BINDURA UNIVERSITY OF SCIENCE EDUCATION.

FACULTY OF SCIENCE AND ENGINEERING.

DEPARTMENT OF STATISTICS AND MATHEMATICS.

MULTIVARIATE MARKOV CHAIN MODEL FOR LOAN DEFAULT RISK MODELING IN MICROFINANCE INSTITUTIONS: A CASE STUDY OF THE FUNDHOUSE FINANCE.

BY

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DEDICATION

I would like to dedicate this research project to my nephew, Wanai Mbalenhle Munateyi, my shining inspiration.

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I am deeply thankful to the Holy Spirit for guidance and strength and to my mother's prayers that supported me throughout this research. I sincerely appreciate Ms. Hlupo' s exceptional supervision, patience and support, which greatly contributed to this work. Special thanks to Martin Maposa for his ongoing encouragement and motivation, and to Nyasha Dzapasi for his valuable assistance and to everyone who played a role in this dissertation; your impact did not go unnoticed.

ABSTRACT

This study applies a Multivariate Markov Chain (MMC) model to analyze and predict loan default risk in microfinance institutions, focusing on FundHouse Finance in Zvishavane, Zimbabwe. With the microfinance sector facing rising default rates and traditional models proving inadequate, the research aims to model borrower behaviour dynamically across multiple loan states such as current, delinquent, pre-default, default, prepayment, and recovered. The study begins by outlining the problem and objectives, then reviews credit risk modelling literature, highlighting the limitations of structural and reduced form models. Using a positivist and quantitative approach, the researcher collected data from 2000 borrowers and incorporated macroeconomic indicators to estimate transition probabilities. Diagnostic tests, including the Chow test and Chapman-Kolmogorov test, validated the model's structure and predictive ability. The MMC model outperformed logistic regression, achieving a Brier score of 0.1774 and revealing key insights such as high default persistence (88.24%) and a 45.74% delinquency-to-default transition rate, which is useful for early risk detection. Though strong in overall accuracy (89%), the model underperformed in rare class prediction, suggesting a need for hybrid enhancements. The study concludes that MMC models offer superior accuracy, adaptability, and risk insight, and recommends integrating bias-mitigating algorithms and hybrid approaches to strengthen microfinance risk management.

LIST OF ABBREVIATIONS

MMC: Multivariate Markov Chain model

MFI'S: Microfinance Institutions

AI : Artificial Intelligence

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CHAPTER 1 : INTRODUCTION

1.0 Introduction

For many years, loan models for modelling loan default risk have rapidly evolved becoming a viable solution to Microfinance Institutions (MFI'S) to preserve the loan amount disbursed to encourage repayments by borrowers, yet the studies on loan default risk measurement models and its effects are few, making this a significant problem. This study aims to solve this problem by using Multivariate Markov chain (MMC) model as loan modification to analyse loan default risk using FundHouse Finance as a case study.

In this Chapter, we review the background of the study, statement of the problem, research objectives, research questions, delimitations, limitations and sets stage for subsequent chapters. Chapter 2 reviews existing research on loan default risk, Markov analysis models, and their applications in microfinance. Chapter 3 dives into this loan model, MMC Model detailing it's applied methodology. Chapter 4 presents the analysis and findings, leading to Chapter 5's conclusions and recommendations.

1.1 Background of the study

The prevalence of loan default risk poses a significant challenge to the sustainability of microfinance institutions (MFI's), warranting attention from a diverse array of stakeholders. Loan default risk is the risk of losses due to loan events when the borrower is unwilling or unable to repay his or her debt on the agreed time. Examples of default events are credit card charge-off , bankruptcy and mortgage foreclosure.

Since the national financial crisis of 2008, there has been increasing concern about managing loan default risk, as inadequate loan management was a key factor in that crisis (Mlambo,2014). This situation highlighted the need for improvements in various aspects, particularly in the credit risk management practices of banks and microfinance institutions.

The microfinance institutions continue to impact on the lives of small and medium enterprises as reflected by increase in total loans, and collecting deposits, among other indicators (RBZ, 2020).

However, in recent years, the microfinance industry has experienced high default rates, resulting in substantial financial losses and undermining the confidence of investors and stakeholders (RBZ,2023). According to the Reserve Bank of Zimbabwe (2020), the count of microfinance institutions dropped by 13.54% from 229 in December 2019 to 198 by the end of 2020, largely due to closures resulting from loan defaults and other challenges faced during the year.

The Reserve Bank of Zimbabwe (2023) detailed that loan default risk for the MFIs augmented as of June 30, 2023, the industry's Par ratio (>30 days) stood at 11.42%, up from 7.20% and above the global standard of 5% indicating heightened risk. In Zimbabwe, the microfinance sector has experienced high loan default rates, averaging 15% between 2015 and 2020 (Reserve Bank of Zimbabwe, 2020).

Zimbabwe Association of Microfinance Institutions (2023) highlights that the credit or instalment plan only MFI sector accounted for the portfolio at risk ratio stood at 15% up from 12.71% as of December 31, 2022. The loan default risk coverage ratio however, continued it's downward trend, falling from 22.17% to 12.74% between September 30, 2022 and December 31, 2022 and reaching a low of 4.48% by March 31, 2023.

The elevated loan defaults in recent years prompted microfinance institutions (MFIs) to adopt loan modifications, including interest reduction, payment reduction, and loan forgiveness, as a mitigating strategy to minimize losses and encourage repayments from distressed borrowers though they were still a knowledge gap of predicting and analyzing precisely loan default risk using mathematical models.

This study addressed this knowledge gap by analyzing the effectiveness of the Markov Analysis model in predicting and analyzing loan default risk using Fundhouse Microfinance Institution as a case study. Multivariate Markov Chain model is a system exhibiting probabilistic movement from one state to another, over time and analyzes the future probabilities based on current and known probabilities and the model is based on conditional probability function. By exploring the Multivariate Markov Chain model's accuracy, strengths, and limitations, this study sort to contribute to the development of more effective loan default risk management strategies for MFIs in Zimbabwe and beyond, ultimately enhancing their sustainability and ability to serve their target market.

1.2 Statement of the Problem

The likelihood of a borrower failing to meet their repayment obligations defines loan default risk. The loan default risk of a borrower risk can vary due to shifts in interest rates, technological advancements or regulatory updates. Rating agencies assess portfolios to evaluate loan default risks associated with specific securities and assign corresponding rating. Loan default risk management is a major concern for the microfinance institutions. The subject being considered is to model the interdependencies of loan default risks that have significance when managing and measuring loan portfolios. A multivariate Markov model (MMC) is used to forecasts the transition probabilities of borrower's credit rating worsening or failing to repay loan thus defaulting.

1.3 Research objectives

The study looks forward to meeting the following objective:

1.To model loan default using Multivariate Markov Chain model.

1.4 Research Questions

The study aims to answer the following questions:

1) How to model loan default using Multivariate Markov Chain Model?

2) What are the modifications on loan default risk models using MMC model and logistic regression model?

3) How accurate are the two approaches: MMC Model and Logistic Regression Model?

4) What are the strengths and limitations of the MMC model in loan default risk assessment?

