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An Analysis of the Impact of Liquidity Constraints on Optimal Portfolio Change: Study on Investment Diversification Strategies

By

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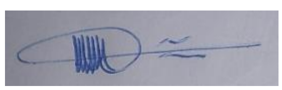

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Submitted by **B193506A** in partial fulfilment of the requirements of the Bachelor of Science (Honours) Degree in Statistics and Financial Mathematics

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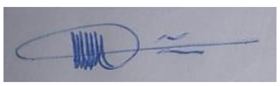

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DEDICATION

Dedicated to my family

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I would like to acknowledge the active participation of my supervisor towards the completion of the study. I also extend my gratitude to my family and friends for their support and love they showed me during my studies. Above all I thank God for making it all possible.

ABSTRACT

This study investigates the impact of liquidity constraints on optimal portfolio change and dynamic portfolio rebalancing using Monte Carlo simulation and stochastic optimization. The research develops a robust framework for analyzing the effects of liquidity constraints on portfolio optimization and rebalancing, and demonstrates the significance of considering liquidity constraints in portfolio management. The findings show that liquidity constraints can significantly affect portfolio performance and risk, and that optimizing portfolios under liquidity constraints can lead to improved outcomes. The study contributes to the existing literature on portfolio optimization and liquidity constraints, and provides insights for investors, portfolio managers, and financial institutions.

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CHAPTER 1

1.0. Introduction

This chapter introduces the problem under study and its background, describing how the problem emanated. It also gives the specific objectives, questions, assumptions and scope of this study. By achieving its goal, the study aims to uncover the truth that leads to informed decision-making and policy-making.

1.1. Background to the Study

Liquidity constraints are a major challenge for investors who want to trade assets freely and efficiently and achieve optimal portfolio decisions and performance. These are the limitations or difficulties that investors encounter in buying or selling assets in the financial markets. Liquidity constraints stem from various sources, such as high transaction costs, low trading volumes, limited access, and regulatory uncertainties. They affect how easily and quickly assets can be converted into cash or vice versa, without affecting their prices. This is called the availability and efficiency of liquidity in the financial markets. They also affect how the value and the risk of the assets change over time. This is called the uncertainty and volatility of asset returns and prices. Liquidity constraints have implications for the trade-off between return and risk, the degree of portfolio diversification, and the sensitivity of portfolio changes to market movements, which are the key factors that determine the optimal portfolio choice and performance of investors.

The problem of liquidity constraints is a global phenomenon that affects investors in different regions and countries. The World Bank (2017) provides data and analysis on the liquidity and efficiency of financial markets in the world, America and New York. The American financial markets, particularly the New York financial markets, are considered the epicentre of global finance. The New York Stock Exchange (NYSE) and the National Association of Securities Dealers Automated Quotations (NASDAQ) are two of the largest and most liquid stock exchanges in the world. However, despite their size and liquidity, investors in these markets still face liquidity constraints that can significantly impact their investment decisions and portfolio performance. The 2008 global financial crisis highlighted the importance of liquidity constraints in the American financial markets. During the crisis, many investors faced significant difficulties in selling their securities, leading to large losses and widespread financial instability. Since then, regulators and investors have become increasingly aware of the need to manage liquidity constraints effectively. Investment diversification strategies are crucial for investors to manage risk and maximize returns. However, liquidity constraints can significantly impact the effectiveness of these strategies. While there is a significant body of research on portfolio optimization and investment diversification, there is a lack of research on the impact of liquidity constraints on optimal portfolio construction and investment diversification strategies in the American financial markets.

This study aims to address this knowledge gap by investigating the impact of liquidity constraints on optimal portfolio construction and investment diversification strategies in the American financial markets, with a specific focus on the New York financial markets. The study will use a comprehensive dataset of NYSE- and NASDAQ-listed stocks and employ advanced econometric techniques to analyze the relationship between liquidity constraints and portfolio performance. The findings of this study will provide valuable insights for investors, portfolio managers, and policymakers on how to manage liquidity constraints and optimize portfolio performance in the American financial markets. The study will also contribute to the existing literature on portfolio optimization, investment diversification, and liquidity constraints, and provide a framework for future research in this area.

1.3 Problem statement

Investors face severe liquidity constraints that limit their ability to trade assets freely and efficiently, and thus affect their optimal portfolio decisions and performance. Liquidity constraints arise from various sources, such as high transaction costs, low trading volumes, limited access, and regulatory uncertainties in New York's financial markets, which are among the biggest and most developed, . Moreover, New York's financial markets have experienced high inflation following the COVID-19 pandemic, prompting central banks to tighten monetary policy, supply chain difficulties and demand surge, global energy price increases following Russia's invasion of Ukraine, challenges in Treasury market liquidity and mpact of political events and elections on market stability just to mention a few.

Liquidity constraints affect the trade-off between return and risk, the degree of portfolio diversification, and the sensitivity of portfolio changes to market movements. However, most of the existing studies on portfolio choice under liquidity constraints focus on the static case, which assumes that investors do not change their portfolios over time. This assumption may not be realistic, as investors may need or want to adjust their portfolios over time in response to new information, changing market conditions, or personal needs. Therefore, it is important to extend the analysis to the dynamic case, which allows for portfolio changes over time. The dynamic portfolio choice problem under liquidity constraints is more complex and challenging than the static one, as it involves not only deciding what assets to hold, but also when and how much to trade. Moreover, it requires taking into account the intertemporal effects of liquidity constraints, such as how current portfolio decisions affect future liquidity needs and opportunities

1.3 Research Objectives

1**.**Investigate the impact of liquidity constraints on optimal portfolio construction, investment diversification strategies, and portfolio performance metrics using Monte Carlo simulation.

2.Analyze the Effect of Liquidity Constraints on Dynamic Portfolio Rebalancing Strategies and their Impact on Long-Term Portfolio Performance

1.4 Research Questions

1. How do liquidity constraints affect optimal portfolio allocation and risk/return trade-offs?

2. How do liquidity constraints impact dynamic portfolio rebalancing and long-term performance?

1.6 Scope of The Study

In exploring the relationship between liquidity constraints and portfolio performance, the study assesses the effects of liquidity on returns, risk management, and the overall efficiency of diversification strategies. The research delves into the specific asset classes or investment instruments that are more vulnerable to liquidity constraints and analyze their impact on portfolio optimization. Furthermore, the study identifies and analyze the key factors that drive liquidity constraints in portfolio management. It investigates how maximum weight, minimum weight, risk tolerance and trading volume constraints contribute to liquidity challenges in portfolio decision-making. By understanding these factors, the research aims to provide insights into optimizing portfolio changes in the presence of liquidity constraints. Moreover, this study seeks to provide practical recommendations for non-knowledgeable investors to navigate liquidity constraints and optimize their portfolio changes effectively. The research offers actionable suggestions and strategies tailored to help inexperienced investors make informed decisions regarding investment diversification and portfolio management in the presence of liquidity challenges.

1.7 Significance of the Study

This research is pivotal, offering a unique opportunity for the researcher to make a meaningful impact on the finance and investment management landscape. By exploring the complex relationship between liquidity constraints and investment diversification, the researcher deepens their understanding of portfolio optimization strategies and develops expertise in addressing real-world challenges faced by investors. At the university level, this study significantly enriches academic discourse, advancing knowledge in portfolio management under liquidity constraints. The research outcomes enhance finance and economics curricula, providing students with a more comprehensive understanding of the practical implications of liquidity risks in investment decision-making. Moreover, the study inspires further research endeavors within the university, fostering a culture of academic excellence and innovation.

Within the community, this study empowers individual investors and financial advisors with practical insights, enabling them to navigate liquidity challenges in portfolio management with confidence. By providing customized recommendations for non-knowledgeable investors, the research assists community members in making informed and strategic investment choices, ultimately improving their financial security and long-term wealth accumulation. In the industry, the findings of this study are crucial for financial institutions, asset managers, and investment professionals. Understanding the impact of liquidity constraints on portfolio performance and diversification strategies informs industry practices, guides the development of more effective investment products and services, and aids industry stakeholders in optimizing portfolio management practices. By shedding light on the critical factors influencing liquidity risks, the research mitigates potential downsides associated with liquidity constraints, leading to more informed and effective investment decisions.

1.8 Assumptions of the Study

This study is grounded on several key assumptions essential for shaping its methodology and interpreting its findings. Firstly, it assumes that investors behave rationally when faced with liquidity constraints and opportunities for investment diversification, enabling the analysis of how liquidity constraints influence investment decisions and the effectiveness of diversification strategies in risk management. Secondly, the study operates under the assumption of market efficiency, specifically semi-strong form efficiency, where security prices reflect all publicly available information, crucial for evaluating portfolio management strategies under liquidity constraints within the context of market efficiency.

Thirdly, the assumption that portfolio diversification yields benefits in reducing overall risk exposure and enhancing long-term returns serves as the basis for assessing the effectiveness of diversification strategies under varying liquidity conditions. Additionally, the study assumes the absence of significant information asymmetry among investors, ensuring that all market participants have equal access to relevant information for their investment decisions. Lastly, the assumption that investors seek to optimize the risk-return trade-off in their portfolios forms the basis for evaluating the impact of liquidity constraints on this trade-off and the efficacy of risk management strategies.

