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BACHELOR OF SCIENCE EDUCATION HONORS DEGREE IN MATHEMATICS

**AN INVESTIGATION ON LEARNER'S ERRORS AND MISCONCEPTIONS ON
OPERATIONS WITH DIRECTED NUMBERS AT FORM ONE LEVEL.**

BY

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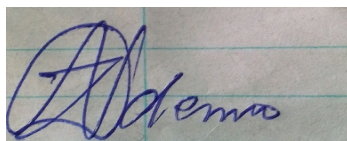
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Dedication

I dedicate this dissertation to my wife Chipu and my three children, Leeroy, Lennox and Leeanna for their patience, love and warmth during the time I needed them most in carrying out this dissertation. I also dedicate this dissertation to God by whose providence I managed to complete it.

Abstract

This study aims to investigate the common errors and misconceptions made by form one learners when performing operations with directed numbers. The research utilized a combination of qualitative and quantitative methods to gather and analyse data, including tasks, questionnaires and interviews conducted on 32 learners and 3 teachers from one school. The findings revealed that learners commonly struggled with addition and subtracting integers, often demonstrating misconceptions related to zero, number lines and the rules of operations. The study identified several error patterns, including “strategic errors”, “calculation errors”, “misconception (direct translation error)”, “conceptual errors” and “procedural errors”. The results suggest that learner’s difficulties with directed numbers largely attributed to inadequate understanding of fundamental concepts and insufficient practice. The study recommends that teachers emphasize conceptual understanding and provide opportunities for learners to engage in meaningful practice to overcome these challenges.

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CHAPTER 1: THE PROBLEM AND ITS SETTINGS

1.1 INTRODUCTION

This chapter focused on the background to the study, statement of the problem, research objectives and research questions. The chapter proceeded to examine the significance of the study, limitations, delimitations and assumption of the study. Thereafter, key words were also defined and the chapter was concluded with a summary of the major points.

1.2 BACKGROUND TO THE STUDY

This study is about the errors and misconceptions learners have when they do tasks on operations on directed numbers. It was noted that learners' at junior and senior level at high schools in Zimbabwe make mistakes and errors when solving mathematical problems. This may be because of misunderstanding of questions by learners or misrepresentation and misinterpretation of a mathematics question, or it can be because of carelessness by learners (Ryan, 2007). Learners perceive directed numbers as a difficult topic to master hence this triggered this research to find out the errors and misconceptions learners made when working out on directed numbers. Directed numbers touches the lives of many in real life, as our lives comprises of negatives and positive fortunes, our businesses had not been spared out as we realize profits(positive) and losses(negative). Furthermore socially we've the positive and negative events that we encounter in real life, for instance the birth of a family member and death of a member respectively is a practical application of directed numbers in our lives. Therefore there is need for learners to understand how to operate directed numbers to have a better understanding of their economic and social environments.

The errors that learners make can be constant, unsystematic and systematic. Constant errors are recurring incorrect responses to a specific number combination. Cox (1975) defines systematic errors as when there is a repeatedly occurring incorrect response that evident in a specific algorithmic computation. Learners make errors constantly and repeatedly which shows that there is a misconception in their thinking about a particular concept. Hence, systematic errors are symptomatic of a faulty line of thinking (luneta & Makonye, 2010). Learners both at junior and senior levels face difficulties in operating with directed numbers. This resulted in low pass rate of mathematics at ordinary level in the past five years yielding to 9, 4% in 2019, 15% in 2020, 13% in 2021, 10% in 2022 and slightly increased to 17, 6% in 2023. It is the

trust of this research to report on the errors and misconceptions that learners' have on operations with directed numbers

1.3 PURPOSE OF THE STUDY

The purpose of this study is to explore the errors and misconceptions that learners have when answering questions on operating with directed numbers.

1.4 STATEMENT OF THE PROBLEM

Most teachers rarely investigate the errors and misconceptions that learners do when solving problems involving directed numbers. If the student fail, they re-teach the topic and give the learners more questions to practice to master the rules of directed numbers. But all these efforts have not yielded much in trying to remedy these challenges. This study seeks to identify these errors learners make and their misconceptions about directed numbers.

1.5 RESEARCH QUESTIONS

The research will be guided by the following research questions;

1. What are the types of errors that the learners have when answering questions on operating integers?
2. What misconceptions do they have regarding their answers?
3. What are the strategies employed to reduce errors and have correct misconceptions in working directed numbers?

1.6 RESEARCH OBJECTIVES

The study seeks to fulfil the following objectives;

1. To identify the types of errors made by learners when operating with directed numbers.
2. To identify the misconceptions learners do have regarding their answers.
3. To identify strategies to reduce errors and remedies for misconceptions in operating directed numbers.

1.7 SIGNIFICANCE OF THE STUDY

Mathematics teachers and learners will be the beneficiaries of this study. This study will help the mathematics teachers to improve their teaching and learning strategies whilst it improves the grades of mathematics learners. Nelson & Powell (2018) argues that the study of errors

does not just involve investigation of pupil's correct, partly correct and incorrect steps towards finding a solution, but also implies the study of the best practices for remediation. Mathematics teachers and learners will be the beneficiaries of this study. This study will help the mathematics teachers to improve their teaching and learning strategies whilst it improves the grades of mathematics learners. Furthermore, errors analysis provides unique learning opportunities for students, allowing them to learn from their mistakes and grow in their mathematical thinking. Basing on the above information, the researcher deemed it fit to study the errors and misconceptions in learning addition and subtraction of directed numbers at form one where the concept of negative numbers was introduced at secondary schools.

1.8 DELIMITATIONS OF THE STUDY

Geographically, the study was confined to one day rural secondary school in Shamva District in Mashonaland Central Province in Zimbabwe. Conceptually, the study was limited to errors and misconceptions made by form one learners in operating integers. Methodologically, the study used mixed methods of study confined to 35 participants.

1.9 LIMITATIONS OF THE STUDY

The research might be affected by the following;

- It may not be possible to control all extraneous variables. The health, mood and life experience of the participants may influence their reactions
- Experimental research can not specify 'why' the outcome occurred
- The researcher may make error/s in the validity of the project
- Shortage of funds to widen the area
- Lack of cooperation from participants who may be suspicious about the motives behind the research.

To do away with some of the limitations of the study, the researcher will;

- Clearly outline the purpose of the study to clear any suspicion that will be in the participants. This will also enhance cooperation

- Try to engage the participants of the study in a day they seem to be happy and healthy especially when administering the test and questionnaire. The mood of the participant is very important as it may influence his/her performance.

1.10 DEFINITIONS OF KEY TERMS

Error

An error is a deviation from accuracy or correctness (Liu et al, 2017)

Mistakes made by learners when solving problems that may be caused by carelessness and misinterpretation of symbols or text

Misconception

Misconception is a false or mistaken view, opinion or attitude (Collins English Dictionary, 2012)

Misconception heavily influence a student's ability to learn and retain mathematics in class and, in many cases, are the cause of great confusion to the student.

Directed numbers

A directed number is a number that can be either positive or negative, depending on its position on the number line. Numbers to the right of zero are positive while numbers to the left of zero are negative and zero is neutral.

Learner

A learner refers to an individual who is engaged in the process of acquiring knowledge or skills through study, experience, or teaching. Julie Cotton (1995)

Mathematics teacher

An instructor who holds a professional qualification to teach mathematics (Clark-Wilson et al, 2014)

Test

(Larsen et al, 2008) defined test is systematic procedure for observing and describing one or more characteristics of person with the aid of a numerical or category system.

At school, performance is usually measured using tests.

1.11 CHAPTER SUMMARY

The chapter introduced the topic under research and highlighted the possible errors faced when operating with directed numbers. Major assumptions of this study were outlined, thereby giving specific objectives of the study. Some important key terms were defined according to how they will be used in the research. The next chapter will dwell on literature review.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

This chapters will review the existing literature on learners' errors and misconceptions on operations with directed numbers at form one level. The literature was reviewed on persistent errors and misconceptions in directed numbers from both empirical and theoretical perspectives. The purpose of this review is to gain a comprehensive understanding of current state of research in this area, identify the gaps in the literature and establish the theoretical frame for the proposed investigation.

