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AN ANALYSIS OF ERRORS MADE BY FORM 2 PUPILS AT FIRM FOUNDATION CHRISTIAN ACADEMY WHEN SIMPLIFYING ALGEBRAIC EXPRESSIONS.

BY

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DEDICATION

This study is dedicated to my lovely wife Stembiso Tshuma who encouraged me through all circumstances to reach beyond the stars I am so thankful. To my son Ethan Edmund Marashe, you are the reason I have so much courage to move on when the road gets tough. To my dear friends and colleagues I am thankful for all the encouragement you have given me.

ABSTRACT

This is a study on "Analysing Pupils' Errors in Learning Algebra at form two level". The main objective of the study was to identify the root cause of pupils' errors when simplifying algebraic expressions at form 2 level and to explore the strategies that reduces pupils' errors in learning algebra. The research questions were how do students perceive learning algebra? What are the reasons behind students' weak in learning algebra? What are the factors behind students' error in learning algebra at basic level? This research was qualitative research which was based on phenomenology research design. Research site was Firm Foundation Christian Academy which is located in Borrowdale area under Harare province. As respondent of this study was from only from form 2. The major tools used for the study were three exercises, an achievement test and interviews conducted on one on one situation with respondents.

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CHAPTER 1: STUDY OVERVIEW

1.1 INTRODUCTION

This chapter aims at establishing the best possible ways of teaching algebraic expressions. It contains some critical aspects that give a pointer to the nature and structure of the study. The researcher will focus much on the errors which pupils make when simplifying algebraic expressions. In this research the researcher will outline major aspects of the research for example the purpose of the study and the significance of the study to various stakeholders like school and the pupils themselves.

1.2 BACKGROUND TO THE STUDY

Algebraic expressions have proved to be a difficult topic for most Junior Certificate level pupils. This is because it is a different concept to what pupils were taught at primary level. During their primary level experience, pupils were taught a combination of digits and boxes whilst in contrast JC level has marked the introduction of a combination of letters and numbers. As a result, the researcher's aim is to help develop strategies that can bridge the gap between primary and secondary level mathematics for smooth understanding of algebraic concepts.

The researcher wants to identify the nature of statement and the status of symbols in statements, considering for example $a + 5b$. Most learners treat a as 1 as a result the answer for the question will be $6ab$ which is incorrect. There is need for teachers to equip learners with the concept of like and unlike terms for effective understanding of the topic. The best way of teaching algebraic expressions is to assign letters something pupils know like fruits. In this case, the teacher can make a letter "a" to stand for apples and a letter "b" to stand for bananas so that $a + 5b$ becomes 1apple, 5 bananas. This has been supported by (Yaftian, 2022) when he argues that junior high school students are in the abstract stage of cognitive development, but many cannot completely think abstractly, and some of them are in operational stage so teachers need to use intuitive and objective teaching aids to foster learners understanding.

(Cockcroft, 2020) emphasises that before pupils go further with the topic of algebra, they should become thoroughly familiar with some new form of algebraic symbolism as this may facilitate their understanding of the topic. Algebra is a language of symbols; as a result, the problem faced by most pupils is to translate from ordinary English to the language of symbols. (Myron, 2021) echoes the same sentiments when he posits that algebra is a source of considerable confusion and negative attitudes among pupils. One can agree with the above claim since most learners seem to develop a negative attitude after failing to master basic concepts in algebraic expressions and it can lead to loss of confidence and low self-esteem amongst learners. It is against this background that the researcher developed an ambition to carry out this research to establish the root causes of errors made by pupils as well as coming up with long lasting solutions.

1.3 PURPOSE OF THE STUDY

The purpose of the study is to investigate root causes of problems and errors made by most pupils in the learning of algebraic expressions at Zimbabwe Junior Certificate level form 2 level to be precise. Another essential aspect of the study is to modify teaching methods and approaches as well as providing opportunities for creative, divergent thinking and inventing new rules and strategies in the learning of algebraic expressions. This is because the learning of algebra at lower secondary school level has been considered to be a base for a better understanding of various topics in mathematics.

1.4 STATEMENT OF THE PROBLEM

Pupils' errors can cause them to perform poorly in any subject. Most mistakes in mathematics come from lack of algebraic skills. To make matters worse, almost all mathematical concepts are developed from algebraic concepts. Therefore, there is need for learners to know common errors associated with simplifying algebraic expressions and the chief causes of the mentioned errors. The researcher is of the view that detecting learners' errors is the stepping stone in eradicating

errors by students. Analysing these errors will help teachers to find effective ways of minimising critical errors in algebra and other branches of mathematics.

1.5 MAIN RESEARCH QUESTION

The main research question of this study is: What are contributory factors behind common errors made by learners when simplifying algebraic expressions.

1.5.1 RESEARCH QUESTIONS

1. What types of errors and misconceptions do students experience in simplifying algebraic expressions?

2. What are the possible reasons behind common errors by pupils in simplifying algebraic expressions?

3. How do secondary school pupils perceive algebra?

1.6 ASSUMPTIONS OF THE STUDY

The following assumptions shall be valid for this study.

1) Selected interviewees will respond timeously.
2) Respondents have a general understanding of algebraic expressions.
3) Information collected from respondents will be accurate, relevant and dependable.

1.7 SIGNIFICANCE OF THE STUDY

The study is of paramount importance to the following stakeholders

The School

The main thrust that drove the researcher's to carry out this research project is to come up with accurate results, findings as far as pupils' performance on simplifying algebraic expression is

concerned. The outcomes of this project will be useful to the school in mapping the way forward on how to eradicate common errors made pupils when simplifying algebraic expressions.

Pupils

The researcher wants to access first-hand information on common errors made by learners when simplifying algebraic expressions. This will enable leaners to benefit from modified teaching techniques that will be recommended after the completion of the research project. In addition to this, the researcher wants to equip pupils with the necessary skills to incorporate real life situations into solving mathematics.

1.8 DELIMITATIONS OF THE STUDY

Delimitations of the study are the boundaries that the researcher sets in a research study, deciding what to include and what to exclude as (Bruggen, 2022) puts it. He further argues that delimitations narrow help to narrow down the study and make it more manageable and relevant to the research goal. The research project is only restricted to Firm Foundation Christian Academy an upcoming school located in Borrowdale. The fact that the research project focused on only one school can somehow discredit the findings since they cannot be generalised because of a limited sample.

1.9 LIMITATIONS OF THE STUDY

To begin with, the problem of in insufficient Mathematics text books for pupils to revise on their own negatively impacted the research progress. This made it difficult for the researcher to come up with a valid conclusion to struggling learners because they were deprived the opportunity to adequate resources which would have afforded them an opportunity to revise thoroughly on their own. However, as a mitigation strategy the researcher improvised by photocopying the few available text books.

1.10 DEFINITION OF TERMS

Project- it is an academic, scientific or professional undertaking to answer a research question.

Simplify- to make easy to understand by reducing to small terms

Expand- unfold, spread out or remove brackets.

Algebraic expression- is a set of terms with letters and numbers that are combined using $addition\ (+), subtraction(-), multiplication(\times)$ and division$(\div)$.

1.11 ORGANISATION OF THE STUDY.

This study has been divided into five chapters. The first chapter gives the background of the study, the statement of the problem, the aim and objectives as well as research questions. It also gives the significance of the study.

Chapter two gives the basis of the study by examining the previous work done by other researchers in the field of algebra. Relevant literature review of errors was carried out. There is also a discussion on the nature of errors and misconceptions within the constructive framework.

The main thrust of chapter three is to discuss the methodological constructs of the research. The chapter gives a detailed explanation of the research design employed, the sample, sampling methods, and the participants of the study, the research instruments, reliability, validity ethical considerations and limitations of the study.

In chapter four the researcher embarked on data analysis and interpretation of the test data. The identified common errors are described, findings are tabled and graphically presented.

Chapter five is the discussion of the finding of the study, recommendations based on the findings as well as the conclusion.

1.12 SUMMARY

This chapter has given the background, the statement of the problem, the aim of the study and its respective objectives as well as the research questions. The chapter has also given the significance of the study and the organisation of the study as well. The next chapter focuses a review of related literature.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

This chapter intends to analyse the views of various scholars on the root causes of challenges faced by most pupils when simplifying algebraic expressions. Many studies have been carried out on the teaching and learning of Mathematics and in particular, algebra. Numerous studies bring many conceptions about algebra. They bring out the fact that many learners have naive theories and misconceptions that interfere with their learning of Mathematics which result in them making errors when solving problems. This chapter presents a review of related literature in an attempt to provide foundation and ground theory for an organized study of errors encountered by learners in algebra.

2.2 BACKGROUND

The review of the empirical literature of the study was essential and necessary. If it was ignored, it was going to be difficult for the researcher to proceed with the project. It provides psychological, sociological, philosophical, contextual, historical knowledge, concept, and traditional way of study. It enabled the researcher to access pre-knowledge of study, pre- techniques, pre- methods, and pre-analysis procedures. So, the researcher reviewed the following literature: (Adhikari, 2021) conducted research entitled **"An error analysis in solving algebraic problems at form 2 level"**. The prime objectives of this study were to identify the types and causes of errors made by students and suggesting possible solutions to resolve the problems. The researcher again managed to review a study entitled **"A constructivist analysis learners' errors and misconceptions in simplifying algebraic expressions at secondary level"** by (Kolitsoe, 2022). The objectives of this study were to find the errors made by students in solving the problem of simplification of algebra and to compare the error made by students in knowledge, skill, and application and problem solving of simplification.

2.3 KEY FINDINGS

The challenge in algebra is that most of the learners fail to understand the main concepts of algebra. Once learners fail to understand the key aspects of algebra, they have difficulties in Mathematics. One of these key aspects is simplifying algebraic expressions. Algebra is a generalised form of arithmetic where letters and both operation and direction signs are used. The use of letters and signs, according to (Foster, 2022), makes it abstract and difficult. This is because algebraic ideas are based on general ideas instead of real facts or events.