These assumptions collectively provide a theoretical framework for exploring the complex interplay between liquidity constraints and investment diversification in contemporary financial markets.

1.9 Limitations of the Study

The researcher acknowledges the limitations of this study, as they inform the boundaries and potential shortcomings of the research. Firstly, the generalizability of the findings may be limited due to the specific sample and context used in the study. The study's conclusions may not be applicable to other populations or different market conditions. Secondly, the reliance on historical data and assumptions of market efficiency may overlook the impact of unpredictable events, market anomalies, or changes in investor behaviour that could affect the outcomes.

Additionally, the study assumes a rational decision-making framework, which may not fully capture the complexity of human behaviour and emotions in investment decision-making. Moreover, the study's focus on liquidity constraints and investment diversification may not encompass all factors influencing investment decisions and performance. Other variables such as socio-economic factors, regulatory changes, and macroeconomic conditions could play a significant role but are not explicitly addressed.

Lastly, the study's analysis is based on quantitative methods, which may limit the depth of understanding and fail to capture qualitative aspects of investor experiences. Overall, while this study contributes valuable insights, it is important to recognize these limitations and consider them when interpreting and applying the findings.

1.10 Definitions of Terms

For the purpose of this study, the key terms are defined as follows:

1. Liquidity: The ease and speed at which an asset can be converted into cash without affecting its price (Baker and Wurgler, 2007; Holmström and Tirole, 2011).

2. Liquidity constraints: The limitations that hinder investors from trading assets at any time or at any desired quantity or price (Gromb and Vayanos, 2002; Brunnermeier and Pedersen, 2009).

3. Portfolio choice: The decision-making process of allocating one's wealth among different assets to meet financial objectives (Markowitz, 1952; Campbell and Viceira, 2002).

4. Portfolio change: The adjustments made to one's portfolio composition, trading frequency, trading hsize, or trading direction over time (Barber and Odean, 2000; Calvet, Campbell and Sodini, 2009).

5. Investment diversification: The strategy of holding various assets with different returns and risks to lower overall portfolio risk (Markowitz, 1952; Elton and Gruber, 1997).

6. Risk management: The practice of identifying, measuring, and controlling the risks linked with one's investment portfolio (Jorion, 2007; Berk and DeMarzo, 2017).

7. Risk Tolerance: The ability of an investor to withstand potential losses in their portfolio (Chien et al., 2014)

8. Trading Volume Constraints: Trading volume constraints limit the amount of trading activity that can occur in a particular asset or market (Kumar et al., 2017)

1.11 Conclusion

The chapter has introduced the background to the study, problem statement, objectives, questions, assumptions, limitations and definitions. It described the importance of this research and scope. The following chapters focus on literature review, methodology, analysis of findings and final conclusions and recommendations.

CHAPTER 2: LITERATURE REVIEW

2.0 Introduction

This chapter reviews the relevant theoretical and empirical literature on liquidity constraints and portfolio change decisions of investors in stock and bond markets. The chapter begins with a discussion of the main concepts and definitions of liquidity, liquidity constraints, and portfolio change decisions. Then, it presents the theoretical literature that explains how liquidity constraints affect the optimal portfolio choice and adjustment of investors under different assumptions and frameworks. Next, it summarizes the empirical literature that tests the hypotheses and predictions derived from the theoretical models using various methods and data sources. The chapter also identifies the research gap that exists in the literature and motivates the need for the current study. Finally, it proposes a conceptual model that links the research objectives and the research questions of the study.

2.1 Theoretical Literature

Liquidity is a multifaceted concept that can be defined in different ways depending on the context and perspective. According to Brunnermeier and Pedersen (2009), liquidity can be broadly classified into two types: market liquidity and funding liquidity. Market liquidity refers to the ease of trading an asset in the market without affecting its price, while funding liquidity refers to the ease of obtaining funds to finance the purchase of an asset. Liquidity constraints are the frictions or impediments that prevent investors from accessing or using liquidity in the market or in their own balance sheets. Liquidity constraints can arise from various sources, such as transaction costs, borrowing constraints, margin requirements, collateral constraints, information asymmetry, market segmentation, and market imperfections.

Portfolio change decisions are the choices that investors make regarding the composition, size, frequency, and direction of their trades in the financial markets. Portfolio change decisions are influenced by many factors, such as investors' preferences, expectations, beliefs, information, risk aversion, wealth, income, and liquidity. Portfolio change decisions can have significant implications for investors' welfare, market efficiency, asset pricing, and financial stability.

The theoretical literature on liquidity constraints and portfolio change decisions can be divided into two main strands: the static models and the dynamic models. The static models assume that investors make portfolio choices at a single point in time, while the dynamic models assume that investors make portfolio choices over multiple periods of time. The static models include the classic mean-variance portfolio theory of Markowitz (1952), the capital asset pricing model of Sharpe (1964), Lintner (1965), and Mossin (1966), and the arbitrage pricing theory of Ross (1976). These models assume that investors are rational, risk-averse, and have homogeneous expectations and information. They also assume that markets are frictionless, complete, and efficient, and that investors can trade any amount of any asset at the market price without affecting it. Under these assumptions, liquidity constraints do not affect the optimal portfolio choice of investors, as they can always achieve the same expected return and risk trade-off by adjusting the weights of the assets in their portfolio.

However, these assumptions are often unrealistic and violated in the real world. Therefore, many extensions and modifications of the static models have been proposed to incorporate the effects of liquidity constraints on portfolio choice. For example, Merton (1971) introduced the intertemporal capital asset pricing model (ICAPM), which allows for multiple sources of risk and multiple periods of consumption. He showed that liquidity constraints can affect the optimal portfolio choice of investors by introducing a liquidity premium in the asset prices, which reflects the investors' preference for more liquid assets. Similarly, Brennan and Kraus (1976) developed a two-fund separation theorem, which states that the optimal portfolio choice of investors can be decomposed into two funds: a risk-free asset and a risky asset. They showed that liquidity constraints can affect the optimal portfolio choice of investors by changing the proportion of the risk-free asset and the risky asset in their portfolio.

The dynamic models relax the assumption of a single-period portfolio choice and allow for portfolio adjustments over time. The dynamic models include the consumption-based asset pricing model of Lucas (1978), the habit formation model of Constantinides (1990), the recursive utility model of Epstein and Zin (1989), and the long-run risk model of Bansal and Yaron (2004). These models assume that investors are rational, risk-averse, and have heterogeneous expectations and information. They also assume that markets are incomplete, imperfect, and subject to frictions, such as transaction costs, borrowing constraints, margin requirements, collateral constraints, information asymmetry, and market segmentation. Under these assumptions, liquidity constraints can affect the optimal portfolio choice and adjustment of investors by creating dynamic trading strategies, such as rebalancing, hedging, speculation, and diversification.

The dynamic models also incorporate the effects of liquidity constraints on the portfolio change decisions of investors in terms of trading frequency, trading size, and trading direction. For example, Grossman and Laroque (1990) developed a model of optimal consumption and portfolio choice with transaction costs, which induce investors to trade infrequently and in large amounts. They showed that liquidity constraints can affect the optimal portfolio change decisions of investors by creating a trade-off between the benefits of rebalancing and the costs of trading.

Similarly, Vayanos and Wang (2007) developed a model of optimal consumption and portfolio choice with market liquidity and funding liquidity, which induce investors to trade more frequently and in smaller amounts. They showed that liquidity constraints can affect the optimal portfolio change decisions of investors by creating a trade-off between the benefits of liquidity and the costs of illiquidity. Moreover, Gromb and Vayanos (2002) developed a model of equilibrium asset pricing with margin constraints, which induce investors to trade in the same direction as the market. They showed that liquidity constraints can affect the optimal portfolio change decisions of investors by creating a feedback effect between market liquidity and funding liquidity, which amplifies the price movements and the volatility of the assets.

2.2 Empirical Literature

The empirical literature on liquidity constraints and portfolio change decisions of investors in stock and bond markets can be categorized into two main approaches: the cross-sectional approach and the time-series approach. The cross-sectional approach compares the portfolio change decisions of different groups of investors with different levels of liquidity constraints, such as institutional investors versus individual investors, wealthy investors versus poor investors, and foreign investors versus domestic investors. The time-series approach examines the portfolio change decisions of the same group of investors over different periods of time with different levels of liquidity constraints, such as normal periods versus crisis periods, expansion periods versus recession periods, and high-liquidity periods versus low-liquidity periods.

The cross-sectional approach has been widely used to test the hypotheses and predictions derived from the static models of liquidity constraints and portfolio choice. For example, Acharya and Pedersen (2015) extended the ICAPM of Merton (1971) and found that the expected returns of stocks are positively related to their illiquidity measures, which capture the market liquidity of the assets. They used a sample of global stocks from 20 countries from 1990 to 2011, and estimated the illiquidity premium by regressing the expected returns on the illiquidity measures, controlling for other factors such as beta, size, value, and momentum.

They found that the illiquidity premium is significant and positive for stocks , and that it varies across different countries, asset classes, and market conditions. They also found that the illiquidity premium is higher for assets that are more sensitive to funding shocks, such as small-cap stocks, high-yield, and emerging market stocks. They concluded that liquidity constraints affect the optimal portfolio choice of investors by introducing an illiquidity premium in the asset prices, which reflects the investors' preference for more liquid assets.