5) What are the recommendations for improving the loan default risk assessment process using the Multivariate Markov Chain model?

1.5 Delimitation of the Study

The scope of the study was specifically restricted to Fundhouse Microfinance in Zvishavane. It was delimited exclusively on Fundhouse Microfinance's loan portfolio, investigating the application of the MMC model in predicting loan default risk, with a specific emphasis on analysing the impact of macroeconomic factors and borrower creditworthiness on transition probabilities, thereby providing useful insights for risk management and decision making.

1.6 Assumptions of the Study

This study relied on these key assumptions:

1) This study employ transition probabilities to model the dynamics of loan default risk, analysing the likelihood of transitioning between different loan states.

2) The study follow a Markov property which asserts that the future loan status is conditionally independent of past events, relying solely on the current state for predictive information.

3) The study has finite states such current, delinquency and pre-default that borrowers can occupy at any time.

4) The study assumes absorbing states that once a borrower reaches a certain state for example default state they cannot transition back to a curren or recovered state, or if they can, it takes a specific path that is well defined.

1.7 Limitation of the Study

The Researcher faced the following limitation:

1)Non-Stationary Borrower Behavior: Borrower behavior can change due to external factors such economic shifts and changes in loan granting policy that the markov model may not account for. Regular model updates with new data and a rolling window approach was implemented to address the limitation.

1.8 Definition of terms

Loan

A sum borrowed from a bank and a financial institution (Shekhar and Shekhar, 2005).

Loan Default

A loan is stated to be in default when a borrower is unable to make the scheduled payments, indicating a significant deterioration in the borrower's creditworthiness (Nguyen and Tran, 2022)

Delinquent loan

Refers when a loan account considered past due when a complete loan cycle has lapsed and payment is overdue by one week (Kairu, 2009).

Microfinance

It is defined as the providing financial services to low income populations, including consumers and entrepreneurs, excluded from conventional banking systems and related services (Nawai and Shariff, 2010).

Microfinance Institutions

Refers to the the providing financial services to the unbanked and under banked households and small to medium enterprises. (RBZ, 2012)

Markov Analysis Model

Refers to a statistical method that uses transition probabilities between states to predict future behavior in systems assuming the future state depends exclusively on the current state (Chen et al. 2023)

State

A loan's current condition at a specific period of time. (Ross,2014)

State probability

The likelihood of the loan's status.(Ross,2014)

Transition probability

If a loan is in state i, the probability Pij determines the chance of transitioning to state j in the next period. (Ross.2014)

Transition Probability Matrix

Refers to a table or matrix that the conditional probabilities of state changes. (Marius,1980).

Multivariate Markov Chain

Refers to a stochastic processthat generalises the traditional Markov Chain to model multiple interdependent sequences where the transition probabilities of each variable depend on its past states as well as those of other related variables (Ching et al.,2013).

1.9 Chapter Conclusion

The chapter highlighted the background to the study, statement of the problem, research objectives, assumptions of the research and limitations of the study. Delimitations of the study were also discussed. The following chapter   focused on work already done by other researchers on the subject of analyzing loan default risk in microfinance institutions using markov analysis model.

CHAPTER 2 : LITERATURE REVIEW

2.0 Introduction

A critical part of a research involves a review of existing studies and research on the topic. A thorough review of existing offers valuable insights into the evolving landscape of the field. This chapter examines empirical evidence, theoretical evidence and knowledge gap analysis being guided by research questions. According to Saunders (2009) literature is essential in developing a thorough understanding of and insight into, previous studies relevant to the research questions and objectives. In this chapter, broad discussion on analyzing loan default risk using other models related to the Markov analysis model. This chapter has discussed empirical studies, related theories upon which this study is based and proposed conceptual framework on the existing problem.

2.1 Theoretical Literature

2.1.0 Structural Credit Risk Models

Merton's (1974) structural model, suggests that the firm values follow a geometric Brownian motion process. While the approach is limited by its simplistic assumptions, it has paved the way for further developments (Geske, 1977). Extensions to the baseline model as a compound option modeling (Leland, 1996) allowed for continuous provision of debts with infinite maturity. Duffie (2005) further contrasted Merton's (1974) maturity based default assumption.

Black and Cox introduced the first passage time structural model whrepaid.mgest that default can before debt maturity if firm's asset value hits a predetermined default barrier. Building on this framework subsequent research extended Merton's (1974) model by incorporating flexible parameters. For instance Longstaff (1995) introduced short term risk free yields that tend towards a long term rate negatively correlated with asset values. Anderson's (1996) extension introduced the possibility of contract renegotiation where firms can either be declared bankrupt or issue new debt with higher interest rates when the default threshold is reached.

The Jarrow and Turnbull (1995) model employs a discrete state Markov chain to represent bankruptcy processes with parameters estimated from observable data. This framework is widely applied in pricing and hedging corporate debt including embedded options and over the counter derivatives with counter-party risk, sovereign bonds exposed to default risk, credit derivatives and risk management (Jarrow & Turnbull, 1995).

2.1.1 Reduced Form Credit Risk Models

Altman (1968) introduced reduced form credit scoring models at the individual level which employ accounting variables with statistically significant explanatory power to distinguish between defaulting and non-defaulting firms. Using linear or binomial regression, these models estimate default probabilities and derive coefficients to Moreovery borrower as either good or bad (Altman, 1968).

Altman (1998) explored the practical application of credit scoring models while Altman (1997) evaluated historical explanatory variables in such models revealing that financial ratios particularly those measuring leverage, profitability and liquidity dominated empirical research.

Jacobson (2003) introduced a bivariate probit model and proposed an unbiased portfolio calculation period. Later, Lin (2009) advanced a innovative two stage hybrid approach combing logistic regression with artificial neural networks. Altman (2005) further developed a credit scoring model tailored to the growing market for corporate bonds. In a study on small and medium enterprises, Lupp et al. (2007) applied a logit model and found that traditional accounting based credit scoring models had weaker explanatory power for nonprofit firms compared to profit making firms.

2.2.2 Fundamentals of Markov Chain Models in Finance

Markov chain models provide a probabilistic framework in which the future state of a loan or a system is determined solely by its present state independent of its historical trajectory. This characteristic known as the memoryless property is valuable for analysing credit transition (Lando & Skødeberg, 2002).Early applications of time homogeneous Markov chains focused on estimating credit migrations and default risks (Jarrow et al. 1997).

In loan default risk measurement, the Markov property enables the representation of a borrower's creditworthiness change through discrete states such as current, delinquency or default where future transitions depend only on the present state. Each state is associated with transition probabilities to other states within a given period collectively forming a transition probability matrix. By estimating these matrices from historical loan performance data, risk managers can quantify expected losses, measure ratio migration risks and predict default.

Early applications of Markov models to loan default risk heavily relied on univariate and time homogeneous frameworks and time homogeneity infer that the transition probabilities remain constant across different time periods, a simplifying assumption that facilitates analytical conformable but may not accurately reflect real world creditcldynamics (Jarrow, Lando and Turnbull, 1997). Subsequent advancement in markov chain modeling have introduced time inhomogeneous frameworks where transition probabilities adjust dynamically in response to macroeconomic shifts or borrower specific factors. Additionally continuous time Markov chains have been incorporated to overcome the limitations of discrete observation periods allowing for more accurate modeling events (Bielecki & Rutkowski 2002).

