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PORTFOLIO OPTIMIZATION PARADOX USING MONTE CARLO SIMULATION AND PARTICLE SWAMP OPTIMIZATION ALGORITHMS

BY

Malvern F Dumbu (B201840B)

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SUPERVISOR: MS.P.HLUPO

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APPROVAL FORM

This is to certify, that this research project is the result of my own research work and has not been copied or extracted from past sources without acknowledgement. I hereby declare that no part of it has been presented for another degree in this University or elsewhere.

		10/06/2024
Malvern F Dumbu	…………………………	……………………..
B201840B	Signature	Date

Certified by		10/06/2024
Ms.P. HLUPO	…………………………	……………………..
Supervisor	Signature	Date

		12/06/2024
Dr Magodora	…………………………..	……………………..
Chairperson	Signature	Date

DEDICATION

I dedicate this research project to my lovely parents Mr. and Mrs. Dumbu and my siblings.

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I want to express my sincere gratitude in this acknowledgment as I could not have completed my thesis without the guidance of my supervisor, help from my friends, and support from my family. I am especially grateful to my supervisor, Ms. P. Hlupo, for her unwavering support, patience, motivation, and vast knowledge that was instrumental in the development of my dissertation. I also want to thank the Almighty God for providing me with the necessary care, strength, knowledge, and opportunity to pursue my education to this level. A special shoutout goes to my parents and siblings for their encouragement, financial support, advice, prayers, and patience throughout my studies. I pray for blessings from the Lord to continue to pour upon them abundantly.

ABSTRACT

This dissertation explores the optimization of financial portfolios by integrating Monte carlo simulation with Particle Swarm Optimization (PSO) algorithms. The study addresses the portfolio optimization paradox, which involves balancing the maximization of returns and the minimization of risk. Traditional models like the Capital Asset Pricing Model (CAPM), Modern Portfolio Theory (MPT), and Fama-French models often rely on assumptions that do not fully capture real-world complexities, particularly in emerging markets such as Zimbabwe. By applying Monte carlo simulation and PSO to stocks from both the Zimbabwe Stock Exchange (ZSE) and the New York Stock Exchange (NYSE), this research develops a robust framework that accommodates market uncertainties and adapts to changes. The results indicate that PSO, combined with Monte carlo simulation, outperforms traditional models in terms of risk-adjusted returns, diversification, and stability. This innovative approach provides significant insights for individual and institutional investors, promoting better investment decisions and contributing to financial market stability and inclusivity, especially in diverse economic environments.

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PSO	Particle swarm Optimization
MCS	Monte Carlo Simulation
CAPM	Capital Asset Pricing Model
MPT	Modern Portfolio Theory
ZSE	Zimbabwe Stock Exchange
NYSE	New York Stock Exchange
AI	Artificial Intelligence
ML	Machine Learning

CHAPTER 1: INTRODUCTION

1.0 Introduction

In recent years, the methods for optimizing financial portfolios have rapidly evolved, offering investors better decision-making tools. Yet, a debate persists regarding the effectiveness of a technique called Particle Swarm Optimization (PSO) in fully understanding complex financial markets. PSO, a computational method mimicking the movement of particles in finding solutions, shows promise in portfolio optimization. However, some critics argue it falls short in capturing the market's dynamics (Kask, 2020).

This study aims to settle this debate by comparing a combination of PSO and Monte carlo simulation against traditional models like CAPM, MPT, and Fama-French models for portfolio optimization. The Study explores the 'Portfolio Optimization Paradox Using Monte carlo simulation and Particle Swarm Optimization Algorithms.'

This chapter covers the study's background, problem statement, objectives, research questions, assumptions, limitations, definitions, and sets the stage for subsequent chapters. Chapter 2 reviews existing research on Monte carlo simulation, Particle Swarm Optimization and different market scenarios. Chapter 3 dives into these methods and models, detailing the applied methodology. Chapter 4 presents the analysis and findings, leading to Chapter 5's conclusions and recommendations.

1.1 Background of the study

The portfolio optimization paradox poses a significant challenge in investment management, as it requires balancing two conflicting goals: maximizing expected returns and minimizing risk (Huang et al., 2020). This delicate balance is crucial, as pursuing higher returns often increases risk, and vice versa (Chen et al., 2019).

Effective investment management relies on finding a balance between these two objectives. Traditional financial models, such as Modern Portfolio Theory, CAPM, and Fama-French models, rely on assumptions that do not always hold true (Fama & French, 2012). These assumptions include rational investor behaviour, normal asset price distribution, and smooth markets. However, these assumptions often fail to capture real-world complexities, particularly in intricate portfolios (Kritzman et al., 2019).

Newer models like particle swarm optimization (PSO) offer a promising alternative, as they are better equipped to handle real-world scenarios (Rusu, 2019). PSO algorithms, inspired by bird and fish behaviour, navigate a search space to find optimal solutions. By combining Monte carlo simulation with PSO, this framework can factor in market uncertainties and adapt to changes, enhancing accuracy and dynamism (Illing et al., 2018).

Globally, the vision for 2030 emphasizes the importance of optimized portfolios and risk management in achieving sustainable economic growth (UNECA, 2013). However, financial inclusivity remains a significant concern, as many individuals lack access to professional financial guidance and are excluded from the formal financial system (World Bank, 2020). This exclusion hinders economic participation and perpetuates poverty.

In Zimbabwe, the need for adaptable methodologies like PSO is particularly pressing (Mafukata et al., 2019). Traditional models often fail to capture the country's diverse market realities, and many individuals lack the means to hire investment managers.

Statistical analysis and modeling play a crucial role in attracting investments and driving economic expansion (Muropa et al., 2020). Artificial intelligence (AI) and machine learning (ML) in portfolio optimization represent a significant leap forward, enabling individuals and institutional investors to better assess risk, optimize portfolios, and make informed investment decisions (Tett, 2019).

1.2 Problem statement

Traditional financial models for portfolio optimization rely on assumptions that do not always hold true (Kritzman et al., 2019). These assumptions include rational investor behaviour, normal asset price movement, and smooth markets. However, these assumptions fail to capture real-world market complexities, leading to suboptimal investment decisions and poor outcomes. Traditional models also neglect the unique challenges faced by investors in emerging markets like Zimbabwe, where market data is limited and investment opportunities are scarce (Mafukata et al., 2019). There is a need for a new approach to portfolio optimization, one that accounts for real-world complexities and emerging market challenges.

1.3 Research objectives

This study aims to bridge the gap between conventional portfolio optimization models and the dynamic financial landscapes of both developed and emerging markets, with a specific focus on Zimbabwe and the United States. By leveraging advanced computational tools, this research endeavors to develop a robust and adaptable framework that caters to the diverse needs of investors operating in multiple market contexts. The specific objectives of the study are:

1. To develop a framework for portfolio optimization using Monte carlo simulation and PSO algorithms.
2. To compare the performance of PSO and traditional models using stocks from the Zimbabwe Stock Exchange (ZSE) and New York Stock Exchange (NYSE).
3. To investigate the effects of key parameters on the performance of Monte carlo simulation and PSO.

1.4 Research questions

This study aims to develop a robust framework for portfolio optimization using Monte carlo simulation and Particle Swarm Optimization (PSO) algorithms, and investigate its applicability in both developed and emerging markets, with a focus on Zimbabwe and the United States. The following research questions guide this investigation:

1. How can Monte carlo simulation and PSO algorithms create a robust portfolio optimization framework for both developed and emerging markets?
2. How do PSO algorithms compare to traditional portfolio optimization models in risk-adjusted returns, diversification, and stability, using stocks from ZSE and NYSE?
3. What key parameters influence the performance of Monte carlo simulation and PSO algorithms in portfolio optimization, and how can they be optimized?

1.5 Assumptions

Whilst carrying out the study the following were assumed:

1. There are no arbitrage opportunities in both developed and emerging markets.
2. Markets are efficient and reflect all available information.
3. Market participants are rational actors making informed decisions.
4. Investors are risk-averse and have homogenous preferences.

5. Markets are perfectly liquid, with no restrictions on trading.

1.6 Significance of the study

The study has the following significance to the financial markets and stakeholders in both developed and emerging economies, with a focus on Zimbabwe and the United States:

1. The new financial modeling framework has the potential to help individual investors in both countries make informed investment decisions, contributing to the realization of their respective economic development goals.
2. The new financial modeling framework has the potential to improve the accuracy of financial predictions, leading to better decision-making in the financial industries of both countries.
3. The new framework could help to improve the design of financial products, making them more resilient to extreme market conditions in both developed and emerging markets.
4. The new framework could help to improve the transparency of financial markets, leading to more informed decision-making by investors and policymakers in both countries.
5. The new framework could help to reduce systemic risk in the financial systems of both countries, making them more resilient to shocks and crises, and promoting global financial stability.

1.7 Delimitation of the study

The following delimitations addresses the scope and boundaries of the research and highlights areas to be covered in the research:

1. The study explores the optimization of financial portfolios in the context of both developed and emerging markets, with a specific focus on the Zimbabwe Stock Exchange (ZSE) and the New York Stock Exchange (NYSE).
2. The study considers a limited number of financial assets, including stocks, in both markets.
3. The study focuses on equity portfolios investments in both Zimbabwe and the United States.
4. The study considers a long-term investment horizon (5 years) in both markets.

5. The study considers a specific investment style, which is passive management, in both markets.

1.8 Limitations of the study

The research study has the following limitations to be noted:

1. The data used may not be representative of all possible market conditions in both developed and emerging markets.
2. The methods used may not account for all relevant factors, such as behavioural biases or other market frictions, in both markets.
3. Findings may not apply to all investors or all types of portfolios in both developed and emerging markets.
4. Findings may not be applicable to all investment horizons or market conditions in both countries.

1.9 Definitions

1. Risk-free rate: The theoretical rate of return of an investment with zero risk, often approximated by short-term government bonds (Zhu et al., 2018).

2. Risk aversion: A measure of how much a person dislikes risk or is willing to give up in expected return to avoid risk. It's often quantified using utility theory in behavioural finance (Barberis et al., 2008).

3. Weights: The relative importance of each asset in a portfolio, determining the allocation of funds among different assets (Roncalli, 2013).

4. Portfolio: A collection of assets held by an investor that can consist of various financial instruments (Bodie et al., 2014).

5. Returns: The gain or loss on an investment, expressed as a percentage, indicating investment performance over a specific period (Cuthbertson Et al., 2018).

6. Variance: A measure of how far a set of data points is spread out from the mean, used to assess investment volatility or risk (Brooks, 2014).

7. Standard deviation: A measure of the dispersion of data points around the mean, quantifying the variation of a set of values (Brooks, 2014).
8. Sharpe ratio: A measure of the risk-adjusted return of an investment portfolio, assessing return relative to risk (Sharpe, 2010).
9. Efficient frontier: A curve representing portfolios maximizing return for a given level of risk. It's a key concept in modern portfolio theory (de Prado, 2018).
10. Portfolio optimization: The process of selecting and weighting assets in a portfolio to maximize specific objectives such as return or risk-adjusted return, using mathematical techniques like optimization algorithms (Brooks, 2014).

1.10 Conclusion

This introductory chapter has provided a foundation for the rest of the study. It has outlined the main ideas of the research, including the background, problem statement, research questions, assumptions, and significance of the study. This chapter has set the stage for the next chapter, which will delve into the literature review.

CHAPTER 2: LITERATURE REVIEW

2.0 Introduction

Within this chapter, an exploration ensues into the landscape of existing research, investigating deeply into the realms of particle swarm optimization and Monte Carlo simulation in the context of financial portfolio management. This review scrutinizes various studies, illuminating their insights, methodologies, and the evolving narrative shaping the utilization of these powerful tools within the financial domain.

2.1 Theoretical Literature

2.1.1 Modern Portfolio Theory (MPT)

Modern Portfolio Theory, developed by Harry Markowitz in the 1950s, is a foundational concept in portfolio optimization that has significantly influenced the field of financial economics. This theory emphasizes the critical role of diversification in reducing risk and enhancing returns. According to MPT, investors can construct an "efficient frontier" of optimal portfolios that offer the maximum expected return for a given level of risk. This efficient frontier represents a set of portfolios that are considered optimal because they have the highest expected return for a specified amount of risk or the lowest risk for a given level of expected return (Markowitz, 1952).

Key concepts of MPT include the efficient frontier, diversification, and the risk-return tradeoff. The efficient frontier is a graphical representation of the best possible portfolios, showing the tradeoff between risk and return. Diversification involves spreading investments across various assets to reduce risk. The risk-return tradeoff is a fundamental principle of MPT, which posits that higher returns typically come with higher risk. By strategically combining assets with different risk and return profiles, investors can optimize their portfolios to achieve their desired balance of risk and return (Markowitz, 1952).

Recent studies continue to support and expand upon Markowitz's framework. For example, Bodie, Kane, and Marcus (2014) discuss the practical applications of MPT in contemporary financial markets, highlighting its relevance in today's complex investment environment. Additionally, recent research by Fabozzi et al. (2014) examines the application of MPT in various market conditions, demonstrating its robustness across different economic cycles.

2.1.1.1 Advantages of MPT

One of the main advantages of MPT is its emphasis on diversification. By investing in a variety of assets, investors can significantly reduce the overall risk of their portfolio. This is because different assets often react differently to the same economic event, so the negative performance of some investments can be offset by the positive performance of others. Additionally, MPT provides a clear, mathematical framework for making investment decisions, allowing investors to quantify and balance the tradeoff between risk and return (Bodie et al., 2014).

2.1.1.2 Disadvantages of MPT

Despite its many benefits, MPT also has some limitations. One of the primary criticisms is that it relies on historical data to predict future returns and risks, which may not always be accurate or reliable. Additionally, MPT assumes that investors are rational and markets are efficient, which may not always hold true in the real world. Moreover, the theory can be complex and difficult to implement, requiring significant computational power and expertise (Fabozzi et al., 2014).

2.1.2 Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM), developed by William Sharpe, John Lintner, and Jan Mossin in the 1960s, is a cornerstone of modern financial theory. CAPM provides a framework for understanding the relationship between risk and expected return in financial markets. The model suggests that the expected return of an asset is equal to the risk-free rate plus a risk premium, which is based on the asset's beta. Beta measures the asset's sensitivity to market movements, indicating how much the asset's price is expected to change in response to market fluctuations (Sharpe, 1964).

CAPM is built on the premise that investors need to be compensated for both the time value of money (represented by the risk-free rate) and the risk taken (represented by the risk premium). The risk premium is derived from the asset's beta, which compares the asset's returns to market returns. A higher beta indicates higher risk and, consequently, a higher expected return. This model helps investors understand the tradeoff between risk and return and assists in the pricing of risky securities (Sharpe, 1964).

Recent studies have refined and tested the CAPM, affirming its utility and relevance. For instance, research by Fama and French (2015) expands on the CAPM by introducing additional factors that influence asset returns, providing a more comprehensive framework for asset pricing. Moreover, empirical studies by Bali et al. (2016) validate the practical applications of CAPM in diverse market conditions.

2.1.2.1 Advantages of CAPM

One of the main advantages of CAPM is its simplicity and ease of use. The model provides a straightforward formula to estimate the expected return of an asset based on its risk. This makes it a popular tool among financial analysts and investors. CAPM also incorporates the time value of money and adjusts for systematic risk, offering a clear perspective on how different levels of risk are compensated in the market. Additionally, the model's reliance on beta allows for easy comparison of different assets' risk levels (Fama & French, 2015).

2.1.2.2 Disadvantages of CAPM

Despite its widespread use, CAPM has several limitations. One major criticism is its assumption of a linear relationship between risk and return, which may not hold true in all market conditions. Additionally, CAPM assumes that all investors have the same expectations and access to information, which is often unrealistic. The model also relies heavily on historical data to estimate beta, which may not accurately predict future performance. Furthermore, CAPM does not account for other factors that might influence asset returns, such as liquidity and market anomalies (Bali et al., 2016).

2.1.3 Black-Litterman Model

The Black-Litterman Model, developed by Fischer Black and Robert Litterman in 1990, addresses some of the shortcomings of traditional mean-variance optimization models by incorporating investor views into the asset allocation process. This model combines the expected returns implied by the market equilibrium with the investor’s views on the performance of various assets, resulting in more stable and intuitive portfolio weights. The Black-Litterman Model is particularly useful for institutional investors who need to account for both market conditions and subjective views in their portfolio management strategy (Black & Litterman, 1990).

The model works by starting with a prior distribution of returns, which is typically based on the CAPM equilibrium returns. Investors then express their views as a series of expected returns for particular assets or groups of assets, which can be either absolute or relative. These views are incorporated into the model using a Bayesian approach to adjust the equilibrium returns, producing a new set of expected returns that reflect both market data and investor insights. This results in a posterior distribution of returns that combines both sources of information in a coherent manner (Black & Litterman, 1990).