Learners possess a serious arithmetic-algebra gap which, as observed by (Seng, 2021), remains a cause for concern as far as learners' understanding of algebra is concerned. Learners that possess a good arithmetic background have slim chances of facing challenges in algebra. This is necessitated by the fact that arithmetic forms a foundation on which algebra is to be built, hence a strong foundation ensures a concrete understanding of abstract concepts. (Thomas, 2020) observed that learners' errors are the symptoms of a combination of misconceptions and misunderstanding. The impacts caused by these two cannot be underestimated as they contribute immensely to learners' poor performance in mathematics. (Clackson, 2022) defines an error as a mistake made during calculations, misinterpretation of data, flawed reasoning in a proof or inaccurate assumptions that lead to incorrect solutions. Errors perhaps result from forgetting, confusion or lack of understanding of key concepts. The idea of learners making errors and or mistakes is a natural part of teach (Fernandez, 2021) puts it. These errors produced by learners as suggested by (Stein, 2023), play a fundamental role in indicating to teachers the stages at which their learners are at as well as showing where there is need for further teaching or study. They assist teachers to advise learners for improvement.

According to (Gomes, 2021) the errors in signs is a serious cause for concern among learners as it has unwanted long lasting effects on learners' performance. This type of error is associated with failure by learners to recognise signs of like terms when grouping them in an attempt to simplify algebraic expressions. A relevant example is the illustration from the question $2x + y - 3x - 2y$. most learners simplify this question the wrong way as shown below

$$2x + y - 3x - 2y$$

$$2x + 3x - y - 2y$$

$$5x - 3y$$

This is a clear indication to show that error of signs poses a serious challenge as far as learners' performance is concerned not only in algebra, but in mathematics at large. The correct way simplifying the above question is

$$2x + y - 3x - 2y$$

$$2x - 3x + y - 2y$$

$$-x - y$$

The distributive error is again an issue of great concern amongst most maths teachers as it affects learner's performance significantly. This marries well with (Watson, 2022) when he likens the distributive law in algebra as the backbone of mathematics understanding. This is undisputed if one considers the effect of failing to master the distributive law when dealing with expansion of brackets in algebra. From the research entitled **Misconceptions and errors in algebra at grade 9 level, the case of two selected secondary schools in Petauke district by (Mbewe, 2013) from the University of Zambia** the following findings were noted from learners that participated.

Simplify the following question $5 - 2(x + 4)$

The three learners that participated all failed to get the correct solution as to be shown below

Solution from learner 1

Stanley Chilifiya

1a) $5 - 2(x + 4)$

$5 - 2x + 4$

$5 + 4 - 2x$

$9 - 2x$

Solution form learner 2

Lushumo Mwape

$5 - 2(x + 4)$

$5 - 2x + 8$

$5 + 8 - 2x$

$13 - 2x$

Solution from learner 3

Salma Chabala

$5 - 2(x + 4)$

$3(x + 4)$

$3x + 12$

It can be noticed that all the three participants failed to get the correct answer as shown from their respective solutions. This can further justify the assertion by (Watson, 2022) when he compares the distributive law in algebra as the backbone of mathematics due to its devastating effect on learner performance if not well grasped. The above errors from the three participants can be attributed to a series of factors chief among them being failure by learners to master the concept of directed numbers as shown from Lushumo Mwape's solution. One can notice that from her working, the concept of distribution law was mastered, but the learner failed to get the correct answer merely because she failed to multiply -2 and $+4$. This clearly supports Suryadi's opinion where he postulates that directed numbers forms the foundation of all mathematical concepts.

(Heeffer, 2022) echoes the same sentiments arguing that a negative sign is more powerful than a positive sign because it can change a positive sign to a negative which cannot be the case if we consider a positive sign. Another possible contributory factor can be the issue of failing to appreciate the concept of order of operations which was invented by a great Greek mathematician by the name Achilles Reselfelt abbreviated as BODMAS. The effect of this error can be clearly seen on Salma Chabala's solution when the learner failed to recognise that there is need to start by distributing -2 inside the bracket. Instead the learner started by subtracting 2 from 5 which was a clear violation of mathematics principles. In addition to this, a combination of errors contributed greatly to Stanley Chilufiya's failure to get the correct answer. Inability to simplify directed numbers can be fingered to be one of the factors as well as the distributive error. The effect of the above mentioned errors can be noticed on first stage of the learner's working. Stanley failed to recognise the impact of -2 outside the bracket as he only effected the impact of -2 to x only instead of affecting $+4$ by -2 using the concept of multiplication in an attempt aimed at removing brackets effectively. The correct solution for the question was supposed to be

$$5 - 2(x + 4)$$

$$5 - 2x - 8$$

$$5 - 8 - 2x$$

$$-3 - 2x$$

$$-(3 + 2x)$$

According to Mbewe's study, errors and misconceptions are habitual and cannot be solved easily. This was evident from the interviews that Mbewe conducted with the participants. In concluding his study, Mbewe recommended that teachers and learners need to talk about errors and misconceptions during the teaching and learning process so that ways of doing away with them could be identified and a lasting solution can be effected.

Another study by a person called Naresh Chaudhary from the thesis entitled **"Student errors in learning algebra at secondary level"**. The following finding were found from the learners that participated. One of the question was,

Simplify the question $(a+b)^2$ and the following responds were noticed.

Solution from learner 1

Dewei Caishen

$$(a+b)^2$$

$$a^2+b^2$$

Solution from learner 1

Chongani Hai

$$(a+b)^2$$

$$(ab)^2$$

$$a^2b^2$$

From the above solutions, one can clearly notice that none of them managed to get the correct answer. An analysis of the two solutions can reveal that the transformation error can be fingered to be the major contributory factor behind the incorrect solutions. According to (Long, 2022), transformation error occurs when a student understands the demands of the question, but fail to choose the appropriate mathematical operation required. This is exactly what transpired, for instance on first solution from Dewei Caishen the learner noticed that there is need to interpret the concept of squaring but failed to execute the process. Instead of writing the two brackets with the same contents and perform expansion of brackets, Dewei squared each term independently. The same can be said as far as the second solution is concerned though the learner seem to have a huge arithmetic gap as compare to the first one. Hai could have missed a lot on simplifying algebraic expressions where the concept of like and unlike terms is given much emphasis. More so, Hai

failed to differentiate the requirements of addition and multiplication which forced the learner to get ab after simplifying $a + b$. Naresh Chaudhary recommended that there is need for teachers to design effective teaching strategies complemented by teaching aids for effective teaching and learning. The correct solution to the question is $(a + b)^2$

$$(a + b)(a + b)$$

$$a^2 + ab + ab + b^2.$$

$$a^2 + 2ab + b^2$$

2.4 THEORETICAL FRAMEWORK

The study employed the use of the theory of constructivism to analyse errors made by learners in simplifying algebraic expressions. Constructivism according to (Saul, 2024) is a learning theory that emphasise the active role of learners in building their own understanding rather than passively receiving information. (Evans, 2023) echoes the same sentiments when he argues that constructivism refers to a process whereby learners actively construct their understanding and knowledge of the world through experiencing things and reflecting on those experiences. Constructivism is a behaviorist theory that originates from Jean Piaget. According to (Piaget, 1970), learning is not a simple passive process of receiving from the surrounding environment. Constructivism is a dynamic process of an individual involving interaction between the individual's existing knowledge and new ideas. According to (James, 2022) the constructivist theory implies that learners do not come to a new grade as empty vessels, tabula rasa, blank slate, but bring pre-knowledge from previous grades as (Blakeley, 2022) puts it. (Resnick, 2022) is of the view that learners are holders of theories constructed from their everyday experiences which is their existing knowledge. The importance of existing knowledge as described by (Olivier, 2023) is that it is stratified in a learner's mind into a series of interconnected concepts called schema. These schemas are retrieved and put into use when a learner encounters a familiar situation. Therefore, the schemas are important tools of each individual learner. The learning of Mathematics truly resembles a constructive process because it is cumulative. Mathematical concepts are interrelated with one another, hence related knowledge which is possessed by a learner is used to

construct new ideas. Mastering of new ideas as described by (Rochelle, 2023) is difficult if basic concepts and skills learnt at early stages were not well understood. During the process of learning, individual thoughts consist of two basic alternative mechanisms, namely; assimilation and accommodation as (Piaget, 1970) puts it. These two processes are of paramount importance in describing how an individual adjust the mind to strange experiences and be in a position to absorb new information. According to (Motley, 2021) assimilation occurs when a new idea is interpreted in terms of an existing schema. The importance of an existing mental structure is that it is a useful tool used to assimilate a new situation. The mathematical demonstration of the process of assimilation can be shown by a learner's working of the question $9^2 - 5^2$ to come up with $(9-5)(9+5) = (4)(14) = 56$. This can largely be attributed to the learner's previous knowledge on how to factorise quadratic expressions, difference of two squares to be precise for example. A question like $(a-b)^2$ = (a – b) (a + b). One can safely argue that the learner's demonstration is enough evidence to show the important role played by prior knowledge. This is because the learner has interpreted the new situation in terms of the already known aspect of factorisation difference of two squares.

Another essential concept in constructive learning is to do with the process of accommodation. According to (Marshal, 2022) accommodation is the cognitive process that involves the development and alteration of mental representations, schemas as individuals encounter new situations. This maries well with (Genevieve, 2022) as he regards accommodation as a process that involves modifying existing mental structures to accommodate new information that does not fit into pre-existing schemas. This process can best be explained by looking at a situation where learners have to find products of algebraic terms. If a learner has the knowledge that $x \times y = xy \; or \; yx$ then when a problem requires the same learner to simplify $x \times 5$ then the learner is more likely to come up with $x5$ as his or her solution he as the learner will be thinking that $x5$ is the same as $5x$. This is a clear indication to show that there is a need for restructuring so that when a learner notices the presence of a number when multiplying algebraic terms, a number must be written first. This shows that it is not always possible to connect new ideas to schemas. The failure to link existing schemas to new situations may result in the creation of a new box in the mind of the individual. The learner may find it difficult to link the knowledge in the box to existing schemas

which may force him or her to memorise the ideas or rules to learn. In the process of recalling, some of the rules are partially remembered resulting in the learner being confused and making errors. These errors, as suggested by (Olivier, 2022), are the natural results of learners' effort to construct knowledge. This concurs well with (Labinowicz, 2022) as he also regards learners' errors as actually natural steps to understanding. According to (Brodie, 2021), understanding learner errors is a way of the picture of the accommodation process can be explained by looking at a situation where learners have to find products of algebraic terms.