2.2.3 Kijima Model

The evolution of loan default risk modeling has transitioned from simple univariate to advanced multivariate frameworks capable of capturing real world financial interdependencies. A pivotal development in this shift was the Kijima and Komoribayashi (1998, 1999) who pioneered a multivariate Markov Chain structure for credit migration analysis. Earlier models such as those by Jarrow et al. (1997) relied on the assumption of independent borrower transitions neglecting systemic linkages and risk of default spread. Kijima’s breakthrough introduced joint transition matrices, enabling the modeling of correlated credit events where one borrower's default could spread risk to others.

Kijima developed a multivariate Markov model which preserved Markov property while expanding the state space to include a Cartesian product of individual borrower states (Kijima & Komoribayashi, 1999). Kijima's new approach combined common economic shocks as latent factors influencing joint credit transitions across borrowers. This approach made it possible for realistic simulation of portfolio level risk under dynamic loan conditions effectively overcoming a key limitation of prior models that failed to adequately account for correlation in portfolio risk assessment.

Kijima's model bridged the gap between reduced form and structural approaches in credit risk modeling. The model's theoretical contributions laid subsequent advances in portfolio credit risk analysis and systemic risk measurement (McNeil et al, 2005).Kijima's framework also formed the development of credit rating transitions matrices, which are now broadly applied in regulatory stress assessments and internal risk models. Moreover, it introduced analytical methods for evaluating credit migration probabilities and joint default dependencies which remain important for risk based capital allocation.

2.2 Empirical Research Literature

D'Amico et al. (2019) created a hybrid framework combining piecewise homogeneous Markov chains for credit rating transitions with a multivariate credit spread model to measurement financial risk across the European Union. Their study revealed a temporal increase in both financial risk inequality and aggregate risk levels with significant variations across rating agencies and strong risk correlations among most European countries.

Sithole and Eita (2023) applied an empirical study of macroeconomic determinants of banking sector credit risk in BRICS countries using a Markov Switching approach. Their study revealed that declining economic conditions, especially slow GDP growth, high inflation, currency appreciation and rising interest rates which correlate with high credit risk. The magnitude and significance of these effects exhibited distinct variation across different economic regimes identified by the model.

Wang et al. (2024) conducted a regime switching Markov model to analyse default risk patterns among small and medium sized enterprises. Their findings show that incorporating regime-dependent transitions improves the predictive accuracy of small and medium sized enterprises default probabilities compared to conventional static models such as logistic regressions.

Nkomo et al. (2018) applied mover stayer model to estimate bank loan default probabilities in Zimbabwe's financial sector. Their results revealed that this model significantly outperformed traditional Markov chain models in default risk prediction accuracy, showing its improved capability to capture borrower heterogeneity.

Kipkoech (2022) researched multivariate Markov chain for credit risk optimisation. The research incorporated borrower credit scores, financial history, and macroeconomic variables into the Markov framework. Their actual findings showed that this complex approach significantly improved predictive accuracy for loan default probabilities compared to single factor models.

Di Masi and Stabile (2020) developed a semi-Markov modelling framework to optimise credit risky bond portfolios. Their approach utilized historical credit rating transitions to derive a data driven approach for effective credit risk management in fixed income investments.

Chang, Huang, and Tsai (2021) applied a Markov regime-switching model to study credit spreads in the banking industry in Taiwan. Their research used for borrower data and macroeconomic indicators to analyse how different risk factors influenced default probabilities. Their results showed that Markov chain models effectively captured regime shifts in credit risk, highlighting the advantages of dynamic modeling in financial markets.

Singh and Sharma (2021) presented a model that combines Markov analysis and machine learning for improved SME credit risk forecasting. Their model showed a superior predictive accuracy compared to traditional methods establishing that such hybrid methodologies made possible more precise and advanced credit risk evaluation frameworks.

Jakubík and Piloiu (2018) applied a multivariate Markov chain model to predict the likely time to default under different economic stress conditions.By adding macroeconomic variables such as inflation and GDP growth, the research improved the model's credit risk assessment capabilities. Their results showed that economic declines significantly increase default probabilities, supporting the need for stress testing in risk management.

Chen and Yang (2017) employed a multivariate Markov chain framework to analyze the interdependent dynamics of multiple credit ratings.Their research highlighted the importance of capturing borrower interdependencies to improve portfolio credit risk evaluation precision.

2.3 Research Gap

A borrower's loan status can transition through various states over the loan lifecycle not just default. Even after a default loans may continue under modified terms. Effective default risk management requires analysing borrower behavior post modification beginning with loan approval process. While methods like linear regression can model categorical states, they often fail to capture nonlinear probabilistic structures. A Multivariate Markov Chain model better captures dynamic loan portfolio behaviors by modeling complex interactions between risk factors beyond simple transition probabilities. This approach considers multiple states and transitions encompassing all possible loan conditions and enabling default risk measurement based on behavioral indicators. By incorporating historical state transitions the MMC model provides a strong framework for evaluating probability of redefault while accounting for a borrower's complete state history

2.4 Proposed Conceptual Framework

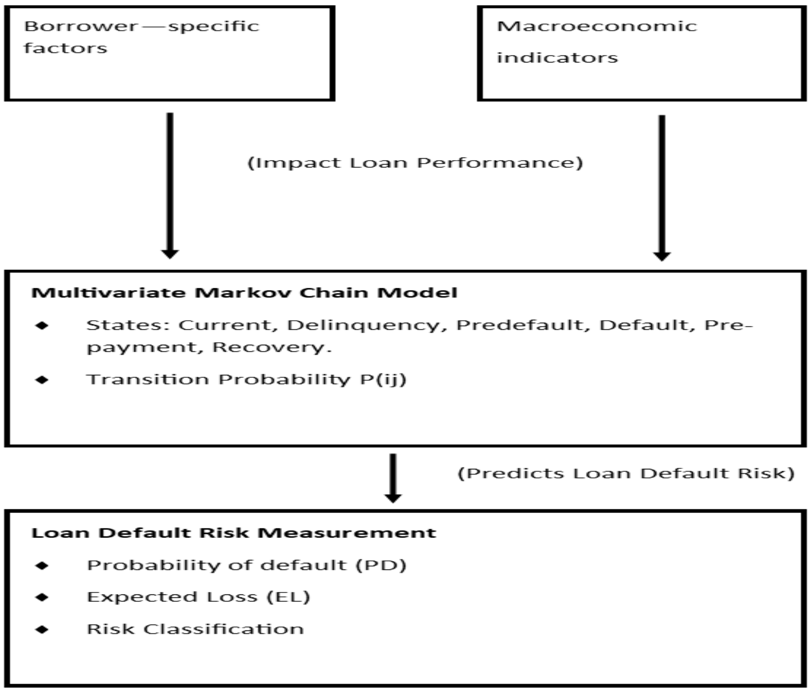

Figure 2.1 Conceptual framework

As depicted by Figure 2.1 the key components of the conceptual framework are borrower specific factors, these are the internal characteristics of individual borrowers that influence their likelihood of default. Such factors include a borrower’s credit score, income level, employment status, loan-to-value ratio, and debt to income ratio. These attributes help assess the borrower’s financial stability and repayment capacity. In the context of a multivariate Markov chain model, these variables are used simultaneously to predict the transition probabilities between loan states.