2.2 ERRORS AND MISCONCEPTIONS: THEORETICAL PERSPECTIVE

An error is regarded as a mistake committed in the process of solving mathematical problems algorithmically, procedurally or by any other method (Mulungye et al, 2016). Errors may also be defined as mistakes made by learners, which can occur for a number of reasons ranging from a data entry or calculation error to a lack of conceptual understanding (Holmes et al, 2013). It is through experience that as teachers we have identified learner's errors at any step in the implemented method or when an incorrect answer is given. The incorrect answer may result either from the proper method incorrectly carried out or an application of an incorrect procedure to a question that can be solved using method (conceptual). Frequent or persistent errors which cannot be corrected through typical instruction are caused by misconceptions, however, such errors need supplementary intervention in order for learners to acquire correct strategies (Boot et al, 2014; Makonye, 2016; Makonye & Fakude, 2016). Ojose (2015), on the other hand, defines misconceptions as "misunderstandings and misinterpretations based on incorrect meanings". Holmes et al (2013) define a mathematical misconception as part of learners' framework that is not consistent with mathematical community which leads such a learner to providing incorrect answers. Central to the above descriptions is resonance of both errors and misconceptions, thus showing misconceptions as being inconsistent with the scientific meaning of concepts and traceable to the persistent resultant errors.

2.3 ERRORS AND MISCONCEPTIONS: EMPIRICAL PERSPECTIVE

Misconceptions are unavoidable stages in the learning process (Moru & Mathunya, 2022). The result that the misconception leads to persistent errors has been shared by the several studies, for example, Iddrisu et al (2017) and Moru and Mathunya (2022). In the study conducted by Aydin-Guc and Aygun (2021) and Kim et al (2016) it was reported that the students made

reversal error where they map the order of words from the question. These results resemble those of Iddrisu et al (2017) where the participants did not mind the subtrahend and the minuend in the question. In this case, the student has the misconception of direct translation (Kim et al, 2016; Mathunya, 2022). The misconception of direct translation is where the students match the order of words that appear in the question (Kim et al, 2016). In the study of Makonye and Fakude (2016), the students carried out operations in the reverse order by matching the terms in the given words order. Based on the feedback from the learners' task and interview of Makonye and Fakude (2016), where the students were asked to subtract -12 from -10, the minuend is -10 and the subtrahend is -12. However, the students took minuend as -12 and subtrahend as -10 and gave the answer as -2. The students did not consider the minuend and subtrahend in the question, and this overlaps with the finding of the studies by Kim et al (2016) and Rababaha et al (2020).

Misconceptions originate from inappropriate generalization of previously learnt content (Aydin-Guc & Aygun, 2021; Im & Jitendra, 2020). This is a misconception of overgeneralization. In dealing with integers, the students make an error called sign error. In the study by Khalid and Embong (2020), the students were given $-6-(-2)$; in this study the students gave their answer as 4. Makonye and Fakude (2016) reported that the students were asked to solve $-7-(+3)$, and the students gave their answer as 4. In both studies, the students ignored the negative sign involved. Misconceptions are the underlying wrong beliefs and principles in one's mind that causes errors (Makonye, 2016). Oliver (1989) opines that misconception of interference is a situation where the previously learnt content interferes with the new knowledge. Rule mixes error, which is a case where the students mixes the rules learnt. For example, in the study of Khalid and Embong (2020), the students were given $-2-6$ and their answer was 8 because negative and negative make positive. Multiplication of integer's knowledge interferes with the addition of integer knowledge.

2.4 THE THEORETICAL FRAMEWORK

This study is underpinned by the Piagetian constructivism theory. Olusegun (2015) describes constructivism as a learning theory found in psychology which explains how people might acquire knowledge. According to Olusegun (2015), this theory suggest that humans construct knowledge and meaning from their experiences. Furthermore, constructivism views learners as actively constructing knowledge rather than passively received it from the environment (Davis et al, 2020). The constructivist view also sees learners as using prior knowledge as a

foundation for building new knowledge. In this view, learners do not come to class as empty vessels; rather, they bring with them their prior knowledge from previous classes and other interactions with the environment. For instance, form one learners had prior knowledge of addition and subtraction of positive numbers from their grade seven classes which is prior knowledge to build on negative numbers introduced at form one level.

Furthermore, related directly to the process of knowledge acquisition is the idea of schema. According to Luneta and Makonye (2013), schemas are valuable intellectual tools in human memory that allow for the generalization, synthesis, storage and retrieval of similar experiences. Schemas are developed on refined through the complementary processes of assimilation and accommodation (Zhiqing, 2015). The process by which individuals incorporate new experience into an already existing schema is termed assimilation (Bhattacharjee, 2015). When a new object is assimilated into an old schema, the schema gets refined (Zhiqing, 2015). Accommodation is a process of restructuring or modifying the existing schema to incorporate new information (Bhattacharjee, 2015; Zhiqing, 2015). When a learner fails to connect the old and new knowledge, this leads to the formation of misconceptions as a result of misinterpretation and poor linkage of assimilation and accommodation (Luneta & Makonye, 2013, Ojose, 2015).

2.5 STRATEGIES TO REDUCE MISCONCEPTIONS

Reducing and correcting misconceptions in adding and subtracting directed numbers is an important aspect of mathematics education. Several literature reviews have explored the strategies employed to achieve this goal. Fong & Ng (2018) asserts that, emphasizing conceptual understanding rather than just procedural fluency. Students understand the meaning of positive and negative numbers and their relationships rather than just memorizing the rules. Moreover, through visual representations students learn to represent numbers on a number line and graphs, making connections between visual and abstract. According to Smith & Johnson (2020), connecting directed numbers to real-world application is another way of marrying theory with practice which enables students learn mathematics beyond abstract context by applying its concepts to solve real-life challenges. In addition to the above, teacher professional development to enhance subject matter knowledge and pedagogical content knowledge help to reduce and correct misconceptions. Teacher deepen their understanding of directed numbers, becoming more confident and accurate in their knowledge, which they can then pass to students. Pedagogical content knowledge enable teachers to learn effective ways to teach

directed numbers, including strategies for addressing common misconceptions, creating engaging lessons, and using appropriate visual aids. Cheng & Wang (2019) recommends practice with feedback and collaborative learning among students as a way to reduce and correct misconceptions. Students identifies misconceptions by receiving feedback on their work, revealing areas where they may be struggling. So students learn to reflect on their own understanding, recognizing areas where they need more practice and support. These studies also found that a combination of these strategies was more effective than any one strategy alone, and that teacher professional development and support were crucial for effective implementation.

2.6 LITERATURE GAPS

Despite the existing body of research, there are still gaps in understanding of learners' errors and misconceptions in operating with directed numbers, including; limited research on form one learners specifically, inadequate exploration of impact of teaching methods on learner's understanding, and a lack of longitudinal studies to track learners' progress over time.

2.7 SUMMARY

This literature review provides a comprehensive overview of current knowledge on learners' errors and misconceptions in operating with directed numbers at form one level. It emphasizes the need to further research to address the existing gaps and to develop effective teaching strategies to support learners' understanding of directed numbers.

CHAPTER 3: RESEARCH METHODOLOGY

3.1 INTRODUCTION

This chapter focused on how the research was carried out and the steps followed during data collection. It consists of research design, and research methodology used in the study. The chapter also outlines the population and sampling strategy in which the researcher looks at the sampling techniques used, samples used and who participated during the research session. The researcher also explored data collection procedures, as well as data presentation and analysis. Research ethics were highlighted as well as validity and reliability of the instruments before summarizing the chapter.

3.2 RESEARCH DESIGN

The design of the research is a plan for data collection. We can minimize the effects of error variance by choosing a design that permits us to calculate the contribution of one or more nuisance variables (Myers et al., 2013). In other words, the design indicates how the research is set up, what happens to participants and what methods of data collection will be used. The research design will enable the researcher to anticipate what appropriate research decisions should be made to maximise validity of the results.