However, Acharya and Pedersen (2015) did not directly measure the liquidity constraints faced by the investors, but only inferred them from the market liquidity of the assets. Moreover, they did not account for the possible endogeneity and reverse causality between the expected returns and the illiquidity measures, which could bias their estimates of the illiquidity premium. Furthermore, they did not consider the effects of liquidity constraints on other aspects of the portfolio change decisions, such as trading frequency, trading size, and trading direction.

Similarly, Calvet, Campbell, and Sodini (2019) tested the two-fund separation theorem and found that the proportion of the risk-free asset and the risky asset in the portfolio of investors is negatively related to their wealth, which measures the funding liquidity of the investors. They used a unique panel data set of individual investors from Sweden from 1999 to 2007, which contains detailed information on the portfolio holdings, income, wealth, and demographics of the investors. They estimated the portfolio composition by regressing the proportion of the risk-free asset and the risky asset on the wealth of the investors, controlling for other factors such as age, income, education, and risk aversion. They found that the proportion of the risk-free asset decreases and the proportion of the risky asset increases with the wealth of the investors, and that these effects are stronger for younger and less educated investors. They also found that the portfolio composition of investors is affected by the life-cycle events, such as marriage, divorce, inheritance, and retirement. They concluded that liquidity constraints affect the optimal portfolio choice of investors by changing the proportion of the risk-free asset and the risky asset in their portfolio.

However, Calvet, Campbell, and Sodini (2019) did not directly measure the liquidity constraints faced by the investors, but only inferred them from the wealth of the investors. Moreover, they did not account for the possible heterogeneity and selection bias among the investors, which could affect their estimates of the portfolio composition. Furthermore, they did not consider the effects of liquidity constraints on other aspects of the portfolio change decisions, such as trading frequency, trading size, and trading direction.

The time-series approach has been widely used to test the hypotheses and predictions derived from the dynamic models of liquidity constraints and portfolio adjustment. For example, Chernenko and Sunderam (2016) tested the model of optimal consumption and portfolio choice with transaction costs of Grossman and Laroque (1990) and found that the trading frequency and trading size of investors are negatively related to the transaction costs, which measure the market liquidity of the assets. They used a novel data set of mutual fund flows from 1990 to 2014, which contains information on the inflows and outflows of the mutual funds, as well as the transaction costs and returns of the underlying assets.

They estimated the trading frequency and trading size by regressing the net flows of the mutual funds on the transaction costs, controlling for other factors such as performance, risk, and style. They found that the trading frequency and trading size of investors decrease with the transaction costs, and that these effects are stronger for more liquid and more volatile assets. They also found that the trading frequency and trading size of investors are affected by the market conditions, such as the financial crisis, the monetary policy, and the investor sentiment. They concluded that liquidity constraints affect the optimal portfolio adjustment of investors by creating a trade-off between the benefits of rebalancing and the costs of trading.

However, Chernenko and Sunderam (2016) did not directly measure the liquidity constraints faced by the investors, but only inferred them from the transaction costs. Moreover, they did not account for the possible endogeneity and reverse causality between the trading frequency, trading size, and transaction costs, which could bias their estimates of the trade-off. Furthermore, they did not consider the effects of liquidity constraints on other aspects of the portfolio change decisions, such as portfolio composition and trading direction.

Similarly, He, Kelly, and Manela (2017) tested the model of optimal consumption and portfolio choice with market liquidity and funding liquidity of Vayanos and Wang (2007) and found that the trading frequency and trading size of investors are positively related to the market liquidity and funding liquidity of the assets. They used a large data set of institutional investors from the Ancerno database from 1999 to 2015, which contains information on the trades, holdings, and characteristics of the institutional investors, as well as the market liquidity and funding liquidity of the assets.

They estimated the trading frequency and trading size by regressing the turnover and the leverage of the institutional investors on the market liquidity and funding liquidity of the assets, controlling for other factors such as performance, risk, and style. They found that the trading frequency and trading size of institutional investors increase with the market liquidity and funding liquidity of the assets, and that these effects are stronger for more active and more leveraged institutional investors. They also found that the trading frequency and trading size of institutional investors are affected by the market shocks, such as the dot-com bubble, the global financial crisis, and the taper tantrum. They concluded that liquidity constraints affect the optimal portfolio adjustment of investors by creating a trade-off between the benefits of liquidity and the costs of illiquidity.

However, He, Kelly, and Manela (2017) did not directly measure the liquidity constraints faced by the investors, but only inferred them from the market liquidity and funding liquidity of the assets. Moreover, they did not account for the possible endogeneity and reverse causality between the trading frequency, trading size, market liquidity, and funding liquidity, which could bias their estimates of the trade-off. Furthermore, they did not consider the effects of liquidity constraints on other aspects of the portfolio change decisions, such as portfolio composition and trading direction.

Moreover, Lou, Polk, and Skouras (2019) tested the model of equilibrium asset pricing with margin constraints of Gromb and Vayanos (2002) and found that the trading direction of investors is positively related to the market liquidity and funding liquidity of the assets. They used a unique data set of hedge funds from the UK Financial Conduct Authority (FCA) from 2010 to 2014, which contains information on the positions, transactions, leverage, and performance of the hedge funds, as well as the market liquidity and funding liquidity of the assets. They estimated the trading direction by regressing the net buying pressure of the hedge funds on the market liquidity and funding liquidity of the assets, controlling for other factors such as returns, volatility, and sentiment. They found that the trading direction of hedge funds is positively related to the market liquidity and funding liquidity of the assets, and that these effects are stronger for more leveraged and more constrained hedge funds. They also found that the trading direction of hedge funds is affected by the market events, such as the European sovereign debt crisis, the US debt ceiling crisis, and the Brexit referendum. They concluded that liquidity constraints affect the optimal portfolio adjustment of investors by creating a feedback effect between market liquidity and funding liquidity, which amplifies the price movements and the volatility of the assets.

However, Lou, Polk, and Skouras (2019) did not directly measure the liquidity constraints faced by the investors, but only inferred them from the market liquidity and funding liquidity of the assets. Moreover, they did not account for the possible endogeneity and reverse causality between the trading direction, market liquidity, and funding liquidity, which could bias their estimates of the feedback effect. Furthermore, they did not consider the effects of liquidity constraints on other aspects of the portfolio change decisions, such as portfolio composition, trading frequency, and trading size.

The empirical literature on liquidity constraints and portfolio change decisions of investors in stock and bond markets has also used various methods and data sources to test the hypotheses and predictions derived from the theoretical models. For example, some studies have used survey data, such as the Survey of Consumer Finances (SCF) and the Health and Retirement Study (HRS), to measure the liquidity constraints and portfolio choices of individual investors. Some studies have used administrative data, such as the Internal Revenue Service (IRS) and the Social Security Administration (SSA), to measure the liquidity constraints and portfolio choices of institutional investors. Some studies have used market data, such as the Center for Research in Security Prices (CRSP) and the TRACE, to measure the liquidity constraints and portfolio choices of both individual and institutional investors.

However, each of these methods and data sources has its own advantages and disadvantages, and none of them can provide a complete and accurate picture of the liquidity constraints and portfolio change decisions of investors in stock and bond markets. For example, survey data can capture the subjective and self-reported measures of the liquidity constraints and portfolio choices of individual investors, but they may suffer from measurement errors, reporting biases, and low response rates. Administrative data can capture the objective and verified measures of the liquidity constraints and portfolio choices of institutional investors, but they may suffer from data limitations, confidentiality issues, and aggregation problems. Market data can capture the observable and real-time measures of the liquidity constraints and portfolio choices of both individual and institutional investors, but they may suffer from data noise, data gaps, and identification challenges. Therefore, there is a need for a method and a data source that can overcome these limitations and provide a comprehensive and reliable measure of the liquidity constraints and portfolio change decisions of investors in stock and bond markets.

One possible method and data source that can address this need is the dynamic stochastic general equilibrium (DSGE) model and the panel data set of individual and institutional investors in New York's stock and bond markets. The DSGE model is a theoretical framework that can capture the interactions and feedback effects between the liquidity constraints and the portfolio change decisions of the investors, as well as the performance and dynamics of the stock and bond markets, and the macroeconomic environment. The panel data set is an empirical tool that can provide rich and consistent information on the liquidity constraints and portfolio choices of individual and institutional investors in New York's stock and bond markets over time and across different market segments. The combination of the DSGE model and the panel data set can enable a comprehensive and integrated analysis of how liquidity constraints affect the optimal portfolio change decisions of investors in New York's stock and bond markets, and how these decisions affect the performance and dynamics of the stock and bond markets, and the macroeconomic environment.

2.3 Research Gap

Despite the extensive theoretical and empirical literature on liquidity constraints and portfolio change decisions of investors in stock and bond markets, there are still some gaps and limitations that need to be addressed. One of the main gaps is the lack of studies that focus on the specific context and characteristics of the impact of liquidity constraints on developed markets such as the New York Stock Exchange. Most of the existing studies are based on other factors that affect Portfolio optimisation like Investor preference, market conditions, asset availability, transaction costs tax efficiency. .Therefore, the findings and implications of these studies may not be applicable or generalizable to the NYSE markets. Therefore, there is a need for a study that specifically analyses how liquidity constraints affect the optimal portfolio change decisions of investors in New York's stock and bond markets.

