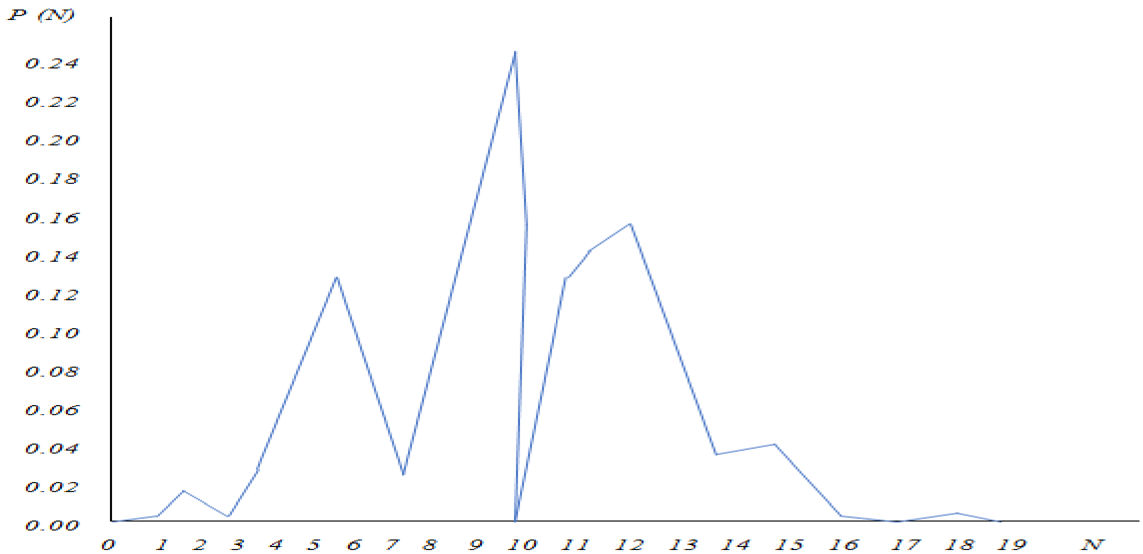

Figure 4.2(b) a graph for $P_{-}(n)$ for $|-\rangle$ with $|\alpha|^2 = 9.0$ *and* $\varphi = 0.3\pi$.

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