3.2.1 RESEARCH PARADIGM

A research paradigm is the set of common beliefs and agreements shared between scientist about how problems should be understood and addressed (Kivunja & Kuyini, 2017). In this research the Positivism research paradigm is going to be used. As a philosophy, positivism adheres to the view that only “factual” knowledge gained through observation (the senses), including measurement, is trustworthy. In positivism studies the role of the researcher is limited to data collection and interpretation in an objective way. In these types of studies research findings are usually observable and quantifiable.

Moreover, in positivism studies the researcher is independent from the study and there are no provisions for human interests within the study. Poni (2014), argue that as a general rule, positivist studies usually adopt deductive approach, whereas inductive research approach is usually associated with a phenomenology philosophy. Moreover, positivism relates to the viewpoint that researcher needs to concentrate on facts, whereas phenomenology concentrates on the meaning and has provision for human interest.

3.2.2 RESEARCH DESIGN

Qualitative and quantitative research approaches has been chosen in this research. In quantitative research, focus is on gathering numerical data and generalising it across groups to explain phenomena (Punch, 2013). Of the challenges in answering tasks on addition of directed numbers with no preconceived hypothesis to be tested, this approach was more preferable than the others because one would be able to describe systematic observation on the study (McMillan & Schumacher, 2014). After that, errors on written tasks on directed numbers were noted. Basing on those errors, in-depth interviews (Mears, 2012) were held with learners' thinking behind those errors. As this research was seeking to find the errors and misconceptions in addition and subtraction of directed numbers, in-depth interviews were viewed to be helpful to gain better insight into the categories of errors and misconceptions that learners exhibited, and find out the causes thereof.

3.2.3 POPULATION AND SAMPLING

A population in statistics and other areas of mathematics is a discrete group of people, animals or anything that can be identified by at least one common characteristic for the purpose of data collection and analysis (Punch, 2013). In this regard, the population consists of all form one learners doing mathematics. All being form ones and mathematics learners, data can be collected for analysis as they have the same characteristics.

Form one learners were chosen to be the participants of this study since the introduction of directed numbers is done at this level. Looking at the time data will be collected; form one learners are the ideal group as they will be always available at the school with minimum disturbances.

A sample is a fraction of the total population to be used in each study. It is a representative of the whole population (Taherdoost, 2016). In this study, the sample consists of thirty-two (32) form one learners and three (3) mathematics teachers at Madziwa high school in Shamva District. Since form one learners at the school total to 180, a sample of thirty two (32) learners constitute eighteen (18) percent of the form one learners' population at the school. This is more than the expected sample size for a quantitative study, that is, ten percent of the population is required for a quantitative study. The probability sampling procedure, called simple random sampling, will be used so that selection of participants depends purely on chance not on personal bias.

3.2.3.1 SIMPLE RANDOM SAMPLING

Simple random sampling is defined as a way of sampling in which participants of a given populace have the same possibility of being selected. In other words, simple random sampling means that every member of the sample is selected from the group of population in such a manner that the probability of being selected for all members in the study group is the same (Jawale, 2012). To get an ideal random sample, the population should be finite, and the members of the populace should be determined and listed to prevent bias. In this regard, the population consists of one hundred and eighty (180) form one learners at a Madziwa High School in Shamva District as indicated by class registers.

To come up with a sample of thirty-two (32) participants, eighty (8) cards written ‘YES’ and thirty-seven (37) written ‘NO’ will be put in a container. Since there are four (4) classes, each class should have eight (8) participants, as such pupils will be asked to pick a card in each of the classes. Those who would have picked a card written ‘YES’ will constitute the participants of the study. Since mathematics teachers at the school are only three, they automatically become part of the sample to determine the accuracy of results and so they are purposively selected (Etikan & Bala, 2017).

3.2.3.2 ADVANTAGES OF SIMPLE RANDOM SAMPLING

Simple random sampling ensures that every element in the sampling frame has an equal chance of being included in the sample. Therefore, it is free from bias and prejudice. Another advantage is that simple random sampling secures a spread of sample across the entire population. Simple random sampling ensures that every individual has equal probability of being selected, thus ensuring a representative sample. Furthermore, it needs minimum knowledge of the study group of population in advance (Etikan & Bala, 2017).

3.2.3.3 DISADVANTAGES OF SIMPLE RANDOM SAMPLING

Simple random sampling is associated with some limitations. White (2015) notes that it can carry large errors from the same sample size than that are found in stratified sampling. This means that random sampling can pick pupils with the same performance. Unlike in stratified sampling where pupils could be grouped according to performance, that is, as best performers, average performers and bad performers. The process of selection can interact with a hidden periodic trait within the population such that the representative from the sample is compromised.

3.3 DATA COLLECTION INSTRUMENTS

Data collection instruments are tools used for collecting information and data required to find solutions to the problem under study. A research instrument is defined as a testing device for measuring given phenomena such as pen and paper test, questionnaire, interviews or a set of guidelines for observation. In other words, it refers to the way in which data is going to be collected. In this study, the researcher is going to use a pen and paper test, interviews and a questionnaire. These instruments will be used to gather data for the purpose of the study and are deemed necessary since the research is quantitative in nature (Zohrabi, 2013).

Written tasks on addition and subtraction of directed numbers were given to learners and their scripts analysed. Marks obtained by pupils in the tasks given will be recorded. Learners who performed poorly in written tasks were interviewed. The researcher needed to understand learners' reasoning within and beyond their written responses on the tasks. This provide learners with the opportunity to say more than they wrote and also check whether the researcher had interpreted their responses correctly or learners had understood their own thinking. Learners will be given questionnaires to respond to after the written tasks. Mathematics teachers will also get their questionnaires. Questionnaires will be collected, and data grouped according to the responses given by participants. The data will then be analysed.

3.3.1 QUESTIONNAIRE

A questionnaire is a list of questions for which answers are being sought. A questionnaire can be defined as a set of printed or written questions with a choice of answers devised for the purpose of a survey or statistical study. For this study, structured questionnaire with closed ended questions will be used. A closed ended questionnaire is the one in which the respondents can fill in answers such as yes or no, agree or disagree. This makes it easy for pupils who are participants to this study to answer quickly and it will simplify the data analysis process. A questionnaire is objective in nature (Taherdoost, 2016).

3.3.1.2 ADVANTAGES OF QUESTIONNAIRE

Wilson (2013) asserts that the questionnaire makes it possible to give similar or standardised questions to the respondents. Hunter (2012) notes that many participants can be reached in a short period of time thereby allowing the collection of vast amounts of information. The results can usually be quickly and easily quantified by either a researcher or using a software package. Questionnaires also reduce the chances of evaluation bias as the same questions are asked to

all respondents. Also, administration is comparatively inexpensive and easy when gathering data even from a large sample. Data on a questionnaire is easy to represent on different methods of representing the data such as bar charts, pie charts and line graphs. The data will show a continuous distribution.

3.3.1.3 DISADVANTAGES OF A QUESTIONNAIRE

Questionnaires have their own limitations. A major setback is that the meaning of the question can be distorted. Patten (2016) argues that people may read differently into each question and therefore respond basing on their own interpretation of the question. Non-structured questionnaires may restrict the respondents to specify answers.

The researcher will, however, ensure that the return rate is high by delivering the questionnaires to pupils for them to respond during lessons time. Pupils will also be told why the information is being collected and how the results will be beneficial so as to erase any suspicion in them. This will make them respond honestly with a positive opinion. The questionnaires will also be anonymous.