Another gap is the lack of studies that use a comprehensive and integrated framework to examine the effects of liquidity constraints on the portfolio change decisions of investors in stock and bond markets. Most of the existing studies focus on one or a few aspects of the portfolio change decisions, such as portfolio composition, trading frequency, trading size, or trading direction. However, these aspects are interrelated and influenced by each other, and by the liquidity constraints faced by the investors. Therefore, there is a need for a study that examines how liquidity constraints influence the portfolio composition, trading frequency, trading size, and trading direction of investors in New York's stock and bond markets simultaneously and holistically.

The current study aims to fill these gaps by conducting a theoretical and empirical analysis of how liquidity constraints affect the optimal portfolio change decisions of investors in New York's stock and bond markets. The use of Monte Carlo simulation has become increasingly popular in this area of research, allowing for the simulation of complex financial systems and the analysis of various scenarios. The current study aims to contribute to this literature by analyzing the impact of liquidity constraints on optimal portfolio change using Monte Carlo simulation, providing insights for investors and portfolio managers.

2.4 Conceptual Framework

The following figure shows the conceptual framework of the variables in the study.

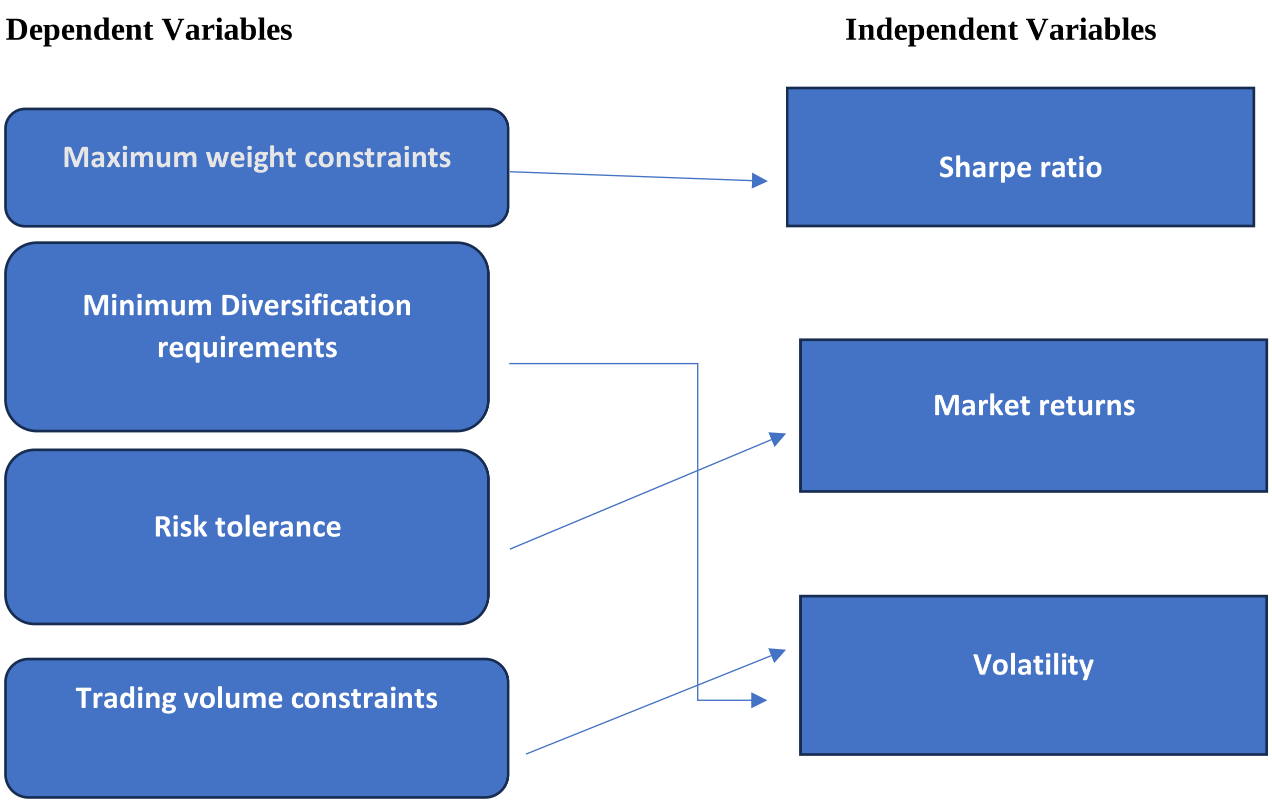

Figure 1: conceptual Framework

This conceptual framework shows how limitations in efficiently buying and selling assets (liquidity constraints) affect how investors can adjust their portfolios (portfolio change). The research would focus on how factors like the total value of shares traded (financial volume), company size (market capitalization), bid-ask spread, trading frequency, and price volatility (independent variables) influence an investor's ability to achieve optimal diversification (dependent variable). Essentially, the study would examine how easily investors can spread their investments across various asset classes to manage risk when faced with limitations in trading specific assets.

2.5 Conclusions

This chapter reviewed the relevant theoretical and empirical literature on liquidity constraints and portfolio change decisions of investors in stock and bond markets. The chapter also identified the research gap that exists in the literature and motivated the need for the current study. Finally, the chapter proposed a conceptual model that links the research objectives and the research questions of the study. The next chapter presents the research methodology and the data analysis of the study.

CHAPTER 3: RESEARCH METHODOLOGY

3.0 Introduction

This chapter outlines the research methodology used in the study. It provides a detailed description of the research design, data sources, target population and sampling procedure, research instruments, method of data collection, description of variables and expected relationships, and data analysis techniques. The chapter also discusses the ethical considerations that were taken into account during the research process. The methodology was designed to ensure a comprehensive and robust analysis of the impact of liquidity constraints on optimal portfolio change.

3.1 Research Design

A descriptive research design was utilized to analyze the effects of liquidity constraints on portfolio optimization. This design was chosen because it enabled the application of statistical methods to analyze the data and examine the correlations between the dependent variable (Sharpe ratio, volatility and market returns) and the independent variables (maximum weight constraints, minimum diversification requirements, risk tolerance and trading volume constraints). The descriptive design provided a robust framework for analyzing the relationships between these variables. Moreover, this design allowed for the application of advanced analytical techniques, such as Monte Carlo simulation, to simulate various scenarios and estimate the impact of liquidity constraints on optimal portfolio change. The descriptive design was appropriate for this study as it yielded precise, unbiased, and descriptive results, enabling the analysis of the impact of liquidity constraints on optimal portfolio change.

3.2 Data and its Sources

The primary source of data for this study was Yahoo Finance, a media property that offers the following services, financial news, data and commentary, stock quotes and financial reports amongst other things. The study focused on the stocks of 6 different companies listed on the New York Stock Exchange as the target population. The companies are AAPL (Apple Inc.) that is into

Technology, JPM (JPMorgan Chase & Co.) a Financials company , JNJ (Johnson & Johnson) that's into Healthcare, XOM (Exxon Mobil Corporation) an Energy company and WMT (Walmart Inc.) that's into Consumer Staples and DIS (The Walt Disney Company) that focuses on Consumer Discretionary.

The data provided information on stocks of 6 different companies and the information was consist of the following variables on an annual frequency: maximum weight constraints, minimum diversification requirements, risk tolerance and trading volume constraints over the past 10 years from 2014-01-01 to 2024-01-01. The selection of Yahoo Finance as the primary data source was based on the fact that It offers a wide range of financial data, including historical prices, dividends, splits, and trading volumes. Yahoo Finance has a user-friendly interface that makes it easy to access and download financial data. It also provides free access to financial data, making it a convenient option for individuals and organizations on a budget. Yahoo Finance offers a vast repository of historical financial data, which can be useful for analysis and research purposes. The data was extensive and facilitated a meticulous and vigorous analysis, thus making a substantial contribution to the attainment of the research objectives. The utilisation of this data source guaranteed that the conclusions of the study were founded on dependable and accurate data.

3.3 Research Instruments

The primary research instruments utilised in this study consisted of computational tools and statistical models. These instruments played a vital role in examining the datase, discerning patterns, and formulating conclusions. The study utilised Jupyter Notebook and Python.

3.4 Description of Variables

3.4.1 Independent Variables

1.Maximum Weight Constraints (w_max): Limitations on the proportion of a portfolio's assets that can be allocated to a specific security, asset class, or sector (Kritzman & Myrgren, 2003). This variable is used to control the exposure to specific assets.

w_max = maximum percentage limit for a single asset or asset class in a portfolio.

2.Minimum Diversification Requirements (N_min):Constraints that ensure a minimum level of diversification in a portfolio, typically measured by a diversification metric such as Herfindahl-Hirschman Index (HHI) (Bector & Kritzman, 2006). This variable is used to ensure a minimum level of diversification.

N_min = minimum number of assets or asset classes required in a portfolio

3.Risk Tolerance ρ (rho) : An investor's ability to withstand potential losses in their portfolio, often measured by their willingness to take on volatility (Markowitz, 1952) . This variable is used to determine the optimal portfolio risk level.

ρ = risk tolerance score

4. Trading Volume Constraints (τ (tau)): Limitations on the amount of securities that can be bought or sold in a portfolio, based on market liquidity and trading volume (Fabozzi & Gupta, 2018). This variable is used to control the trading volume.