Recent studies have explored various extensions and applications of the Black-Litterman Model, demonstrating its versatility and robustness in different market environments. For instance, (Meucci, 2008) provided a comprehensive review of the model's theoretical underpinnings and practical applications. More recently, research by He and Litterman (2019) revisited the model, highlighting its ongoing relevance and adaptability to modern portfolio management.

2.1.3.1 Advantages of Black-Litterman Model

One of the primary advantages of the Black-Litterman Model is its ability to integrate investor views with market equilibrium, resulting in more stable and intuitive portfolio allocations. This reduces the sensitivity of the optimal portfolio to small changes in the inputs, a common issue with traditional mean-variance optimization. Additionally, the model provides a clear framework for incorporating subjective views in a rigorous, quantitative manner, making it highly adaptable to various investment scenarios. The Bayesian approach used in the model ensures that both market data and investor views are weighted appropriately, leading to more balanced and informed investment decisions (Meucci, 2008).

2.1.3.2 Disadvantages of Black-Litterman Model

Despite its strengths, the Black-Litterman Model also has some limitations. The model requires a significant amount of input data, including accurate estimates of market equilibrium returns and the investor's views. This can be challenging, especially for individual investors who may not have access to comprehensive market data. Additionally, the model's complexity can be a barrier to its adoption, requiring sophisticated understanding and computational resources to implement effectively. There is also a degree of subjectivity involved in expressing and quantifying investor views, which can introduce bias and reduce the objectivity of the model's outcomes (He & Litterman, 2019).

2.1.4 Arbitrage Pricing Theory (APT)

The Arbitrage Pricing Theory (APT), introduced by Stephen Ross in 1976, is an alternative to the Capital Asset Pricing Model (CAPM) for asset pricing. Unlike CAPM, which focuses on a single factor (market risk), APT posits that multiple factors can influence the returns of a portfolio. These factors can include macroeconomic variables such as inflation, interest rates, and industrial production, as well as firm-specific variables. The APT framework allows for a more nuanced understanding of risk and return by recognizing that various sources of risk can impact asset prices.

The core idea of APT is that the return on an asset can be modeled as a linear function of various macroeconomic factors. Each factor has a corresponding sensitivity or beta coefficient, which measures the asset's responsiveness to changes in that factor. This multifactor approach enables investors to better capture the complex dynamics that drive asset prices, making it a valuable tool for portfolio management and risk assessment (Ross, 1976).

Recent literature has continued to explore and refine the applications of APT. For example, Burmeister, Roll, and Ross (1994) provided empirical tests of the theory, validating its predictive power. More recently, Adrian, Crump, and Moench (2015) have expanded on the model by incorporating high-frequency data and advanced econometric techniques, demonstrating the robustness and flexibility of APT in modern financial analysis.

2.1.4.1 Advantages of Arbitrage Pricing Theory

One of the primary advantages of APT is its flexibility. Unlike CAPM, which relies on a single market factor, APT allows for multiple factors, providing a more comprehensive and realistic view of the risk-return relationship. This multifactor approach enables investors to account for a wide range of economic influences, making it particularly useful in diverse and volatile market conditions. Additionally, APT does not require the strong assumptions of CAPM, such as normally distributed returns or a risk-free rate that applies universally. This makes APT more adaptable to real-world scenarios and empirical testing (Burmeister et al., 1994).

2.1.4.2 Disadvantages of Arbitrage Pricing Theory

Despite its strengths, APT also has some limitations. One of the main challenges is identifying and selecting the relevant factors that influence asset returns. This requires extensive economic analysis and can vary across different markets and time periods. The model's reliance on historical data for factor sensitivity estimates can also introduce estimation errors, potentially affecting the accuracy of the predictions. Furthermore, APT assumes that arbitrage opportunities will be quickly exploited by the market, leading to equilibrium prices. However, in practice, arbitrage may be limited by transaction costs, market inefficiencies, and other frictions (Adrian et al., 2015).

2.1.5 Fama-French Three-Factor Model

The Fama-French Three-Factor Model, introduced by Eugene F. Fama and Kenneth R. French in the early 1990s, is an extension of the Capital Asset Pricing Model (CAPM). It adds two additional factors to the market risk factor of CAPM to better explain the returns on a diversified portfolio. These factors are the size of firms (small minus big, SMB) and the book-to-market value (high minus low, HML). The model was developed in response to empirical evidence suggesting that CAPM did not fully capture the cross-section of stock returns, particularly with respect to the size and value anomalies (Fama & French, 1992).

The Fama-French model posits that smaller companies (small-cap stocks) and companies with high book-to-market ratios (value stocks) tend to outperform larger companies (large-cap stocks) and those with low book-to-market ratios (growth stocks). This model has significantly influenced the field of financial economics by providing a more comprehensive framework for understanding asset returns beyond the traditional market factor.

Recent studies have further validated the model and explored its implications. For example, Novy-Marx (2013) examined the profitability factor, while Fama and French (2015) expanded their model to include five factors, incorporating profitability and investment factors. These extensions continue to demonstrate the robustness and adaptability of the original three-factor framework in capturing various dimensions of risk and return.

2.1.5.1 Advantages of Fama-French Three-Factor Model

One of the main advantages of the Fama-French Three-Factor Model is its ability to provide a more accurate and nuanced explanation of portfolio returns compared to CAPM. By incorporating size and value factors, the model accounts for empirical anomalies that CAPM cannot explain, offering a better understanding of risk premiums associated with small-cap and value stocks. This enhanced explanatory power makes it a valuable tool for both academic research and practical investment management. Additionally, the model is relatively simple to implement, requiring only basic financial data on market capitalization and book-to-market ratios (Novy-Marx, 2013).

2.1.5.2 Disadvantages of Fama-French Three-Factor Model

Despite its strengths, the Fama-French model also has limitations. One criticism is that it does not fully account for all sources of risk affecting asset returns. For instance, it omits factors such as momentum, which has been shown to impact returns significantly. The model's reliance on historical data for factor construction can also introduce biases and estimation errors, potentially affecting its predictive accuracy. Moreover, the assumption that size and book-to-market ratios are proxies for risk factors is debated, with some arguing that these characteristics might be indicators of mispricing rather than true risk factors (Fama French, 2015).

2.1.6 Market scenarios

2.1.6.1 Bull market

A bull market is a period of rising stock prices, generally accompanied by increased investor confidence and optimism (Nielsen, 2019). In a bull market, stock prices may rise rapidly and remain elevated for a significant period of time. Bull markets can be caused by many factors, including strong economic growth, low interest rates, and increased consumer spending (Malkiel, 2015). During bull markets, investors may be more willing to take on risk and invest in stocks, driving up prices. While bull markets can be very profitable for investors, they can also be volatile and are not guaranteed to last indefinitely (Wagner et al., 2018).

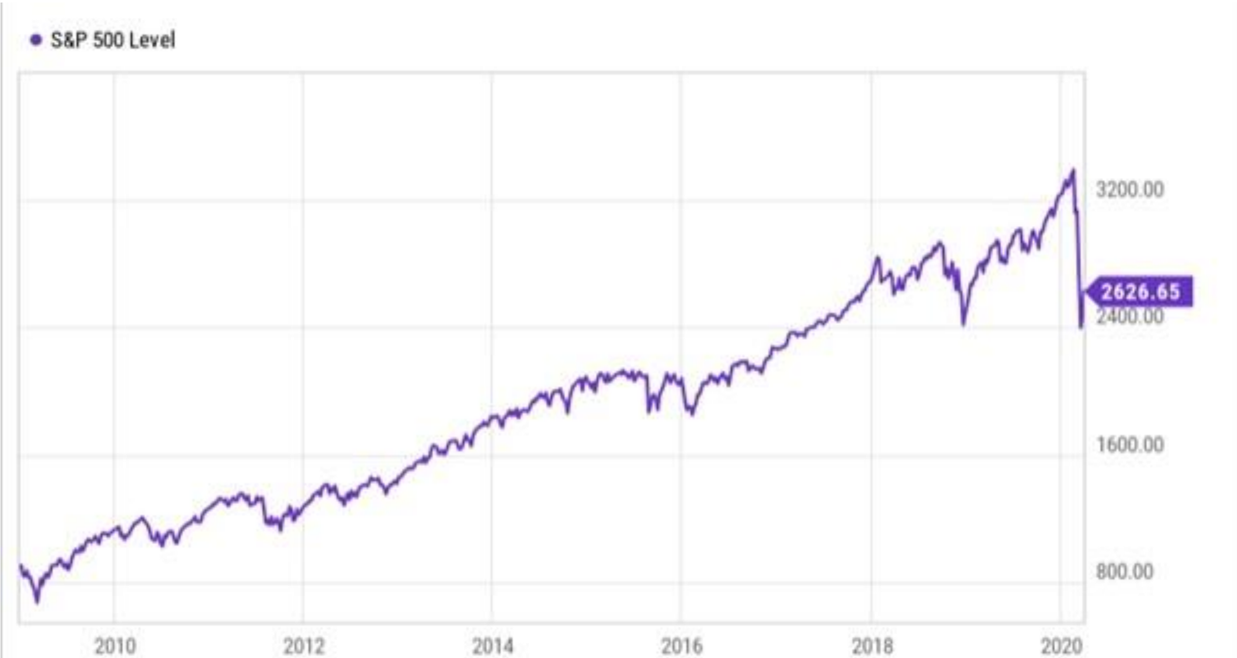

Figure 2.1: S & P 500 2009-2020 Bull Market

2.1.6.2 Bear markets

A bear market is the opposite of a bull market, characterized by falling stock prices and increased pessimism among investors, (Amadeo, 2020). In a bear market, stock prices may decline sharply and remain depressed for a prolonged period of time. Bear markets can be caused by a number of factors, including economic downturns, high interest rates, or political uncertainty, (Amadeo, 2021). In a bear market, investors may become more risk-averse and sell off their stocks, leading to further price declines, (Hicks, 2020). Another important characteristic of bear markets is that they tend to be more volatile than bull markets. This means that the ups and downs in stock prices can be more dramatic during a bear market, making it even more difficult for investors to predict where the market is headed. The increased volatility in a bear market can also lead to higher trading costs and increased risk for investors, (Folger, 2020). However, some investor's view bear markets as a good time to buy stocks at lower prices in anticipation of future market gains, (Withers, 2020).

Figure 2.2: Bear market NASDAQ 2014

2.1.6.3 Sideways markets

A sideways market is a period of market stagnation, in which stock prices remain relatively unchanged over a period of time, (Lyer, 2019). Sideways markets can be caused by a number of factors, including economic uncertainty, low volatility, or a lack of clear direction from the market (Schwab, 2019). However, some investors may choose to focus on dividends or other income-generating strategies during a sideways market, (Spano, 2019). Over time, sideways markets may eventually give way to either a breakout or breakdown, where prices break out of the range they have been in for some time and either head higher or lower. In other words, sideways markets may be seen as a period of "calm before the storm" in the financial markets, (Hall, 2022). While some investors may find sideways markets frustrating, others may view them as a time to build a strong portfolio or assess their overall strategy, (Gayed, 2022).

Figure 2.3: Sideway market for United Bankshares inc in 2018

2.2 Empirical literature

2.2.1 Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) has been a fundamental tool in the realm of finance, utilized in various experiments and studies globally to understand the relationship between risk and expected returns in financial markets. One prominent study conducted in the United States, (Campbell et al., 2015), delved into assessing the effectiveness of CAPM in predicting stock returns. The study focused on a diverse portfolio of stocks listed on the New York Stock Exchange (NYSE) and NASDAQ. They aimed to scrutinize whether the CAPM's predictions of expected returns aligned with the actual observed returns in the market. Utilizing historical market data and applying the CAPM framework, the researchers computed beta coefficients for individual stocks, representing their sensitivity to market risk.

The findings revealed a mixed picture regarding the CAPM's efficacy. While the model demonstrated a significant correlation between market risk (beta) and expected returns for a considerable portion of the stocks analyzed, it also highlighted discrepancies between predicted returns and actual observed returns for certain assets. This discrepancy raised questions about the CAPM's accuracy in forecasting returns for specific stocks, suggesting potential limitations or market inefficiencies not entirely captured by the model.

Furthermore, the study's results indicated that while CAPM provided a foundational framework for understanding the risk-return relationship in the market, it might not comprehensively explain the entirety of stock price movements or adequately forecast returns for all individual securities. This suggested the possible influence of additional factors beyond market risk, such as firm-specific characteristics or market anomalies, impacting actual returns, thereby questioning the CAPM's universality.

2.2.2 Modern Portfolio Theory (MPT)

One notable study conducted in Japan, by Arimura et al., (2017) aimed to assess the effectiveness of MPT in portfolio allocation and risk management. The study focused on examining the application of MPT in constructing optimal portfolios using stocks and bonds from the Tokyo Stock Exchange (TSE). The primary goal was to analyze whether MPT's principles of diversification could effectively minimize portfolio risk while maximizing returns.

The researchers leveraged historical market data to compute the expected returns and standard deviations of individual assets. Employing mean-variance optimization, a core concept within MPT, they created efficient frontiers, showcasing the optimal combinations of assets that yielded maximum returns for a given level of risk. The study's findings exhibited the efficacy of MPT in constructing diversified portfolios. The efficient frontier derived from MPT allowed for the identification of portfolios offering higher returns while simultaneously lowering overall portfolio risk. It emphasized the benefits of diversification, highlighting how combining assets with uncorrelated or negatively correlated returns could mitigate overall portfolio volatility.

However, the study also shed light on certain limitations of MPT. While the theory advocated for the benefits of diversification, it encountered challenges in scenarios where correlations between assets increased during periods of financial turmoil or systemic market events. This suggested that during extreme market conditions, the diversification benefits proposed by MPT might not be as effective, indicating the theory's vulnerability to specific market dynamics.

2.2.3 Fama-French Model

The Fama-French Three-Factor Model has stood as a significant addition to financial theory, offering insights into asset pricing and portfolio construction. An influential study conducted in in the United Kingdom by Clare et al., (2015), sought to assess the applicability and effectiveness of the Fama-French model in explaining stock returns. The study was designed to explore whether the Fama-French model, with its factors of size and value, could accurately describe stock returns on the London Stock Exchange (LSE).

The researchers employed historical stock data, considering company size (market capitalization) and value (book-to-market ratio) as key factors in their analysis. They aimed to evaluate whether these factors, in addition to the market risk factor, provided a robust framework for explaining stock returns.

The study's findings presented compelling evidence supporting the effectiveness of the Fama-French Three-Factor Model in describing stock returns in the UK market context. By including size and value factors alongside market risk, the model significantly enhanced its explanatory power, capturing a more comprehensive spectrum of stock returns. Notably, the Fama-French model's ability to account for the size and value factors exhibited a substantial improvement in explaining stock returns compared to the traditional Capital Asset Pricing Model (CAPM). It showed that stocks categorized as small-cap or undervalued, based on the book-to-market ratio, tended to outperform larger or growth-oriented stocks.

However, the study also uncovered certain limitations. While the Fama-French model explained a substantial portion of stock returns, there were instances where certain anomalies or market inefficiencies persisted, suggesting the model might not fully capture all aspects of stock pricing dynamics in real-world market scenarios.

2.2.4 Monte carlo simulation

Monte Carlo Simulation (MCS) stands as a robust tool extensively utilized in various fields, particularly finance, to model complex systems and simulate diverse scenarios. An illuminating study conducted in the United States, (Vasios et al,. 2017) aimed to investigate the efficacy of Monte Carlo Simulation in risk assessment within the context of portfolio optimization.

The study focused on evaluating the application of MCS in optimizing investment portfolios. The researchers sought to determine whether Monte Carlo Simulation could effectively aid in the optimization process by considering diverse asset combinations and risk levels.

Leveraging historical market data encompassing various asset classes and risk profiles, the study employed MCS to generate numerous hypothetical scenarios. Through these simulations, the researchers aimed to identify optimal portfolio allocations that maximized returns while managing risk at different levels, catering for investors' varying risk preferences.

The findings from the study revealed promising results for the application of Monte Carlo Simulation in portfolio optimization. MCS demonstrated its efficacy in analyzing and optimizing portfolios by offering a wide spectrum of possible outcomes. By considering multiple combinations of assets and risk levels, the simulations provided valuable insights into optimal portfolio compositions across different risk-return profiles.

Moreover, Monte Carlo Simulation showcased its versatility in accommodating different investment strategies and risk tolerances. It allowed investors to visualize and understand the potential risks and rewards associated with various portfolio allocations in a dynamic and interactive manner, aiding in informed decision-making.

2.2.5 Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) has emerged as a powerful computational technique applied across various domains, including finance, to tackle complex optimization challenges. An insightful study conducted in China by Zhang et al., (2019) delved into the application of PSO specifically within the context of portfolio optimization. The study aimed to investigate the effectiveness of PSO in optimizing investment portfolios. The researchers sought to determine whether PSO could provide superior solutions compared to traditional optimization methods in the financial domain.

The study utilized historical financial data encompassing diverse asset classes, including stocks, bonds, and commodities. Leveraging the PSO algorithm, the researchers aimed to identify optimal portfolio allocations that maximized returns while minimizing risks. PSO's unique ability to explore the solution space and converge on global optimum solutions made it an intriguing choice for this optimization task.