Mildred Ncube carried out a research entitled "**An analysis of errors made by learners in simplifying algebraic expressions at grade 9"** by (Ncube, 2021). The essence of the study was to have a deep understanding of learners' errors when simplifying algebraic expressions and finding out the possible causes of the errors. She applied the use of a sequential explanatory design which is a mixed-method approach that is both quantitative and qualitative method. This study is based on the theory of constructivism. She selected a total of eight two students from three hundred students randomly grade 9 learners from secondary school in Ga-Sekgopo village, Mopani district in Limpopo Province in South Africa as the sample for this study. In this study, she found six common types of errors in simplifying algebraic expressions. The errors are: Conjoin error (combining unlike terms) - 16%, misapplication of rules-370, misinterpretation of symbolic notation-60, misuse of distributive property-22%, substituting letters by numbers-10%, and sign error-9%. She concluded that learners mostly misapplied rules when they were simplifying algebraic expressions. They are failing to deal with direction, operation, sign, and algebraic rules. The main reasons behind the errors are poor arithmetic background, lack of arithmetic skills, lack of awareness and understanding of the meaning of expressions, and lack of conceptual knowledge.

(Laudari,2021) conducted a study entitled "**An error analysis of form 2 students in simplifying algebraic expressions"** with an objective of finding out common errors made by learners, root causes of the said errors as well as designing a lasting solution to the errors. This study was quantitative followed by qualitative methods and procedures. All form 2 students enrolled in government schools running classes situated in rural location Tanahu district were considered as the population of this study. 10 students from each school including an equal number of boys and

girls were selected as the sample of this study. The research design of this study was a small survey. The sample for this study consisted of 40 students by random sampling and 4 teachers for interview. From this study researcher extended the following conclusion that students committed maximum errors in transformation after that comprehension, process, skill, and encoding respectively but didn't commit any error in reading level, and the role of gender is less important to committing the errors it means there is no significant difference between boys and girls to simplify algebraic expressions.

(Egodawatte, 2021) conducted a study entitled **"Secondary school students misconception in algebra at form 2 level"**. This study aimed at investigating secondary school students' errors and misconceptions in algebra focusing on form 2 learners. The intention was to expose the nature and origin of those errors and to make suggestions as a way of eradication misconceptions and errors for effective teaching and learning. A mixed method research design was used, an algebra test that was pilot-tested for its validity and reliability was given to a sample of grade 9 students in an urban secondary school in Ontario. The results indicated several error categories under each area. Some errors emanated from misconceptions for example a question like $5a + 2b - a$ was simplified as $7ab - a = 7b$. From the working by one of the students, one can notice that the learner missed essential algebraic skill under the concept of grouping of like and unlike terms. Again the other contributory factor behind learners' errors was the lack of understanding of the basic concept of the variable in different contexts. The abstract structure of algebraic expressions posed many problems to students such as understanding or manipulating them according to accepted rules, procedures, or algorithms.

(Matt, 2021) carried out a research project entitled **"The analysis of students' errors in learning of quadratic expression"**. The purpose of the study was to determine learners' errors in learning quadratic expressions. A samples of 30 from three different secondary schools in Jambi district of Jakarta town in Indonesia. A diagnostic test was used as the instrument of this study that included three components: Simplifying algebraic expressions through grouping of like terms, factorisation of algebraic expressions by grouping and factorisation of quadratic expressions involving

difference of two squares. A diagnostic interview was also used to identify at which level students' errors occur in simplifying problems. The types of errors that were observed were based on Newman's error Hierarchy Model which includes reading type error, comprehension, transformation, process skill, and an encoding error. Analysis of data was carried out using descriptive statistics: percentage and frequency. The findings showed that most students make an error in transformation and process skills in simplifying algebraic expressions. The number of students who made encoding errors and carelessness was small. The students' error in simplifying algebraic expressions were a result of a huge arithmetic gap dating back to learners' failure to master directed numbers particularly the effect of signs especially negative signs to be precise for example $3a + 2b - 8a$ was simplified go give $3a - 8a + 2b = 5a + 2b$. This is enough evidence to prove that learners had a huge arithmetic gap that needs to be bridged. The recommendations from the study emphasised the need for teachers to apply the use of relevant teaching aids when introducing the topic of directed numbers as it forms the backbone of all mathematics concepts as shown in the illustration above.

(Sharma,2020) conducted a study entitled "**An error analysis on solving verbal problems of algebra by grade 10 students"** with an intention of finding the patterns of errors committed by the grade 10 students while solving verbal problems in algebra. The prime objective of this study was to identify and make a comprehensive analysis of common errors made by learners when interpreting algebraic expressions involving word problems. In order to achieve the intended goal, the researcher developed an achievement test. The sample of this study consisted of twenty students and the school was selected purposively for the convenience of the researcher. From this study, the researcher extended the conclusion those students commit more errors while solving the verbal problem in algebra, the concentration of errors was seen in the phase of transformation, process skill, and comprehension of the problem.

(James, 2022) conducted a study **"An error analysis in solving algebraic problems of sixty-five students" at form 2 level.** The prime objectives of the study were to find the errors made by students in simplifying algebraic expressions as well as designing solutions on how best can the

errors be eradicated. This is because failure to understand the basic principles of algebraic expressions by learners at foundation level, form 1 and two to be precise has contributed immensely to learners' failure of the subject at ordinary level according to researches that have been conducted by various authors. This research was qualitative and was based on descriptive design. The sample of this study was Janata secondary school of Surkhet and all sixty-five students of form 2 were selected. Interview schedules were the main tool of this study. Major findings were as: 75% of errors occurred at the comprehension and transmission stage, 12% of errors occurred at the process skill stage, 5% errors occurred encoding stage, 8% errors occurred due to carelessness of students, and 40%,34%, and 26% errors on knowledge level, skill and application level and problem-solving level. The study concludes that poor performance of students simplifying algebraic expressions were rooted in their algebraic background which shows a huge arithmetic gap that needs to be bridged. He went on to advocate for a cooperative approach between students and teachers in order to achieve the ultimate goal, Teachers were advised to employ interactive teaching methods like group work and pair work where discussions can be done freely. The use of effective teaching aids by teachers was also advocated for as it plays a critical role as far as learners' retention capacity is concerned. On the other hand, learners were encouraged to be actively involved during the learning process. He put much emphasis to his suggestion by quoting Benjamin Franklin which reads "Tell me and I forget, teach me and I remember, involve me and I learn.

2.5 CONCLUSION

From the whole chapter, the researcher deduced that various researches that were conducted regarding the topic of algebraic expressions were centred on finding out common errors made by learners when simplifying algebraic expressions. The researcher has noticed that arithmetic gap contributes mostly to learners' failure as such, the researcher is of the opinion that there is need for all stakeholders involved to work together to arrest the problem from its early stages. It is against this background that the researcher felt that the study on simplifying algebraic expressions at form 2 level was worth carrying out.

In the next chapter, the researcher outlines the design and methodology used in this study

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

The thesis of this chapter is to give a well detailed description of the design and methodology used in the study. It contains issues to do with the targeted population, sample and sampling techniques, research instruments, validity, reliability and data analysis procedures. In other words, the chapter will give a framework of data collection and an overview of the research design adopted. The population from which the researcher selected the sample will be put into consideration.

3.2 THE RESEARCH DESIGN

The prime objective of this research was to analyse errors in simplifying algebraic expressions by form 2 learners at Firm Foundation Christian Academy. This concurs well with (Luka's 2021: 20) view which reads, "The purpose of a research design is to specify a plan for generating empirical evidence that will be used to answer the research questions. The intention is to use a design that will result in drawing the most valid, credible conclusions from the answers to the research questions. In this study the researcher wanted to quantify errors made by form 2 learners in simplifying algebraic expressions as well as gaining an understanding of the reasons behind the said errors. (Singh, 2021) echoes the same sentiments arguing that a research design is a detailed plan of investigation and a blueprint of the research work with the mandate to formulate a roadmap on how collection and analysis of data has been carried out.

The researcher opted to carry out the study based on both quantitative and qualitative due to their effectiveness. This marries well with (Jason, 2022) as he posits that combining qualitative and quantitative data can provide more complete picture of the phenomenon being studied as qualitative data can provide rich detail and context while quantitative data can provide broader patterns and trends. This can clearly show that combining qualitative and quantitative data is more effective as enables the researcher to come up with accurate findings and well detailed conclusion and recommendations as well. This research is based on phenomenology research design.

Phenomenological research according to (Natalie, 20220 involves trying to understand the universal experience through assigning assessment tasks of the topic of the research. One can argue that the phenomenological study describes the meaning for several individuals of their lived experiences concepts or phenomena. In other words, it focuses on describing what all participants have in common as they experience a phenomenon. The main purpose of phenomenology is to reduce individuals' experience with a phenomenon to a description of the universal essence as (Manen, 2020) puts it. Phenomenology rejects scientific practicality and the view that the empirical sciences have a privileged position in identifying and explaining any phenomenon in the world. This philosophy refocuses inquiry, concentrating not on descriptions of worldly objects but descriptions of experience. The phenomenological perspective encourages careful description of ordinary conscious experience of everyday life which includes perception, believing, remembering, deciding, feeling, judging, and evaluating.

According to (Patton, 2021 p. 69), phenomenological inquiry focuses on the question: What is the structure and essence of the experience of this phenomenon for these people? While the phenomenon being experienced could be an emotion such as loneliness, jealousy, or fear. Phenomenology asks for the very nature of a phenomenon, for that which a "thing 1s what it is and without which it could not be what it is (Merleau-Ponty, 2022). By phenomenology, he meant the study of how people describe things and experience them through their senses. This most basic philosophical assumption was that we can only know what we experience. Thus, phenomenologists focus on how individuals put together the phenomena they experience in such a way as to make sense of the world and, in so doing, develop a world view. There is no separate or objective reality for people. There is only what they know their experience is and means. The subjective experience incorporates the objective thing and a person s reality (Moustakas, 2022).