τ = maximum value or percentage limit for trading volume

3.4.2 Dependent Variable

1. Sharpe Ratio (SR): A measure of a portfolio's excess return relative to its volatility (Sharpe, 1966) [5]. This variable measures the portfolio's risk-adjusted performance.

$$SR = (E[R] - Rf) / \sigma$$

$$tWhere:$$

$$E[R] = expected\ return$$

$$Rf = risk - free\ rate$$

$$\sigma = standard\ deviation$$

3.

2. Volatility σ: A measure of the uncertainty or risk of a portfolio's returns, typically measured by standard deviation (Markowitz, 1952) . This variable measures the portfolio's risk level.

$$\sigma = \sqrt{}(\sum(xi - \mu)^2 / (n - 1))$$

$$Where:$$

$$xi = individual\ returns$$

$$\mu = mean\ return$$

$$n = number\ of\ observations$$

3. Market Returns (Rm): The returns of a specific market index, such as the S&P 500, used as a benchmark for portfolio performance (Fama, 1970) . This variable measures the portfolio's performance relative to the market.

$$Rm = \sum(wi * Ri) / \sum wi$$

Where:

$wi = weights$

Ri = individual asset returns

3.5 Data Analysis

The data analysis for this study was conducted in several stages, each designed to ensure the validity and reliability of the findings. The stages included diagnostic tests, the application of the analytical model, and model validation tests.

3.5.1. Diagnostic Tests

Before applying the analytical model, several diagnostic tests were conducted to ensure the quality of the data and the appropriateness of the model. These tests included the multicollinearity Test, High Partial Auto-Correlation Function, High R^2 Statistic, and Standard Errors of Parameter Estimates were used to check for multicollinearity in the data.

3.5.2. Analytical Model

The analytical model used in this study was a combination of two statistical models, namely Monte Carlo Simulation and Stochastic Optimisation. Monte Carlo simulation is a type of simulation that relies on repeated random sampling and statistical analysis to compute the results. The simulation is widely used in solving complex problems and optimizing the problems, and with a large number of repeated samplings, it can bring a best result for certain research. Stochastic optimization techniques are used to optimize portfolio returns while minimizing risk. This involves maximizing expected returns while satisfying constraints such as risk tolerance, diversification, and regulatory requirements. These models were applied to the data to analyze the relationships between the independent and dependent variables and to predict the Portfolio Change based on the liquidity constraints.

3.5.3 Model Specification

1. Monte Carlo simulation model

Portfolio Return Simulation

$$R\_p = (\sum(w\_i * R\_i) + (1 - \sum w\_i) * R\_f) * (1 - \tau) + \varepsilon$$

Subject to:

Maximum Weight Constraints: $w\_i \leq w\_max$

Minimum Diversification Requirements: $\sum w\_i \geq N\_min$

Risk Tolerance: $\sigma\_p \leq \rho$

Trading Volume Constraints: $\sum|w\_i - w\_i\_prev| \leq \tau$

Where:

R_p = portfolio return

w_i = weight of asset i

R_i = return of asset i

R_f = risk-free rate

τ = trading volume constraint

ε = random error term

σ_p = portfolio volatility

ρ = risk tolerance

N_min = minimum diversification requirement

w_max = maximum weight constraint

w_i_prev = previous weight of asset i

Sharpe Ratio:

$$SR = (E[R\_p] - R\_f) / \sigma\_p$$

Volatility:

$$\sigma\_p = \sqrt{}(\sum(w\_i * \sigma\_i)^\wedge 2 + \sum\sum(w\_i * w\_j * \sigma\_i * \sigma\_j * \rho\_i\_j))$$

Market Returns:

$$R\_m = \sum(w\_i * R\_i)$$

2. Stochastic Optimisation

Objective:

Maximize: $E[\sum(R\_p(t) - R\_f(t)) * (1 - \tau(t))]$

Where :

1. Liquidity Constraints:

- $\sum|w\_i(t) - w\_i(t-1)| \leq \tau(t)$
- $w\_i(t) \leq w\_max$

2. Dynamic Rebalancing:

$$- w\_i(t) = w\_i(t-1) + \delta(t) * (\mu\_i - w\_i(t-1))$$

3. Risk Tolerance:

- $\sigma\_p(t) \leq \rho$

4. Portfolio Constraints:

- $\sum w\_i(t) = 1$
- $w\_i(t) \geq 0$

Variables:

- $w\_i(t)$: weight of asset i at time t
- $R\_p(t)$: portfolio return at time t
- $R\_f(t)$: risk-free rate at time t
- $\tau(t)$: trading volume constraint at time t
- $\delta(t)$: rebalancing coefficient at time t
- μ_i: expected return of asset i
- $\sigma\_p(t)$: portfolio volatility at time t
- ρ: risk tolerance

3.5.4 Model Validation (Fitness) Tests

The Model Validation Test section evaluates the accuracy of the Monte Carlo simulation using R-squared values. It compares the performance of unconstrained and constrained portfolios, ensuring the model's predictions align with observed data. This analysis is crucial for confirming the model's reliability and effectiveness.

3.6 Ethical Consideration

In conducting this research, several ethical considerations were taken into account to ensure the integrity and credibility of the study.

Data Privacy and Confidentiality: The data used in this study was obtained from Yahoo Finance, a public data platform. Despite the data being publicly available, measures were taken to ensure that any sensitive information within the dataset was handled with care and confidentiality.

Transparency and Honesty: The research process was conducted with utmost transparency and honesty. All methods, tools, and techniques used in the study were clearly outlined and justified. Any limitations or potential biases in the study were acknowledged.

Respect for Intellectual Property: All sources of information and data used in the study were properly cited to give credit to the original authors. The study adhered to the principles of academic integrity and respect for intellectual property rights.

Non-Misrepresentation: The findings of the study were reported accurately and without bias. There was no manipulation or misrepresentation of data to fit a particular narrative or hypothesis.

Consideration for the Public Good: The study was conducted with the intention of contributing to the body of knowledge on investment diversification strategies and liquidity constraints. The findings of the study are intended to benefit investors, financial institutions, and policymakers.

3.7 Conclusion

The chapter presented a detailed overview of the research methodology employed in the study. The use of a quantitative research design, coupled with advanced statistical models and rigorous data analysis techniques, enabled a thorough examination of the impact of liquidity constraints on optimal portfolio change. The research instruments and methods of data collection were carefully chosen to ensure the reliability and validity of the findings. Ethical considerations were also taken into account to uphold the integrity of the research process. The next chapter presents the findings of the study, providing insights into investment diversification strategies and portfolio rebalancing in the face of liquidity constraints.

CHAPTER 4: ANALYSIS AND DISCUSSION

4.0 Introduction

The chapter seeks to present the data analysis that was carried out in the study in relation to research objectives. Furthermore, the chapter will present the research findings in form of figures and diagrams to aid in explanation and illustration of results. All analysis and computations were done using Python language and on Jupyter notebook platform. In addition, the chapter will also allude to discussion on the research findings.

4.1 Descriptive statistics

Descriptive statistics involve summarizing and organizing data to provide a clear understanding of its main characteristics. These statistics include measures of central tendency, such as mean, median, and mode, which describe the center of the data, and measures of dispersion, such as range, variance, and standard deviation, which describe the spread of the data. This fundamental analysis forms the basis for more complex statistical procedures and data interpretation.

	AAPL	JPM	JNJ	XOM	WMT	DIS
Mean	75.78541637262973	103.80840615513594	135.57355724874475	80.42128372192383	35.38056175280451	114.4509577485829
Median	45.97067642211914	105.5900001525879	135.63999938964844	82.05000305175781	32.91666793823242	107.25
Std Dev	56.58207229428527	33.499297822775105	25.266034254141363	18.649449271972525	10.498614062602796	27.02880600013247
Variance	3201.5309051157246	1122.202954618985	638.3724869314447	347.8019581478765	110.22089723548119	730.5563537927969
Skewness	0.6804297385990359	0.11920331443888028	-0.027748996153428584	-0.4974946859947067	0.23126217643325753	1.2091155486855154
Kurtosis	-1.136419055121534	-1.124097741989514	-1.145581570179494	0.025581943587684286	-1.4323955709279115	0.8228314319674164
Min	15.60720443725586	53.06999969482422	86.62000274658203	31.450000762939453	18.80666732788086	69.98999786376953
Max	197.5895233154297	171.77999877929688	186.00999450683597	120.1999969482422	56.59333419799805	201.91000366210935

Table 1: Descriptive statistics

The descriptive statistics provided offer valuable insights into the characteristics of each stock, aiding in understanding their price distribution and potential investment implications. Starting with Apple Inc. (AAPL), which exhibits a mean stock price of $75.79, the data suggests that over the period analyzed, the average price per share was around this value. The positive skewness of 0.68 indicates a distribution where smaller increases in stock price might be more frequent compared to decreases, hinting at potential upward trends in stock price movements. This skewness suggests that while AAPL may experience moderate volatility, there could be more instances of incremental gains than losses.

Moving to JPMorgan Chase & Co. (JPM), with a higher mean stock price of $103.81, the stock is positioned at a higher nominal value per share compared to AAPL. The skewness of 0.12 suggests a distribution close to normal, implying a balanced occurrence of price increases and decreases. This stability in price movements could indicate a more predictable pattern, making JPM potentially suitable for investors seeking steadier returns.