3.3.2 INTERVIEWS

An interview is a conversation for gathering information. A research interview involves an interviewer, who coordinates the process of the conversation and asks questions, and an interviewee, who responds to those questions. Interviews can be conducted face-to-face or over the telephone. The internet is also emerging as a tool for interviewing. In this study the researcher used interviews to collect data from the teachers. Qualitative information is obtained from the teachers through interviewing them. (Nashon, 2021)

3.3.2.1 ADVANTAGES OF INTERVIEWS

Interviews are an appropriate method when there is a need to collect in-depth information on people's opinions, thoughts, experiences, and feelings. Interviews are useful when the topic of inquiry relates to issues that require complex questioning and considerable probing. Face-to-face interviews are suitable when your target population can communicate through face-to-face conversations better than they can communicate through writing or phone conversations (e.g., children, elderly or disabled individuals). (Nol, 2021)

3.3.2.2 DISADVANTAGES OF INTERVIEWS

Conducting interview studies can be very costly as well as very time-consuming. An interview can cause biases. For example, the respondent's answers can be affected by his reaction to the interviewer's race, class, age or physical appearance. Interview studies provide less anonymity, which is a big concern for many respondents. There is a lack of accessibility to respondents (unlike conducting mailed questionnaire study) since the respondents can be in around any corner of the world or country. (Nol, 2021)

3.4 DATA COLLECTION PROCEDURES

Upon obtaining clearance from the university and permission from the Provincial Education Director and District Offices of the Ministry of Primary and Secondary Education (MoPSE), the researcher also sought permission from the school head where the research is being carried out. Subsequently, the researcher made appointments of dates for interviews to be conducted in person at the schools with learners and teachers that were included in the sample of the study (Saunders, 2016). The researcher also distributed and retrieved questionnaires in person from one learners under the study. The researcher will administer tasks to learners on operating directed numbers and collect scripts after the exercise. The data collection was done over a week's period to afford adequate time for conducting of interviews at the schools

3.5 DATA ANALYSIS METHODS

Data collected will be presented in tables, charts, bar graphs and analysed. Firstly, learners' errors in addition and subtraction of directed numbers will be presented in the form of a frequency table. This means the first research question will be analysed by identifying and noting the frequencies of each of the types of errors identified. The responses from questionnaires of arranging directed numbers in ascending order will also be tabulated and graphs produced. These will be analysed and commented on. The responses on interviews on the highlights of the errors and misconceptions that learners have, will be recorded on a table, presented on a pie chart and analysed.

3.6 RELIABILITY AND VALIDITY

Reliability refers to the extent to which the same answers can be obtained using the same instruments more than one time (Babbie, 2012, p.158). According to Wilson (2015) reliability issues are most of the time closely associated with subjectivity and once a researcher adopts a

subjective approach towards the study, then the level of reliability of the work is going to be compromised.

Validity of research can be explained as an extent at which requirements of scientific research method have been followed during the process of generating research findings. Oliver (2012) considers validity to be a compulsory requirement for all types of studies. There are different forms of research validity and main ones are specified by Cohen et al (2014) as content validity, criterion-related validity, construct validity, internal validity, external validity, concurrent validity and face validity.

Measures to ensure validity of a research include, but not limited to the following points: Appropriate time scale for the study must be selected; Appropriate methodology is to be chosen, considering the characteristics of the study; The most suitable sample method for the study is to be selected; The respondents must not be pressured in any ways to select specific choices among the answer sets.

It is important to understand that although threats to research reliability and validity can never be eliminated, however the researcher will strive to minimize this threat as much as possible.

3.7 ETHICAL ISSUES

Ethical considerations are one of the critical issues to be addressed when carrying out research. Ethics can be viewed as moral principles and standards that govern a person's behaviour in conducting a research (Punch & O'Donoghue, 2013). The researcher ensured that respondents fully understand the nature of the study, the risks and benefits, so that they make informed decisions on whether to participate or not. Confidentiality was also assured so that questions were answered freely without any fear of victimization (Brundrett & Rhodes, 2013). Basically, the researcher assured participants of their rights to privacy, right to anonymity, right to fairness and right to informed consent. The researcher also obtained permission from Bindura University authorities, Provincial Education Director, District Schools Inspector as well as school heads to carry out this study in the sampled school.

3.8 CHAPTER SUMMARY

The chapter focused on research methodology, whereby the research design was formulated. The sampling procedure was clearly explained. The data collection procedure and data presentation and analysis methods were outlined. The next chapter will focus on data presentation, analysis and interpretation.

CHAPTER 4: DATA PRESENTATION, ANALYSIS AND INTERPRETATION

4.1 INTRODUCTION

In this chapter the researcher reports several findings from the research project in which thirty-two(32) form one pupils and three (3) mathematics teachers were involved. Data was collected by means of tasks, interviews and questionnaires. Pupils were given tasks for the researcher to be able to identify knowledge gaps. Closed ended questionnaires and open ended interviews were administered to teachers. Similar questions on questionnaires and interviews were administered to find the views of the teachers on the same issues.

The return rate of questionnaires administered to pupils was 100% and that of those administered to teachers was 100%. The researcher presented and analysed data collected from the participants of the study.

4.2. DATA PRESENTATION, ANALYSIS AND INTERPRETATION

This section of the chapter covers the presentation and analysis of data collected by the researcher for the purpose of the study. The researcher viewed 32 learners' scripts on how they answered tasks on addition and subtraction of directed numbers to answer the question "What are the types of errors that learners made when answering questions on operating integers"

4.2.1 LEARNERS' SCRIPTS ANALYSIS

Table 4.1: learners' scripts analysis on addition and subtraction of directed numbers

Addition of directed numbers	Analysis
Transcript from learner 1(female) 1. Simplify a) $(-3) + 5 = +8$ (wrong) b) $7 + (-9) = -16$ (wrong) c) $(-2) + (-4) = +6$ (wrong) d) $(-6) + (-3) = +9$ (wrong) e) Add (-10) to $(+13) = +23$ (wrong)	Addition of numbers with different signs require that you subtract the smaller one from the bigger one and assign the sign of the bigger one for question (a), (b) and (e) but learner 1 just added the two numbers and assigned the sign of the bigger number thus <i>misconception of the concept</i> . In question (c) and (d) addition of numbers with same sign implies that you add the two numbers together and assign the common sign, but learner1 had <i>misconception of the concept</i> in

	that regard.
<p>Subtraction of directed numbers</p> <p>2. Simplify</p> <p>a) $8 - (-3) = -5$ (wrong)</p> <p>b) $(-5) - 2 = -3$ (wrong)</p> <p>c) $(-4) - (-7) = -3$ (wrong)</p> <p>d) $3 - 9 = -6$ (correct)</p> <p>e) Subtract 6 from -8</p> <p>$6 - 8 = -2$ (wrong)</p>	<p>The concept of subtraction of negative numbers require that you must change sign of the second and add, but Learner1 had not done what is said on (a) and (c) the learner shows no correct procedure thus ended up in committing <i>procedural errors</i>.</p>
<p>3. $-6 - (-3) = +3$ (wrong)</p>	<p>The concept of changing the sign of the second has not been adhered to resulting in making <i>conceptual errors</i>.</p>
<p>4. $17^{\circ}\text{C} - 12^{\circ}\text{C} = 5^{\circ}\text{C}$ (wrong)</p> <p>5. Moderately confident</p>	<p>The word problems on integers require anyone to have good language proficiency. When considering the learners' performance, which is on the left side, it shows that this learner had a language problem and <i>strategic errors</i> because the learner have not been able to formulate the correct setup of the numbers.</p>

Fig 1 showing learner 1's transcript on addition and subtraction of directed numbers