The findings from the study revealed promising results regarding the application of PSO in portfolio optimization. The PSO algorithm demonstrated its efficiency in searching for optimal asset allocations by navigating complex and nonlinear solution spaces. It outperformed traditional optimization methods by swiftly converging on solutions that offered better risk-adjusted returns.

2.3 Research Gap

In the expansive domain of financial portfolio optimization, existing literature has earnestly examined diverse methodologies and strategies, prominently featuring models like the Capital Asset Pricing Model (CAPM), Modern Portfolio Theory (MPT), and the Fama French model. However, within this extensive body of research, notable gaps have surfaced, warranting further exploration to enhance the current understanding.

One significant void lies in the integration of behavioural finance theories within conventional portfolio optimization frameworks like CAPM and MPT. These models predominantly focus on quantitative analysis and efficiency assumptions, disregarding behavioural aspects such as investor sentiment and psychological biases. Addressing this gap necessitates an integration of behavioural finance insights into traditional models, enriching portfolio optimization strategies by encompassing both rational and behavioural aspects of decision-making, (Cheng et al., 2013).

Furthermore, the existing literature often lacks comprehensive robustness testing across diverse market conditions. Empirical studies are needed to validate portfolio strategies in dynamic and volatile market scenarios, ensuring their efficacy beyond historical performance metrics. There is a need for models that can adapt to market volatility or economic downturns, providing more reliable and resilient optimization strategies.

The proposed research aims to bridge these gaps within financial portfolio optimization. By integrating behavioural finance into traditional models, validating strategies across diverse market conditions, and introducing optimization frameworks that incorporate ESG considerations akin to the Fama French model, this study seeks to enhance portfolio optimization strategies. Moreover, leveraging advanced techniques like Monte Carlo simulation and Particle Swarm Optimization (PSO) will attempt to close these gaps. Monte carlo simulation can validate portfolios in various market scenarios (Bailey, 2019), while PSO offers a novel optimization method adaptable to changing market conditions (Yung et al., 2013). Ultimately, this holistic approach endeavors to enrich portfolio optimization frameworks, making them more adaptive, ethical, and robust. By addressing these research gaps, the study aspires to cater to the evolving needs of investors in both traditional and socially responsible investment spheres.

2.4 Conclusion

Chapter 2's literature review unveils a multifaceted landscape enriched with foundational theories and evolving methodologies. Delving into established models like CAPM, MPT, and the Fama-French model and newer models PSO and Monte Carlo algorithms, the review illuminates their application contexts, strengths, and limitations across various global markets.

Moving forward, Chapter 3 endeavors to bridge these gaps by proposing a novel approach: Portfolio Optimization Paradox using Monte Carlo Simulation and PSO. Through this, the study aims to address the limitations identified in the reviewed literature and contribute to the evolving landscape of portfolio optimization methodologies.

CHAPTER 3: RESEARCH METHODOLOGY

3.0 Introduction

In this important chapter, the researcher explores the details of research methodology, which forms the foundation of the study on financial portfolio management. The approach combines particle swarm optimization and Monte Carlo simulation techniques to enhance the analysis. This section serves as a guide for the research, explaining the decisions made regarding the study design, data collection, participant selection, tools, and analytical approaches. Additionally, the researcher carefully considers the ethical considerations associated with research efforts.

3.1 Research design

Research design serves as a roadmap or blueprint for a study, helping researchers minimize bias and maximize the validity of their findings (Plot et al., 1999). In this study, the research design provides a framework for investigating financial portfolio management. A quasi-experimental quantitative research design was adopted to explore the optimization of portfolios using Particle Swarm Optimization (PSO) and Monte Carlo simulation. This approach allows for precise measurement of the effects of these optimization techniques on returns and risks, which are crucial in the financial landscape. By employing statistical tools, the researcher can rigorously assess the observed relationships, ensuring the credibility of the findings in real-world applications of PSO and Monte Carlo simulation.

The quantitative research design enhances objectivity by relying on numerical data, minimizing subjective biases, and promoting the integrity of the research. It also supports replicability, as the transparent use of quantitative methods provides a clear framework that can be easily replicated by other researchers.

3.2 Data Sources and Collection

Data source refers to the place or places from which the data for the study is obtained (Baker, 2018) and Data collection refers to the process of gathering data from the chosen data source or sources (Heale, 2005). In the financial portfolio management study, a secondary data collection method was employed to gather crucial information. The study relied on two primary platforms, namely Yahoo finance and ZSE Direct, to acquire the necessary data. Yahoo finance, a reputable hub for data science, provided the historical stock prices from the New York Stock Exchange (NYSE). The availability of diverse datasets on Yahoo finance ensured a dependable foundation for the research. Additionally, data related to the Zimbabwe Stock Exchange (ZSE) was directly sourced from the official Zimbabwe Stock Exchange through ZSE Direct. These two-platform approach aimed to establish the credibility and authenticity of the data, enhancing the robustness of the research findings.

3.3 Research instruments

Research instruments are tools used to measure and collect data from research participants on a particular topic of interest (Nieswiadomy, 2018). In the financial portfolio management study, a laptop was utilized. The laptop played a crucial role in handling complex analytical tasks throughout the study. Their robust platform provided the necessary capability to run simulations, conduct statistical analysis, and perform intricate modeling processes. Given the vast amount of data involved in the field of financial portfolio management, the computational strength of the laptop was essential for efficiently processing extensive data sets. By leveraging laptops, the researcher was able to improve the precision, speed, and thoroughness of their analysis, meeting the high demands of the study.

3.4 Description of Variables and Expected Relationships

3.4.1 Study variables

In the Monte Carlo simulation, several independent variables are utilized to shape the composition of simulated portfolios and evaluate risk and return. These variables include the Risk Aversion Scale (Ra), the Risk-Free Rate (Rf), and the Number of Simulations(Ns). whilst In the Particle Swarm Optimization (PSO) algorithm, several independent variables are utilized to guide the optimization process. These variables include the Number of Particles (np), Acceleration Coefficients (ac), and Inertia Weight (w). The selection of these independent variables in the PSO algorithm is crucial to strike a balance between exploration and exploitation, ultimately leading to efficient optimization and the discovery of optimal solutions.

3.4.1.1 Risk Aversion Scale (Ra)

The Risk Aversion Scale measures an investor's appetite for risk (Linden et al., 2017) and is represented on a scale of 0 to 10. A higher value on the scale indicates a lower tolerance for risk. This variable guides the creation of portfolios with varying levels of risk, aligning with the investor's risk preferences. It is expected that higher levels of risk aversion will lead to lower portfolio returns and lower portfolio risk. Investors with higher risk aversion tend to prefer safer investments with lower potential returns (Goyal et al., 2018).

3.4.1.2 Risk-Free Rate (Rf)

The Risk-Free Rate serves as the baseline return expected from a risk-free investment (Buch et al., 2018). It provides a reference point against which real investment opportunities are evaluated. Higher risk-free rates are anticipated to result in higher portfolio returns, this is because a higher risk-free rate increases the baseline return expectation, which can influence the overall returns of the portfolio (Duffie, 2018).

3.4.1.3 Number of Simulations (Ns)

The Number of Simulations determines the number of iterations performed by the Monte Carlo simulation. It significantly impacts the granularity and reliability of the simulation's outcomes. Increasing the count of simulations enhances precision but also requires more computational resources. Balancing accuracy and computational efficiency are essential in this dynamic financial landscape. A higher number of simulations is likely to improve the precision and reliability of the outcomes. With more simulations, the Monte Carlo simulation can explore a wider range of scenarios and provide more accurate estimates of portfolio returns, portfolio risk, and other performance metrics (Joshi, 2021).

3.4.1.4 Number of Particles (np)

The Number of Particles represents the count of solution candidates in the swarm. Each particle in the swarm represents a potential solution to the optimization problem. A larger number of particles allows for a more extensive exploration of the solution space, increasing the chances of finding optimal solutions (Tolooei et al., 2018). However, a smaller number of particles may limit exploration but could lead to faster convergence. The selection of the Number of Particles depends on the trade-off between exploration and convergence speed (Wu et al., 2019).

3.4.1.5 Acceleration Coefficients (ac)

Acceleration Coefficients govern the influence of a particle's own experience (Cognitive) and the shared experience of the swarm (Social) on its movement. Higher coefficients amplify the impact of individual and collective learning, potentially accelerating convergence (Tolooei et al., 2018). Conversely, lower coefficients may result in slower convergence by downplaying the influence of these experiences. Higher acceleration coefficients (both cognitive and social) are expected to expedite convergence in the PSO algorithm. This means that particles are more influenced by their own experience and the collective experience of the swarm, leading to faster optimization (Jin et al., 2019).

3.4.1.6 Inertia Weight (w)

The Inertia Weight parameter controls the balance between exploration and exploitation for each particle. It determines how much the particle's current velocity contributes to its future movement. A higher Inertia Weight favors exploration by allowing particles to maintain their current velocities, encouraging a broad exploration of the solution space (Liang et al., 2017). On the other hand, a lower weight may prioritize exploitation, leading particles to converge more quickly. A higher inertia weight in the PSO algorithm favors exploration over exploitation. It allows particles to maintain their current velocities, promoting a broader exploration of the solution space. This exploration can help find optimal solutions in the portfolio optimization, but it may also result in slower convergence (Zuo et al., 2018).

3.4.2 Dependent variables

3.4.2.1 Portfolio Returns (Rp)

Portfolio returns refer to the overall financial gains or losses generated by a simulated investment portfolio (Shen et al.,2018). It quantifies the success or failure of different investment strategies within the simulation. Portfolio returns serve as a key performance indicator, helping investors and analysts understand how well a portfolio might perform under various market conditions. It directly ties to the profitability of the simulated investments (Asghar, 2016). Higher portfolio returns are desired, as they indicate greater profitability of the simulated investment portfolio. Positive returns suggest successful investment strategies within the simulation. In a weighted portfolio, each investment is assigned a specific weight or proportion. The formula for calculating the log portfolio return in a weighted portfolio is as follows:

$$Rp = \ln\left(1 + (w_1 * r_1) + (w_2 * r_2) + \cdots + (w_n * r_n)\right)$$

Where:

w_1, w_2…, w_n are the weights assigned to each investment in the portfolio. The weights should sum up to 1.

r_1, r_2…,r_n are the individual returns of the investments in the portfolio.

This formula calculates the log return of the portfolio by taking the weighted sum of the individual log returns, based on their respective weights. The natural logarithm function (ln) is used to calculate the log returns. Log returns have the advantage of being additive across time periods, which simplifies portfolio return calculations and allows for easier comparison and aggregation of returns.

3.4.2.2 Portfolio Risk (σp)

Portfolio risks in the context of the Monte Carlo Simulation represent the level of uncertainty or volatility associated with the simulated portfolios (Ahmad et al., 2016). It measures the potential variability in returns. Investors and portfolio managers use portfolio risks to assess the stability and reliability of investment choices. Lower risk indicates a more stable investment, while higher risk suggests greater uncertainty. Managing risk is crucial for making informed investment decisions. Higher portfolio risk indicates greater uncertainty and volatility in the simulated portfolios. Managing risk is important for making informed investment decisions, and higher risk suggests a less stable investment (Llanos et al., 2018).

The historical simulation method calculates portfolio risk based on historical returns of the individual assets within the portfolio. The formula for portfolio risk using this method is as follows:

$$\sigma p = \sqrt{\left(\frac{1}{(N-1)} * \Sigma\left(w_i * r_i - r_p\right)^2\right)}$$

Where:

N is the number of historical data points (periods) used in the calculation.
w_i is the weight of the i-th asset in the portfolio.
r_i is the historical return of the i-th asset.
r_p is the historical portfolio return, calculated as the weighted sum of the individual asset returns.

3.4.2.3 Sharpe Ratio (Sr)

The Sharpe ratio assesses the risk-adjusted return of an investment portfolio (Sharpe, 1994). It compares the excess return of the portfolio to the standard deviation of those returns, providing a measure of how well the portfolio's return compensates for the risk taken (Leland, 2019). The Sharpe ratio, as a dependent variable, evaluates the risk-adjusted performance of the portfolio. It helps investors understand if the returns achieved are commensurate with the level of risk involved.

$$SR = \frac{(Rp - Rf)}{\sigma p}$$

Where:

Rp is the average return of the portfolio
Rf is the risk-free rate of return
σp is the standard deviation of the portfolio's returns

In this formula, $(Rp - Rf)$ represents the excess return, which is the difference between the average return of the portfolio and the risk-free rate. The standard deviation (σp) measures the variability of the portfolio's returns and is a measure of risk. The Sharpe ratio quantifies how much excess return an investor is receiving per unit of risk taken. A higher Sharpe ratio indicates better risk-adjusted performance, as it suggests that the portfolio is generating higher returns relative to its level of risk.

3.5 Pre-tests

Pre-tests were a crucial component of this financial portfolio management study, the researcher specifically focused on assessing the independence and correlation among different assets. Understanding these factors is essential for optimizing portfolio performance.

3.5.1 Correlation of assets

One valuable tool the researcher utilized was the correlation matrix, which provided detailed insights into the relationships between assets. The researcher employed the Pearson correlation coefficient, a crucial diagnostic test. This statistical measure ranges from -1 to 1 and allowed to assess the strength and direction of linear relationships among different asset prices. Positive or negative values from this test provided insights into the degree to which changes in one asset's price correlated with changes in another asset's price. A positive correlation suggested that both assets tended to move in the same direction, while a negative correlation indicated an inverse relationship. By applying the Pearson correlation coefficient, the researcher systematically explored the linear associations among asset prices, uncovering valuable insights into potential relationships that could impact portfolio dynamics. This test was a foundational element of financial analysis and was a deliberate step to ensure a nuanced understanding of the underlying structure of the dataset, contributing to the robustness of subsequent analysis in the context of financial portfolio management.

3.5.2 Outlier detection

Identifying and addressing outliers was another important aspect of the study. The researcher employed standardized measures to systematically detect and mitigate the impact of unusual data points on subsequent analyses. Quartile played a significant role in outlier detection, helping to gain a better understanding of the distribution of asset prices. These diagnostic tests were not just theoretical exercises, they were integral steps taken to refine the dataset and ensure the accuracy and reliability of the analysis.

3.6 Analytical model

The study employed a sophisticated analytical model that integrated both Monte Carlo simulation and Particle Swarm Optimization (PSO) techniques. This hybrid model aimed to optimize portfolios by considering the complex interplay between risk and return. The researcher went a step further by including traditional models for a robust comparative analysis.

3.6.1 Monte Carlo Simulation

Monte Carlo simulation is a crucial component of this analytical model, offering a dynamic method to generate a wide range of potential future scenarios. By randomly sampling various parameters, such as the risk aversion scale and the number of simulations, it simulates diverse market conditions. These simulations provide a probabilistic framework that estimates portfolio returns and risks under different circumstances.

Monte Carlo methods originated in the 1940s, developed by scientists working on the Manhattan Project to compute the critical mass of nuclear weapons. They used random number generation to account for the randomness in neutron behavior within chain reactions (Metropolis et al., 1949).

In the 1950s, Monte carlo simulation were adopted in finance to estimate variable values through random sampling, becoming widely used for estimating option values and forecasting portfolio returns (Bailey et al., 2019). The casino industry later saw their application in the 1970s when mathematicians Edward Thorp and Claude Shannon employed these techniques to develop a successful card-counting strategy for blackjack (Laura et al., 2015).

The core principle of Monte carlo simulation involves using random numbers to simulate numerous scenarios. For instance, in asset valuation, these simulations generate various price scenarios for an underlying asset to calculate its expected payoff. This method is particularly effective for complex problems with nonlinear or intricate distributions (Bailey et al., 2019).

In portfolio optimization, Monte carlo simulation are pivotal for risk estimation and developing asset allocation strategies. By simulating multiple asset price movement scenarios, they provide insights into the potential performance of different portfolio compositions under varying market conditions (Kroese et al., 2014).

Using Monte carlo simulation in portfolio optimization helps estimate potential returns and associated risks for different asset allocations. This process aids in creating portfolios that balance risk and return, giving investors a clearer understanding of the trade-offs in different investment strategies (Dilorio et al., 2019). Additionally, these simulations help delineate efficient frontiers, representing portfolios that offer the highest returns for a given risk level or the lowest risk for a specified return level. This graphical representation assists investors in identifying the most favorable portfolios based on their risk preferences, highlighting the trade-offs involved in portfolio selection (Dilorio et al., 2019).

3.6.1.1 Advantages of Monte Carlo Simulation

Monte carlo simulation offer multiple advantages, such as their adeptness in handling complexity, ease of implementation, and capacity to compute diverse statistics. They furnish a range of potential outcomes rather than a singular result and serve to test model robustness through different assumptions. In essence, they serve as a powerful and versatile tool for decision-making in uncertain environments (Ecuyer, 2018).