Another important dimension of a phenomenological approach is the assumption that there is an essence to shared experience. According to (Patton, 2021), these essences are the core meanings mutually understood through a phenomenon commonly experienced. The experiences of different people are bracketed, analysed are compared to identify the essence of the phenomenon. A

phenomenologist thus assumes a commonality in human experiences and uses the method of bracketing to search for these commonalities.

Unlike descriptions that depict things and happenings as they exist independently of a person's experience of them, phenomenological research requires descriptions of an experience as appears in a person's consciousness. This further requires the researcher to redirect a person's awareness toward their own experiences. According to (Polkinghorne, 2022), the problem with such requirements is that consciousness being activity is in a state of continuous flux and therefore differs from natural objects. Consciousness is moreover an integration of perception, memory, and imagination. Access to consciousness is also problematic because the data a researcher collects are several times removed from the actual event. Furthermore, we have direct awareness of only one consciousness, our own. As phenomenological research is based on descriptions of experience, another difficulty in its acceptance seems to be the fact that the whole enterprise seems too straightforward and too superficial. A common assumption Seemed to be that flimsy things such as descriptions were not worthy of being accepted as scientific knowledge.

However, according to (Giorgi, 2021), despite the still unsolved problems surrounding the use of descriptions in phenomenological search, a quantitative analysis of such descriptions can be more useful and more valuable since a combination of quantitative and qualitative approaches yield a more comprehensive outcome. He goes on to remind us that descriptions pervade science and form the basis of all scientific reports which contain such precise descriptions that the experiment described can be replicated in any branches of science as psychology is dependent upon descriptions

According to (Polkinghorne, 2022), the general format for phenomenological study entails gathering several descriptions from people who are having or have had the experience under investigation, analysing these descriptions to get a grasp of the constituents that make the experience what is and producing a report that gives an accurate and articulate description of the

experience. The sum of such a procedure was to ensure that a reader of the report could come away and win the feeling that he or she had understood better what it was like for someone to undergo that particular experience. Thus the major obstacle seemed to be not the first-hand descriptions supplied by the research participants but the post-descriptive analysis that the researcher developed using a qualitative perspective. According to (Giorgi 2021), although there are no universally accepted models for such analyses, the bottom line is that the analysis should seem credible.

Phenomenology is a philosophical movement attributed to Edmund Husserl (1859-1938). The phenomenological tradition, like any other philosophical tradition, spans many different positions and perspectives. However, there exists a universal understanding that phenomenological studies describe the meanings of lived experiences about the human phenomenon. The underlying assumption of phenomenological research is that every human has a personal and unique perception and perspective of the world, life, and experience. Phenomenology has been characterised as a theory of the unique (Manen, 2020). However, the goal of phenomenological research is the understanding of what is common in the unique manifestations of the phenomenon. As a methodology, phenomenology might be criticised as being Eurocentric. Indeed, historically it has been so, yet the fact that the focus is to uncover the essential nature of human experience means that its methods and techniques can be fruitfully applied to the broadest range of human experience, across cultures and ethnicities. This study employs phenomenology as the methodology design because provides mechanisms.

However, the methodological perspectives of phenomenology are not the sole reason for its use. An equally compelling reason is that this methodology aligns closely with own epistemological and pedagogical beliefs (Patton, 2021). In other words, phenomenological research comes known through the significant world and raised the consciousness of the human being. Pedagogy refers to the art of teaching, in a phenomenological study, participants are selected based on their experience of a common phenomenon. The selection criteria were based on accessibility and the student's willingness to participate. It is this overarching interest that prompted this study. The purpose of the phenomenological inquiry is to uncover the essence of human phenomena and is premised on

the idea that within the uniqueness of each individual there lays the common essence of the phenomenon under study.

In this study, the researcher used Newman's techniques as the theoretical base of the study and based on them, the results were analysed. In this study, the researcher opted to use exercises, a test and face to face discussions with participants.

3.3 RESEARCH METHODOLOGY

The learners' thinking processes and procedures used in simplifying algebraic expressions had been obtained. Their underlying concepts were revealed through execution of these procedures. In the quantitative phase, a test instrument was used to identify and classify errors. The findings of the quantitative study were used to determine the type of data that were gathered in the qualitative phase. The study used qualitative data to explain and explore quantitative data and it provided the researcher with information on how learners came up with their answers. Interviews came in the qualitative phase when learners were asked to answer some questions to justify the procedures they had used in obtaining their solutions. Interviews helped to expose learners' thinking processes that were not clear in their working.

3.3.1 POPULATION AND SAMPLING

(James, 2022 p70) defines a population as, "…. all individuals, units, objects or events that will be considered in a research project." In this study, the target population was all form 2 learners form Firm Foundation Christian Academy in Borrowdale. The researcher identified the population on which data collection methods were to be applied to gather information. The researcher noticed that the population was very large to handle and decided to work with a sample of the population. (Schumacher, 2021) define a sample as a group of individuals from whom data is collected. Therefore, a sample is part of the entire population which usually represents the whole group under study. Sampling is necessary because it is not always possible or practical to study the whole population. (Brink, 2021) argues that sampling refers to a process of selecting the sample from a population to obtain information regarding phenomena. Once the general problem has been

identified, the task becomes to select the unit of analysis. As described by (Schumacher, 2021), the unit of analysis is the object which is to be studied in terms of research variable that constitute the constructs of interest. In this study, learners' errors and the reasons why they made these errors were the units of analysis. The participants in this study were randomly selected. A sample of 20 learners was randomly selected from a population of 80 form 2 learners. The researcher tried to be fair enough when conducting the sampling process, he opted to use the concept of inscribing yes on 20 cards and the 40 were written no. The cards were mixed in a hat randomly and all the form 2s were ordered to pick the cards from the hat. All the learners who managed top pick the cards that are written yes were fortunately considered. The researcher chose random sampling since it takes in to account fairness due to its openness and free from bias to pupils of mixed ability. This concurs well with (Schumacher, 2021) when he argues that random selection gives every member of the population equal chances of being selected. The justification of selecting students from only one secondary school was as follows: convenient to the researcher because of easy access and the researcher could relate well with the sample resulting in quality and credibility of research data.

3.3.2 DATA COLLECTION METHODS

Tests and exercises are procedures which are used to evaluate students' progress. (Trevor, 2022) defines a test in mathematics as an assessment given to evaluate learners' understanding of mathematical concepts, problem- solving skills and ability to apply mathematical principles. He goes further to assert that tests may include a variety of question types such as short answers and problem-solving questions. (Johnson, 2023) defines an exercise in mathematics as a specific problem or task that is given to students to practice and apply the concept they have learnt. Exercises often involve calculations, proofs or problem-solving skills related to the topic being studied. The chief reason behind the use of tests and exercises is that it is the easiest method for measurement of achievement of objectives for a lesson or topic. Tests and exercises provide a quick feedback which is easy to analyse whether one performs as expected by the teacher or not. This enable teacher to notice the kind of help pupils' need at different levels. In this study, the researcher opted to administer a total of three exercises, a test and one on one interview with participants which saved a dual purpose of assessing learners' errors as well as analysing the root cause when doing one to one interviews with participants. All critical concepts pertaining the topic

of algebra were incorporated in order to come up with a valid and clear reflection of learners' performance on the entire topic. The first exercise comprises of simplifying algebraic expressions by means of grouping like and unlike terms. The researcher chose to start with this concept since it forms the basic principles of algebra for example if one is tasked to simplify $3x - 2y - 2x$. The skill that is vital on the above question is to identify like terms, group them and finally simplify. For instance in the case of the above question $3x\ and\ -2x$ must be grouped together and simplified to give x so that the final answer will be $x - 2y$. The second exercise involves a combination the previous concept and that of expansion of a single bracket for example a question like simplify $3x - 2(x + 2y)$. The idea was to marry the previous concept with the idea of bracket expansion. The researcher concluded the task of exercises by assigning a 5 question exercise regarding expansion of two brackets for example $(x + 2)(2x - 3)$, then a test that took everything in to consideration was administered to assess and evaluate the overall performance of learners.

The solutions provided by learners gave the researcher a chance to identify their errors. The errors made by learners in written exercises and test were identified by the researcher according to type. They were then quantified according to occurrence frequencies. From the identified errors, a representative for each type of error was deliberately selected for a one on one question and answer session which resembles an interview between the teacher and learners. This task was useful in the sense that it afforded the researcher to have access to first-hand information regarding the root cause of errors made by learners so as to design an appropriate strategy to provide a lasting solution to the identified errors. This was achieved through asking participants to explain the procedures they had followed in coming up with their solutions. The question and answer session provided an understanding of the reasons that lead learners to make errors. The session was conducted two days after the administering of a test for a quick feedback to participants. This is in line with the principle of time interval as coined by the behaviourist theorist by the name B. F Skinner in his theory of operant conditioning.

However, the disadvantages of using tests and exercises for education are that, some pupils may become frustrated if they continue to fail and become demotivated towards learning. Again it may

not be reflective as pupils may pass an exercise soon after learning a concept, but fail the same to manipulate the same concept under examination conditions owing to memorisation, but the idea of assigning exercises and a test can be credited for its effectiveness. Pupils who were selected to participate in the research wrote the three assigned exercisers and a test as required by the researcher. This gave a platform from which the researcher collected data from which he later analysed through learners' performance.

3.4 RELIABILITY

Reliability reflects the degree to which a measurement instrument produces consistent results when applied repeatedly to the same phenomenon as (Warren, 2023) asserts. (Gertz, 2023) echoes the same sentiments when he postulates that reliability deals with the ability to come up with the same review of a given phenomenon if and when the review is conducted under same conditions. On the same note, (Opie, 2021) views reliability as being synonymous with worthy of trust and unambiguousness of the representation of the total population under study. Therefore, one can define reliability as an indication of the extent to which results can provide formal assurance in qualitative research. A pilot test was administered prior to the main study with the mandate to ascertain validity and reliability of instruments used. Pilot testing is of great importance as it helps the researcher to determine ways to identify learners for the face to face question and answer session with the researcher. Data can be interpreted differently by different individuals. Therefore, the researcher needed to try his level best to have dependable data for the consistency of the study.