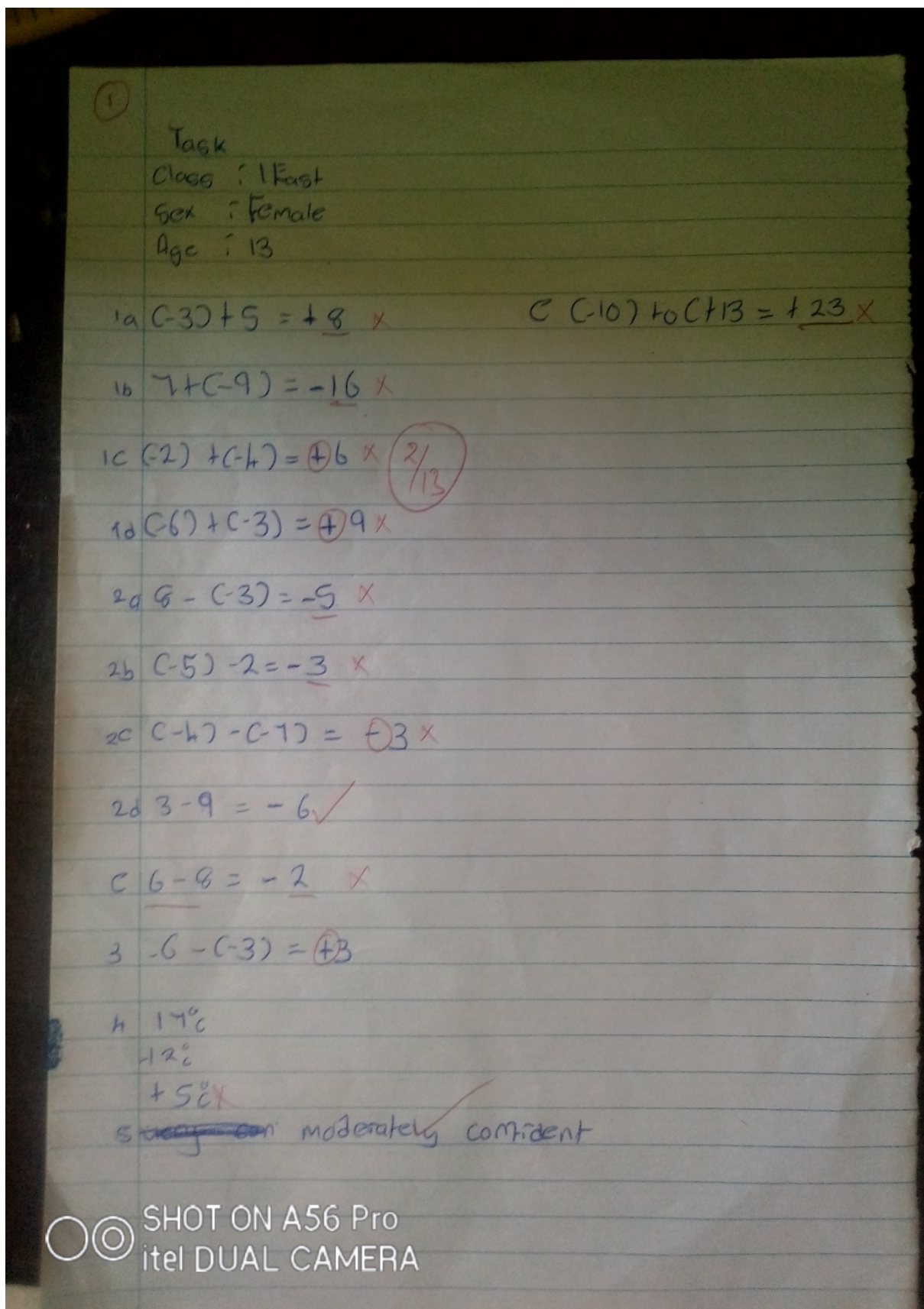
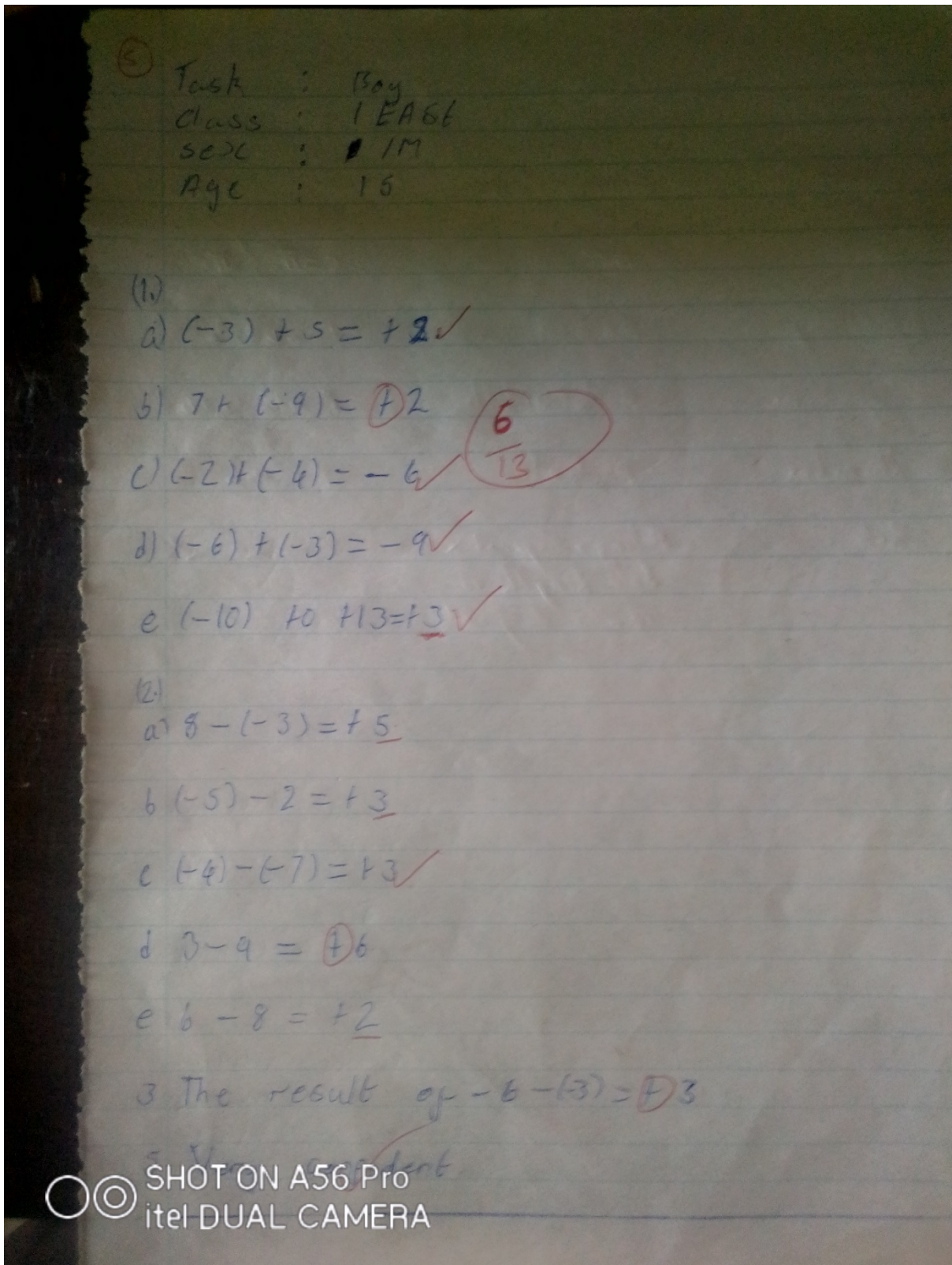


Table 4.2 Showing learners' scripts analysis.

Addition of directed numbers	Analysis
<p>Transcript from learner 5 (male)</p> <p>1. Simplify</p> <p>a) $(-3) + 5 = +2$ (correct)</p> <p>b) $7 + (-9) = +2$ (wrong)</p> <p>c) $(-2) + (-4) = -6$ (correct)</p> <p>d) $(-6) + (-3) = -9$ (correct)</p> <p>e) $(-10) + 13 = +3$ (correct)</p>	<p>Number 1 (a), (c), (d) and (e) have been done correctly, the learner shows that he understands the concepts very well. The calculation $(-9) = +2$ instead of -2.</p>
Subtraction of directed numbers	Analysis
<p>2. Simplify</p> <p>$8 - (-3) = \text{a) } 5$ (wrong)</p> <p>$(-5) - 2 = \text{b) } 3$ (wrong)</p> <p>$(-4) - (-7) = \text{c) } 3$ (correct)</p> <p>$3 - 9 = \text{d) } 6$ (wrong)</p> <p>$6 - 8 = \text{e) } 2$ (wrong)</p>	<p>Learner 5 ignore the negative signs when subtracting directed numbers in (a), (b), (d) and (e). He treated negative large numbers as positive and subtract smaller numbers getting positive answers. The learner did not shows correct procedure of subtracting negative numbers resulting in making <i>procedural errors</i>.</p>
<p>3. The result of $-6 - (-3) = +3$ (wrong)</p>	<p>The concept of changing the second sign had not been adhered to resulting in making <i>conceptual errors</i>.</p>
<p>4. No answer</p> <p>5. Very confident</p>	<p>The learner did not answer question 4 due to poor language proficiency. He had no strategy to answer this question so he suffered from <i>strategic errors</i> which shows that the learner is struggling with application of mathematical concepts to solve real-world problems.</p> <p>Question 5 the learners' respond is not matching his performance of 6/13.</p>

Fig 2 Showing learner 5's transcript on addition and subtraction of directed numbers



As the learners' tasks were being marked, the researcher noted the types of errors that were made by pupils. The frequency of the errors is shown in table 3 below.