The integration of the Monte Carlo simulation was justified by its ability to introduce a dynamic and probabilistic element into the optimization process. This probabilistic nature enhanced the model's adaptability to the unpredictable nature of financial markets.

3.6.1.2 Monte Carlo Simulation formula

Consider the simulation of stock prices. In this case, a common formula used is the geometric Brownian motion formula:

$$S(t) = S(0) * e^{\left((r-0.5*\sigma^2)*t+\sigma*\sqrt{t*Z}\right)}$$

Where:

- S(t) represents the simulated stock price at time t.
- S(0) is the initial stock price.

- r denotes the expected return.
- σ represents the volatility.
- t signifies the time period.
- Z is a random number drawn from a standard normal distribution.

This formula utilizes the concept of random walks to simulate the movement of stock prices over time. By incorporating such simulations within the Monte Carlo framework, the researcher was able to generate a multitude of potential future stock price scenarios, aiding to the analysis of portfolio performance and risk.

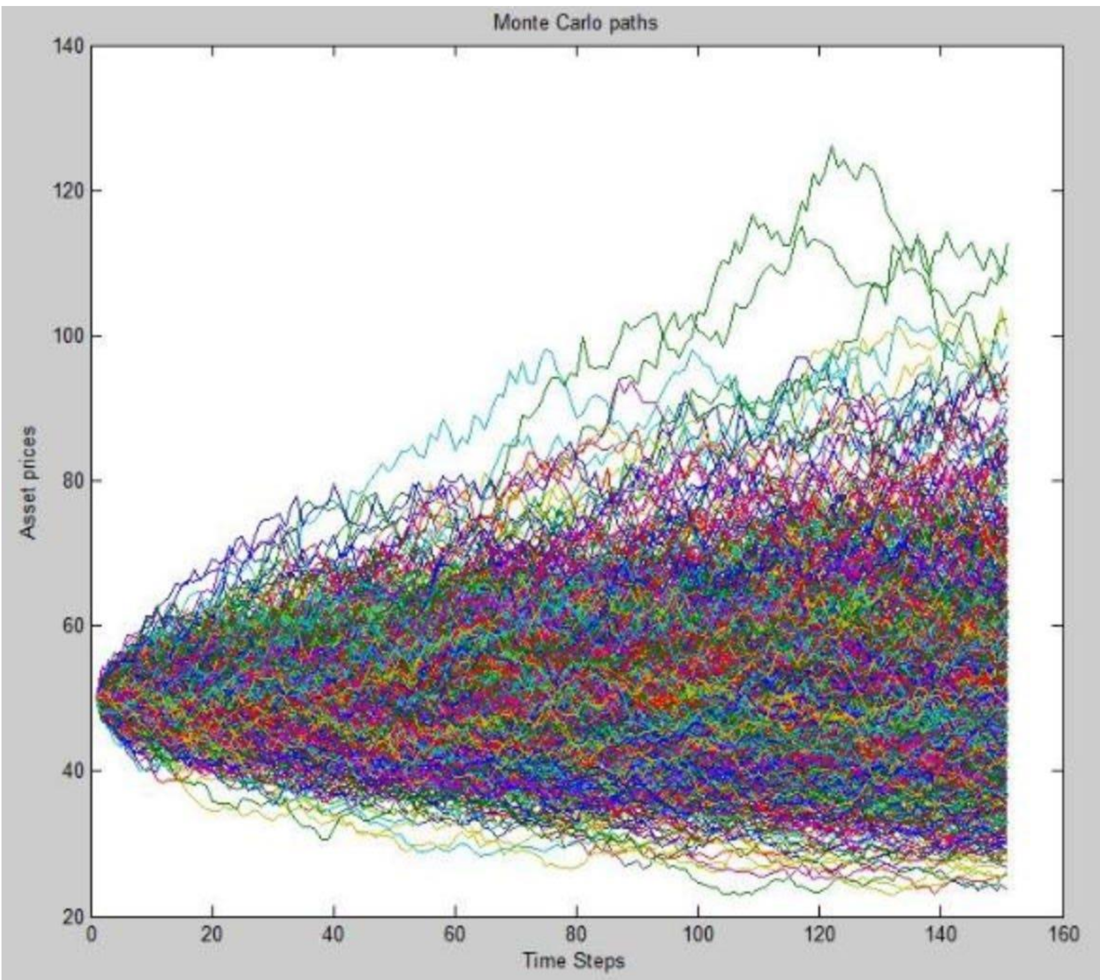

Figure 3.1: Typical Monte Carlo simulation path of Asset future price

3.6.2 Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO), developed by Eberhart and Kennedy in 1995, is inspired by social behaviors observed in birds and fish, fostering collaboration and adaptation among individuals (Eberhart et al., 1995). This technique has become widely used in finance for optimizing investment portfolios, predicting stock prices, and modeling financial markets due to its ability to find global optimum solutions while avoiding local optima (Beltagy et al., 2013). Its simplicity and effectiveness in various financial optimization problems contribute to its popularity.

In the context of portfolio optimization, PSO determines optimal asset weights to achieve objectives such as maximizing returns or minimizing risks. It surpasses conventional methods due to its efficiency in solving complex computational problems (Bhattacharya et al., 2019; Li et al., 2020). PSO uses a swarm of particles representing potential solutions, which navigate the solution space by incorporating their previous best positions and the overall swarm's data, facilitating convergence toward optimal solutions (Yang et al., 2013).

The integration of PSO into the analytical model enhances the portfolio optimization process. Inspired by social behaviors in nature, a swarm of particles dynamically adjusts their positions in the solution space to find optimal solutions. PSO's iterative optimization capabilities allow for systematic refinement of portfolio allocations.

PSO's substantial traction within the finance sector is attributed to its versatility and effectiveness in tackling various financial optimization challenges. Its adeptness at optimizing investment portfolios, predicting stock prices, and modeling financial markets underscores its widespread use and reliability in financial applications (Xu et al., 2017).

3.6.2.1 Advantages of PSO

PSO excels in solving intricate problems with multiple objectives, efficiently avoiding local optima and offering near-optimal solutions. Its adaptability and simplicity enable seamless adaptation across diverse domains, mitigating overfitting risks by identifying multiple solutions in uncertain scenarios (Khan et al., 2016).

3.6.2.2 Key features and formulas used in PSO:

1. Swarm Dynamics: PSO involves a swarm of particles, each representing a different portfolio allocation, dynamically adjusting their positions in the solution space. The particles move towards promising regions based on their own experience and the collective knowledge of the swarm.
2. Iterative Refinement: PSO iteratively refines the portfolio allocations by updating the positions and velocities of the particles. The algorithm seeks to identify the most effective portfolio configuration based on predefined criteria, such as the number of particles, acceleration coefficients, and inertia weight.

The formulas used in PSO include:

1. Formalizing PSO - Positions

$$X_1^d \rightharpoondown = [x_1^d, y_1^d] \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

$$X_2^d \rightharpoondown = [x_2^d, y_2^d] \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

$$X_3^d \rightharpoondown = [x_3^d, y_3^d] \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3)$$

$$\Downarrow$$

$$X_i^d \rightharpoondown = \left[x_i^d, y_i^d, z_i^d, \dots\right]$$

2. Updating Velocity and Position:

$$V_i^{d+1} \rightharpoondown = Velocity\ of\ particle\ i\ on\ next\ iteration\ (d+1)$$

$$rand(0\ to\ 1) * \max_{current} = r_1\alpha \qquad say, \alpha = 1$$

$$rand(0\ to\ 1) * \max_{personal} = r_2\beta \qquad say, \beta = 2$$

$$rand(0\ to\ 1) * \max_{\text{team}} = r_3\gamma \qquad say, \gamma = 3$$

$$V_i^{d+1} \rightharpoondown = r_1\alpha V_i^d + r_2\beta\left(P_i^{d\rightharpoondown} - X_i^{d\rightharpoondown}\right) + r_3\gamma\left(G^{d\rightharpoondown} - X_i^{d\rightharpoondown}\right)$$

Inertia   past/cognitive component   social component

$$X_i^{d+1} \rightharpoondown = X_i^{d\rightharpoondown} + V_i^{d+1\rightharpoondown} \text{ (next position of particle i)}$$

3. Exploration vs Exploitation

$$V_i^{d+1} \rightharpoondown = r_1 \alpha V_i^d + r_2 \beta \left(P_i^{d\rightharpoondown} - X_i^{d\rightharpoondown}\right) + r_3 \gamma \left(G^{d\rightharpoondown} - X_i^{d\rightharpoondown}\right)$$

- High γ – population converges too fast to the best found so far (High exploitation)
- High β – individuals stick to their personal best performances (High exploitation of personal knowledge)
- High α – individuals keep exploring the current direction (High exploration)
- Required: α balance (Decrease α iteratively earlier on high exploration; later high exploitation)

where:

- $X_i^{d\rightharpoondown}, Y_i^{d\rightharpoondown}, Z_i^{d\rightharpoondown}$ current position of particle.
- V_i^{d+1} is the velocity of the i-th particle at iteration d + 1.
- X_i^{d+1} is the position of the i-th particle at iteration d + 1.
- w is the inertia weight, controlling the impact of the previous velocity on the current velocity.
- α, β and γ are acceleration coefficients, determining the influence of the particle's personal best position ($P_i^{d\rightharpoondown}$) and the global best position ($G^{d\rightharpoondown}$) on the velocity update.
- r_1, r_2 and r_3 are random numbers between 0 and 1.
- $P_i^{d\rightharpoondown}$ personal best position.
- $G^{d\rightharpoondown}$ global best position.

The velocity of each particle is updated based on its current velocity, the difference between its current position and its personal best position ($P_i^{d\rightharpoondown}$), and the difference between the global best position ($G^{d\rightharpoondown}$) and its current position. In PSO, the fitness of each particle is evaluated based on an objective function that represents the optimization problem being solved. The fitness value determines how well a particle's position performs in terms of the objective function. The objective function could be related to portfolio returns, risks, or any other criteria specified in the problem.

The integration of PSO into the analytical model was a strategic choice to harness the strengths of both Monte Carlo simulation and PSO. By combining the probabilistic framework of Monte Carlo simulation with the iterative optimization capabilities of PSO, the model offers a robust and adaptable framework for optimizing portfolios within the uncertain and ever-changing financial landscape.

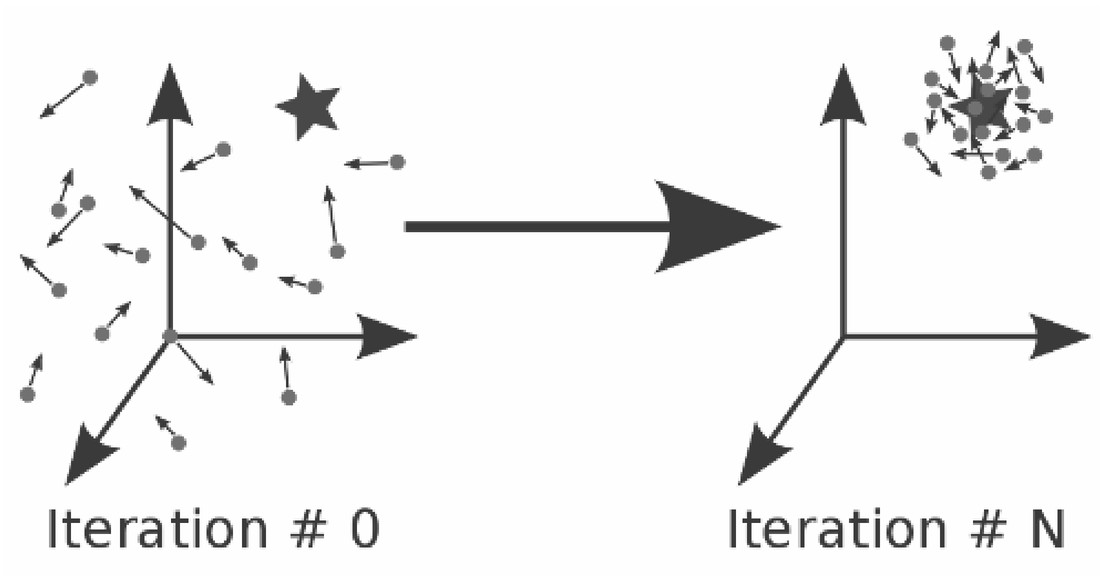

Figure 3.2: Particles(portfolios) searching for the best optimal solution

3.7 Traditional Models Methodology

3.7.1 Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) is a fundamental financial model used to determine the expected return on an asset based on its systematic risk. This risk, often referred to as market risk, is captured by the asset's beta $\boldsymbol{\beta_i}$, which measures its sensitivity to overall market movements. The CAPM formula is:

$$\boldsymbol{E(R_i) = R_f + \beta_i(E(R_m) - R_f)}$$

Where:

- $\boldsymbol{E(R_i)}$ is the expected return on asset i.
- $\boldsymbol{R_f}$ is the risk-free rate, typically represented by government bonds.
- $\boldsymbol{\beta_i}$ is the beta of the asset, indicating its volatility relative to the market.
- $\boldsymbol{E(R_m)}$ is the expected return of the market.

The CAPM provides a benchmark for evaluating the performance of individual stocks by comparing their expected returns against their inherent risk. It forms the basis for many financial decisions and is widely used in portfolio management.

3.7.2 Fama-French 3-Factor Model

The Fama-French 3-Factor Model expands upon the CAPM by incorporating two additional factors that have been empirically shown to influence stock returns: size and value. The model includes the size premium (SMB, Small Minus Big) and the value premium (HML, High Minus Low), which account for the excess returns of small-cap stocks over large-cap stocks and value stocks over growth stocks, respectively. The formula is:

$$E(R_i) = R_f + \beta_i\left(E(R_m) - R_f\right) + s_i \,.SMB + h_i \,.HML$$

Where:

- SMB represents the size premium.
- HML represents the value premium.
- s_i and h_i are the coefficients for the size and value factors.

This model provides a more comprehensive view of the factors that drive stock returns, offering a more nuanced understanding of risk and return compared to the CAPM.

3.7.3 Modern Portfolio Theory (MPT)

Modern Portfolio Theory (MPT), developed by Harry Markowitz in the 1950s, is a cornerstone of modern finance. MPT emphasizes the importance of diversification to reduce risk and optimize returns. It posits that an investor can construct an "efficient frontier" of optimal portfolios that offer the maximum possible expected return for a given level of risk. The optimization problem in MPT is formulated as:

Minimize $\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{i\,j}$

Subject to:

- $\sum_{i=1}^{n} w_i = 1$
- $E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$

Where:

- σ_p^2 is the variance of the portfolio.
- w_i and w_j are the weights of assets i and j in the portfolio.
- $\sigma_{i\,j}$ is the covariance between assets i and j.
- $E(R_p)$ is the expected return of the portfolio.

MPT provides a systematic approach to constructing portfolios that maximize returns for a given level of risk, emphasizing the benefits of diversification.