The researcher is solely responsible for ensuring that there is reliability as far as the instruments used are concerned. In this study, the reliability of the study was ascertained by a pilot study. According to (Cohen, 2022), the purpose of a pilot study is to impart knowledge of the main study about the quality of the questions in the research task. The pilot study gave an idea of the relevance of the task; it helped to identify the quality of question or instruction and also the context of the question. The pilot study, as suggested by (Opie, 2021), assisted the researcher in getting rid of inapplicable data to the study.

3.5 Validity

Test validity as suggested by (Schumacher (2021) refers to the extent to which inferences based on instrument are reasonable. (McGrath, 2022 p 89) asserts that "validity is not an article that can be purchased with a skill; instead, validity is like nobility, character and quality to be assessed relative to purpose and circumstances. It is a measure of the degree to which explanations of an event match reality". Validity can be viewed as not depending on the data but the interpretation of the data. A test can be viewed as valid if it serves its intended purpose well (Thomas, 2021). In this study, the test questions were constructed in line with form 2 questioning techniques as well as relying on common errors encountered by the researcher in the teaching and learning situation.

3.6 Ethical issues

Participating in the research was voluntary and the sampling technique employed by the researcher took into consideration the aspect of fairness. The researcher informed parents of participants and also sought permission from the school principal which he was granted. The researcher achieved this through preparing formal documents seeking for permission from parents, from learners and from the school head as well. The documents included informed invitation letters to learners for their participation and consent forms for parents or guardian for their children to participate. Anonymity of participants' information was greatly ensured. The results from the participants were confidential. The researcher kept copies of all participants' responses in a secure place so that they could be accessed at any time for recall and analysis whenever the need arose.

3.7 Conclusion

The researcher has given explanations of the research design and methodology, target population, sampling and sampling techniques and the research instruments used. Having also presented how issues regarding validity, reliability and ethical issues were dealt with as well as briefly outlining time frame and data analysis procedures, the researcher in the next chapter is going to analyse the key findings of the study.

CHAPTER 4: DATA PRESENTATION, ANALYSIS AND INTERPRETATION

4.1 Introduction

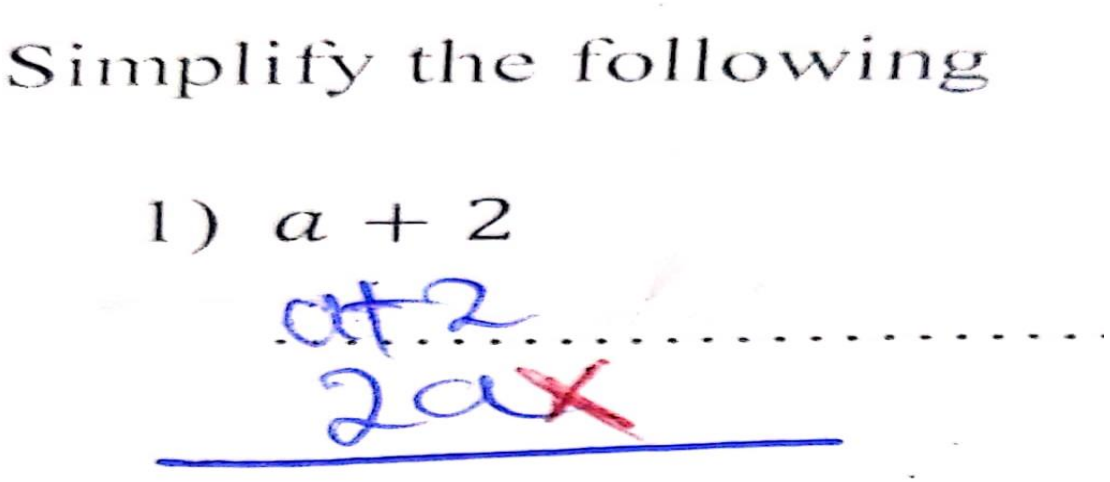
This chapter provides the findings of the research study in terms of data collected from form 2 learners at Firm Foundation Christian Academy. In other words, the chapter focused much on how the researcher summarised collected numerical data from learners' exercises and a test. Results of learners' performance were organised and analysed to form suitable statistical tables and graphs showing their performance.

4.2 ANALYSIS OF LEARNERS' PERFOMANCE

The researcher administered a total of three exercises and a test with the intention of finding out common errors from learners, their root cause as well as designing a strategy of eradicating errors amongst learners as far as the topic of algebra is concerned. The researcher noticed a total of six common types of errors displayed by participants. The main errors observed were: conjoining, misapplication of rules, misinterpretation of symbolic notation, misusing the distributive property, sign error and the error of substituting letters by numeric values. The researcher analysed errors from each task that he administered from the exercise to the test.

Learner's performance on the first exercise is a clear indication to show that they did not understand the concept of like and unlike terms. This is a very important concept under the topic of basic processes of algebra in form 1. The effect of failing to understand this concept contributes greatly to learners' failure not only at form 2 level but also their ordinary level final exams as (Myron, 2022) puts it. Nearly 85% of participants were combining unlike terms which is a clear indication that the concept of basic algebra was not well grasped, A relevant example is the case from the first question on exercise one which reads: Simplify $a + 2$. The responses from several learners is enough evidence to show that there is a huge algebraic gap amongst learners. Most learners failed to notice that $a \; and \; 2$ are unlike terms, as such they cannot be added and at the as a result learner one incorrectly presented her working as follows.

Learner 1's solution



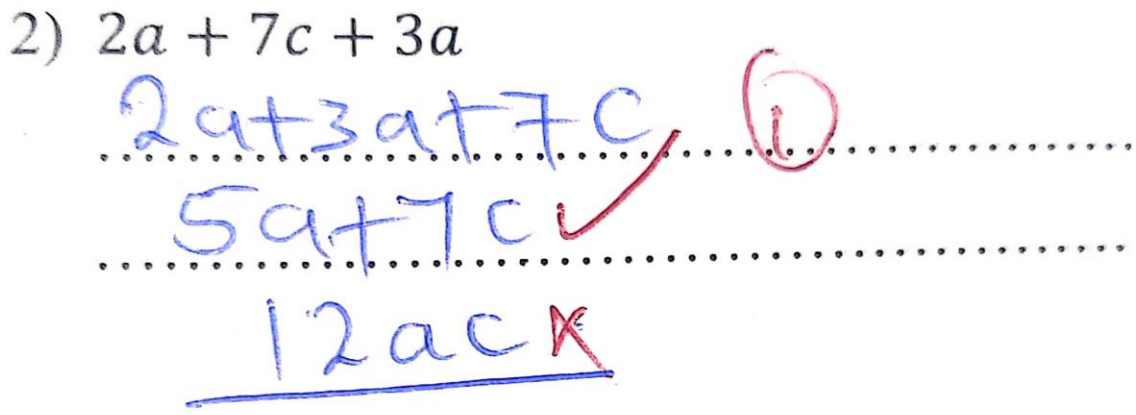
From the above demonstration, it can be noticed that the concept of algebra is of paramount importance and it requires maximum attention from both the teacher and learners. This marries well with (Kuchemann's, 2022) view as he likened the topic of algebra to the backbone of mathematics. This is because all the topics in mathematics are intertwined and they seem to require the application of algebra. The same mistake was noticed again on the second question which reads simplify $2a + 7c + 3a$. The question was simplified as shown below

$$2a + 3a + 7c$$

$$5a + 7c =$$

$$12ac$$

Learner 5' solution



The illustration above is enough evidence to support (William's view, 2023) who argues that learners' understanding of algebraic concepts must start with his or her ability to distinguish between like and unlike terms. The learner managed to identify $2a$ and $3a$ as like terms and correctly grouped them together to obtain $5a$ after simplifying. The problem started when the learner failed to notice that $5a$ and $7c$ are unlike terms, hence they cannot be combined by any mathematical operation.

(Thomas, 2021), classified the above type of errors under the conjoin error bracket where lack of information on like and unlike terms by students contributes immensely to failure to simplify algebraic expressions. This has forced Learners to add coefficients and constants and also coefficients of unlike terms. The expression $a + 2$ was given as 2a by 85% of the learners. The other expression where learners added unlike terms was when they were asked to expand and simplify $2a + 7c + 3a$ and the final answer from 95% of participants was 12ac. The two learners had similar explanations as to how they came up with the demonstrated responses.

In the excerpts **R** = researcher and the **L** = learner.

Excerpt 1: Learner 1 R: Look at question 1. Go through the question and the solution that you provided.

L: I am done.

R: How did you come up with your answer?

L: The question told me to add $a$and 2 and I did.

R: How did you do it?

L: I added a and 2 the I got $2a$

R: If you look at a and 2are they like terms?

L: No they are not.

R: So why did you add them?

L: The question said simplify that's why I added to get single value.

The learner seemed to have taken the addition sign as an action verb. This was due to the fact that there was misinterpretation of the question. The word 'simplify' had a different meaning to the learner. The question seemed to suggest that the terms were to be reduced to a single term. The learner in this case might have followed (Mamba's,2022) view that learners may anticipate the behaviour of algebraic expressions to be similar to that of arithmetic expressions.

Excerpt 2: learner 5

R: Go to question 2 and read the question and the solution that you provided.

L: I am done

R: How did you come up with your answer?

L: The instruction was "simplify the following expressions" and I simplified as required.

R: Look at the given terms from the question, are they all like terms?