4.2.2 Frequency of errors from learner's tasks

Table 4.3 Frequency of types of errors

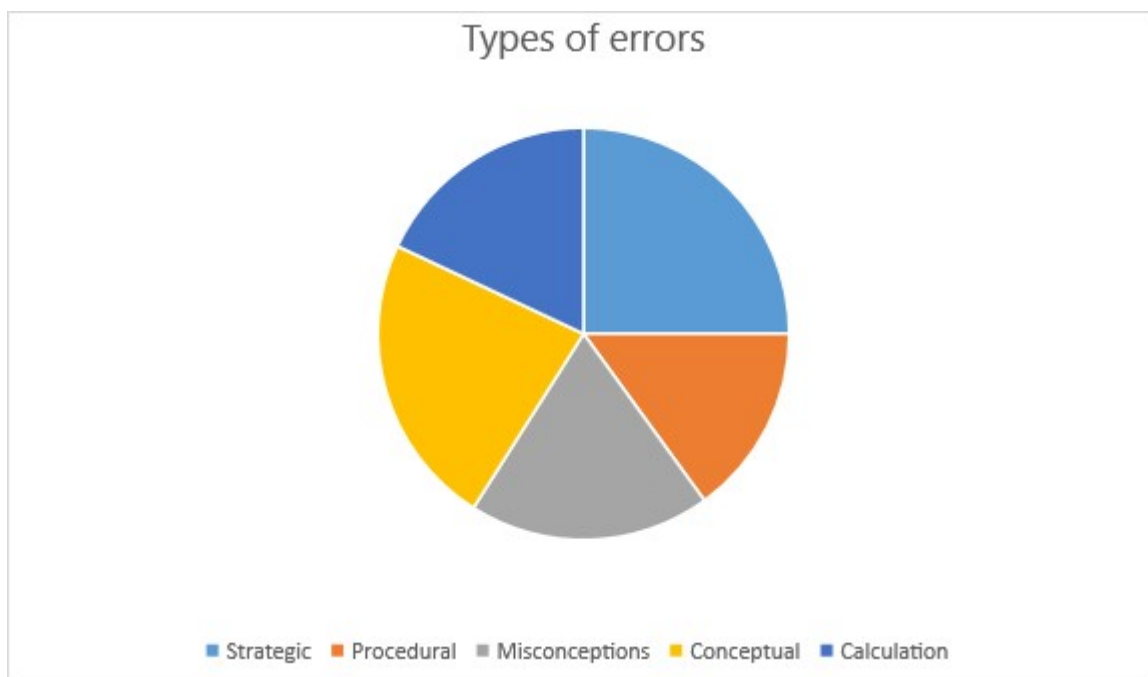
Types of errors	Misconceptions	Procedural	Conceptual	Calculation	Strategic
Number of Learners	23	18	28	22	31

From the learners' tasks taken, a total of 31 strategic errors were noted. This was the highest frequency of errors noted which translates to 25% of the total errors that were made by learners. It shows that most learners failed to solve real-life problems by applying the concept of directed numbers. The question 4 on their task demanded that learners *find the difference between temperatures of 17°C above zero and 12°C below zero*. Learners failed to understand that numbers above zero are positive and numbers below zero are negative so they simply subtracted 12°C from 17°C and got 5°C as the answer instead of doing their working $17^{\circ}\text{C} - (-12^{\circ}\text{C})$ which give a result of 29°C.

On the other hand, procedural errors had the lowest frequency of 18 learners among the errors noted. This constitutes 15% of the total errors made by learners in the study. Learners lack understanding of some procedures to take when answering for instance question 2(c) $(-4) - (-7)$ where learners were supposed to change the sign between the directed numbers to $(-4) + 7$ and simplify to get +3. They added 4 to 7 and got +11 most of them as their solution, which is wrong.

Misconceptions made on the tasks administered had 23 learners on the frequency table these were noted on question mostly on question 1 (c) $(-2) + (-4)$ and (d) $(-6) - (-3)$. Here learners simply added the two numbers and give their answer as positive numbers as +6 and +9 respectively thereby missing the concept that adding two negative numbers will result in a negative number. Conceptual errors which has the second highest frequency had 28 learners occur in $(-6) - (-3)$? Most learners gave $-6 - (-3) = +3$ as their respond instead -3 . Lastly calculation errors were done by 22 learners

Fig 3 Pie-chart showing the types of errors made by learners on tasks administered.



4.2.3. Demographic data of teachers

Table 4. Demographic information of teachers

Types of respondents	Gender	Age	Experience	Qualifications
Teacher :A	Female	34 years	6 years	Diploma in education
Teacher : B	Male	37 years	10 years	Bachelor of Commerce in Economics
Teacher : C	Male	47 years	17 years	Post Graduate Diploma in Education

There were three teachers who participated in this study as respondents. These teachers were naturally selected as they were the all the teachers in the department for the selected school. They comprised of two males and one female, experienced teachers teaching mathematics at different levels at a high school.

4.2.4 Teacher's responses to the questionnaires

The questionnaires that were administered to teachers were collected and the findings tabulated

Table 4.4

Question	Responses: Teacher A	Responses: Teacher B	Responses: Teacher C
1. How often do you observe students struggling with adding and subtracting directed numbers?	Very often	Very often	Very often
2. Which of the following errors or misconceptions have you observed in students?	Confusion between addition and subtraction. Inability to understand the zero concept. Difficulty with negative signs	Difficulty with negative signs	Confusion between addition and subtraction. Difficulty with negative signs. Calculation errors.
3. Do you think students understand the concept of adding and subtracting directed numbers as	Not yet.	Yes.	It depends on the student and the instruction they receive.

opposed to just following rules?			
4. Which of the following teaching strategies do you find effective in helping students grasp the concepts of adding and subtracting directed numbers.	Visual aids. Real-world applications.	Visual aids.	Visual aids. Real-world applications. Practice with mixed signs.
5. How confident are you in your ability to address errors and misconceptions related to addition and subtracting directed numbers?	Very confident.	Very confident.	Very confident.
6. How often do you incorporate real life examples or scenarios when	Rarely.	Very often.	Very often.

teaching the concepts of adding and subtracting numbers?			
7. Do you find that students struggle more with adding or subtracting directed numbers, and why do you think this is the case?	Equally with both.	Subtracting.	Equally with both.
8. When teaching about directed numbers, do you emphasize the understanding of the concept or focus more in the rules and procedures?	Understanding the concept.	Understanding the concept.	Understanding the concept.
9. In your experience, do students tend to view adding and subtracting numbers as challenging or straight	Challenging.	Challenging.	Mixed responses.

forward?			
10. What level of importance do you place on addressing errors and misconceptions related to adding and subtracting directed numbers in your teaching?	Very high.	Very high.	Very high.

The questionnaires were administered to all the three teachers and there was 100% return. The researcher managed to collect data from the three teachers involved. All the three teachers share the same sentiments on question 1, 5, 8 and 10 on the questionnaire. This shows that operations of directed numbers is a cause of concern to any mathematics teachers, teaching mathematics at any level. On the research question 1: *How often do you observe students struggling with adding and subtracting directed?* All the teachers agree that they observe learners struggling with operation of directed numbers very often. This means directed numbers are a challenge to learners regardless of the level they are at school. On question 5: *How confident are you in your ability to address errors and misconceptions related to adding and subtracting directed numbers?* All the teachers agree that they are very confident to address issues to do with directed numbers implying that they are ready to take this bull by its horns and liberate the learners from this dilemma. They also concur with each other that, they teach learners to understanding of the concept on question 8: *When teaching about directed numbers, do you emphasize the understanding of the concept or focus more on the rules and procedures?* This shows that these teachers' focus is not only concerned with teaching learners only to pass the exam but they teach learners for life such that they can apply the concept to solve real-life problems. Lastly, all the three teachers agree that it is of paramount importance to *address errors and misconceptions related to adding and subtracting directed numbers in their teaching*, which is on question 10.