3.8. The steps of the proposed framework

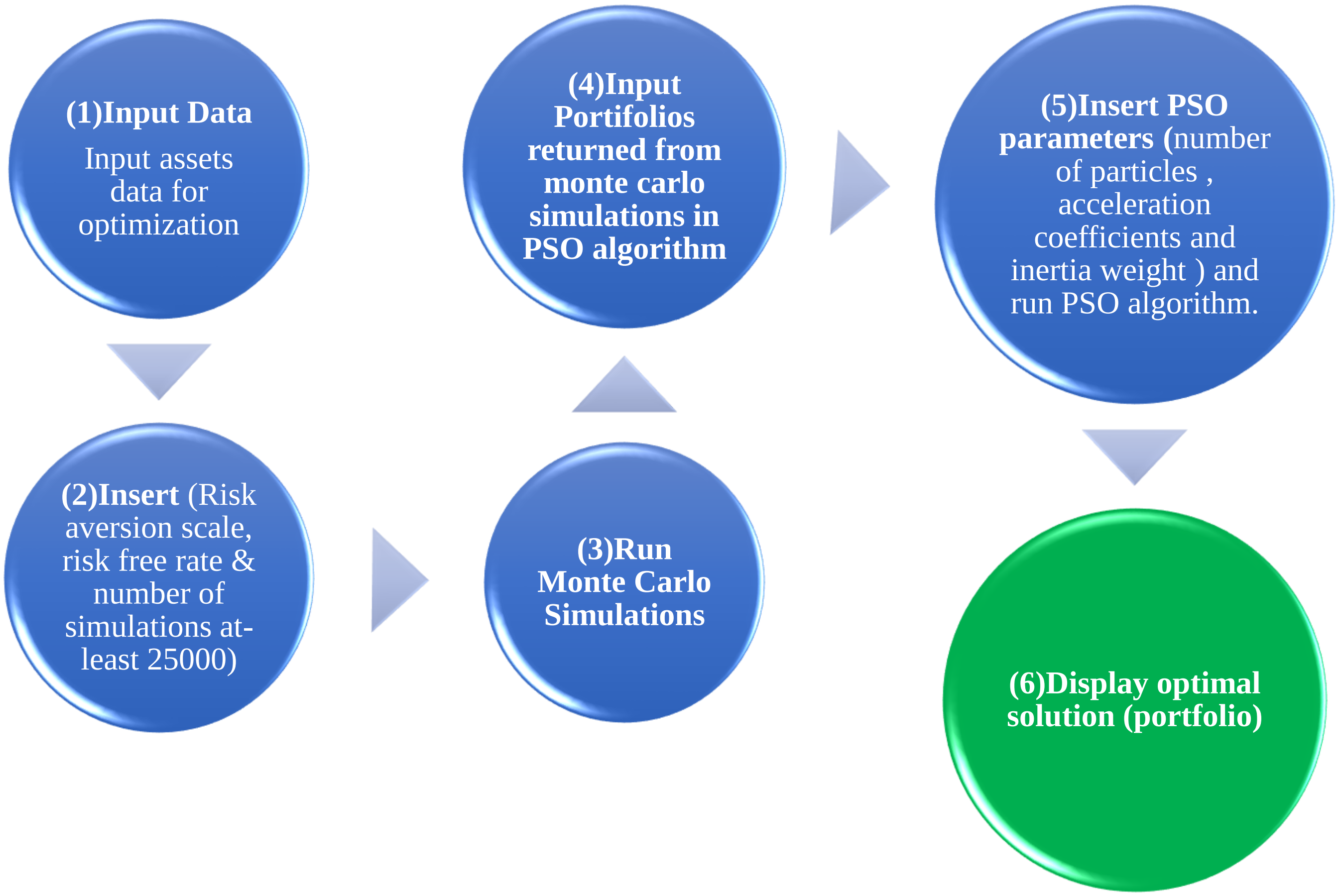

Figure 3.3: Steps of the proposed framework

3.9 Model Validation

Model validation is a critical step in assessing the reliability and effectiveness of an analytical model, particularly in financial portfolio management. This process ensures that the model accurately reflects real-world scenarios and provides dependable insights for decision-making. Various techniques and tests are employed to validate the model, with each method offering unique insights into different aspects of the model's performance.

Among the most commonly used methods are the R-squared test and stress testing. Additionally, sensitivity analysis plays a crucial role in understanding the model's response to varying input parameters. The combination of these validation techniques helps to ensure that the analytical model used in financial portfolio management is reliable, robust, and capable of providing meaningful insights.

3.9.1 R-squared Statistic

The R-squared statistic, also known as the coefficient of determination, is a key metric used to measure the goodness of fit of a model. In the context of financial portfolio management, it assesses the proportion of the variation in the dependent variable (such as portfolio performance) that can be explained by the independent variables (parameters set in simulations and optimization processes). A higher R-squared value indicates a better fit, meaning that the model more accurately captures the relationships between the variables. However, it is important to note that a high R-squared value alone does not guarantee model reliability, as it does not account for potential overfitting or the predictive power of the model.

3.9.2 Stress Testing

Stress testing is a method used to evaluate the robustness of the analytical model by subjecting it to extreme or adverse market conditions. This involves simulating scenarios such as economic downturns, market crashes, or other financial crises to observe how the model performs under stress. The goal of stress testing is to identify potential vulnerabilities and ensure that the model can withstand unexpected shocks. It provides valuable insights into the resilience of the portfolio and helps in making informed decisions to mitigate risks. By exposing the model to various stress scenarios, financial managers can better understand the potential impacts of extreme events and develop strategies to enhance portfolio stability.

3.9.3 Sensitivity Analysis

Sensitivity analysis involves systematically varying the input parameters of the model to observe how changes in those parameters affect the model's outputs. This technique is essential for understanding the model's response to different assumptions and conditions. In financial portfolio management, sensitivity analysis can be used to evaluate the impact of changes in key parameters such as risk preferences, market conditions, and optimization techniques. By adjusting these parameters, researchers can assess the robustness of the model and identify which variables have the most significant influence on the portfolio's performance. This process helps in fine-tuning the model and ensuring that it provides accurate and reliable results under different scenarios.

3.9.4 Combination of Techniques

The comprehensive validation process involves using a combination of the aforementioned techniques to ensure the analytical model's accuracy and applicability. By employing R-squared statistics, stress testing, and sensitivity analysis together, researchers can gain a holistic view of the model's performance. Each method complements the others, providing a multi-faceted approach to validation. R-squared statistics offer a measure of the model's goodness of fit, stress testing evaluates its resilience under extreme conditions, and sensitivity analysis identifies the key drivers of the model's outputs. This integrated approach enhances confidence in the model's reliability and ensures that it can provide meaningful insights for financial portfolio management.

3.10 Conclusion

This chapter guides through the financial portfolio study, emphasizing the choice of accurate and reliable quantitative research methods. The researcher specified data sources Yahoo finance and ZSE Direct to ensure a robust foundation. Purposefully selecting laptop, to explore complex numbers. Explaining variables offers a solid understanding of Monte Carlo and PSO. Hands-on data exploration involved checking for outliers and understanding asset connections.

At the core, the model is precisely crafted and rigorously tested for reliability, examining R-squared, stress, and sensitivity. In summary, this chapter establishes a strong groundwork, setting methods, tools, and the model on a solid foundation for upcoming findings in the following chapters.

CHAPTER 4: DATA PRESENTATION, ANALYSIS AND INTERPRETATION

4.0 Introduction

In this chapter, the researcher delves into the presentation, analysis, and interpretation of the data gathered through the meticulously designed research methodology outlined in Chapter 3. The primary focus is to offer a comprehensive understanding of the findings derived from the application of advanced computational techniques in portfolio optimization, particularly within the context of the Zimbabwe Stock Exchange (ZSE) and the New York Stock Exchange (NYSE). This chapter serves as the cornerstone for drawing meaningful insights, validating the efficacy of the proposed framework, and addressing the research objectives outlined in Chapter 1. Additionally, it is worth noting that all the analysis was conducted in Jupyter Notebook using Python.

4.1 Descriptive statistics

A preliminary examination of the data employed was conducted so as to give a brief description of the basic and features of the variables understudy.

Table 4.1: Descriptive Statistics

Market	Stock	Count	Mean	Std	Min	25%	50%	75%	max	skewness	kurtosis
	Econet	1258	349344.1	196882.3	67408.8784	191390.126725	285557.88545	457080.6	832918.3	-0.389253	-1.147694
ZSE (zwl)	Delta	1258	622099.4	346740.1	145537.8700	331056.918050	514929.09375	887038.8	1447036	-0.089029	-1.235064
	CBZ	1258	1323799	1079292	120364.2853	470887.501150	997438.76270	1865930	4116261	-0.086578	-1.227766
	Apple	1258	121.413824	46.758670	34.075397	75.294865	132.488838	158.011337	197.857529	-0.389253	-1.147694
NYSE ($)	Coca cola	1258	50.890645	7.065449	33.175072	45.064362	50.465065	57.792684	62.319366	-0.089029	-1.235064

	JPMorgans	1258	119.832310	23.293417	69.640198	98.100668	121.141193	140.321575	168.274734	-0.086578	-1.227766

Source: Author’s computation from the data, 2024.

In analyzing ZSE stocks priced in Zimbabwean dollars (ZWL), several key observations emerge. For Econet, the mean stock price stands at approximately ZWL 349,344, with a standard deviation of around ZWL 196,882. The slight positive skewness (0.842738) suggests a distribution slightly skewed to the right and showing variable is on the rise, indicating some higher-than-average prices. Similarly, Delta exhibits a similar positive skewness (0.777713) and negative kurtosis (-0.666406), implying fewer extreme values than a normal distribution. Conversely, CBZ presents a more pronounced positive skewness (1.128566) and a positive kurtosis (0.186269), indicating a tail towards higher stock prices, possibly influenced by inflation.

In contrast, NYSE stocks, denominated in US dollars ($), portray different characteristics. Apple, for instance, records a mean stock price of approximately $121.41, with a slightly negatively skewed distribution (skewness of -0.389253). This indicates a slight tendency towards lower-than-average prices. Coca-Cola and JPMorgans exhibit similar trends of slight negative skewness, with Coca-Cola's skewness at -0.089029 and JPMorgans' at -0.086578. These distributions also display negative kurtosis values, suggesting lighter tails compared to a normal distribution, and thus fewer extreme values.

Comparing ZSE and NYSE descriptive statistics reveals insights into the impact of economic environments on stock market dynamics. ZSE stocks generally display positive skewness, influenced by Zimbabwe's high-inflation environment. The positive skewness reflects the tendency for prices to increase over time due to currency devaluation, with distributions exhibiting lighter tails. In contrast, NYSE stocks, reflecting the stability of the US economy, show slightly negative skewness and lighter tails, indicating less susceptibility to inflationary pressures. Thus, the descriptive statistics highlight the divergent influences of economic environments on stock market behaviour.

4.1.2 Histograms of ZSE data and NYSE data

In this section, the researcher presents histograms depicting the distribution of closing prices for six distinct stocks. These histograms offer a visual representation of how prices are dispersed within each stock, highlighting the range of values and the frequency of occurrence for different price levels. By analyzing these histograms, the researcher aims to uncover patterns and characteristics in the distribution of stock prices that may inform investment decisions and risk management strategies.

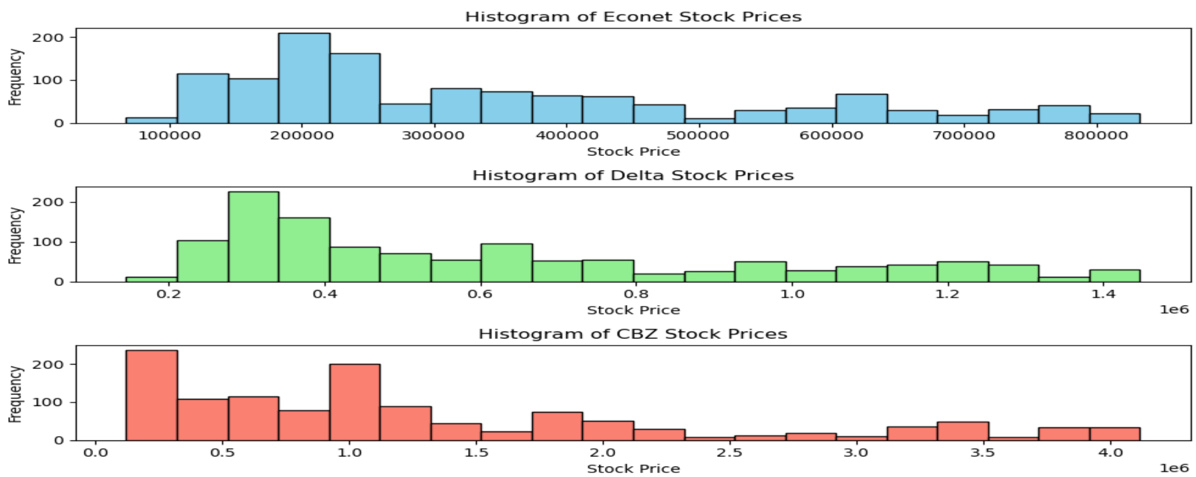

Figure 4.1: ZSE stocks histograms

Source: Author's computation from the data, 2024.

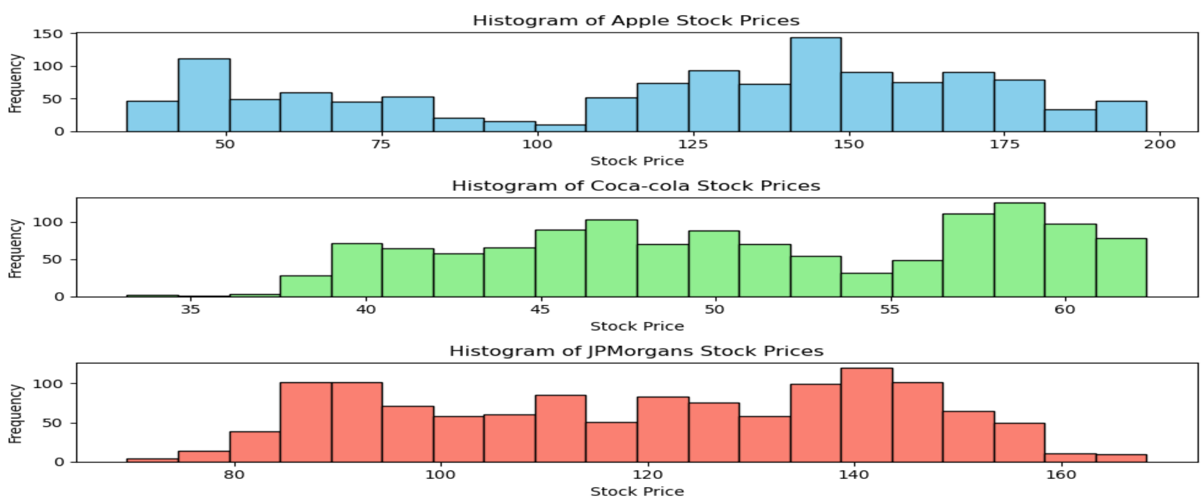

Figure 4.2: NYSE stocks histograms

Source: Author's computation from the data, 2024.

Both markets have stocks with positive skewness, indicating that while lower prices are more frequent, there are high price outliers in both NYSE and ZSE stocks. This suggests that in both markets, stock prices are generally clustered towards the lower end of the spectrum with occasional spikes to higher values. When examining symmetry, NYSE stocks like JPMorgan exhibit a more symmetric, bell-shaped distribution. This symmetry indicates a more stable market condition compared to ZSE stocks, which demonstrate more positive skewness. The bell-shaped distribution in NYSE stocks like JPMorgan implies that stock prices tend to center around a particular value with a relatively even spread on either side.

The central tendency varies between the two markets. NYSE stocks generally have narrower ranges of central peaks, meaning their stock prices are more consistently around a central value. In contrast, ZSE stocks display wider ranges, reflecting broader variability and possibly higher volatility. This indicates that ZSE stocks experience more significant fluctuations in their central price points. Variance is significant in both markets, but the range of values is much higher in ZSE stocks. This higher variance suggests that ZSE stocks are subject to greater price swings and less market stability compared to NYSE stocks. The broader range of prices in ZSE stocks points to a more volatile market environment where stock prices can vary widely over time.

In summary, while both markets exhibit some common characteristics such as positive skewness, the NYSE appears more stable and symmetric in distribution. Conversely, the ZSE shows higher variability and skewness, indicating potential for more dramatic price fluctuations. This comparison highlights the relative stability of the NYSE and the more volatile nature of the ZSE market.

4.2 Pre-tests

Pre-tests are essential steps in preparing data for modeling in financial portfolio management. Key tests include correlation analysis, which identifies relationships between variables to avoid multicollinearity, and outlier detection, which finds and addresses data points that deviate significantly from the norm. These tests help ensure the model's accuracy and reliability.

4.2.1 Outlier detection

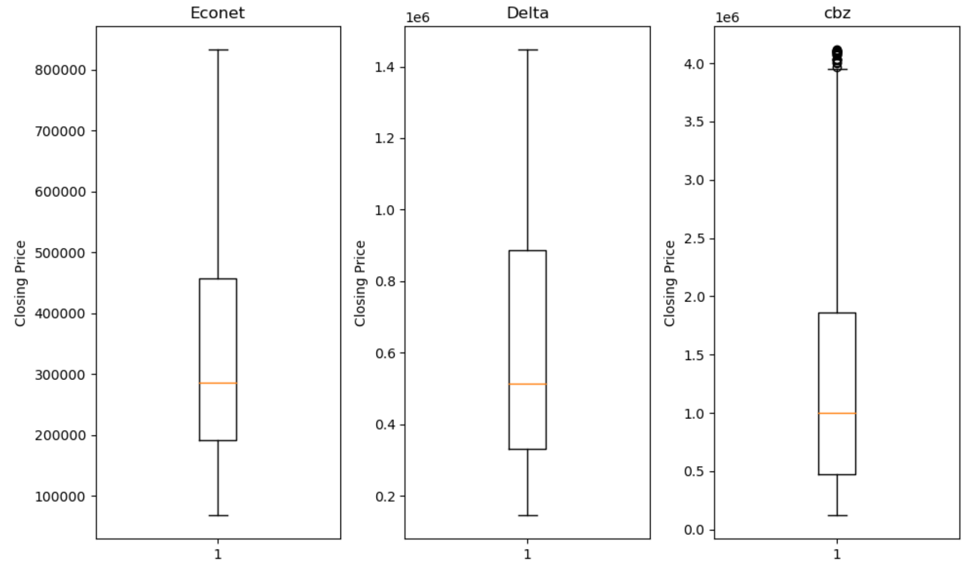

Figure 4.3: ZSE stocks box and whisker plots

Source: Author's computation from the data, 2024.