L: No only $2a$ and $3a$ have something in common not 7c

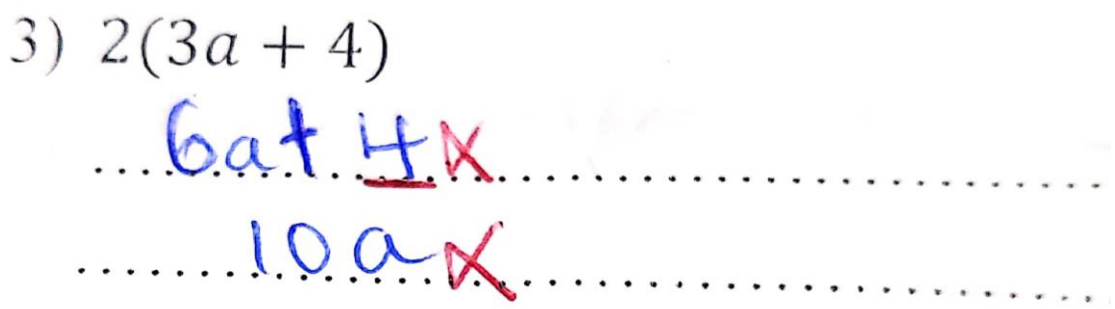
R: So why did you combine all the three terms to come up with 12ac

L: It was a mistake which I have taken heed of

The learner had an idea on how to go about the process of simplifying the algebraic expression. This can be supported by the learner's second stage where $2a$ and $3a$ have been grouped together. This is a clear indication to show that the leaner new about the concept of like and unlike terms. The researcher assumed that the learner misinterpreted the word simplify and this forced the learner to add the coefficients of a and c which are 5 and 7 respectively to get $12ac$.

Furthermore, a combination of two errors were noticed from the third and final question of the first exercise. The first error has to do with incorrect bracket expansion where only the first term was affected by the outside term leaving the outside term unaffected. In addition, the learner failed to recognise that $6a$ *and* 4 are unlike terms which cannot be added.

Learner 3's solution



In analysing the above working, one can notice that the concept of unpacking brackets is very crucial and it deserves a lot of attention and emphasis from educators to avoid unnecessary loss of marks. The researcher decided to embark on a question and answer session with learner 3 to find out how the learner managed to come up with such a solution as a way of trying to figure out the root causes of errors in algebraic expressions.

Excerpt 3: learner 3

R: Go to question 3 and read the question and the solution that you provided. What is your solution?
L: My solution is *10a*
R: How did you come up with that answer?
L: I removed brackets first and got *6a + 4* and since the question said I must remove brackets and simplify, I went on to add the two terms and obtained *10a*.

R: How did you come up with 4 as your second term after bracket expansion?

L: I just wrote the second term inside the bracket as it is on the question since it is far from 2 the outside term.

R: Are you aware of the meaning of the word "bracket" and its interpretation in mathematics?

L: I only remember that bracket means multiplication and that can be answered by my first term $6a$ which is a result of the product of 2 and $3a$.
R: The correct solution after expanding the bracket was supposed to be $6a + 8$ since 4 is inside the bracket and must be affected by 2 using the multiplication operation to interpret bracket concept. In addition $6a + 8$ was supposed to be the final solution since they are unlike terms.

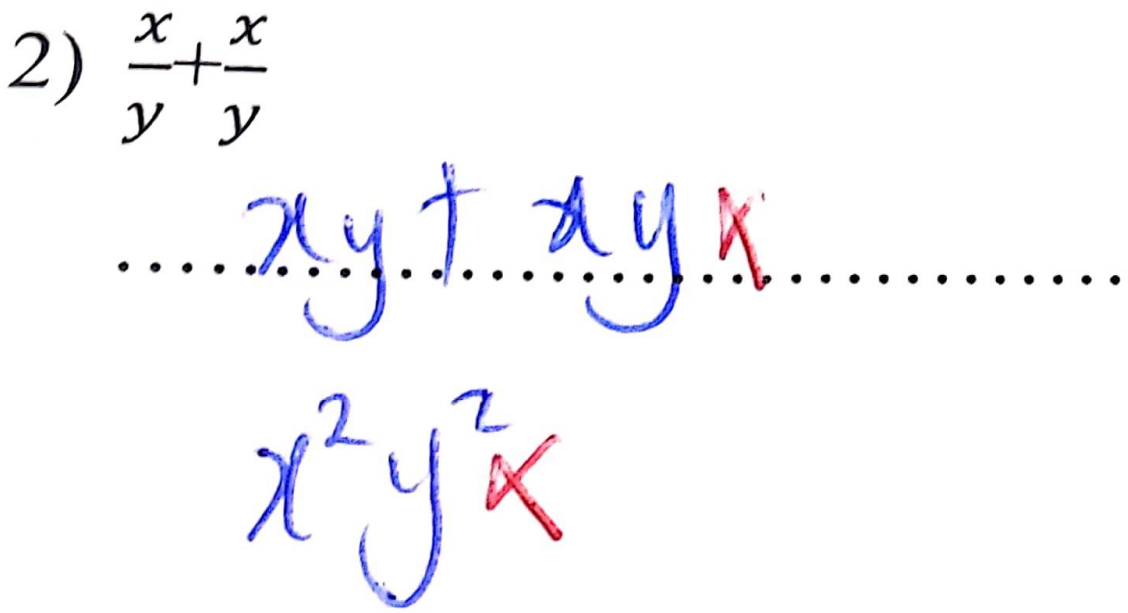
The second exercise was characterised by a continuation of errors from most student. The first question reads $4m + m$ and the solution from most learners was $4m^2$ as shown below. The errors were mainly a result of misapplication of rules as shown from the following responses. Learners used rules inappropriately in simplifying algebraic expressions. Considering *4m + m* where the most common answer was given as $4m^2$, learners showed confusion between addition and multiplication. Instead of adding the like terms learners tended to multiply *4m* by *m*.

Learner 6's solution

The above illustration is a clear indication to show that some learners needs assistance in distinguishing operation is mathematics. The demonstration show that the learner could not figure out that the two terms $4m$ *and* m are combined together by an addition operation and since they are like terms, it means that adding them to come up with a single answer is achievable. The learner erroneously multiplied m *and* m to come up with m^2. This can be a result of learner's failure to understand the difference between addition and multiplication of algebraic terms.

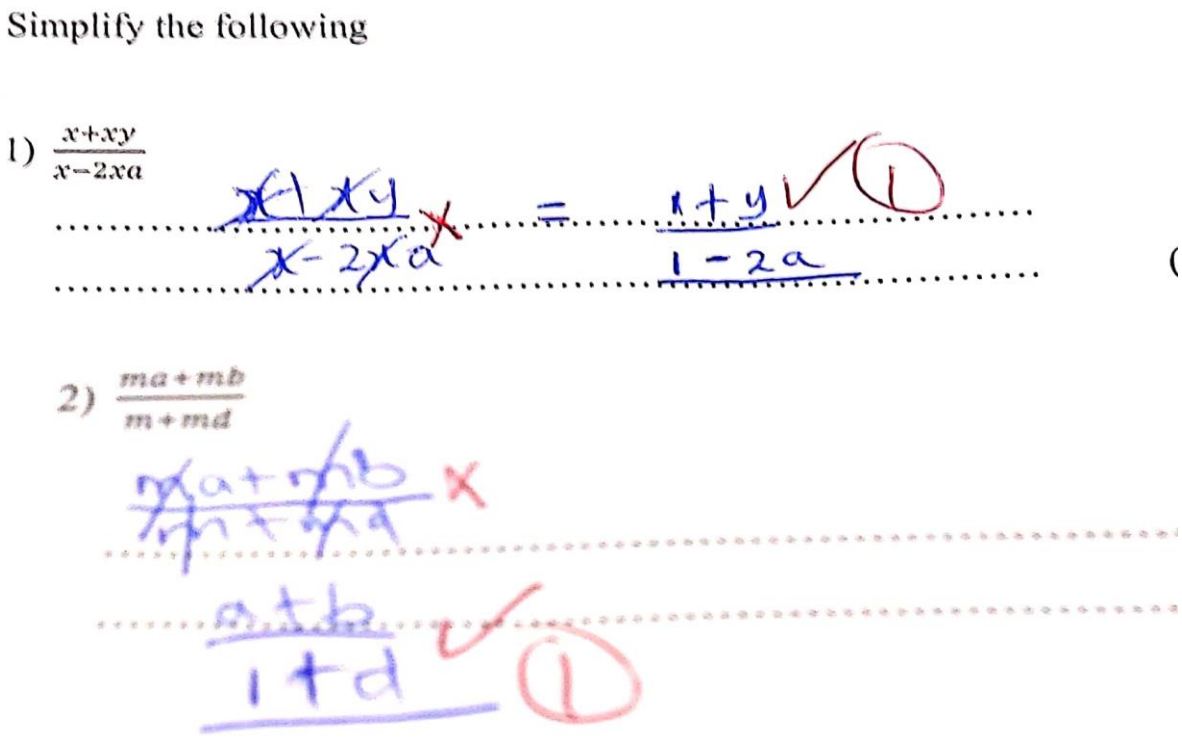
The second question again was not spared from errors, the learner was supposed to add an algebraic fraction which reads $\frac{x}{y}+\frac{x}{y}$. A series of errors were again noted as shown below.

Learner 7's solution



This can be a result of failure to understand the topic of addition and subtraction of fractions a concept whose basics are developed as early as grade five primary level. The disappearance of the denominator clearly show that the learner missed the concept of simplifying fractions at basic level. This is because when simplifying fractions either by adding or subtracting, the only situation that will allow a denominator to disappear is when the denominator is factor of the numerator. If it is not that case, the denominator will always appear up to the final answer. Another error that is worth mentioning has to do with failure to recognise that $xy\ and\ xy$ are like terms that can be added not multiplied. Instead of getting $2xy$ after adding the two terms, the learner multiplied which is completely wrong.

In addition to this, more errors were again noticed on exercise three on both questions as illustrated below.



From both questions, it can be noticed although the learner managed to get the correct answer, but working was not properly executed. The proper working for such questions involves factoring out of the common factor from both the numerator and the denominator then divide where necessary. Errors of this nature can be addressed by revisiting the topic of factorization from its basics in form one and emphasise on the main principles of the concept. The idea of taking out the common factor and introduce a bracket must be reiterated by the teacher for leaners to understand well.

The researcher administered a five-question test to assess whether learners rectified the mistakes they made during exercise sessions. The occurrence of errors from learners' work was still on though some mistakes were rectified as to be shown from learners' demonstrations.