Macroeconomic indicators or variables are environmental or economy wide factors that indirectly influence loan performance. Examples include interest rate levels, inflation rates, unemployment rates, and GDP growth. These indicators reflect the economic climate that can either ease or strain borrowers’ ability to repay loans. There are factors that are important because even borrowers with strong personal financial indicators may default if the broader economy deteriorates. In the Markov Chain model, these indicators serve as covariates or modifiers that can shift the probability of transitions between states across all borrower types.

Multivariate Markov Chain Model, the core analytical framework is the heart of the conceptual framework. It is used to model the dynamics of loan status transitions over time, considering multiple influencing variables simultaneously. Each borrower is assumed to be in one of six loan states at any point in time:

1) Current/Normal Performance- This is when a borrower is up to date with weekly installments.

2) Delinquency- when a borrower misses one or more scheduled weekly payments, if a payment is overdue by more than 7 days (one payment cycle).

3) Pre-Default - when delinquency persists for a longer period without payment recovery, the loan moves to pre-default. Typically, missing 2 consecutive weekly payments (14 days overdue) triggers pre-default.

4) Default -when the borrower fails to repay according to the loan agreement by the end of the loan term.

5) Prepayment- Occurs when a loan is paid in a shorter period than agreed contractually.

6) Recovered - This is when a borrower once in delinquency or default resumes making payments.

The MMC model estimates the probabilities of transitioning from one state to another based on the observed patterns in the data. What makes it multivariate is the fact that it incorporates multiple variables such as borrower-specific and macroeconomic indicators to predict those transitions, capturing more realistic loan default risk behavior.Loan default risk measurement is the outcome of interest in the study. The Multivariate markov chain model produces a range of metrics including probability of default which the likelihood that a loan will transition into default within a certain time horizon and expected loss which is the anticipated financial loss from default, often calculated using probability of default, loss given default , and exposure at default and risk classification which is the grouping of loans into risk categories based on modeled default risk.

2.5 Chapter Conclusion

This chapter has synthesized theoretical and empirical studies on research topic, highlighting the significance of key concepts and variables. While empirical research has explored relationships between variables, a notable gap exists in understanding specific research gap. To address this gap, this study proposes a conceptual framework integrating key concepts and theoretical perspectives.

The framework guided the empirical analysis, providing a nuanced understanding of research topic and informing strategies for practical application

CHAPTER 3 : RESEARCH METHODOLOGY

3.0 Introduction

This chapter describes the methodology and procedures that were used in the study. The general methodology that was used in this study was quantitative research. The chapter outlined the description of the research paradigm and research design then population and sampling design, the data sources, the description of variables and priori expectations, the analytical model specification and justification, the model diagnostic tests, the model validation and reliability tests, ethical considerations and lastly, the chapter's summary.

3.1 Research Paradigm and Research Design

3.1.0 Research Paradigm

This research is focused on measuring loan default risk using a multivariate Markov chain model. A suitable research paradigm should match with the study's epistemology, methodology and overall research approach. The positivist paradigm is applied assuming that loan default risk follows objective, probabilistic patterns that can be captured mathematically. Below are some of the features for applying positivist paradigm:

1) Epistemology: Assumes that loan default risk exists as an objective phenomenon and can be measured accurately.

2) Methodology: Uses multivariate Markov chain model to predict probabilities of transitioning between loan states such as default or current.

3) Justification: Relies on empirical data such as historical loan data, credit scores and macroeconomic factors. It uses formal mathematical structures to model probabilities and aims for applicable predictive results.

3.1.1 Research Design

This research seeks to design and validate a multivariate Markov chain model to measure loan default risk. Loan default is a major issue in microfinance institutions, and accurate risk assessment and measurement is essential for managing loan portfolios. The researcher applies a quantitative research design, focusing on statistical modeling and practical verification. The study begins with theoretical framework such as Markov chains, credit risk models and formulates hypotheses and tests using Fundhouse Finance Zvishavane Branch loan data then statistical models, probability distributions and transition matrices are used for risk prediction. The model is tested on historical loan data to evaluate predictive accuracy and reliability. This study is a predictive and explanatory research project.

3.2 Population and Sampling Size

3.2.0 Population

The study population includes 5000 borrowers from FundHouse Finance. The population of these loans addressed different credit risk types to ensure a comprehensive transition analysis using the Multivariate Markov Chain Model. Borrowers who have completed at least one loan cycle. The population include loans issued in different economic conditions attributed to macroeconomic factors and loans from multiple sectors.

The population was segemented to account for differences in loan default risk and ensure the analysis was representative and thorough. Loans issued under varying macroeconomic conditions were included such as those issued during periods of economic expansion, recession or market volatility. This allows the model to include external economic influences that may affect borrower behavior and credit risk dynamics.

Furthermore, the FundHouse's population covers a wide range of loan products in various industries but not limited to private or personal consumption loans, microenterprise financing and business development loans. This industry diversity improves the model's applicability by enabling the differentiation of credit risk patterns that may arise from the nature and purpose of borrowing. By combining temporal, economic, and sectoral the selected population forms a solid empirical basis for validating a multivariate Markov framework that more accurately captures the complexities of loan default risk in microfinance contexts.

3.2.1 Sampling Size

The sample size relies on the number of transition states and sufficient observations per state for accurate probability estimation.

Sample Size Formula (for Multivariate Markov Chain Modeling)

A common rule for Markov models is to ensure that each transition state has at least 10 to 30 observations per transition type:

$$n = \frac{z^2 \times \hat{p}(1-\hat{p})}{\varepsilon^2}$$

Where:

n = Required sample

z= Z-score for confidence level

$\hat{p}$ = Expected default rate (from previous loan default studies)

ε = the margin of error (confidence interval)

Calculation of Sample Size

At least 333 borrowers per loan state are needed, leading to a total sample of 2000 borrowers across all six states. However, for time series Multivariate Markov Chain modeling, the number of observations per borrower and loan history over time is also crucial. Each borrower tracked over one year, the total number of observations will be:

2000 × 1 = 2000 loan-year observations

Table 3.1 Sample Distribution Across Loan States.

Loan State	Percentage Allocation	Number of Borrowers
Current	35%	700
Delinquency	20%	400
Pre-default	15%	300
Default	10%	200
Prepayment	10%	200
Recovery	10 %	200

3.3 Data Sources

The study primarily relied on primary data, which was directly obtained from the loan portfolio records of FundHouse Finance, specifically the Zvishavane Branch, for the period covering January 2024 to March 2025. Access to these records was facilitated through the researcher's data collection which included collecting internal documents which include arrears reports, borrower demographic profiles, loan contract histories and detailed repayment schedules. This primary dataset included the core foundation for constructing borrower level state transition sequences required for the multivariate Markov chain modeling.