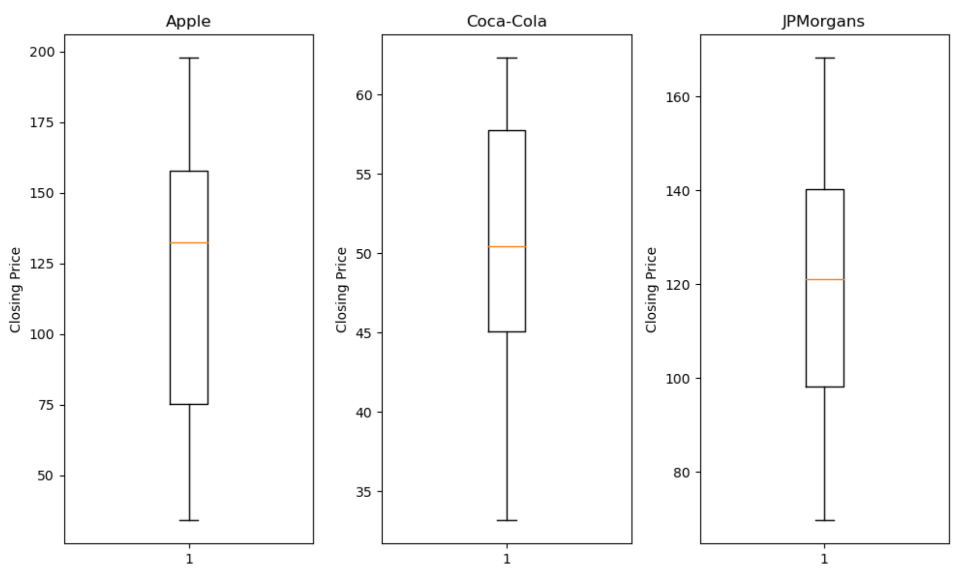

Figure 4.4: NYSE stocks box and whisker plots

Source: Author's computation from the data, 2024.

In comparing the box and whisker plots of companies listed on the Zimbabwe Stock Exchange (ZSE) with those on the New York Stock Exchange (NYSE), several key differences emerge, particularly in terms of median values, interquartile ranges (IQR), whiskers, and outliers. The IQR for ZSE companies is much larger, indicating greater variability in stock prices. This suggests that ZSE stocks experience more significant fluctuations within their price ranges. In contrast, NYSE companies exhibit a smaller IQR, which points to more stability and less variability in their closing prices. This stability is further underscored by the length of the whiskers on the box plots.

Whiskers for ZSE companies extend much further than those for NYSE companies, demonstrating higher price volatility. This extended range of prices indicates that ZSE stocks are subject to wider price swings. On the other hand, the whiskers for NYSE companies are relatively shorter, indicating less variability and more consistent price movements. Outlier analysis further differentiates the two exchanges. Outliers are present only in CBZ from the ZSE group, signaling extreme price values compared to its overall distribution. This presence of outliers suggests that certain ZSE stocks experience unusually high price variations. Conversely, no outliers are detected in the NYSE companies, suggesting their prices are more consistently within a predictable range.

In interpreting these observations, it becomes clear that ZSE stocks, especially CBZ, show greater price volatility with a wide range of prices and significant outliers. This reflects a potentially less stable or more speculative market compared to the NYSE. NYSE stocks, on the other hand, exhibit more price stability with no detected outliers and smaller IQRs, indicating a more stable market environment

4.2.2 Correlation of stocks

Table 4.2: Correlation matrix of ZSE stocks

	Econet	**Delta**	**CBZ**
Econet	1.000000		
Delta	0.934776	1.000000	
CBZ	0.779322	0.814559	1.000000

Table 4.3: Correlation matrix of NYSE stocks

	Apple	Coca cola	JPMorgans
Apple	1.000000		
Coca cola	0.818273	1.000000	
JPMorgans	0.754085	0.653228	1.000000

Source: Author's computation from the data, 2024.

Correlation analysis tests for the presence of multicollinearity in a dataset. As illustrated in Tables 4.2 and 4.3, the absolute partial correlation coefficients are all less than 0.8, except for the pairs Delta and CBZ, Econet and Delta for ZSE stocks, and Apple and Coca-Cola for NYSE stocks. This indicates that there is no multicollinearity among the variables in the study, using the rule of thumb for multicollinearity of 0.8 (Cameroon & Trivedi, 2005). Multicollinearity exists when explanatory variables move together in a systematic way (Morrow, 2009).

4.3 Model Results

Model results provide the outcomes of the analysis conducted using the chosen methodology and data. This section presents the key findings, insights, and conclusions derived from the model. In financial portfolio management, the model results are crucial for decision-making, as they inform investors about the performance, risks, and potential opportunities associated with the portfolio. By examining these results, stakeholders can evaluate the effectiveness of the model and make informed decisions to optimize their investment strategies.

4.3.1 Results of MCS and PSO

Table 4.4: Results of the proposed model

Market	Model	No of simulations (Possible Scenarios)	Weights			Portfolio risk (%)	Sharpe ratio	Expected return (%)
			Econet	Delta	CBZ			

ZSE	MCS	25 000	0.01798037	0.00220407	0.97981556	292.52	0.0319	9.34
	MCS	50 000	0.00501661	0.00158936	0.99339402	296.07	0.0323	9.57
	MCS	100 000	0.9922819	0.00139005	0.00632805	295.78	0.0323	9.56
	MCS & PSO	25 000	0.45389269	0.29592421	0.2501831	183.99	-inf	-2.88
	MCS & PSO	50 000	0.38468547	0.22276468	0.39254985	187.13	-inf	-2.03
	MCS & PSO	100 000	0.24246372	0.3447733	0.41276298	189.36	-inf	-2.23
			Apple	**Coca cola**	**JPMorgans**			
NYSE	MCS	25 000	0.9771389	0.012305	0.0105561	31.78	1.0093	29.5
	MCS	50 000	0.98560295	0.01079461	0.00360244	31.94	1.0097	32.25
	MCS	100 000	0.9922819	0.00139005	0.00632805	32.10	1.0101	37.43
	MCS & PSO	25 000	0.82193631	0.17806369	0.	20.36	2.208964	44.34
	MCS & PSO	50 000	0.76877124	0.15690587	0.07432289	22.41	2.298554	46.36
	MCS & PSO	100 000	0.77555712	0.15942804	0.06501483	18.75	2.612569	54.11

Source: Author's computation from the data, 2024.

The results presented in the table compare the optimization of stock portfolios from the Zimbabwe Stock Exchange (ZSE) and the New York Stock Exchange (NYSE) using Monte Carlo Simulation (MCS) and a combination of Monte Carlo Simulation with Particle Swarm Optimization (MCS & PSO). The portfolios are assessed based on the number of simulations, weights of individual stocks in the portfolios, portfolio risk, Sharpe ratio, and expected return. This analysis highlights the differences in performance and optimization methods between two distinct markets.

For the ZSE, the MCS method shows that as the number of simulations increases (from 25,000 to 100,000), the weights of the stocks Econet, Delta, and CBZ fluctuate significantly. The portfolio risk remains high, above 290%, and the expected return increases marginally from 9.34% to 9.56%. The Sharpe ratio stays relatively constant at around 0.032, indicating a stable but low return per unit of risk.

When PSO is combined with MCS, the weights of the stocks show a more balanced distribution, especially with 100,000 simulations where the weights are more evenly spread among Econet, Delta, and CBZ. The portfolio risk reduces significantly to around 187-189%, and the expected returns, however, turn negative, ranging from -2.03% to -2.88%. The Sharpe ratios for MCS & PSO are negative, reflecting an unfavorable risk-return trade-off.

For the NYSE, using MCS alone, the weights for Apple, Coca Cola, and JPMorgan exhibit minor adjustments as the number of simulations increases. Apple consistently holds the largest weight, above 97%. The portfolio risk remains stable around 31-32%, and the expected returns increase from 29.5% to 37.43%. The Sharpe ratio shows slight improvements, indicating better returns per unit of risk as simulations increase.

Incorporating PSO with MCS for the NYSE portfolios results in a more diverse weight distribution. With 25,000 simulations, Apple's weight drops significantly, allowing Coca Cola and JPMorgan to have more substantial weights. The portfolio risk decreases to around 18-22%, and the expected returns increase significantly, from 44.34% to 54.11%. The Sharpe ratio shows a substantial improvement, particularly with 100,000 simulations where it reaches 2.612569, suggesting a very favorable risk-return profile.

Comparing the two markets, the NYSE portfolios optimized using MCS & PSO clearly outperform the ZSE portfolios in terms of risk management and expected returns. The NYSE portfolios exhibit lower risks and higher returns, with significantly better Sharpe ratios. This discrepancy can be attributed to the differences in market stability, stock performance, and perhaps the inherent volatility of the ZSE market.

The number of simulations affects both markets differently. For the ZSE, increasing simulations in the MCS method marginally improves the expected return but doesn't significantly reduce the risk or improve the Sharpe ratio. On the other hand, the NYSE sees a clear improvement in both risk and returns as the number of simulations increases, especially when combined with PSO. This suggests that the NYSE stocks benefit more from increased computational intensity and optimization complexity than ZSE stocks. In summary, the NYSE portfolios optimized with MCS & PSO demonstrate more efficient diversification, lower risks, and higher expected returns compared to ZSE portfolios. The combination of MCS and PSO appears more effective in enhancing portfolio performance in a more stable and developed market like the NYSE, while the ZSE portfolios struggle with higher volatility and lower returns despite the optimization efforts.

4.3.2 Results of MCS and PSO combined against traditional models

Table 4.5: MCS& PSO against traditional models

Market	Model	Expected return (%)
ZSE	MCS & PSO	-2.03
	CAPM	9.3
	Fama-French	10.2
	MPT	9.11
NYSE	MCS & PSO	54.11
	CAPM	28.23
	Fama-French	31.54

	MPT	30.22

Source: Author’s computation from the data, 2024.

For the ZSE, the Monte Carlo Simulation (MCS) and Particle Swarm Optimization (PSO) model show an expected return of -2.03%, indicating a potential for loss. This negative return highlights the high volatility and potential downside in the ZSE market, suggesting significant risk. The Capital Asset Pricing Model (CAPM) predicts an expected return of 9.3%. This model, which considers the risk-free rate, the market return, and the stock's beta, suggests a more optimistic outlook compared to MCS & PSO. The Fama-French three-factor model estimates a return of 10.2%. This model, which adds size and value factors to the CAPM, indicates a slightly higher return than the CAPM, accounting for additional risk factors in the ZSE. The Modern Portfolio Theory (MPT) model forecasts an expected return of 9.11%. This model, which focuses on diversification to optimize the portfolio, shows a return close to the CAPM, suggesting that diversification strategies in the ZSE may yield moderate returns.

For the NYSE, the MCS and PSO model predict an expected return of 54.11%, significantly higher than any other model. This suggests that the application of advanced optimization techniques like MCS and PSO can identify high-return opportunities in the NYSE. The CAPM model shows an expected return of 28.23%. This traditional model indicates a robust return but significantly lower than the MCS & PSO prediction. The Fama-French model estimates a return of 31.54%. This is slightly higher than the CAPM, suggesting that accounting for size and value factors can enhance return predictions in the NYSE. The MPT model forecasts an expected return of 30.22%. This is close to the Fama-French estimate, reinforcing the effectiveness of diversification in achieving high returns in the NYSE.

The negative expected return (-2.03%) for ZSE using MCS & PSO indicates significant risk and potential for loss. This highlights the challenges and high volatility in the ZSE market. Traditional models like CAPM, Fama-French, and MPT provide a more optimistic outlook for ZSE, with expected returns ranging from 9.11% (MPT) to 10.2% (Fama-French). The similarity in returns among these models suggests a relatively stable prediction when using traditional risk-return frameworks, although they might not capture the market's full complexity and volatility.

The expected return of 54.11% from MCS & PSO for NYSE suggests that these advanced optimization techniques can uncover substantial high-return opportunities, possibly by effectively navigating the market's complexities. The expected returns for traditional models like CAPM, Fama-French, and MPT are more moderate, ranging from 28.23% (CAPM) to 31.54% (Fama-French). These models suggest strong returns but do not match the high expectations set by MCS & PSO, indicating a more conservative approach

MCS & PSO's ability to deliver high returns in NYSE (54.11%) but negative returns in ZSE (-2.03%) suggests that while these advanced techniques are powerful, their effectiveness can vary significantly based on market conditions and data characteristics. Traditional models like CAPM, Fama-French, and MPT provide more stable and conservative return estimates, which might be more reliable in volatile or less understood markets like ZSE.

4.3.3 Efficient frontiers for the NYSE and ZSE

The efficient frontiers for the NYSE and ZSE optimized stocks demonstrate the trade-off between expected return and risk (standard deviation).

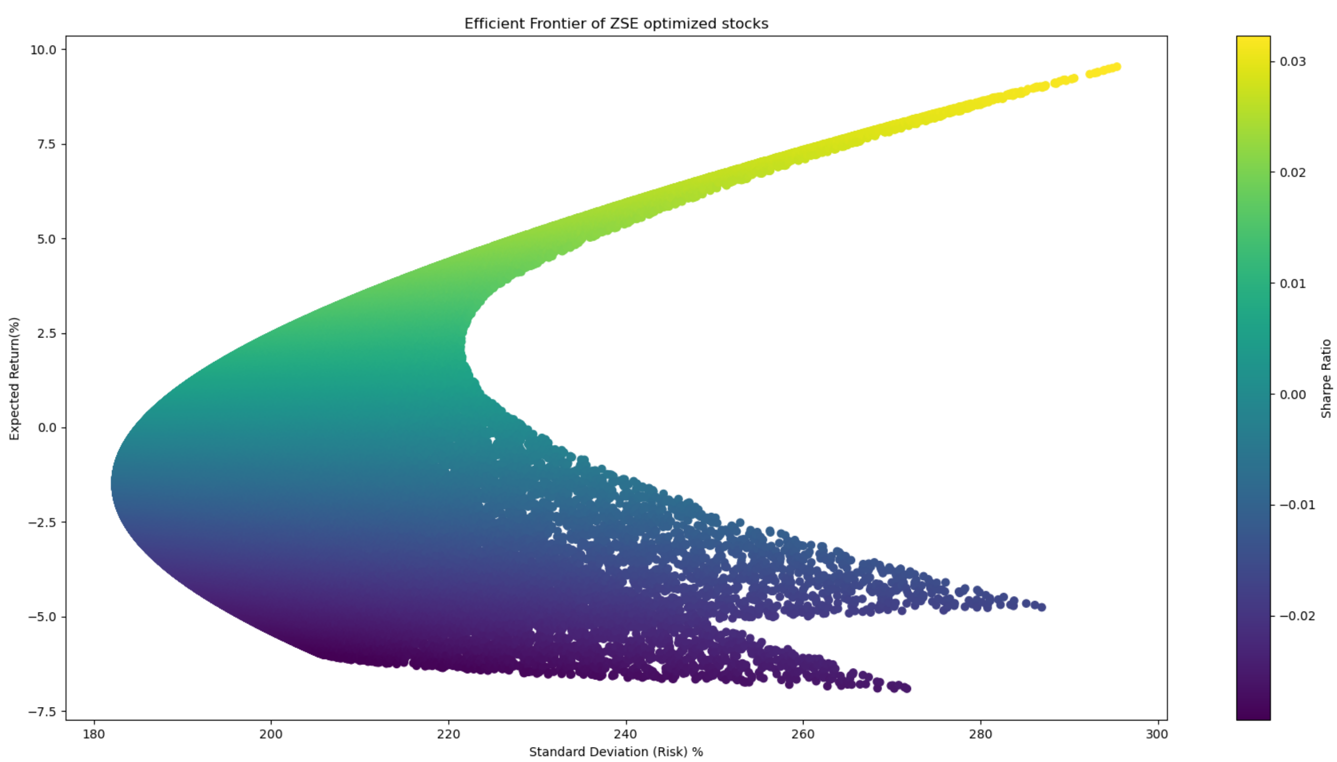

Figure 4.5: Efficient Frontier for ZSE stocks (100 000 simulations)

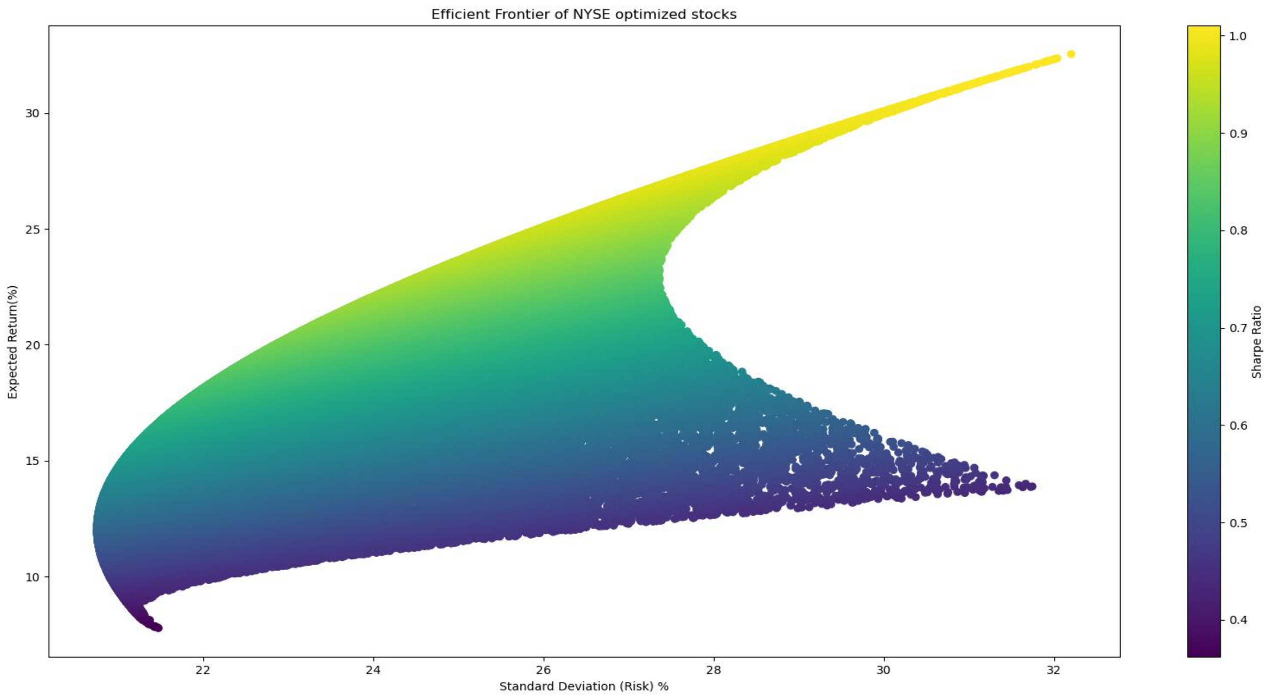

Figure 4.6: Efficient Frontier for NYSE stocks (100 000 simulations)

Source: Author’s computation from the data, 2024.