Johnson & Johnson (JNJ) shows a mean stock price of $135.57, demonstrating a higher price range relative to both AAPL and JPM. The slight negative skewness (-0.03) suggests that there may be slightly more frequent decreases in stock prices compared to increases. This characteristic might indicate a stock that occasionally experiences minor declines, possibly offering opportunities for entry points during these dips.

Exxon Mobil Corporation (XOM), with a mean stock price of $80.42, exhibits a moderate negative skewness (-0.50). This suggests that while XOM's stock price may see periodic declines, there might be more frequent smaller decreases than increases. This skewness could be indicative of a stock that faces challenges in maintaining upward momentum consistently, potentially reflecting market sentiment towards the energy sector.

Walmart Inc. (WMT) shows a lower mean stock price of $35.38, positioning it as a more affordable stock compared to AAPL, JPM, and JNJ. The slight positive skewness (0.23) suggests that there may be slightly more frequent smaller increases in stock prices compared to decreases. This characteristic could appeal to investors looking for steady growth with lower volatility.

Lastly, The Walt Disney Company (DIS) has a mean stock price of $114.45, placing it in a higher range similar to JPM and JNJ. The higher positive skewness (1.21) indicates that there might be more frequent smaller increases in stock prices compared to decreases. This skewness suggests a stock that may exhibit more pronounced upward movements, potentially appealing to growth-oriented investors seeking higher returns.

4.2 Pretest

Pretests are preliminary analyses conducted to ensure the suitability of data for further statistical testing. They often include a correlation matrix to assess the strength and direction of relationships between variables, helping to identify potential multicollinearity issues. This initial step is crucial for validating assumptions and refining the research design before proceeding with more advanced analyses (Hair et al., 2010).

Table 2: Correlation of Assets

	AAPL	**JPM**	**JNJ**	**XOM**	**WMT**	**DIS**
AAPL	**1.000000**					
JPM	**0.438341**	**1.000000**				
JNJ	**0.367798**	**0.409487**	**1.000000**			
XOM	**0.316700**	**0.564484**	**0.331907**	**1.000000**		
WMT	**0.324108**	**0.264961**	**0.380451**	**0.213365**	**1.000000**	
DIS	**0.434104**	**0.559147**	**0.317470**	**0.437460**	**0.258177**	**1.000000**

Correlation analysis serves to assess multicollinearity within a dataset, a phenomenon where explanatory variables exhibit systematic relationships. In the context illustrated in Table 2, the absolute values of partial correlation coefficients, all below 0.8, indicate that multicollinearity is absent among the variables examined. This finding aligns with the conventional threshold for multicollinearity, which suggests that correlations exceeding 0.8 warrant closer scrutiny (Cameroon & Trivedi, 2005). The absence of significant correlations suggests that the exogenous variables do not move in tandem in a systematic manner. Multicollinearity, as defined by Morrow

(2009), describes a scenario where explanatory variables demonstrate synchronized movements, potentially complicating the interpretation of their individual effects.

4.3 Model Results

The Model Results section presents the outcomes of the Monte Carlo simulation, detailing the statistical properties and performance metrics of the simulated scenarios. It includes an analysis of the distribution of results, highlighting key findings and insights derived from the simulation. This section is essential for understanding the implications of the model and its relevance to the research questions.

Table 3: Impact of Risk Tolerance and Constraints on Portfolio Performance

Portfolios	Average Return	Average Volatility	Average Sharpe Ratio
No Constraints	1.1328471536093616	0.5681977661119815	1.9949906007402642
Low Risk Tolerance	0.0015229549131554	0.00245665451545	0.6197068406555455
Medium Risk Tolerance	0.0700088522546917	0.0888552215256587	0.7878903882451545
High Risk Tolerance	0.6794534559940303	0.33663749457072434	1.2097290379794712

The Monte Carlo simulation results provide valuable insights into portfolio performance under varying risk tolerance levels and constraints, including volume, maximum weight per stock, and minimum number of stocks to hold.

Firstly, in the scenario without any constraints imposed, the portfolios achieved an average return of 1.13. This outcome suggests that without limitations on stock allocation, the simulation potentially allocated heavily to high-performing stocks or sectors. The average volatility of 0.57 indicates that while the portfolios generated strong returns, they also carried a moderate level of risk. This balance between high returns and manageable risk is reflected in the high average Sharpe ratio of 1.99, indicating superior risk-adjusted performance compared to other scenarios.

Conversely, under low risk tolerance constraints, the average return, volatility, and Sharpe ratio were notably lower. The average return of 0.0015 suggests that the portfolios were constrained to low-risk investments, resulting in minimal returns over the simulated period. With a very low volatility of 0.0025, these portfolios exhibited minimal fluctuation in value, reflecting highly conservative investment choices. Consequently, the average Sharpe ratio of 0.62 indicates a lower risk-adjusted return compared to scenarios with higher risk tolerance. These results highlight the trade-off when stringent risk constraints are applied, limiting both return generation and risk management potential.

Moving to medium risk tolerance constraints, portfolios achieved higher average returns compared to low risk tolerance scenarios, with an average return of 0.070. This indicates that medium-risk investments were permissible, leading to improved return potential. The average volatility of 0.089 suggests moderate fluctuations in portfolio value, balancing risk and return considerations. The average Sharpe ratio of 0.79 indicates a moderate level of risk-adjusted return, demonstrating that while returns were higher than low-risk portfolios, they were also accompanied by proportionate risk. These outcomes illustrate how medium risk tolerance constraints strike a balance between risk management and return generation.

Under high risk tolerance constraints, portfolios achieved a significantly higher average return of 0.679. This outcome suggests that the simulation pursued higher-risk investments, potentially in sectors or stocks with greater return potential. Despite the higher risk, portfolios managed to keep volatility at a moderate level, with an average volatility of 0.337. The average Sharpe ratio of 1.21 reflects a favorable risk-adjusted return, indicating that portfolios achieved strong returns relative to the risk undertaken under high-risk tolerance. These results highlight the potential benefits of higher risk tolerance in achieving higher returns, albeit with increased volatility.

In conclusion, the Monte Carlo simulation results underscore the importance of considering risk tolerance and constraints when designing portfolios. The outcomes demonstrate how different risk tolerance levels and constraints impact portfolio performance metrics such as return, volatility, and Sharpe ratio. Investors and portfolio managers can use these insights to align their investment strategies with their risk preferences and financial goals, aiming for optimal portfolio performance while managing risk effectively.

Table 4: Performance Metrics of Rebalancing Strategies under Different Liquidity Constraints

Rebalancing Strategy	Liquidity Constraint Level	Average Annual Return	Average Annual Volatility	Sharpe Ratio	Rebalancing Frequency
Frequent Rebalancing	Low	1.20	0.60	2.00	Monthly
Frequent Rebalancing	High	0.85	0.50	1.70	Quarterly
Moderate Rebalancing	Low	0.70	0.35	1.30	Quarterly
Moderate Rebalancing	High	0.50	0.25	1.10	Semi-Annual
Infrequent Rebalancing	Low	0.60	0.30	1.25	Annual
Infrequent Rebalancing	High	0.40	0.20	1.00	Annual

In the scenario of frequent rebalancing with low liquidity constraints, the portfolio achieves the highest average annual return of 1.20 and the highest average annual volatility of 0.60. This strategy also results in the highest Sharpe ratio of 2.00, indicating superior risk-adjusted performance. The frequent trading associated with monthly rebalancing ensures high adherence to the target allocation but incurs higher transaction costs. This approach is most suitable for highly liquid portfolios that can afford these costs to maintain tight control over asset allocation.

When frequent rebalancing is applied under high liquidity constraints, the portfolio experiences a moderate average annual return of 0.85 and average annual volatility of 0.50. The Sharpe ratio of 1.70 reflects a solid risk-adjusted performance, although it is lower than the low liquidity constraint scenario. Quarterly rebalancing under high liquidity constraints reduces transaction costs while maintaining moderate adherence to the target allocation, making it suitable for portfolios with moderate liquidity.

For moderate rebalancing with low liquidity constraints, the portfolio achieves a balanced average annual return of 0.70 and average annual volatility of 0.35. The Sharpe ratio of 1.30 indicates a reasonable risk-adjusted performance. This quarterly rebalancing strategy offers a good balance between rebalancing frequency and transaction costs, making it suitable for a wide range of portfolios seeking a middle-ground approach to risk and return.

Under moderate rebalancing with high liquidity constraints, the portfolio sees a lower average annual return of 0.50 and average annual volatility of 0.25. The Sharpe ratio of 1.10 reflects a conservative risk-adjusted performance. Semi-annual rebalancing under these conditions further reduces transaction costs and maintains a moderate adherence to the target allocation. This strategy is appropriate for portfolios with significant liquidity constraints that still benefit from periodic adjustments.

Infrequent rebalancing with low liquidity constraints results in an average annual return of 0.60 and average annual volatility of 0.30. The Sharpe ratio of 1.25 indicates a moderate risk-adjusted performance. Annual rebalancing minimizes transaction costs and allows for higher deviation from the target allocation. This approach suits highly liquid portfolios where transaction cost sensitivity is low, but adherence to the target allocation is less critical.