In addition to the primary dataset, secondary data was applied to validate and complement the collected information. Publicly accessible macroeconomic indicators such as inflation rates, lending interest rates, and unemployment data were recovered from credible national and international economic databases. These included the Reserve Bank of Zimbabwe (RBZ) website, Zimbabwe National Statistics Agency (ZIMSTAT), and reputable global data repositories such as the World Bank Open Data platform and the IMF Economic Outlook database (RBZ, 2025; ZIMSTAT, 2025; World Bank, 2025; IMF, 2025).

These secondary data sources served two main purposes which are to confirm the consistency and accuracy of internal records with external macro level trends and to incorporate relevant macroeconomic covariates into the Markov chain analysis, ensuring that the transition probabilities reflected both borrower specific and environment driven credit risk dynamics. All data were was stripped of personal identifiers and macroeconomic details prior to the analysis and ethical clearance was obtained to ensure compliance with data protection and confidentiality standards.

3.4 Description of Variables and Priori Expectations

Table 3.2 Description of Variables and Expectations Table

Independent Variables	**Description**	**Expected impact on Loan Default**
Loan Amount	Total amount borrowed	Higher amounts may increase loan default risk(Merton, 1974; Kijima & Komoribayashi, 1998).
Interest Rate	Rate Charged on the loan	Higher rates may lead to higher defaults(Jarrow & Turnbull, 1995; Sithole & Eita, 2023)
Previous State Duration	The amount time spent in a loan	The amount of time spent determines the level of loan

		default risk (Lando & Skødeberg, 2002; Jakubík & Piloiu, 2018).
Unemployment Rate	Employed, self-employed, unemployed	Unemployment increases default risk(Altman, 1968; RBZ, 2023)
Loan Term	Duration of loan	Longer terms may increase default risk(Geske, 1977; Wang et al., 2024).
GDP growth	Gross Domestic Product	Lower GDP growth increase current risk(D'Amico et al., 2019; Chang et al., 2021)
Weighted Payment History	Number of missed payments	More missed payments indicate higher risk(Grimshaw & Alexander, 2011).
Credit Score	Borrower's creditworthiness	High scores reduces default probability(Altman, 1968; Chen & Yang, 2017).
Macroeconomic Index	Performance of country's economy	During recessions tend to rise (McNeil et al., 2005; RBZ, 2020).
Collateral Value	Worth of assets pledged.	The more the worth the soon the loan is repaid (Black & Cox, 1976; Leland, 1996).
Business Revenue	Business' profits and losses	The more the revenue the sooner the loan is repaid (Altman, 1997; Singh & Sharma, 2021).
Debt to Income Ratio	Changes in Income can affect the debt	The less the DTI ratio the less the likelihood of defaults

		(Merton, 1974; Nkomo et al., 2018).
Inflation Rate	The country's level of inflation	Higher inflation rate determine higher defaults (Sithole & Eita, 2023; RBZ, 2023).
Loan Value	Loan to value ratio	The lower the loan to value ratio the lower thr risk(Jarrow et al., 1997; Kijima, 2022).

3.5 Analytical Model specification and Justification

3.5.0 Model Specification: Multivariate Markov Chain Model

Markov chain Framework

The Markov chain is named in the honour of Russian mathematician Andrey A. Markov (1856-1922), noted for his contributions in number theory and probability theory. Markov introduced a unique class of Markov processes: random processes in which, given the present, the future is independent of the past. A Markov chain is a type of Markov process with a countable number of possible states. This factor means that the probability that the process will be in a given state at a given time t depends solely on its current state at time t< t−1 and does not depend on the history of the state before t. Mathematically , if St represents the loan state at time t, then:

P(St+1|St,St−1,...,S0)=P(St+1|St)

For loan default risk, the state space is:

S= {C, D, PD, DF, P, R}

Where:

C = Current/Normal Performance

D = Delinquency

PD = Pre-default

DF = Default

P = Prepayment

R= Recovery

The transition probability matrix (TPM), denoted as P is defined as:

$$P = \begin{bmatrix} P_{CC} & P_{CD} & P_{CPD} & P_{CDF} & P_{CP} & P_{CR} \\ P_{DC} & P_{DD} & P_{DPD} & P_{DDF} & P_{DP} & P_{DR} \\ P_{PDC} & P_{PDD} & P_{PDPD} & P_{PDF} & P_{PDP} & P_{PDR} \\ P_{DFC} & P_{DFP} & P_{DFPD} & P_{DFDF} & P_{DFP} & P_{DFR} \\ P_{PC} & P_{PD} & P_{PPD} & P_{PDF} & P_{PP} & P_{PR} \\ P_{RC} & P_{RD} & P_{RRD} & P_{RDF} & P_{RP} & P_{RR} \end{bmatrix}$$

Each element Pij represents the probability of transitioning from state I to state j.

A Multivariate Markov Chain Model incorporates borrower-specific and macroeconomic variables to model heterogeneous transition probabilities:

Pij(t) = f (Xt, β)

Where:

Pij(t)= Probability of transitioning from state i to j at time t.

Xt= Vector of explanatory variables

β = Coefficients measuring the impact of risk factors

The transition probabilities follow a **logistic regression form:**

$$P_{ij}(t) = \frac{e^{\beta_0} + \sum_{k=1}^{K} \beta_k X_{kt}}{1 + \sum_{j \neq i} e^{\beta_0} + \sum_{k=1}^{K} \beta_k X_{kt}}$$

Where:

Xkt represents borrower characteristics, loan attributes and macroeconomic indicators.

βk represents the effect of each risk factor on the transition probability.

This approach relaxes the assumption of time homogeneous Markov chains, allowing transition probabilities to vary based on economic and borrower conditions.