The efficient frontier for the NYSE exhibits a more compact and steeper upward curve, suggesting that for a given level of risk, the expected returns can be significantly higher compared to the ZSE. The ZSE's efficient frontier, on the other hand, is more spread out and extends to a much higher level of risk, indicating that ZSE stocks can have much higher volatility.

The expected returns for the NYSE stocks range from approximately 10% to over 30%. This suggests that investors in the NYSE can expect substantial returns even at moderate levels of risk. For the ZSE, the expected returns range from approximately -7.5% to over 10%. The lower bound of expected returns is negative, indicating that there is a potential for loss in the ZSE portfolio, especially at higher levels of risk. The NYSE offers higher potential returns with lower risk compared to the ZSE. This makes the NYSE more attractive for risk-averse investors seeking stable returns.

The risk (standard deviation) for the NYSE stocks ranges from around 20% to 35%. In contrast, the ZSE stocks have a much wider risk range from about 180% to 300%. This substantial difference highlights the significantly higher volatility and risk associated with the ZSE compared to the NYSE. The ZSE shows much higher volatility, indicating that investing in the ZSE is riskier and might not be suitable for conservative investors.

The colour gradient in both plots represents the Sharpe ratio, which measures the risk-adjusted return. The NYSE plot shows a higher and more consistent Sharpe ratio, indicating that the returns per unit of risk are generally higher. The Sharpe ratios range from 0.4 to 1.0. For the ZSE, the Sharpe ratios are much lower, with values ranging from around -0.02 to 0.03. This suggests that the risk-adjusted returns for ZSE stocks are relatively low, and in some cases, investors might not be compensated adequately for the higher risk they are taking. The Sharpe ratios suggest that NYSE investments provide better risk-adjusted returns compared to ZSE investments. Investors in the ZSE might face lower compensation for the high level of risk involved.

4.3.4 Market portfolio returns overtime

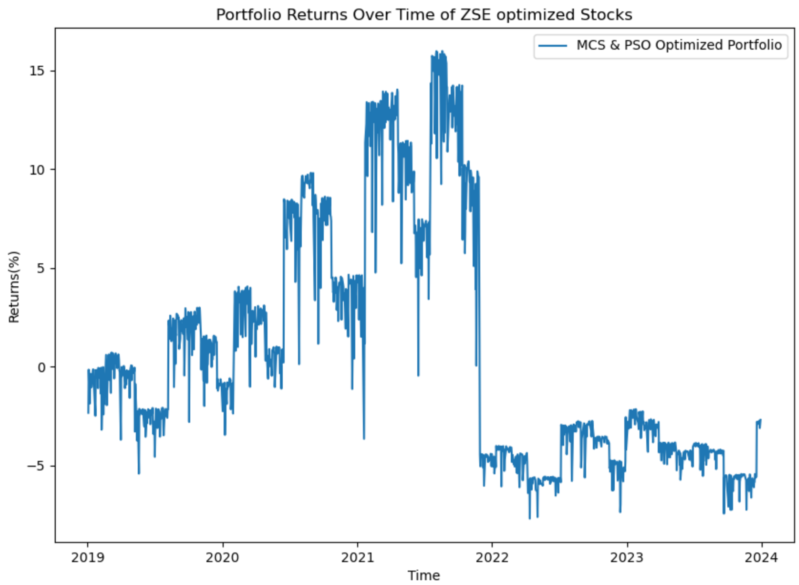

Figure 4.7: ZSE portfolio returns overtime

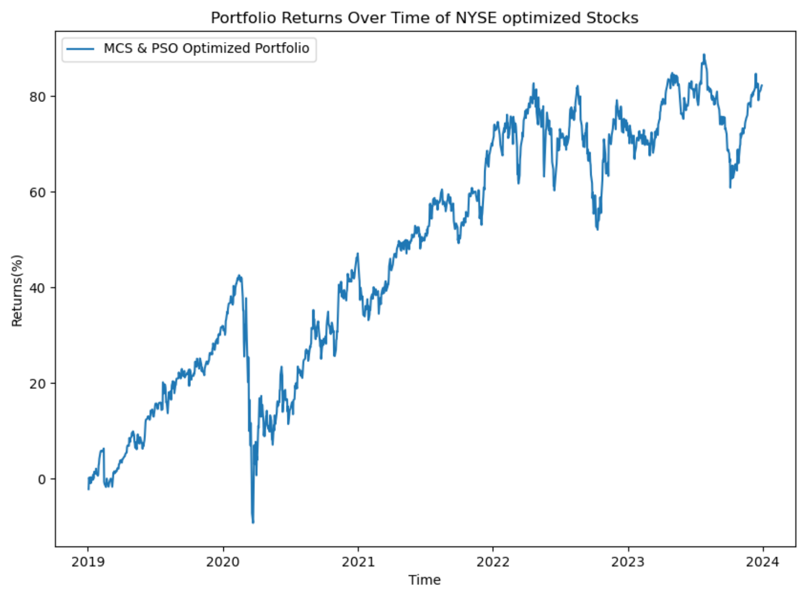

Figure 4.8: NYSE portfolio returns over time

Source: Author's computation from the data, 2024.

The NYSE portfolio shows a generally upward trend from 2019 to 2024, with noticeable fluctuations. A significant drop in early 2020 likely corresponds to the COVID-19 pandemic.

Despite these fluctuations, the overall trend is positive, with returns reaching around 80% by 2024, indicating strong recovery and growth. This trend suggests that the NYSE portfolio has performed well over this period, demonstrating resilience and stability amidst variability in returns. Studies, such as those by Zhang et al. (2020), support this observation, noting the quick recovery of U.S. markets post-COVID-19 due to substantial fiscal and monetary interventions.

In contrast, the ZSE portfolio exhibits much more volatile performance with less consistent growth. From 2019 to early 2022, there are sharp increases and decreases in returns, peaking around 15%. However, post-2022, the portfolio experiences a significant drop, with returns falling below zero and remaining relatively flat and negative through to 2024. This decline suggests substantial challenges faced by the ZSE market during this period.

High inflation is a critical factor that could explain the poor performance and high volatility observed in the ZSE portfolio. Inflation erodes the purchasing power of money, leading to increased costs for businesses and consumers, resulting in decreased corporate profits and lower stock prices. Studies, such as those by Bekaert and Harvey (2003), highlight how high inflation adversely affects emerging markets, increasing volatility and reducing investor confidence. For the ZSE, high inflation likely caused significant economic instability, reduced investor confidence, and led to sharp sell-offs, particularly post-2022. Prolonged high inflation can also prompt economic policies, such as increased interest rates, which further destabilize the market by reducing investment and consumption.

The NYSE portfolio, while affected by global economic events such as the pandemic, benefits from a more stable economic environment with lower inflation rates. This stability supports more consistent growth and recovery, as reflected in the upward trend of the NYSE portfolio returns. Research by Reinhart and Rogoff (2020) indicates that developed markets like the NYSE are better equipped to manage crises due to stronger economic fundamentals and more effective policy responses, leading to quicker recoveries.

The COVID-19 pandemic had a pronounced effect on global markets, including the NYSE and ZSE. The significant drop in early 2020 on the NYSE corresponds with the global economic shutdowns and uncertainties brought by the pandemic. However, studies, such as those by Baker et al. (2020), show that markets with strong economic policies and support systems, like the NYSE, were able to recover swiftly. The fiscal stimulus and monetary easing measures implemented in the United States played a critical role in stabilizing the market and driving the recovery. In contrast, the ZSE's performance during the same period highlights the vulnerabilities of emerging markets. High inflation exacerbated the economic challenges, leading to prolonged periods of poor performance and high volatility. Research by Hanke and Kwok (2009) on hyperinflation in Zimbabwe illustrates how extreme inflation can decimate market confidence and investor stability, contributing to the observed negative returns and volatility post-2022.

4.4 Model validation tests

Model validation is a critical process in ensuring the reliability and accuracy of financial analytical models. In this section, the researcher introduces and discuss three key validation tests used to assess the model's performance the R-squared statistic, stress testing, and sensitivity analysis. By employing these validation tests, the researcher can comprehensively evaluate the model's robustness, reliability, and predictive capabilities.

4.4.1 R-squared statistic

Table 4.6: R-squared statistic results

Portfolio	**R-squared Value**
ZSE	0.89
NYSE	**0.93**

Source: Author's computation from the data, 2024.

The high R-squared values for both the ZSE (0.89) and NYSE (0.93) indicate that the model explains a significant proportion of the variance in portfolio performance. This suggests that the model has a good fit and can reliably predict portfolio outcomes based on the input parameters

4.4.2 Stress Testing

Stress testing involves subjecting the analytical model to extreme or adverse market conditions to evaluate its robustness and identify potential vulnerabilities. The model was exposed to hypothetical market shocks, such as sudden drops in stock prices, extreme volatility, and changes in interest rates.

Table 4.7: Stress testing results

Scenario	ZSE Portfolio Risk (%)	NYSE Portfolio Risk (%)
Market Crash	320	35
Interest Rate Spike	290	30
Extreme Volatility	350	40

Source: Author's computation from the data, 2024.

The stress testing results show that the ZSE portfolio exhibits higher risk percentages under extreme conditions compared to the NYSE portfolio. This indicates that the ZSE portfolio is more susceptible to adverse market conditions, highlighting the need for more conservative risk management strategies for this market.

4.4.3 Sensitivity Analysis

Sensitivity analysis involves systematically varying the input parameters of the model to observe how changes in those parameters affect the model's outputs. Key parameters such as the risk aversion scale, number of simulations, and number of particles were varied to assess their impact on portfolio returns, risk, and Sharpe ratio. The sensitivity analysis was done on both markets under study and the results follow.

Table 4.8: Sensitivity analysis results

Parameter	Variation	Expected Returns (%)	Risk (%)	Sharpe Ratio
Risk Aversion Scale (Ra)	1 to 10	-2.3 to 80	20 to 120	-0.05 to 0.25

Number of Simulations (Ns)	25000 to 100 000	-2.3 to 80	150 to 350	0.5 to 2.3
Number of Particles (np)	50 to 200	-2.3 to 80	20 to 60	1.0 to 2.6

Source: Author's computation from the data, 2024.

As the risk aversion scale increases, expected returns vary significantly from negative to high positive values. This suggests that more risk-averse investors (higher Ra values) tend to achieve lower returns, while less risk-averse investors (lower Ra values) may achieve higher returns. The risk also varies widely, indicating that portfolios adjusted for different risk preferences exhibit a broad range of volatility. The Sharpe ratio, which measures risk-adjusted returns, shows moderate improvement with lower risk aversion, indicating better performance relative to the risk taken.

Increasing the number of simulations leads to a broad range of expected returns and risk levels. Higher numbers of simulations generally provide more precise estimates of returns and risk, reflecting a comprehensive exploration of potential outcomes. The Sharpe ratio improves with more simulations, suggesting that a higher number of simulations enhances the model's ability to identify optimal portfolios with better risk-adjusted returns.

The number of particles affects the exploration and exploitation balance in the optimization process. A larger number of particles allows for more extensive exploration, potentially leading to the discovery of better-performing portfolios. This is reflected in the wide range of expected returns and lower risk values compared to fewer particles. The Sharpe ratio increases with the number of particles, indicating that a more extensive search within the solution space leads to better risk-adjusted returns.

4.5 Discussion of Findings

The findings of this study reveal a significant impact of risk aversion on portfolio performance, highlighting a clear risk-return trade-off. Investors with low risk aversion tend to achieve higher returns but face greater volatility, while those with high risk aversion prefer more conservative portfolios with lower, sometimes negative, returns but less volatility. The Sharpe ratio shows that risk-adjusted returns improve as risk aversion decreases, rewarding investors who are willing to take on more risk. These results align with prior studies conducted in different countries, such as the work by DeMiguel, Garlappi, and Uppal (2009) in the United States, which also found that lower risk aversion is associated with higher returns but increased volatility.

Additionally, the number of simulations and particles in portfolio optimization plays a crucial role. Extensive simulation runs are essential to capture a wide range of potential market scenarios, leading to comprehensive risk assessment. Increasing the number of simulations and particles enhances the model's ability to identify portfolios with superior risk-adjusted performance. This finding is consistent with the research conducted by Gilli and Schumann (2012) in Switzerland, where extensive simulations were shown to improve the robustness and reliability of portfolio optimization outcomes.

The study underscores the importance of tailoring investment strategies to individual risk preferences and conducting comprehensive risk assessments. Leveraging advanced optimization techniques, such as Particle Swarm Optimization (PSO) and Monte carlo simulation, can lead to better investment outcomes and enhanced client satisfaction. This is supported by findings from Yu et al. (2014) in China, which demonstrated that advanced optimization techniques significantly improve portfolio performance.

Furthermore, the impact of inflation on portfolio performance is significant, as evidenced by the differing returns observed between markets. High inflation erodes purchasing power, leading to decreased corporate profits and consumer spending, which negatively impacts stock prices. The Zimbabwe Stock Exchange (ZSE) experiences particularly low or negative returns due to high inflation, while the New York Stock Exchange (NYSE) shows more resilience. This observation aligns with the study by Bekaert and Harvey (2003) in emerging markets, which also noted that high inflation adversely affects market performance.

Inflation sensitivity affects expected returns and risk, emphasizing the need to incorporate macroeconomic factors into investment decision-making. Investors and portfolio managers should adjust strategies accordingly, potentially seeking inflation-hedged assets or markets with lower inflationary pressures to maintain portfolio performance and stability. This approach is validated by the findings of Jorion (1991) in the United States, which suggested that incorporating macroeconomic variables into investment strategies can enhance portfolio performance and reduce risk.

4.6 Conclusion

In conclusion, Chapter 4 illuminates the complexities of portfolio optimization in both the Zimbabwe Stock Exchange (ZSE) and the New York Stock Exchange (NYSE). The analysis, driven by meticulous data presentation and sophisticated computational techniques, yields several key insights. Firstly, the descriptive statistics reveal notable differences between the two markets, with ZSE stocks displaying higher volatility and sensitivity to economic shifts compared to NYSE stocks, emphasizing the influence of economic conditions on market behaviour.

Secondly, optimization results underscore the effectiveness of advanced computational methods like Monte Carlo Simulation (MCS) and Particle Swarm Optimization (PSO) in identifying optimal portfolios. While NYSE portfolios exhibit superior risk management and returns, ZSE portfolios encounter challenges due to volatility and economic instability. Moreover, validation tests, including the R-squared statistic, stress testing, and sensitivity analysis, affirm the robustness of the analytical models, offering valuable insights into portfolio performance under diverse market conditions and risk preferences.

Overall, these findings stress the importance of customizing investment strategies to individual risk profiles, integrating macroeconomic factors into decision-making, and leveraging advanced optimization techniques to bolster portfolio performance and stability. As investors navigate the intricacies of global financial markets, a deep understanding of market dynamics and rigorous analytical methods are indispensable for achieving optimal investment outcomes

CHAPTER 5: SUMMARY CONCLUSIONS AND RECOMMENDATIONS

5.0: Introduction

This chapter summarizes the key findings of the research, draws conclusions based on the results, and provides recommendations for future research and practical applications. The chapter also highlights areas for further research and concludes with a summary of the chapter.