Simplify the following fully

1) $2(6a + 9)$

$12a + 18$ (1) ..... (1)

2) $3(2m + 3) + (2 + 5m)$

$6m + 6 + 2 + 5m$ ✗

$6m + 5m + 6 + 2$ ✗

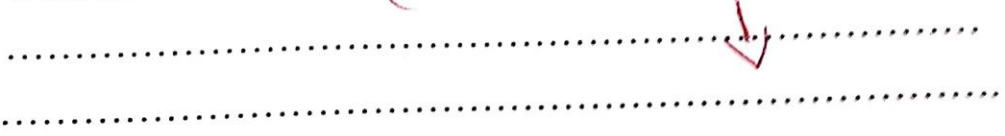
$11m + 8$ ✗ ..... (2)

From the demonstration above, one can notice that the learner rectified invalid bracket expansion as shown on question 1. The learner effectively multiplied all the two terms inside the bracket by 2 which is the outside bracket. On question 2 the learner made a silly mistake form the first stage by getting 6 after multiplying 3 and 3 instead of getting g 9. The mistake distorted everything and as a result, the learner failed to get a correct answer. This can be addressed by reminding learners to pay maximum attention when doing school work since silly contribute greatly to learners' failure.

Improper expansion of brackets has contributed to the learners' failure to expand a squared bracket as illustrated below.

3) $(x + b)^2$

$x^2 + b^2$ ✗ $(x+b)^2 = (x+b)(x+b)$

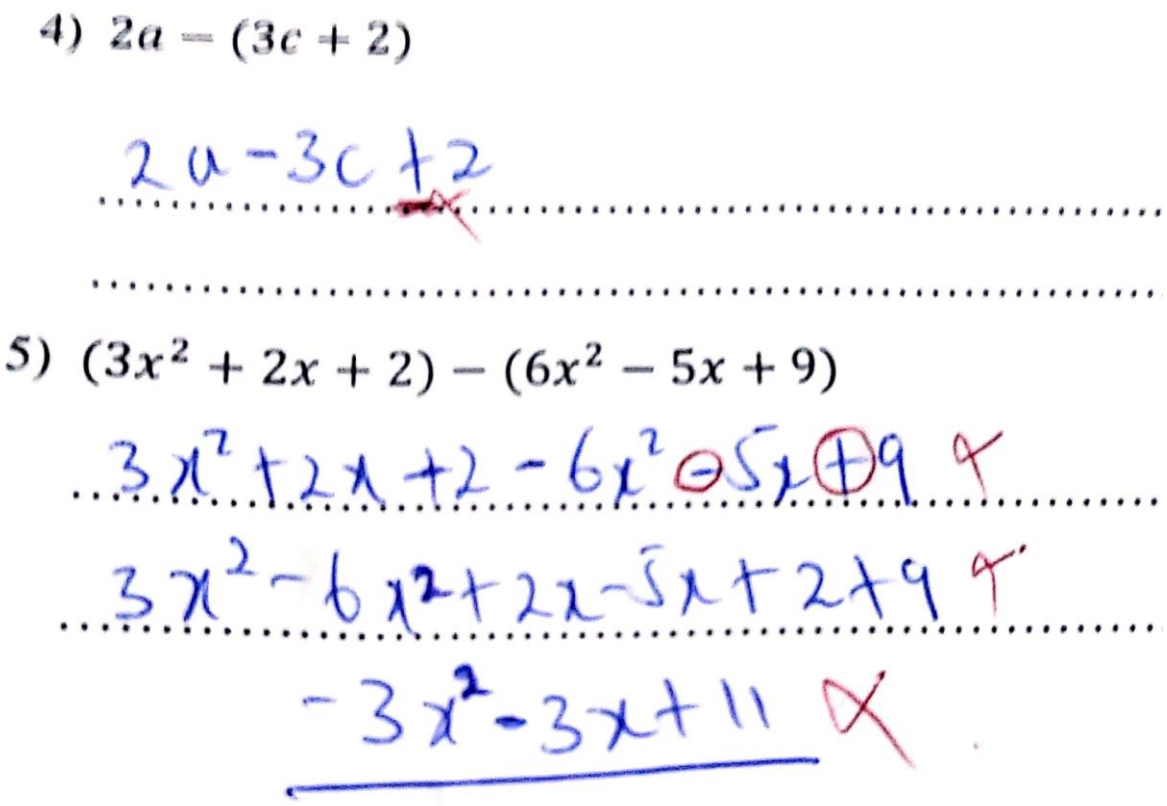


This is a common error not only amongst learners in junior secondary, but it can also affect learners as far as advanced level if not properly managed at basic level. Most learners mistakenly apply the law of indices which is inapplicable and as a result, they end up getting incorrect answers. The

proper way of working that question out was to write two brackets as shown and expand the normal way as shown below

$$(x+b)^2$$
$$(x+b)(x+b)$$
$$x(x+b)+b(x+b)$$
$$x^2+xb+bx+b^2$$
$$x^2+2bx+b^2$$

Learner 5' solutions



The above demonstration from learner 5 show is an indication to show that the sign error is very critical owing to its devastating effects on learners' performance not only in algebraic expressions, but in mathematics as a subject. The root cause of the shown mistakes might have emanated from lack of understanding of the concept of directed numbers a critical concept as far as introduction to secondary mathematics is concerned.

4.2.1: DESCRIPTION OF IDENTIFIED COMMON ERROPRS

Conjoin error: When learners were simplifying expressions, they combined unlike terms. This was mainly seen in item 1 and 2 from the first exercise in which learners were asked to simplify $a + 2\ and\ 2a + 7c + 3a$ respectively. The common solutions from at least 18 participants were $2a\ and\ 12ac$ which represent 80% of the sample size. On third question, nearly half of the participants failed to expand the bracket which is a point of concern since bracket expansion is nof paramount significance in algebra.

Misapplication of rules: This error prevailed in most of the questions in the test. Learners misapplied rules in many items. In the first item of the test, learners were asked to simplify $4m + m$ and surprisingly most of them could not get the correct answer. The most common answer was $4m^2$ instead of adding terms learners multiplied the terms. This error was committed by 60% of the learners. On second question of exercise 2 linked to adding algebraic fractions learners just cross multiplied and ended up without a denominator. In that item, learners reduced the expression $xy + xy$ them multiplied again as the case with the previous question instead of adding Learners' thinking might have been influenced by previous learnt concepts of exponents and solving of equations with fractions respectively.

Misinterpretation of symbolic notation: This error was committed by 65% of the learners. Learners made misinterpretations of terms with invisible coefficients. Learners assumed that 0 was the coefficient of terms with invisible coefficients as there were no numbers before the letters. As a result, the coefficients of terms like m were taken as 0. That is why some of the learners when asked to simplify $4m + m$ gave their answer as 4m.

Invalid distribution of brackets: The distributive error in bracket expansion was also a very common error especially in Question 2. This error was committed by 45% of the learners in the study. Some of the learners expanded one part of the bracket and left the other part. This was evidenced in the expansion of *2(3a+4)* where the learners gave 6*a*+4 as their solution. According to (Martz, 2020), learners use known rules in appropriate situations but incorrectly adapt the known rules. When expanding the expression $(x + b)^2$ some learners just gave their answer

as x^2+b^2. In this case, learners failed to retrieve the correct expansion of a binomial as was found by Mbewe (2013) in his study on misconceptions and errors in algebra at Grade 11 level. Learners relatively executed the distributive rule. The literature attributes invalid distribution of brackets to learners' failure to master basic facts and concepts.

Sign errors: Learners had problems working with integers and operation signs. The percentage of errors due to inappropriate use of signs was 55%. Sign errors were mainly due to failure to combine operation and direction signs. When simplifying the expression $2a - (3c + 2)$, some learners gave *2a -3c + 2* as the answer. It can be deduced that the minus sign was not considered as affecting the second term inside brackets. In the question where learners were asked to subtract the second expression from the first expression, learners understood what they were supposed to do. They correctly wrote $(3x^2 + 2x + 2) - (6x^2 - 5x + 9)$ but failed to work with the signs properly and had $-3x^2 - 3x + 11$ as their answer. Learners failed to simplify the middle terms. The other minus sign was ignored as the term to be subtracted already possessed a negative sign. Learners showed too many misunderstandings in working with signs.

4.2.2 Presentations of results

The researcher coded these five common errors and also added two more codes. The other two codes were for "correct answer and for question not answered." The researcher used the coding system for easy capturing of the learners' results. The explanation of the coding system used by the researcher is given in the table below.

Table 4.1: Coding system

Description	Correct answer	Conjoin error	Misapplication of rules	Misinterpretation of symbolic notation	Invalid bracket expansion	Sign error	No answer
Code	1	2	3	4	5	6	7

As the table displays, there were 7 answer codes used. The first code represented correct answer. The second code was for the error due to conjoin of terms. Code 3 represented the error due to misapplication of rules. Code 4 stood for error due to misinterpretation of algebraic notation. The error due to wrong application of the distributive property was represented by code 5. Code 6 represents errors a result of incorrect signs and lastly, code 7 represented situations where the learner did not attempt to answer the question.

Using the above-mentioned codes, the researcher recorded learners' results and tabled them. The table below is an extract of the tabulated results.

Table 4.2: An extract of learners' response codes

Item → / Learner ↓	Exercise1			Exercise 2		Exercise 3		Test				
	1	2	3	1	2	1	2	1	2	3	4	5
L1	3	2	3	3	7	1	1	1	1	3	3	6
L2	2	4	4	1	3	3	2	1	1	2	6	4
L3	1	3	2	1	4	3	5	1	3	1	4	5
L4	3	4	2	1	6	4	4	2	1	1	3	6
L5	4	1	5	2	2	1	2	1	1	1	4	3
L6	1	2	4	1	4	1	3	1	2	2	4	4
L7	1	3	4	1	3	1	5	2	1	5	3	2
L8	2	5	3	2	4	2	4	1	2	4	2	6
L10	1	2	4	5	2	1	3	1	5	3	4	3
L12	1	3	2	3	1	3	3	2	4	2	6	2
L13	2	1	1	3	4	1	4	1	5	3	4	3
L14	1	2	3	1	2	2	3	1	4	2	6	5
L15	2	1	4	3	4	3	2	4	3	5	6	2
L16	3	4	1	2	3	2	4	5	2	1	3	6
L17	1	3	4	1	5	6	1	3	1	4	4	1
L18	2	3	1	4	1	5	2	3	3	2	1	6
L19	2	1	3	1	5	6	1	4	4	6	2	4

L20	1	3	4	2	3	5	4	3	2	3	3	5

Table 2 is an extract of the results of all learners with all their answers indicated in the appropriate answer codes.