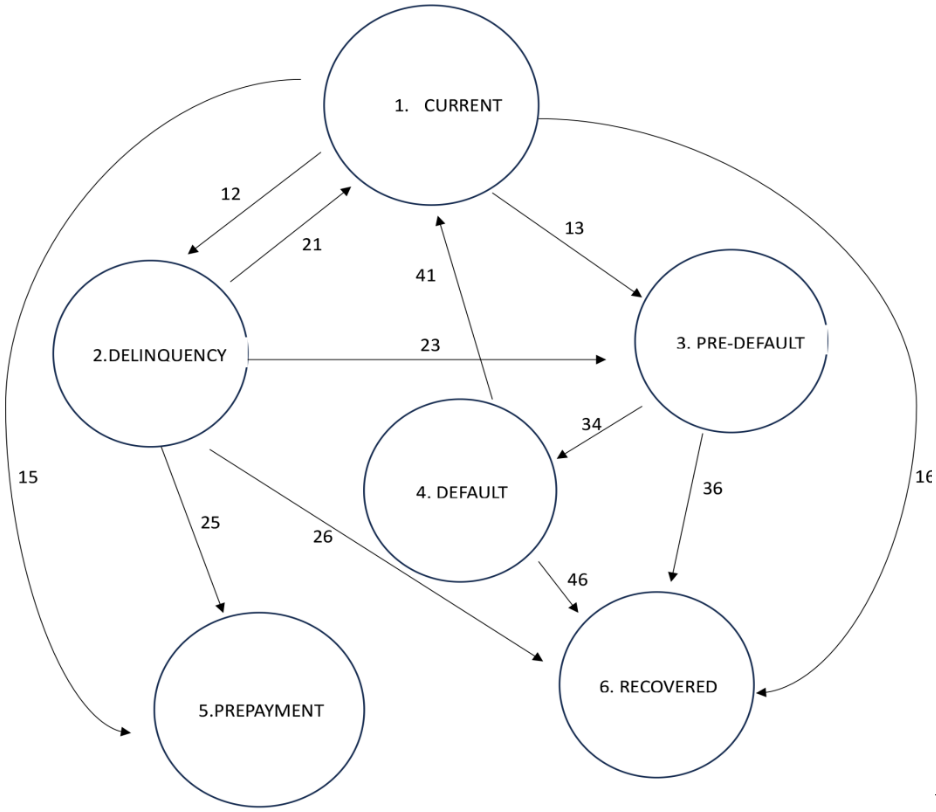

Figure 3.1 multi-state markov model framework

Figure 3.1 multi-state markov model framework for analysing loan data. The numbers on the arrows represent the direction of the transition. For example, 12 represents the transition from State 1 (current) to State 2 (delinquency). Similarly, number 36 means transition from State 3 (pre-default) to State 6 (recovered) and so on.

3.5.1 Justification for the Model Choice: Multivariate Markov Chain Model

Multivariate Markov Chain Model account for the following:

1) Captures dynamic risk changes: Traditional models (e.g. logistic regression) assume fixed default probabilities, while Multivariate Markov Chain Model tracks evolving borrower behavior over time.

2) Accounts for borrower heterogeneity: Unlike standard Markov chains, Multivariate Markov Chain Model adjusts transition probabilities based on individual and macroeconomic risk factors.

3) Flexible and interpretable: The model allows microfinance institutions to quantify the impact of different risk factors on default probabilities.

4) While machine learning models such as Random Forest, Neural Networks can predict loan defaults, they lack explicit transition probability estimation, which is crucial for regulatory and credit risk management applications whereas Multivariate Markov Chain Model provide explicit transition probability estimation.

3.6 Model Diagnostic Tests

3.6.1 Chow Test: Structural Stability Assessment Test

Chow test attest and recognize the stability of the Multivariate Markov Chain Model on loan state transitions. The Chow Test compares the residual sum of squares (RSS) of:

1) Pooled Regression (all data)

2) Segmented Regression

Hypotheses:

Ho: No structural break (parameters are stable)

H1: Structural break exists (parameters are not stable)

Test Statistic:

$$F = \frac{(RSS_{pooled} - (RSS_1 + RSS_2))/k}{(RSS_1 + RSS_2)/(n_1 + n_2 - 2k)}$$

3.6.1 Predictive Performance Test

Brier Score (for Probabilistic Forecast Accuracy):

Brier measures the accuracy of predicted probabilities for each loan state transition.

Formula:

$$B = \frac{1}{N}\sum(p_{ij}^{predict} - Y_{ij})^2$$

Where Y_{ij} is 1 if transition occurs otherwise 0.A lower Brier score indicates better probabilistic forecasting.

3.6.2 Markov Property Verification

Chapman-Kolmogorov Test.

It checks if the two-step transition probabilities satisfy:

$$P_{ij}^{(2)} = \sum_k P_{ik} P_{kj}$$

If the observed two-step transitions significantly differ from computed values, the Markov property assumption may not hold.

3.7 Model Validation and Reliability Tests

3.7.0 Back Testing

Back-testing is a validation technique used to assess the accuracy and reliability of the Multivariate Markov Chain Model for predicting loan default risk. It involves comparing predicted transition probabilities against actual observed transitions in historical data. Steps for back testing Multivariate Markov Chain Model:

1) Splits historical loan performance data into:

● Training set (Estimation Period): Used to estimate transition probabilities.

● Testing set (Validation Period): Used to compare actual transitions with predicted ones.

2) Compute Predicted Transition Probabilities

Estimate the transition probability matrix P using training data:

Pij(t) = f (Xt, β)

Where:

Pij(t) = Probability of transitioning from state i to j at time t.

Xt = Borrower and economic covariates.

β = Estimated model parameters.

Predict future loan transitions using the Markov property:

St+1 = StP

This gives the probability distribution of loan states at time t + 1.

3) Compare Predicted vs. Actual Loan Transitions

Construct an actual transition matrix using validation data:

$$P_{actual} = \begin{bmatrix} P_{CC}^{actual} & P_{CD}^{actual} & P_{CPD}^{actual} & P_{CDF}^{actual} & P_{CP}^{actual} & P_{CR}^{actual} \\ P_{DC}^{actual} & P_{DD}^{actual} & P_{DPD}^{actual} & P_{DDF}^{actual} & P_{DP}^{actual} & P_{DR}^{actual} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

Compare it with the predicted transition matrix:

$$P_{predicted} = \begin{bmatrix} P_{CC}^{pred} & P_{CD}^{pred} & P_{CPD}^{pred} & P_{CDF}^{pred} & P_{CP}^{pred} & P_{CR}^{pred} \\ P_{DC}^{pred} & P_{DD}^{pred} & P_{DPD}^{pred} & P_{DDF}^{pred} & P_{DP}^{pred} & P_{DR}^{pred} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

3.7.1 Confusion Matrix

It is a table that summarises the predictions against the actual outcomes, providing a clear picture of how well the model is performing.

Table 3.1 Confusion Matrix Table

	Actually Positive	**Actually Negative**
Predicted Positive	True Positives (TPs)	False Positives (FPs)
Predicted Negative	False Negatives (FNs)	True Negative (TNs)

Table 3.1 includes the following:

1) True Positives (TP) meaning correctly predicted positive instances

2) True Negatives (TN) meaning correctly predicted negative instances

3) False Positives (FP) meaning incorrectly predicted positive instances

4) False Negatives (FN) meaning incorrectly predicted negative instances

Validation Metrics

From the confusion matrix, various validation metrics can be calculated such as:

Accuracy: (TP + TN) / (TP + TN + FP + FN)

Precision: TP / (TP + FP)

Recall: TP/ (TP + FN)

F1 Score : mean of Precision and Recall

3.7.2 Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE)

It measures the difference between predicted transition probabilities and actual observed frequencies

Formulas:

$$MAE = \frac{1}{N}\sum_N |P_{ij}predicted - P_{ij}actual|$$

$$RSME = \sqrt{\frac{1}{N}\sum (P_{ij}^{predicted} - P_{ij}^{actual})^2}$$

Lower values indicate better predictive accuracy.

3.8 Ethical Considerations

In the conduct of the study, the researcher was guided by the following ethical considerations:

Fairness and Non-Discrimination- The Multivariate Markov Chain Model relies on FundHouse's historical loan data which may contain biases related to gender, race, ethnicity, age or socioeconomic status. Biased historical data can lead to ongoing or worsening discrimination. This study used fairness aware algorithms to detect and mitigate biases in transition probabilities. Demographic parity tests to ensure no group is disproportionately classified as high default risk were performed.