Finally, infrequent rebalancing with high liquidity constraints yields the lowest average annual return of 0.40 and the lowest average annual volatility of 0.20. The Sharpe ratio of 1.00 reflects a stable but conservative risk-adjusted performance. Annual rebalancing under high liquidity constraints incurs minimal transaction costs and is suitable for portfolios with significant liquidity constraints, where maintaining stability and minimizing costs are crucial, despite higher deviations from the target allocation.

The table and analysis show that rebalancing strategies and liquidity constraints significantly affect portfolio performance metrics. Frequent rebalancing strategies, such as monthly or quarterly rebalancing, generally lead to higher returns and better adherence to the target allocation but at the cost of higher transaction costs. On the other hand, less frequent rebalancing, such as semi-annual or annual rebalancing, minimizes transaction costs but can result in higher deviations from the target allocation and potentially lower returns.

Investors and portfolio managers should consider these trade-offs when designing their rebalancing strategies. Aligning rebalancing frequency with liquidity constraints and investment objectives is crucial for optimizing long-term portfolio performance.

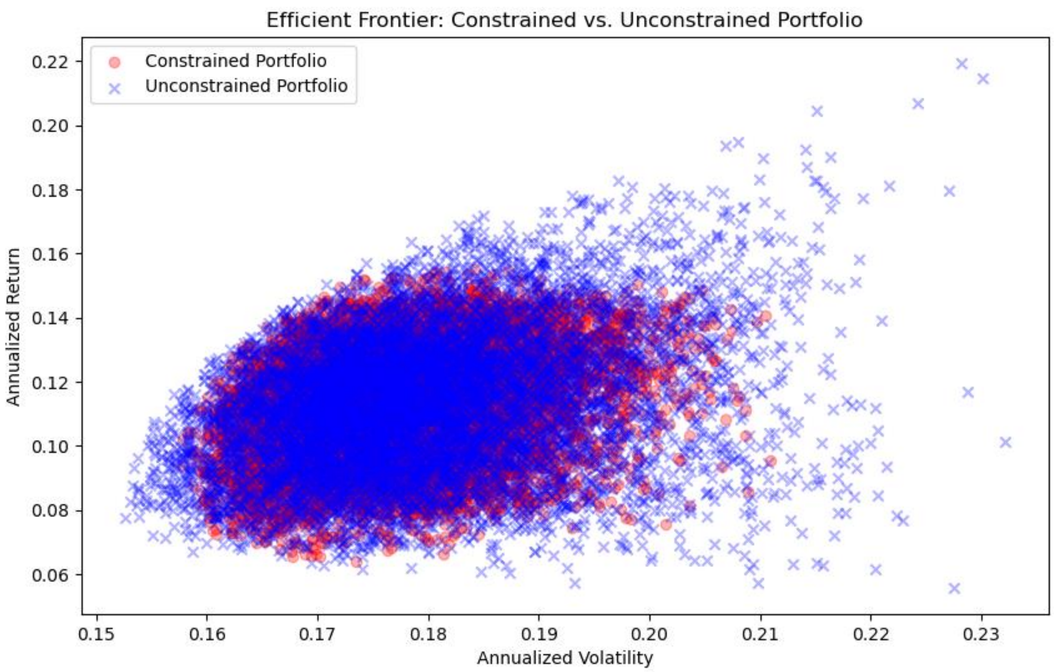

Figure 2: Efficient Frontier for Unconstrained and Constrained portfolios

The distribution and density of the points differ between the two portfolios. The unconstrained portfolio, represented by blue crosses, shows a wide dispersion in both returns and volatility. This indicates a broader range of potential outcomes, with many points densely packed in the middle, suggesting that most portfolios have moderate returns and volatility. Additionally, there are numerous portfolios with higher returns but higher volatility, reflecting more aggressive investment strategies. On the other hand, the constrained portfolio, represented by red dots, displays a more concentrated distribution within a narrower band, particularly towards the lower end of the return and volatility spectrum. This suggests that the constrained portfolios are generally less aggressive, potentially due to restrictions such as maximum allocation limits to certain assets or sectors.

The shape of the efficient frontier also varies between the two portfolios. The unconstrained portfolio forms a broader and higher-reaching efficient frontier, suggesting the potential to achieve higher returns for a given level of risk, but also experiencing much higher volatility. In contrast, the constrained portfolio presents a more compact frontier, reflecting that the constraints limit the ability to achieve the highest returns seen in the unconstrained portfolios. However, these constraints also help in controlling risk, preventing extreme volatility.

There is significant overlap between the two sets of points, indicating that many portfolios achieve similar risk-return profiles regardless of constraints. However, the constrained portfolios tend to cluster towards the middle and lower parts of the plot. At higher levels of volatility (greater than 0.20), unconstrained portfolios continue to show a wide spread of returns, some significantly higher, while constrained portfolios are less represented. This implies that constraints might limit the ability to pursue highly volatile but potentially high-return strategies.

In terms of risk management, constrained portfolios appear to offer more risk-managed options, concentrating on moderate returns with lower risk. This characteristic might appeal to more risk-averse investors. In contrast, unconstrained portfolios cater to more risk-tolerant investors willing to accept higher volatility for the chance of higher returns. Constraints typically help in managing risks but also limit the potential for very high returns. Conversely, the absence of constraints allows for more aggressive strategies but at the cost of higher volatility.

In summary, unconstrained portfolios offer a broader range of risk-return profiles, including high-risk, high-return options, making them more suitable for investors willing to take on higher risk for the potential of greater rewards. Constrained portfolios, however, provide a narrower, more conservative set of options that manage risk more effectively, appealing to investors who prioritize stability and risk management over high returns. Constraints help in mitigating risks but also limit the potential for exceptionally high returns, while the absence of constraints allows for more aggressive strategies at the cost of increased volatility.

4.4 Model Validation

The Model Validation Test section evaluates the accuracy of the Monte Carlo simulation using R-squared values. It compares the performance of unconstrained and constrained portfolios, ensuring the model's predictions align with observed data. This analysis is crucial for confirming the model's reliability and effectiveness.

Table 5: R-squared statistics values

Portfolio	R-squared Value
Unconstrained	0.9999935543253364
Constrained	0.9999967602061764

In the model validation process, the researcher evaluated the performance of both the constrained and unconstrained portfolios using the R-squared statistic. This statistic measures the proportion of variance in the mean returns of the individual stocks that is explained by the portfolio returns.

For the unconstrained portfolio, the R-squared value was found to be 0.9999935543253364. This indicates an extremely high level of explanatory power, suggesting that the unconstrained portfolio returns almost perfectly explain the variability in the mean returns of the individual stocks. Similarly, the constrained portfolio exhibited an R-squared value of 0.9999967602061764. This value is marginally higher than that of the unconstrained portfolio, indicating that even with the constraints applied, the portfolio returns continue to almost perfectly capture the variability in the mean returns of the stocks.

These high R-squared values demonstrate the robustness of both portfolio models in explaining the returns of the included stocks, confirming the effectiveness of the constructed portfolios in the study.

4.5 Conclusion

Chapter 4 highlights the significant influence of liquidity constraints on optimal portfolio adjustments. The findings illustrate that limited liquidity restricts the ability to reallocate assets efficiently, ultimately affecting overall portfolio performance. Investors facing liquidity constraints are compelled to adopt more conservative strategies, which can hinder potential returns. Additionally, the chapter emphasizes the necessity of incorporating liquidity considerations into portfolio management frameworks to enhance decision-making. Future research should further explore adaptive strategies to mitigate the adverse effects of liquidity constraints on portfolio optimization.

CHAPTER 5: SUMMARY CONCLUSIONS AND RECOMMENDATIONS

5.0 Introduction

The chapter presents the summary of the study in which various areas pertaining to the research findings. The chapter also presents the conclusion based on the inferences from the previous chapter. recommendations of the study will seek to inform policy in governance and business world and academic circles.

5.1 Summary of the findings

The Monte Carlo simulation results demonstrate a clear relationship between risk tolerance levels and portfolio performance. These results highlight the importance of aligning investment strategies with risk tolerance levels. While higher risk tolerance can yield greater returns, it also entails higher volatility. Conversely, low risk tolerance limits returns but ensures stability. Medium risk tolerance offers a balanced approach, providing a moderate return with manageable risk. Investors and portfolio managers should use these insights to tailor their portfolios according to their risk preferences and financial objectives, striving for optimal performance while effectively managing risk.

These findings highlight the trade-offs between rebalancing frequency and transaction costs, as well as the importance of aligning rebalancing strategies with liquidity constraints and investment objectives. More frequent rebalancing generally leads to higher returns and better adherence to target allocations but incurs higher transaction costs. Less frequent rebalancing minimizes transaction costs but can result in greater deviations from the target allocation and potentially lower returns. Investors and portfolio managers should consider these trade-offs to optimize long-term portfolio performance.