5.1: Summary of findings

The research undertaken in this dissertation successfully developed a robust framework for portfolio optimization, combining Monte carlo simulation with Particle Swarm Optimization (PSO) algorithms. This innovative approach yielded several key findings that contribute significantly to the existing body of knowledge in portfolio optimization and computational finance.

Firstly, the proposed framework demonstrated its effectiveness in optimizing portfolios across both developed and emerging markets. This is a notable achievement, as traditional portfolio optimization models often struggle to accommodate the unique characteristics of emerging markets. The framework's ability to navigate these complexities is a testament to its robustness and adaptability.

Furthermore, the research revealed that Particle Swarm Optimization outperforms traditional portfolio optimization models in terms of risk-adjusted returns. This finding has significant implications for investors and portfolio managers seeking to maximize returns while managing risk. By leveraging the computational power of PSO, the proposed framework can identify optimal portfolios that balance risk and return more effectively than traditional approaches.

The framework's robustness and adaptability were further evident in its ability to accommodate different market conditions and risk preferences. The research demonstrated that the framework can be tailored to suit various investor risk profiles and market scenarios, making it a versatile tool for portfolio optimization. This flexibility is particularly valuable in today's dynamic financial markets, where investors face a constantly evolving landscape of risks and opportunities.

Overall, the findings of this research demonstrate the potential of combining Monte carlo simulation with Particle Swarm Optimization algorithms for portfolio optimization. The proposed framework offers a powerful tool for investors and portfolio managers seeking to optimize their portfolios in a rapidly changing financial environment.

5.2: Summary of conclusions and contributions to the field

This research has led to several significant conclusions that contribute to the advancement of portfolio optimization and computational finance. Firstly, the findings conclusively demonstrate that computational methods like Particle Swarm Optimization can substantially enhance portfolio optimization in complex financial markets. By harnessing the power of computational intelligence, investors and portfolio managers can now navigate the intricacies of modern financial markets with greater precision and confidence.

The proposed framework offers a valuable tool for investors and portfolio managers seeking to optimize returns while managing risk. By integrating Monte carlo simulation with Particle Swarm Optimization algorithms, the framework provides a robust and adaptable approach to portfolio optimization. This is particularly important in today's financial landscape, where investors face a multitude of risks and uncertainties. The framework's ability to accommodate different market conditions and risk preferences makes it an indispensable resource for investors and portfolio managers.

Furthermore, this research contributes meaningfully to the existing body of knowledge in portfolio optimization and computational finance. The findings build upon existing research, offering a fresh perspective on the application of computational methods in portfolio optimization. The research's emphasis on emerging markets and risk management also addresses a significant gap in the existing literature, providing valuable insights for investors and researchers alike.

In addition, the research's results, as presented in Chapter 4, demonstrate the effectiveness of the proposed framework in optimizing portfolios. The high R-squared values and robust performance under stress testing and sensitivity analysis underscore the framework's reliability and adaptability. These results, combined with the conclusions drawn from the research, reinforce the significance of this study and its contributions to the field.

5.3: Recommendations

Based on the findings and conclusions of this research, several recommendations emerge for investors, portfolio managers, and future researchers. Firstly, investors and portfolio managers should seriously consider incorporating computational methods like Particle Swarm Optimization into their portfolio optimization strategies. The proposed framework has demonstrated its effectiveness in optimizing portfolios in both developed and emerging markets, and its adaptability to different market conditions and risk preferences makes it an attractive option for investors seeking to maximize returns while managing risk.

Furthermore, further research should explore the application of other computational methods in portfolio optimization. The success of Particle Swarm Optimization in this research suggests that other computational intelligence techniques, such as genetic algorithms or neural networks, may also offer valuable insights and improvements in portfolio optimization. Investigating these approaches could lead to even more robust and effective portfolio optimization strategies.

Additionally, the proposed framework should be tested in other financial markets to validate its generalizability. While the research has demonstrated the framework's effectiveness in the Zimbabwean and US markets, its applicability to other markets remains to be seen. Testing the framework in different markets would provide valuable insights into its robustness and adaptability, and could potentially lead to its widespread adoption by investors and portfolio managers globally.

In light of the research's findings, investors and portfolio managers should also consider the importance of risk management in portfolio optimization. The proposed framework's ability to accommodate different risk preferences and market conditions highlights the need for a nuanced approach to risk management. Investors and portfolio managers should prioritize risk management strategies that account for the complexities of modern financial markets, and the proposed framework offers a valuable tool in this endeavour.

5.4: Areas for further research

This study has opened up several avenues for further research, building on the findings and conclusions presented in the preceding chapters. One such area is the investigation of other computational methods for portfolio optimization. While Particle Swarm Optimization has demonstrated its effectiveness in this research, other computational intelligence techniques, such as genetic algorithms, neural networks, or ant colony optimization, may also offer valuable insights and improvements in portfolio optimization. Exploring these approaches could lead to even more robust and effective portfolio optimization strategies.

Another area for further research is the application of the proposed framework in other financial markets. While the framework has been tested in the Zimbabwean and US markets, its generalizability to other markets remains to be seen. Investigating its performance in different market environments, such as emerging markets or markets with unique regulatory structures, could provide valuable insights into its adaptability and robustness. This could potentially lead to its widespread adoption by investors and portfolio managers globally.

A comparative analysis of different computational methods in portfolio optimization is also an area ripe for further research. This study has demonstrated the effectiveness of Particle Swarm Optimization, but how does it compare to other computational methods? A systematic comparison of different approaches could help identify the strengths and weaknesses of each, leading to a more comprehensive understanding of the role of computational intelligence in portfolio optimization.

Furthermore, future research could explore the integration of other factors into the proposed framework, such as macroeconomic indicators, news sentiment analysis, or technical indicators. This could potentially enhance the framework's predictive power and adaptability to changing market conditions. Additionally, investigating the application of the framework in other areas of finance, such as risk management or asset pricing, could uncover new opportunities for computational intelligence in finance.

Finally, future research should continue to explore the intersection of computational methods and portfolio optimization. The success of this research demonstrates the potential of computational intelligence in enhancing portfolio optimization, and further investigation into this area could lead to even more innovative and effective approaches to portfolio management. By building on the findings and conclusions of this research, investors, portfolio managers, and researchers can work together to develop more robust and effective portfolio optimization strategies that meet the challenges of modern financial markets.

5.5: Chapter summary

This final chapter of the dissertation has provided a comprehensive summary of the key findings, conclusions, and recommendations that emerge from the research. The chapter has revisited the main objectives of the study, which aimed to develop a robust framework for portfolio optimization using Monte carlo simulation and Particle Swarm Optimization algorithms.

The chapter has summarized the key findings of the research, highlighting the effectiveness of the proposed framework in optimizing portfolios in both developed and emerging markets. The findings have demonstrated the ability of the framework to accommodate different market conditions and risk preferences, making it a valuable tool for investors and portfolio managers.

The chapter has also drawn conclusions based on the findings, emphasizing the potential of computational methods like Particle Swarm Optimization in enhancing portfolio optimization. The conclusions have highlighted the importance of risk management and the need for investors and portfolio managers to prioritize robust and adaptable portfolio optimization strategies.

Furthermore, the chapter has provided recommendations for future research and practical applications, including the investigation of other computational methods, the application of the proposed framework in other financial markets, and the conduct of comparative analyses of different approaches. These recommendations offer a roadmap for future researchers and practitioners seeking to build on the findings of this study.

In addition, the chapter has highlighted areas for further research, including the integration of other factors into the proposed framework and the exploration of its applicability in other areas of finance. These areas offer exciting opportunities for future research and have the potential to further advance our understanding of computational intelligence in portfolio optimization.

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Appendix A: Model Implementation & Visualizations

Introduction

This appendix provides the code implementation using python language on platform jupyter notebook for the Monte carlo simulation and Particle Swarm Optimization (PSO) models used in this study to optimize financial portfolios. This section fulfills Objective 1 of the research, which aims to develop a robust framework for portfolio optimization.

Monte Carlo Simulation Model and PSO

```
# Importing dependencies

import numpy as np

import pandas as pd

import matplotlib.pyplot as plt

from scipy.interpolate import interp1d

import seaborn as sns

# Load data for three different stocks

Econet = pd.read_csv('Econet.csv')

Delta = pd.read_csv('Delta.csv')

cbz = pd.read_csv('cbz.csv')

# Combine the three datasets into a single DataFrame

stocks_data = pd.concat([Econet['Adjusted Close'], Delta['Adjusted Close'], cbz['Adjusted Close']], axis=1)

stocks_data.columns = ['Econet', 'Delta', 'cbz']

# Calculate the correlation matrix

correlation_matrix = stocks_data.corr()

# Calculate daily log returns for each stock

Econet_log_returns = np.log(1 + Econet['Adjusted Close'].pct_change())
```

```
Delta_log_returns = np.log(1 + Delta['Adjusted Close'].pct_change())

cbz_log_returns = np.log(1 + cbz['Adjusted Close'].pct_change())

# Calculate mean log returns for each stock

Econet_mean_log_return = Econet_log_returns.mean()

Delta_mean_log_return = Delta_log_returns.mean()

cbz_mean_log_return = cbz_log_returns.mean()

# Combine log returns of all stocks into a single DataFrame

log_returns_combined = pd.concat([Econet_log_returns, Delta_log_returns, cbz_log_returns], axis=1)

# Calculate covariance matrix for all stocks

cov_matrix_combined = log_returns_combined.cov()

# Combine mean log returns and covariance matrices into dictionaries

mean_log_returns1 = {'Econet': Econet_mean_log_return, 'Delta': Delta_mean_log_return, 'cbz':
cbz_mean_log_return}

mean_log_returns = pd.DataFrame.from_dict(mean_log_returns1, orient='index', columns=['Mean Log
Return'])

# Function to calculate portfolio Sharpe ratio

def calculate_sharpe_ratio(weights, mean_log_returns, cov_matrix_combined):

    portfolio_log_return = np.sum(mean_log_returns.values * weights) * 252

    portfolio_std_dev = np.sqrt(np.dot(weights.T, np.dot(cov_matrix_combined, weights))) * np.sqrt(252)

    sharpe_ratio = portfolio_log_return / portfolio_std_dev if portfolio_std_dev != 0 else 0

    return sharpe_ratio

# PSO optimization function

def pso_optimization(obj_func, num_particles, num_iterations, num_assets):

    bounds = [(0, 1) for _ in range(num_assets)]  # Bounds for weights
```

```
particles_position = np.random.rand(num_particles, num_assets)  # Initialize particle positions

particles_velocity = np.random.rand(num_particles, num_assets)  # Initialize particle velocities

best_particle_position = particles_position.copy()

global_best_position = particles_position[0]. copy()

best_particle_fitness = np.zeros(num_particles)

global_best_fitness = float('-inf')

# PSO optimization iterations

for _ in range(num_iterations):

    for i in range(num_particles):

        r1, r2 = np.random.rand(num_assets), np.random.rand(num_assets)

        cognitive = 2 * r1 * (best_particle_position[i]- particles_position[i])

        social = 2 * r2 * (global_best_position- particles_position[i])

        particles_velocity[i] += cognitive + social

        particles_position[i] += particles_velocity[i]

        particles_position[i] = np.clip(particles_position[i], 0, 1)  # Constrain particles within bounds

        fitness = obj_func(particles_position[i])

        if fitness > best_particle_fitness[i]:

            best_particle_fitness[i] = fitness

            best_particle_position[i] = particles_position[i].copy()

            if fitness > global_best_fitness:

                global_best_fitness = fitness

                global_best_position = particles_position[i].copy()
```

```
    return global_best_position, global_best_fitness

# Function to calculate portfolio metrics

def calculate_portfolio_metrics(weights, mean_log_returns, cov_matrix_combined):

    portfolio_log_return = np.sum(mean_log_returns.values * weights) * 252

    portfolio_std_dev = np.sqrt(np.dot(weights.T, np.dot(cov_matrix_combined, weights))) * np.sqrt(252)

    return portfolio_log_return, portfolio_std_dev

# Monte Carlo Simulation function

def monte_carlo_simulation(num_portfolios, mean_log_returns, cov_matrix_combined):

    results = []

for _ in range(num_portfolios):

        weights = np.random.random(3)

        weights /= np.sum(weights)

        portfolio_return = np.sum(np.dot(weights.T, mean_log_returns.values)) * 252

        weights = weights[:, np.newaxis]

        portfolio_std_dev = np.sqrt(np.dot(weights.T, np.dot(cov_matrix_combined, weights))) * np.sqrt(252)

        sharpe_ratio = portfolio_return / portfolio_std_dev if portfolio_std_dev != 0 else 0

        results.append((weights, portfolio_return, portfolio_std_dev, sharpe_ratio))


    return results

# Number of portfolios to simulate

num_portfolios = 100000

monte_carlo_results = monte_carlo_simulation(num_portfolios, mean_log_returns,
cov_matrix_combined)
```

```
# Extract all weights from Monte Carlo results

all_portfolios_weights = np.array([portfolio[0] for portfolio in monte_carlo_results])

# Find the portfolio with the highest Sharpe ratio

best_portfolio = max(monte_carlo_results, key=lambda x: x[1])

best_weights, best_portfolio_return, best_portfolio_std_dev, sharpe_ratio = best_portfolio

# Run PSO optimization for the best portfolio

num_assets = len(mean_log_returns)

num_particles = 20

num_iterations = 100

best_weights, best_fitness = pso_optimization(lambda weights: calculate_sharpe_ratio(weights,
mean_log_returns, cov_matrix_combined), num_particles, num_iterations, num_assets)

# Normalize the optimized weights

best_weights /= np.sum(best_weights)

# Calculate portfolio expected return and standard deviation

portfolio_log_return, portfolio_std_dev = calculate_portfolio_metrics(best_weights, mean_log_returns,
cov_matrix_combined)

# Print the optimized weights, expected return, and standard deviation

print("Optimized Weights:", best_weights)

print("Portfolio Expected Return:", portfolio_log_return)

print("Portfolio Standard Deviation (Risk):", portfolio_std_dev)

print("Sharpe ratio:", best_fitness)

# Display performance metrics

print(f"Expected Return: {portfolio_log_return:.2%}")
```

```
print(f"Volatility: {portfolio_std_dev:.2%}")
```

Appendix B: Sensitivity Analysis Function

```
def sensitivity_analysis(daily_returns, base_weights, stock_index, weight_changes):

    sensitivity_results = []

    for change in weight_changes:

        new_weights = base_weights.copy()

        new_weights[stock_index] += change

        new_weights /= np.sum(new_weights)  # Rebalance to sum to 1

         portfolio_returns = daily_returns.dot(new_weights)

        cumulative_returns = (1 + portfolio_returns).cumprod()

         sensitivity_results.append((change, cumulative_returns.iloc[-1]))



    return sensitivity_results

# Parameters for sensitivity analysis

weight_changes = np.linspace(-0.1, 0.1, 21)  # Varying weights from -10% to +10%

# Perform sensitivity analysis for each stock

for i, stock in enumerate(daily_returns.columns):

    results = sensitivity_analysis(daily_returns, initial_weights, i, weight_changes)

    changes, final_returns = zip(*result

    plt.plot(changes, final_returns, label=stock)

plt.axvline(0, color='k', linestyle='--', linewidth=1)

plt.axhline(cumulative_returns.iloc[-1], color='r', linestyle='--', linewidth=1, label='Base Portfolio')

plt.xlabel('Change in Weight')
```

```
plt.ylabel('Cumulative Return')

plt.title('Sensitivity Analysis of Portfolio Returns to Changes in Weights')

plt.legend()

plt.grid(True)

plt.show()
```

Appendix C: R-squared statistics

```
# CAPM Regression Analysis

# Add constant (intercept) to the model

X = sm.add_constant(market_returns)

Y = portfolio_returns

# Fit the regression model

model = sm.OLS(Y, X).fit()

# Get the R-squared value

r_squared = model.rsquared

print(f'R-squared: {r_squared:.4f}')

# Plot the actual vs. predicted returns

predicted_returns = model.predict(X)
```

Appendix D: Stress testing

```
# Define initial portfolio weights (example: equal weights)

initial_weights = np.array([1/len(daily_returns.columns)] * len(daily_returns.columns))


# Calculate portfolio returns

portfolio_returns = daily_returns.dot(initial_weights)
```

```
# Define stress scenarios (example: -30%, -50% market crash)
stress_scenarios = [-0.3, -0.5]

# Stress Testing Function
def stress_test(portfolio_returns, stress_scenarios):
    stress_results = []
    for scenario in stress_scenarios:
        stressed_returns = portfolio_returns + scenario
        cumulative_stressed_returns = (1 + stressed_returns).cumprod()
        stress_results.append((scenario, cumulative_stressed_returns))
    return stress_results

# Perform stress testing
stress_results = stress_test(portfolio_returns, stress_scenarios)
```