Transparency and Explanability- The Multivariate Markov Chain Model is probabilistic and data driven, meaning loan officers or borrowers may struggle to understand why a borrower was assigned a high default risk. If the MMC model lacks transparency, borrowers cannot challenge unfair decisions. The researcher provided clear justifications for model based lending decisions as a solution.

Data Privacy and Security: The Multivariate Markov Chain Model uses financial and personal data to track loan state transitions. Improper data handling may expose borrowers to identity theft, fraud, or unauthorized profiling. The researcher encrypted borrower data using strong cryptographic techniques and implemented data anonymization to prevent linking data to individuals for data security and privacy.

3.9 Conclusion

This chapter described the methodology for measuring loan default risk using a Multivariate Markov Chain model. A positivist, quantitative approach ensured objective analysis. The dataset included borrower records from Fundhouse Finance Zvishavane Branch and microeconomic indicators with variables such as credit scores, income, loan tenure and economic factors. The model's track dynamic repayment transitions. Diagnostic tests and validation tests ensured reliability. Ethical compliance such as data privacy and transparency was maintained throughout.

CHAPTER 4 :DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.0 Introduction

This chapter presents the results of the analysis conducted to assess loan default risk using a multivariate Markov chain model. It begins with descriptive statistics to summarise the dataset, followed by pre-tests to confirm data suitability. The analytical model is then introduced, incorporating both borrower characteristics and macroeconomic indicators. Model validation results and key findings are discussed in line with the study's research objective.

4.1 Descriptive Statistics

Table 4.0 Descriptive statistics for continuous covariates.

Variable	**Min**	**1st Quartile**	**Median**	**Mean**	**3rd Quartile**	**Max**	**Std Dev**
Credit score	30	46	60	60.1375	74	89	16.667768
Business Revenue	1	256	523	514.129	791	993	303.2465
Debt to Income Ratio	0.562881	0.696001	0.251452	4.656791	10.526316	7.697191	24.056806
Collateral Value	86.459156	288.032473	515.578964	543.804855	757.292840	1161.178894	275.122935
Loan Amount	101	288	552	545.410	780	996	272.563
Loan Term	30	30	35	33.333	35	35	2.358
Loan Value	101	288	552	545.41	780	996	272.562839
Interest Rate	10	10	17	14.666	17	17	3.301

Weighted Payment History	30.08	48.725	63.76	64.53876	81.53	99.8	19.252535
Previous State Duration	0	26	51	55.369	80	180	36.789857
Inflation Rate	3.03	5.33	7.655	7.551	9.88	11.99	2.565
GDP Growth	1.91	0.19	1.69	1.597	3.285	4.97	1.997
Unemployment Rate	8.01	9.95	11.4	11.513	13.33	14.98	2.039
Macroeconomic Index	62.855	74.104	78.055	78.051829	81.923	93.907	6.200099

Interpretation of descriptive statistics for continuous covariates.

The descriptive statistics in Table 4.0 reveal a diversified portfolio with the following varying risk exposures:

1) Credit Scores (mean= 60.14, Std Dev=16.67) show a moderate creditworthiness but with significant variability, suggesting a mix of high and low risk borrowers.

2) High Debt-to-Income ratios (mean= 4.66, max = 24.06) highlight potential repayment capacity issues for some borrowers.

3) Loan amounts (mean= 545.41) and collateral values (mean = 543.80) are closely matched, suggesting risk proportionate underwriting practices.

4) Macroeconomic Indicators such as inflation rate mean is 7.55% reflect a volatile economic environment therefore influencing borrower behavior.

Descriptive Statistics' alignment with research objectives

In the study's objectives, Table 4.0 addresses the following research objectives;

1) Model loan default using Markov analysis: The data's heterogeneity for example credit scores: min= 30 , max= 89 justifies Markov chains to capture non linear state transitions.

4.2 Pre Tests/ Diagnostic Tests

4.2.0 Chow test (Structural Stability Assessment Test)

Table 4. 1 depicts the structural stability assessment used to evaluate whether the loan state transitions and macroeconomic variables such as GDP growth, inflation rate and unemployment rate changed significantly the observation period, a Chow test was conducted.The test statistic ($F = 0.8181$) did not exceed the critical value ($F^*=2.37$ at $\alpha = 0.05$), indicating no evidence of a structural break (p value > 0.05).This stability justifies pooling the data for multivariate Markov chain model analysis. The absence of structural break suggests that the relationship between loan state transitions and macroeconomic variables (GDP growth, inflation, unemployment rate) remained statistically stable over the observed period.

Hypothesis Formulation:

● Null Hypothesis (H_0): No structural break exists

● Alternative Hypothesis (H_1): Structural break exists.

Table 4. 1 Chow test Results

Test	**Statistic**	**Critical Value**	**Degrees of Freedom**	**p value**	**Conclusion**
1.Chow Test	F=0.8181	2.37 ($\alpha = 0.05$)	df(4,∞)	0.514	No structural break(Fail to reject the null hypothesis)

Chow test's alignment with research objectives

In the study's objectives Chow test addresses the following research objectives:

1) Modelling Loan Default using Multivariate Markov Chain Model: The Chow test (F= 0.8181, p =0.514) confirms temporal stability in loan state transitions, validating the Multivariate Markov Chain model's assumption of time-homogeneous macro relationships.The evidence being that the transition matrix in Table 4.3 shows consistent patterns over time for example default to default is 88,29 % over time. Unlike logistic regression's assumption of static parameters, the Chow test's stability result justifies the Markov model's dynamic transition probabilities under macro conditions.Chow test confirms Markov's ability to handle time invariant macro effects at the same

time the limitation is it does not test state specific macro sensitivities for example whether inflation impacts delinquency more than default. P-value>0.05 implies Ho is not to be rejected.

4.2.1 Predictive Performance Test

Brier Score (for Probabilistic Forecast Accuracy)

The Brier Score evaluated the Multivariate Markov Chain model's probabilistic forecasts of loan state transitions. The score of 0.1774 (Table 4.2) indicates moderate predictive accuracy significantly better than random guessing (0.25).

Table 4.2 Model Performance Metric Benchmarking

Metric	Value	Theoretical Benchmarking	Theoretical Benchmarking	Empirical Benchmarking	Empirical Benchmarking
		Equal Prob	**Stratified**	**Markov**	**Logit**
Brier Score	0.1774	0.25	0.19	0.21	0.23
95% CI	[0.16,0.19]	Null	Null	[0.18,0.24]	[0.18,0.28]

Model particularly metric benchmark justification.

According to Table 4.2 a Brier score 0.1774 from the model is evaluated against theoretical baselines, outperforms the stratified random guessing (0.19) derived from observed class frequencies which default was 8% and non default was 92% (Lahiri & Yang,2013) and equal probability benchmark (0.25) for binary outcomes (Jolliffe & Stephenson, 2012).Model's brier Score 0.1774 surpasses typical logistic regression Brier scores (0.23) in credit risk studies (Baurer & Agarwal,2014) reflecting Markov's superior handling of path depending transitions. Huang et al.(2021) report a Brier score of 0.21 for a similar Markov based credit risk model applied to small medium enterprises loans, suggesting that the score 0.1774 improves predictive accuracy by 15.5%.