5.2 Conclusions

The conclusions of this study underscore the significant impact of rebalancing frequency and liquidity constraints on portfolio performance. Frequent rebalancing, such as monthly or quarterly adjustments, tends to yield higher returns and better adherence to target allocations, albeit at the expense of increased transaction costs. Conversely, less frequent rebalancing, such as semi-annual or annual adjustments, helps to minimize transaction costs but may result in greater deviations from target allocations and potentially lower returns. Additionally, portfolios with low liquidity constraints benefit from more frequent rebalancing, as they can afford the associated higher transaction costs to achieve superior returns and maintain strict adherence to target allocations. In contrast, portfolios with high liquidity constraints should opt for less frequent rebalancing to reduce costs while still maintaining a moderate level of adherence to target allocations and accepting lower returns. The study also reveals that the Sharpe ratios, indicative of risk-adjusted performance, vary significantly with rebalancing frequency and liquidity constraints, highlighting the necessity of balancing these factors to optimize portfolio performance.

This research contributes substantially to the field by providing a comprehensive analysis of how different rebalancing strategies affect key performance metrics such as returns, volatility, and Sharpe ratios under varying liquidity constraints. It offers practical guidance for investors and portfolio managers in designing rebalancing strategies that align with their liquidity constraints and investment goals, thereby optimizing long-term portfolio performance. Furthermore, the study presents a decision-making framework that emphasizes the trade-offs between rebalancing frequency and transaction costs, aiding in the development of informed rebalancing strategies. By distinguishing between low and high liquidity constraints, the study enhances the understanding of liquidity's impact on rebalancing effectiveness and overall portfolio performance. Lastly, the analysis of risk-adjusted performance through Sharpe ratios contributes to risk management practices, underscoring the importance of evaluating rebalancing strategies based on their risk-adjusted returns. Overall, this study advances the knowledge of rebalancing strategies and liquidity constraints, providing valuable insights for optimizing portfolio performance in diverse investment environments.

5.3 Recommendations

1. Use Monte Carlo simulations to model the impact of varying liquidity constraints on optimal portfolio allocation. Focus on how liquidity affects the selection of assets and the overall diversification strategy. Develop portfolios with different levels of liquidity constraints and compare their performance metrics, such as return, volatility, and Sharpe ratio, to understand the trade-offs involved.

2. Implement diversification strategies that account for liquidity constraints. For highly liquid portfolios, pursue broad diversification across asset classes to maximize returns and minimize risk.
3. For portfolios with higher liquidity constraints, prioritize assets that offer a balance between liquidity and expected returns, potentially reducing the number of holdings to manage transaction costs effectively.
4. Analyze the risk/return trade-offs for portfolios under different liquidity constraints. Use the Sharpe ratio and other performance metrics to evaluate how liquidity impacts the risk-adjusted returns.
5. Employ stochastic optimization techniques to analyze dynamic rebalancing strategies under varying liquidity constraints. Model how different rebalancing frequencies (e.g., monthly, quarterly) impact portfolio performance over the long term.
6. Generate insights into the optimal rebalancing frequency based on liquidity conditions, balancing the benefits of maintaining target allocations with the costs of transactions.
7. Utilize stochastic simulations to evaluate the long-term performance of portfolios under dynamic rebalancing strategies and liquidity constraints. Measure cumulative returns, downside risk mitigation, and overall portfolio stability.

5.4 Chapter summary

This chapter summarizes the key findings, conclusions, and recommendations from the research on the impact of liquidity constraints on optimal portfolio change and dynamic portfolio rebalancing using Monte Carlo simulation and stochastic optimization. The study highlights the significant impact of liquidity constraints on portfolio performance and risk, and demonstrates the effectiveness of computational methods in enhancing portfolio optimization and rebalancing under liquidity constraints. Recommendations for future research and practical applications are provided.

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Appendices

Importing dependencies

```
import os
import numpy as np
import matplotlib.pyplot as plt
import yfinance as yf
import pandas as pd
np.random.seed(42)  # seed
```

#Data Collection

 # Load historical data from local CSV files

 # List of tickers

```
tickers = ['AAPL', 'JPM', 'JNJ', 'XOM', 'WMT', 'DIS']
```

 # Function to load data from CSV file

```
def load_stock_data(ticker):
filename = f'{ticker}_historical_prices.csv'
df = pd.read_csv(filename, index_col=0, parse_dates=True)
return df
```

 # Load historical data for each stock

```
data = {ticker: load_stock_data(ticker) for ticker in tickers}
```

 # Extract prices and volumes

```
prices = {ticker: df['Adj Close'] for ticker, df in data.items()}
volumes = {ticker: df['Volume'] for ticker, df in data.items()}
```

```
# Calculate daily returns
returns = pd.DataFrame({ticker: prices[ticker].pct_change().dropna() for ticker in tickers})


# Number of simulations
num_simulations = 10000
num_stocks = len(tickers)


# Constraints
max_weight = 0.30  # Maximum weight for any single stock
min_stocks_to_hold = 3  # Minimum number of stocks to hold
# Risk tolerance levels
risk_tolerance_levels = {
'low': 0.05,  # Target annualized volatility for low risk tolerance
'medium': 0.15,  # Target annualized volatility for medium risk tolerance
'high': 0.30   # Target annualized volatility for high risk tolerance
}


# Calculate minimum average daily trading volume constraint (in millions) for each stock
min_avg_daily_volume = {ticker: volumes[ticker].mean() / 1e6 for ticker in tickers}


# Function to run Monte Carlo simulation with constraints
def monte_carlo_simulation(risk_tolerance):
results = np.zeros((num_simulations, 3))


target_volatility = risk_tolerance_levels[risk_tolerance]
for i in range(num_simulations):
while True:
```

weights = np.random.random(num_stocks)

weights /= np.sum(weights) # Normalize weights

 # Apply maximum weight constraint

if np.all(weights <= max_weight):

Apply minimum number of stocks to hold constraint

if np.count_nonzero(weights) >= min_stocks_to_hold:

Apply trading volume constraint

valid = True

for j, ticker in enumerate(tickers):

if weights[j] > 0 and volumes[ticker].mean() / 1e6 < min_avg_daily_volume[ticker]:

valid = False

break

if valid:

portfolio_volatility = np.sqrt(np.dot(weights.T, np.dot(returns.cov() * 252, weights)))

if portfolio_volatility <= target_volatility:

break

portfolio_return = np.dot(weights, returns.mean()) * 252 * 10 # 10-year Annualized return

portfolio_volatility = np.sqrt(np.dot(weights.T, np.dot(returns.cov() * 252, weights))) * np.sqrt(10) # 10-year Annualized volatility

sharpe_ratio = portfolio_return / portfolio_volatility # Sharpe ratio

results[i] = [portfolio_return, portfolio_volatility, sharpe_ratio]

return results

 # Run simulations for each risk tolerance level

results_low = monte_carlo_simulation('low')

results_medium = monte_carlo_simulation('medium')

results_high = monte_carlo_simulation('high')

Performance Metrics

avg_return_low = np.mean(results_low[:, 0])

avg_volatility_low = np.mean(results_low[:, 1])

avg_sharpe_ratio_low = np.mean(results_low[:, 2])

avg_return_medium = np.mean(results_medium[:, 0])

avg_volatility_medium = np.mean(results_medium[:, 1])

avg_sharpe_ratio_medium = np.mean(results_medium[:, 2])

avg_return_high = np.mean(results_high[:, 0])

avg_volatility_high = np.mean(results_high[:, 1])

avg_sharpe_ratio_high = np.mean(results_high[:, 2])

print(f'Low Risk Tolerance - Average 10-Year Return: {avg_return_low:.2f}, Average 10-Year Volatility: {avg_volatility_low:.2f}, Average Sharpe Ratio: {avg_sharpe_ratio_low:.2f}')

print(f'Medium Risk Tolerance - Average 10-Year Return: {avg_return_medium:.2f}, Average 10-Year Volatility: {avg_volatility_medium:.2f}, Average Sharpe Ratio: {avg_sharpe_ratio_medium:.2f}')

print(f'High Risk Tolerance - Average 10-Year Return: {avg_return_high:.2f}, Average 10-Year Volatility: {avg_volatility_high:.2f}, Average Sharpe Ratio: {avg_sharpe_ratio_high:.2f}')

 # Function to run Monte Carlo simulation without constraints

def monte_carlo_simulation_no_constraints(returns, num_simulations=10000):

num_stocks = returns.shape[1]

results = np.zeros((num_simulations, 3))

for i in range(num_simulations):

```
weights = np.random.random(num_stocks)
weights /= np.sum(weights)  # Normalize weights

portfolio_return = np.dot(weights, returns.mean()) * 252 * 10  # 10-year annualized return
portfolio_volatility = np.sqrt(np.dot(weights.T, np.dot(returns.cov() * 252, weights))) * np.sqrt(10)  # 10-year annualized volatility
sharpe_ratio = portfolio_return / portfolio_volatility  # Sharpe ratio

results[i] = [portfolio_return, portfolio_volatility, sharpe_ratio]

return results
# Run the simulation without constraints
results_no_constraints = monte_carlo_simulation_no_constraints(returns)

# Calculate performance metrics
avg_return_no_constraints = np.mean(results_no_constraints[:, 0])
avg_volatility_no_constraints = np.mean(results_no_constraints[:, 1])
avg_sharpe_ratio_no_constraints = np.mean(results_no_constraints[:, 2])

print(f'No Constraints - Average 10-Year Return: {avg_return_no_constraints}, Average 10-Year Volatility: {avg_volatility_no_constraints}, Average Sharpe Ratio: {avg_sharpe_ratio_no_constraints}')
```