The total number of learners for each answer code was calculated and summarized in a table. The next table illustrates the total number of learners for each of the six common errors per item together with the total number of correct and no answers.

Table 4.3: Number of learners per answer code per item.

Answer code → Item ↓	1	2	3	4	5	6	7
Exercise 1							
1	8	6	3	1	0	0	0
2	4	4	4	3	1	0	0
3							
Exercise 2							
1	8	4	4	1	1	0	0
2	2	3	4	5	2	1	1
Exercise 3							
1	6	3	4	1	2	2	0
2	3	4	4	5	2	0	0
Test							

1	9	3	3	2	1	0	0
2	6	4	3	2	2	0	0
3	4	5	4	2	2	1	0
4	1	2	5	6	0	4	0
5	1	3	3	3	3	5	0
CODE TOTAL	52	41	42	31	16	13	1
% OF CODE	27	21	21	16	8	6	1

Overall, learners gave 27% of correct answers, 1% of unattended items of questions and 72% of the answers had errors. Also, 21% of answers with errors were in the category of "misapplication of rules" while 8% of the answer codes were associated with "invalid distribution of brackets". Conjoin error had 2% of the distribution, sign error 6% and misinterpretation of symbolic notation with 16%. The table above indicates the distribution of errors among the five error codes together with the other two non-error codes, one for correct answers and the other for missing answer. It can be seen that Question number 1of exercise 1 was dominated by code 1 which is a reflection that learners mastered how to simplify basic algebraic expressions.

Question 3 from exercise 1 required learners to remove brackets and then simplify the expression. The most prevalent error in this question was due to misusing the distributive rules. The distributive property was wrongly applied in almost all items of this question. Question 2 from exercise 2 had most learners substituting the unknown by numeric values. Learners had problems with direction and operation signs in question 2 from the test. Learners failed to deal with integers as a result missed the correct answer for question from test. Question 5 from also found learners mostly substituting letters by numbers as in question 3.

The overall contribution of each error to this study was also calculated, presented on a pie chart as shown below.

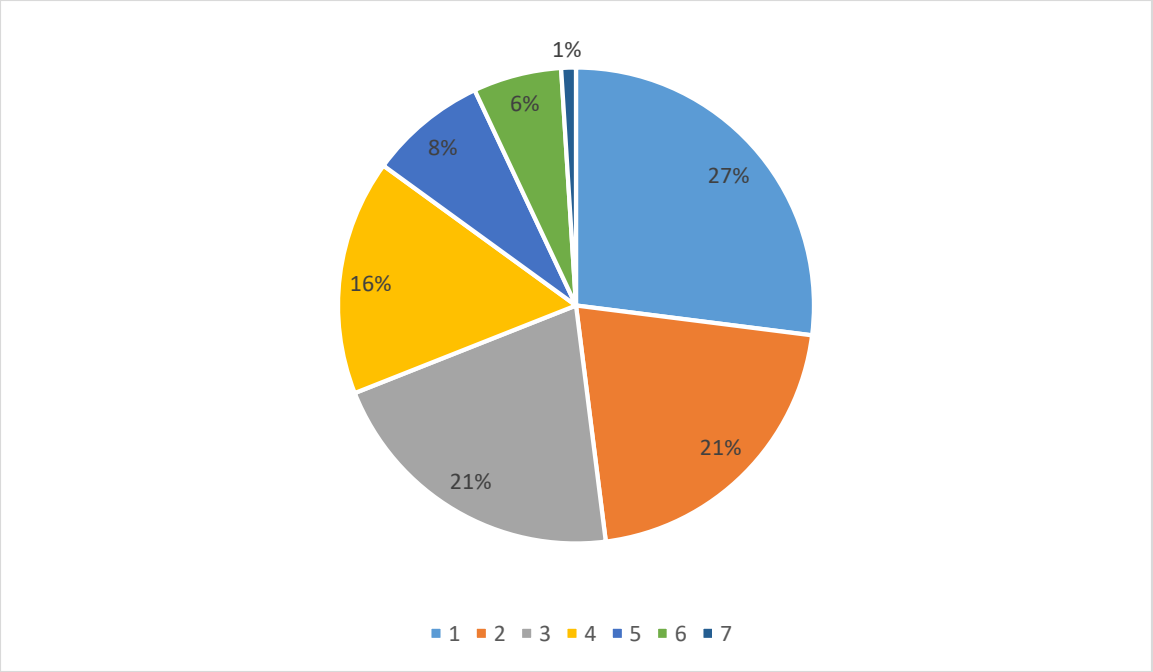


Figure 4.1: Overall contribution of each error

4.3 Conclusion

The two phases of the investigation have allowed the researcher to discover the most common errors made by learners in simplifying algebraic expressions. It also enabled the researcher to discover the possible causes of those errors. The researcher came up with five types of errors as the common errors made by learners in simplifying algebraic expressions. These errors include: misapplication of rules, conjoin error, misinterpretation of symbolic notation, misusing the distributive property and sign errors. The error due to misapplication of rules was the most common error in this study. Learners misapplied rules mostly because of the fact that algebra has many rules which confuse learners. Learners in many situations learn manipulation of rules without reference to their meaning.

CHAPTER 5: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

The chapter's main focus is to provide a brief summary of the research project, major challenges faced by the form 2 learners at Firm Foundation Christian Academy when simplifying algebraic expressions and recommendations by the researcher.

5.2 RECOMMENDATIONS

The results have indicated that learners' errors when simplifying algebraic expressions have their root causes. The researcher has learnt more from identifying learners' errors and their causes. Any study is fruitless if the findings of the study are not useful for future (Mamba, 2022). The researcher is of the opinion that the findings of this study are going to benefit Mathematics Department at form 2 level by making teachers aware of the common errors made by learners when simplifying algebraic expressions.

5.2.1 RECOMMENTATIONS FOR TEACHING/ LEARNING

The findings from this study mainly showed that learners lacked the basics of algebra, and therefore teachers should assist learners to grasp the basics of algebra like: collecting like and unlike terms; bracket expansion, addition and subtraction of directed algebraic terms. Knowing the basics of algebra will go a long way in understanding the procedural and conceptual aspects of algebra. Teachers should take the constructivist perspective into consideration and be in a position to create a strong arithmetic background for learners so that the arithmetic background could be applied to algebra. Teachers are encouraged to use teaching methods that enable learners to gain both procedural and conceptual knowledge. The teaching methods should allow learners to give explanations for their answers. Teachers should listen carefully to learners' explanations and be able to identify learners' misconceptions and find ways of helping learners to understand algebraic concepts.

In addition to this, there is need for teachers to create a classroom environment that allows learners to come up with their own conceptions from the procedural and conceptual knowledge taught by the teacher. Learners should also be encouraged to share their successes and problems in algebra in a way trying to clear misconceptions. At times, learners should receive individual attention in order to address the issue of individual differences. Learners should be given a variety of algebraic expressions to simplify. According to (Fallen, 2021) giving learners a variety of algebraic expressions, makes learners experience the different ways in which algebraic expressions are supposed to be simplified. Learners will get used to algebraic manipulation and algebraic representation.

5.2.2 RECOMMENTATIONS FOR FURTHER RESEARCH

The findings of this study showed that the methods and approaches used by the teacher to teach algebraic concepts have an effect on the way learners grasp the concepts; therefore, the researcher recommends that there is need to identify the role of the teacher in the errors produced by learners in simplifying algebraic expressions. It is good to understand the way the teacher delivers the concepts to the learners. This will enable identification of the teacher's contribution to the commitment of the errors by learners. The study also suggests broadening the research by not only concentrating on simplifying algebraic expressions but on algebra as a whole. This may improve the relevance of the research on the teaching/learning situation. The researcher also suggests the use of a bigger sample including participants from several schools

5.3 LIMITATION OF THE STUDY

A relatively small sample was used in this study and all participants were from one school. This is hard to claim to have captured the entire performance in algebra. However, the mixed methods design approach was used in analysing data in an attempt to promote richness and triangulation.

5.4 SUMMARY

The purpose of the study was to assess the challenges faces by form 2 pupils in the teaching and learning of algebraic expressions. The researcher also aimed at modifying teaching strategies and approaches so as to ensure the process of teaching and learning of algebraic expression is well conducted. The study was carried out using twenty pupils who were randomly selected from all the form 2s.

In chapter 2, the researcher consulted literature where different authors observed some of the problems which are encountered by pupils in the learning of algebraic expressions, root causes and methods that can be used to minimize errors. The researcher chose to use three exercises and a test when carrying out data collection on chapter three. Questioners were designed for learners to carry out one on one question and answer session with the researcher. The usefulness of this exercise is that it provided first-hand information regarding the root cause of learners' errors and how best can they be minimised.

Information gathered from pupils' exercises, a test and interviews was tabulated and presented on a pie chart in chapter 4. The researcher went further to analyse errors made by pupils, sources of errors and possible remedies in the learning of algebra by form 2 pupils

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APPENDICES

APPENDIX 1: Exercise 1

Simplify the following

1) $a + 2a$

…….. (1)

2) $2a + 7c + 3a$

……..

…….. (2)

3) $2(3a + 4)$

……..

…….. (2)

APPENDIX 2:Exercise 2

Simplify the following

1) $4m + m$

…….. (1)

2) $\frac{x}{y}+\frac{x}{y}$

…….. (2)

APPENDIX 3 :Exercise 3

Simplify the following

1) $\frac{x+xy}{x-2xa}$

……

…… (2)

2) $\frac{ma+mb}{m+md}$

……

…… (2)

APPENDIX 4:Revision Test

Simplify the following fully

1) $2(6a + 9)$

…… (1)

2) $3(2m + 3) + (2 + 5m)$

……

……

……. (2)

3) $(x + b)^2$

……

……

……. (2)

4) $2a - (3c + 2)$

……...

…… (2)

5) $(3x^2 + 2x + 2) - (6x^2 - 5x + 9)$

…… (2