Contrary to the above motion, the three teachers had different views on *how they think students understand the concept of adding and subtracting directed numbers as opposed to just following the rules*, question 3. Teacher A's respond is "not sure", teacher B say "yes" while teacher C says "it depends on the student and the instruction they receive". This shows that there is no one best solution suitable to any problem (contingency approach). On question 2: *Which of the following errors and misconceptions have you observed in student when adding and subtracting directed numbers?* Confusion between addition and subtraction, inability to understand the concept of zero, difficulty with negative numbers and calculation errors were all selected by the three teacher. Similarly, on question 4: *Which of the following teaching strategies do you find effective in helping students grasp the concepts of adding and subtracting directed numbers?* Visual aids, real-world applications and practice with mixed signs were selected as responses by either teacher except formative assessments and feedback. This means it is not any effective way to help learners grasp the concept of operating directed numbers though it may be used.

Furthermore, the researcher have noted that on question 6, 7 and 9 two of the respondents agree with each other on their responses while one respondent offers a different view. For instance on question 7: *Do you find students struggle more with adding or subtracting directed numbers.* Teacher (A) and Teacher(C) says learners struggle *equally with both* while Teacher (B) pointed out that learners struggle more with *subtraction*. This means that both adding and subtracting directed numbers is a real challenge to the learners although subtraction is troublesome since on the foundation of education of form one learners negative numbers didn't exists.

4.2.5 TEACHERS' RESPONSES TO THE INTERVIEW QUESTIONS

The questionnaires that were administered to teachers were also collected and their findings were recorded. Three mathematics teachers responded to the interview as follows

INTERVIEW RESPONSES FOR TEACHERS

1. In your experience, what are the main hurdles that students face when learning about adding and subtracting directed numbers?

Teacher A: *most students are unable to understand the concept*

Teacher B: *dealing with negative signs*

Teacher C: *failing to take away a bigger number from a smaller one and to take away a negative /positive number from another negative number.*

2. Can you share any specific examples of common errors or misconceptions that students demonstrate when working with directed numbers?

Teacher A: *Use of the wrong sign on the answer for example $-58 + 50 = +8$. Use of the wrong rule/law for example $-25 - 13 = +12$*

Teacher B: *$-3 - (-2) = -5$ learners forget to change sign, $-3 + 2 = 1$ learners forget to give negative sign on the answer, $-3 + 8 =$ and most learners sometimes do not give the answer.*

C: *$-3 - 5 = 8$ or -2 , $9 - 13 = 22$ or 4 , $-3 \times -2 = (-54) + (-9)$ reduced to $-4 + 9$ failing to remove brackets or subtract directly, $9 - 3$ when asked to rearrange learners give $3 - 9$ as they fail to see that negative sign belongs to 3.*

3. How do you assess students' understanding of adding and subtracting directed numbers, and what strategies do you use to address any misconceptions or difficulties?

Teacher A: *Most students fail to understand the concept. To address difficulties I revisit or reteach the concept each and every term*

Teacher B: *Learners got confused, I usually use visual aids for quick thinking and also trying to repeat the concept in most topics for them to absorb*

Teacher C: *$-3 - 5$ is addressed as taking 3 and then taking away 5 or taking away -8 at once which is given as the answer.*

$12 - 9$ Is now being given as -3 after the introduction of the topic, learners told to use "everyday primary school concepts"

4. What additional support or resources do you believe would benefit you in teaching the concepts of adding and subtracting directed numbers more effectively?

Teacher A: *Since directed numbers are used to describe changes in temperature in real life situation, availability of thermometers to use as media would benefit us in the teaching the concepts.*

Teacher B: *Number line for beginners, for writing candidates, topics like equation of a line, vectors and velocity time graphs are of importance.*

Teacher C: *If possible learners to verify answers with use of calculators, but learners must not use the calculators for sole calculation purposes.*

All the three teachers involved in the research were interviewed. The interview process was a success, and the researcher came out with the information from the teachers involved. On

question 1: *What are the main hurdles that students face when learning about directed numbers?* The response from all the three teachers interviewed review that learners fail to understand the concept, dealing with negative signs are the major stumbling blocks of learners' progress in mathematics. *Common errors and misconceptions in working with directed numbers* were note in question 2; subtraction of negative numbers and addition of negative and positive numbers dominated from the three responses obtained. For instance, $-25 - 13$ and $-3 + 8$ which show misconceptions errors. $-3 - (-2)$ and $(-4) - (+9)$ Which are examples of procedural errors were also outlined. Use of visual aids, repeating the concept and language for instructions were measures outlined as *strategies used to address misconceptions and difficulties faced by learners in understanding the concept of adding and subtracting directed numbers* on question 3. Lastly respondents advocated for use of visual aids like thermometers, number lines and calculators *as additional resources needed to enhance effective teaching of directed numbers* on question 4.

4.3 SUMMARY

The information gathered shows that learners make errors in adding and subtracting directed numbers. The errors were classified as, procedural, conceptual, calculation, strategic and misconceptions of the concept. The occurrence of one type of error can lead to numerous errors. Strategic errors had the highest frequency whilst procedural errors had the lowest frequency. The language of instruction proved not to be compatible with many learners. As such the learners try to solve a problem without understanding what it demands leading to strategic errors. It has been unveiled also that most teachers re-teach the topic when learners fail. There are mixed feelings from teachers on whether students understanding of the concept of adding and subtracting directed numbers as opposed to just following the rules. The next chapter will have a glance at the research summary, conclusion and recommendations related to the findings of this study.

CHAPTER 5: SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

The study began with the choice of topic. The background to the study, purpose of the study, statement of the problem, research questions, and research objectives, significance of the study, delimitation and limitations of the study were outlined.

Literature review was done in which different errors and misconceptions were deliberated on. Different types of errors which include calculation, conceptual, procedural, strategic errors and misconceptions were discussed. Views of various authorities were taken note of and commented on. The reasons for doing error analysis were also stated. The research methodology was explained. In this, instruments used for data collection were stated with advantages and disadvantages of each being outlined. Data collection plan and data analysis was put forward. The researcher then went ahead to collect data.

5.2 SUMMARY OF THE STUDY

Learners' tasks made it possible for the researcher to identify learners' errors and misconceptions in adding and subtracting directed numbers. The researcher designed instructional activities in such a way that pupils were able to complete the tasks administered without the help of the researcher. The learners' scripts were all collected to avoid losing vital information.

In addition, the use of questionnaires and interviews to the teachers under the study made it possible for the researcher to get first-hand information from the educators. Integrating quantitative and qualitative data in research made the research a success and the findings were accurate. Thereby mixed method research method was used in the research. (Halcomb & Hickman, 2015)

This study shows that the process of investigating errors and misconceptions made by learners in adding and subtracting directed numbers was successful since the errors and misconceptions were identified on the tasks administered. These results are a motivating factor to both educators and learners.

Teachers' responses to questionnaires and interviews revealed that errors and misconceptions in adding and subtracting directed numbers are challenges faced by learners at form one level and later levels as well. Most of the teachers indicated that pupils do not understand the concept of subtracting negative numbers and that they have problems in presenting the correct procedure to solve such problems. As such they are bound to make errors. The study yields the results of the problems which have been analysed by the researcher.

5.3 CONCLUSION

The following conclusions were drawn in line with the research process and analysis. Findings from research question one concluded that learners make errors and had misconceptions regarding addition and subtraction of directed numbers at form one level. The errors identified from learners' tasks administered included procedural, conceptual, calculation and strategic. The following misconceptions of concepts were also noted, adding a positive and negative number always results in a positive answer $7 + (-9) = 16$, adding two negative numbers always results in a positive answer $(-6) + (-3) = +9$ and subtracting two negative numbers always result in a positive answer $(-5) - 2 = +3$. Chief amongst all the misconceptions was on temperature problem that integers only apply to abstract math problems, not in real-world situations. The last research question on strategies that can be employed to reduce errors and correct misconceptions will be addressed on recommendations.

5.4 RECOMMENDATIONS

- ❖ The use of visual aids in teaching such as number lines, thermometers, graphs and diagrams can help learners visualize the concepts of directed numbers
- ❖ Real-world applications through the use of everyday examples, such as temperature changes, financial transactions, elevations to illustrate the practical relevance of directed numbers
- ❖ Practice with mixed signs, provide ample opportunities for learners to practice adding and subtracting directed numbers with different signs
- ❖ Teaching of conceptual understanding of directed numbers to be foregrounded before procedural knowledge
- ❖ Use of Concrete-representational –abstract (CRA) approach, introduce concepts using concrete objects like number lines, then move to representational models like graphs and finally abstract representations like equations
- ❖ Delay the use of calculators, and encourage critical thinking among students where they can question and justify their answers
- ❖ Emphasize zero, clarify zero's role as a reference point separating positive and negative numbers.

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APPENDICES

LEARNERS' TASKS

SECTION A: Addition of directed numbers

1. Simplify the following by adding

a) $(-3) + 5 =$

b) $7 + (-9) =$

c) $(-2) + (-4) =$

d) $(-6) + (-3) =$

e) Add (-10) to $(+13)$

SECTION B: Subtraction of directed numbers

2. Simplify the following by subtracting

a) $8 - (-3) =$

b) $(-5) - 2 =$

c) $(-4) - (-7) =$

d) $3 - 9 =$

e) Subtract 6 from -8

3. What is the result of $-6 - (-3)$?
4. What is the difference between temperatures of 17°C above zero and 12°C below zero?
5. On a scale of 1 to 5, how confident are you in adding and subtracting directed numbers?

- 1-Not confident at all
- 2-Slightly confident
- 3-Moderately confident
- 4-Very confident
- 5-Extremely confident.

SECTION A: Introduction

I am Mutsenu Frank a student at Bindura University of Science Education studying towards a Bachelor of Science Honors Education Degree in Mathematics. I am carrying out a study on the errors and misconceptions made by form one learners on operating with directed numbers.

Please take your time to answer this questionnaire to the best of your knowledge and abilities. The data you provide is strictly for academic purposes and you are not required to provide your name.

SECTION B: Demographic Data

SEX:

AGE:

EXPERIENCE:

QUALIFICATION:.....

SECTION C: Circle the most appropriate answer from the given options

1. How often do you observe students struggling with adding and subtracting directed numbers?
 - A. Very often
 - B. Often
 - C. Occasionally
 - D. Never

2. Which of the following errors or misconceptions have you observed in students when adding and subtracting directed numbers? (select all that apply)
 - A. Confusion between addition and subtraction
 - B. Inability to understand the concept of zero
 - C. Difficulty with negative signs
 - D. Calculation errors

3. Do you think students understand the concept of adding and subtracting directed numbers as opposed to just following rules?
 - A. Yes
 - B. No
 - C. Not sure
 - D. It depends on the student and the instruction they receive.

4. Which of the following teaching strategies do you find effective in helping students grasp the concepts of adding and subtracting directed numbers? (select all that apply)
 - A. Visual aids
 - B. Real-world applications
 - C. Practice with mixed signs
 - D. Formative assessments and feedback

5. How confident are you in your ability to address errors and misconceptions related to adding and subtracting directed numbers?
 - A. Very confident
 - B. Moderately confident
 - C. Slightly confident
 - D. Not confident

6. How often do you incorporate real-life examples or Scenarios when teaching the concepts of adding and subtracting directed numbers?
 - A. Very often
 - B. Often
 - C. Rarely
 - D. Never

7. Do you find that students struggle more with adding or subtracting directed numbers, and why do you think this is the case?

- A. Adding
 - B. Subtracting
 - C. Equally with both
 - D. Not sure
8. When teaching about directed numbers, do you emphasize the understanding of the concept or focus more on the rules and procedures?
- A. Understanding of the concept
 - B. Rules and procedures
 - C. Both equally
 - D. Not sure
9. In your experience, do students tend to view adding and subtracting directed numbers as challenging or straightforward?
- A. Challenging
 - B. Straightforward
 - C. Mixed responses
 - D. Not sure
10. What level of importance do you place on addressing errors and misconceptions related to adding and subtracting directed numbers in your teaching?
- A. Very high
 - B. High
 - C. Moderate
 - D. Low

INTERVIEW SCHEDULE FOR TEACHERS

SECTION A: Introduction

In-Depth interview questions for teachers on Errors and Misconceptions in Addition and Subtraction of Directed Numbers, particularly in teaching form one learners.

Answer all questions honestly and openly. Your responses will remain anonymous and confidential.

SECTION B: Demographic Data

SEX:

AGE:

EXPERIENCE:
.....

QUALIFICATION:
.....

SECTION C: Research Items

1. In your experience, what are the main hurdles that students face when learning about adding subtracting directed numbers?

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2. Can you share any specific examples of common errors or misconceptions that students demonstrate when working with directed numbers?

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3. How do you assess students' understanding of adding and subtracting directed numbers, and what strategies do you use to address any misconceptions or difficulties?

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4. What additional support or resources do you believe would benefit you in teaching the concepts of adding and subtracting directed numbers more effectively?

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SAMED

P Bag 1020
BINDURA
ZIMBABWE

Tel: 0271 - 7531 ext 1038
Fax: 263 - 71 - 7616



BINDURA UNIVERSITY OF SCIENCE EDUCATION

Date: 15/05/2024

TO WHOM IT MAY CONCERN

NAME: MUSENU FRANK REGISTRATION NUMBER: B 225056 B
PROGRAMME: HBSCED-MATHS PART: 2.2

This memo serves to confirm that the above is a bona fide student at Bindura University of Science Education in the Faculty of Science Education.

The student has to undertake research and thereafter present a Research Project in partial fulfillment of the Bachelor of Science Education programme. The research topic is:

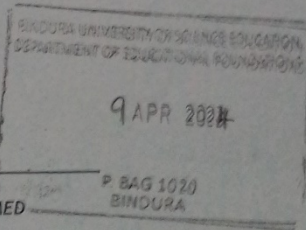
An investigation on learners' errors and misconceptions on operations with directed numbers.

In this regard, the department kindly requests your permission to allow the student to carry out his/her research in your institutions.

Your co-operation and assistance is greatly appreciated.

Thank you

Zindemo
Zindemo (Dr.)
CHAIRPERSON - SAMED



EC NUMBER 0139733 H

REF: C/426/3 Mash Central

All communications should be addressed to
"The Provincial Education Director
Mashonaland Central Province"
Telephone: 0271- 6948/6996/7134/6994
Fax: 0271-6997



Ministry of Primary and Secondary Education
Mashonaland Central Province
P.O Box 340
Bindura
Zimbabwe

20 MAY 2024

MUTSENU FRANK
MASHUWA HIGH SCHOOL
P.BAG 1023 MASHUWA

RE: PERMISSION TO COLLECT DATA FOR RESEARCH PURPOSE:

AT MASHUWA H.S.H.

SCHOOLS: SHAMVA DISTRICT: MASHONALAND CENTRAL PROVINCE.

Reference is made to your application to carry out research at the above district in Mashonaland Central Province on the title:

"AN INVESTIGATION ON LEARNERS' ERRORS AND MISCONCEPTIONS ON OPERATIONS WITH DIRECTED NUMBERS"

I am pleased to inform you that the Provincial Education Director has granted you permission to carry out your research in SHAMVA DISTRICT. You should, however, liaise with D.S.I SHAMVA DISTRICT before you start. You should ensure that your research work does not disrupt the normal operations of the school. Where students are involved, parental consent is required.

Finally, you are advised to submit a copy of your findings to the Provincial Education Director (PED) Mash Central for Primary and Secondary Education.

C. M. Mutwesho

FOR: PROVINCIAL EDUCATION DIRECTOR
MASHONALAND CENTRAL PROVINCE

DISCIPLINE
MINISTRY OF PRIMARY AND
SECONDARY EDUCATION
20 MAY 2024
MASHONALAND CENTRAL PROVINCE
P.O. BOX 340 BINDURA