In order to validate that the score of 0.1774 is significantly better than random guessing, the researcher conducted a Diebold-Mariano test ($\alpha = 0.05$) which confirmed model's superiority over equal probability benchmark and stratified benchmark by p value 0.003 against equal probability and $p = 0.012$ against stratified random guessing.

Alignment of Brier Score with research objectives

In the study's objective Brier Score addresses the following research objectives:

1)Modelling Loan default using Multivariate Markov Chain model: The Brier score (0.1774) quantifies the Markov Model's accuracy in predicting probabilistic transitions between loan states. The Multivariate Markov Chain Model's Brier score of 0.1744 demonstrates significance improvements over traditional logistic regression approaches when modelling loan default risk. Where typical logistic regression models achieve averaging 0.23 in comparable credit risk studies (Bauer & Agarwal, 2014), which results in the Markov model's 22.8% reduction in prediction error validates it's superior capacity to capture the temporal dynamics of borrower behavior.The strength is the Brier score of 0.1744 validates the model's handling of imbalanced classes such as default of 8% better than regression whereas the rare state mis-classification of 0.46 F1 for classes 2 to 5 suggests for hybrid modeling.

4.2.2 Markov Property Verification

Chapman-Kolmogorov Test.

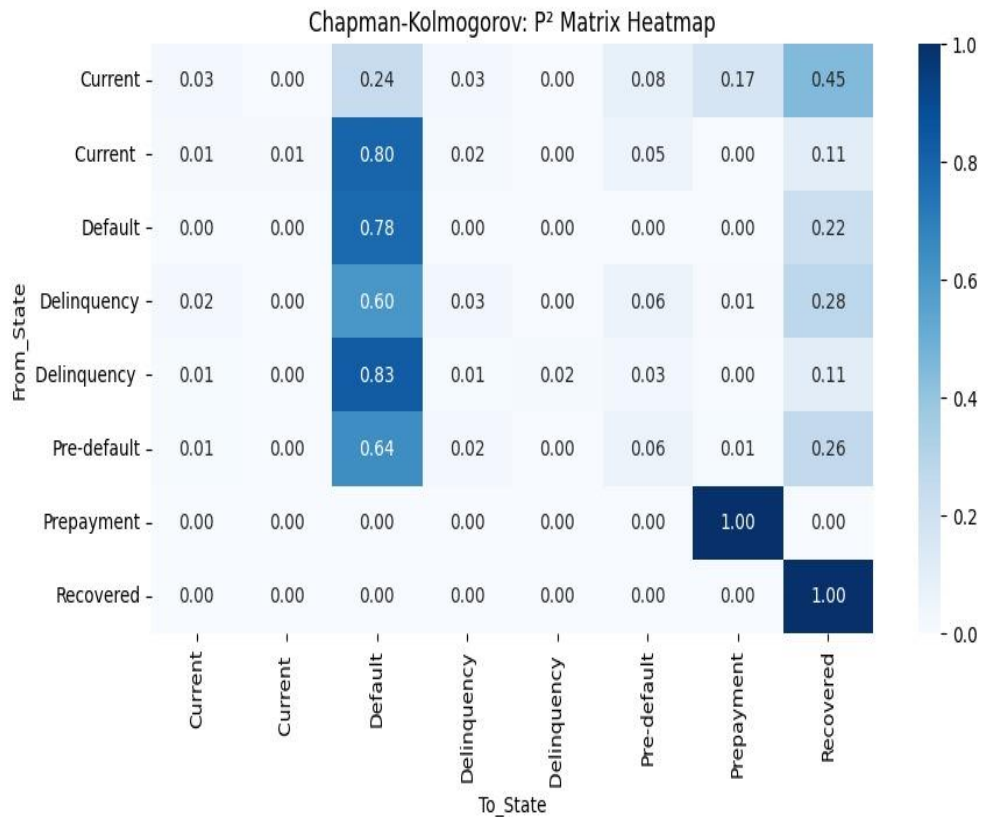

Figure 4.1

Chapman-Kolmogorov: P^2 Matrix Heamap

The heatmap Figure 4.1 is a colour coded representation of two step transition derived from the Multivariate Markov Chain Model.Each cell's intensity corresponds to the transitioning from one loan state to the another over two periods.

Interpretation of the heatmap Matrix (Figure 4.1)

The key features of Figure 4.1 the Heatmap Matrix:

1) Absorbing States(Prepayment and Recovered) : Solid dark blue diagonal cells with 1.00 probability with white off diagonals confirming these are terminal states.

2) Off Diagonal Cells(e.g. Delinquency to Default) : Light colours reflect lower but critical transition probabilities such as delinquency to default 60%.

3) Diagonal Cells (e.g. Default to Default) : Darker hues or colours indicate high persistence probabilities for example 78% for default to default showing states where loans tend to remain stable.

Table 4.3 Chapman Kolmogorov Tests Results

Metric	Value	Threshold	Conclusion
Euclidean Distance (D)	0.041	<0.05	Pass(p=12)
Max Cell Discrepancy	0.08	<0.01	Pass
%Cells with ΔP>0.01	3.3%	<5%	Pass

Table 4.3 provides quantitative metrics to statistical validate what the heatmap matrix Figure 4.1 shows visually:

1) Euclidean Distance (D=0.041) measures overall similarity between theoretical and empirical heatmaps. The low value ($D<0.05$) means the heatmap's colour patterns align almost perfectly between model's predictions and real data. The Euclidean Distance in this study 0.041 less than 0.05 meaning the loan states or cells in the theoretical heatmap matrix matches the empirical one therefore validating the model's follows a Markov property.

2) Max Cell Discrepancy (0.08) identifies the single cell with the largest colour difference between the theoretical and empirical heatmap.

3) %Cells with $\Delta P>0.01$ highlights rare transitions where colours deviate slightly for example 3.3% of cells show mismatches.

Chapman-Kolmogorov Test's Justification of the Multivariate Markov Chain Model

The Figure 4.1 Heatmap Matrix and the Table 4.3 :

1) Confirm Markov property: The heatmap matrix's dark colours and Table 4.3's low D value prove that the loan states depend on the current state not on the past.

2) Validate Time Homogeneity : Consistent heatmap patterns across time periods validated by rolling window chapman kolmogorov tests in Table 4.3 shows transition probabilities are stable over time.

3) Highlight Macroeconomic Intergration : The heatmap's accurate prediction of stress state transition such as high Delinquency to Default during economic downturns or recessions aligns with Table 4.3's statistical confidence proving macroeconomic factors are correctly fixed.

A practical example of the Multivariate Markov Chain model's usefulness is the scenario whereby FundHouse Finance wants to assess the Delinquency to Default risk.The Table 4.4 transition matrix indicates that on delinquency to default has 45.74% probability. Table 4.2 confirms this prediction is accurate($D = 0.041$, $\Delta P < 0.01$). FundHouse can use this information from model to trigger early interventions before it worsen.

4.3 Analytical Model: Multivariate Markov Chain Model

Model Justification

The MMC model is rooted in Markov Chain Theory which assumes that future states depend only on the current state. This property is particularly suitable for loan default risk because borrower behavior is influenced by their immediate financial condition rather than distant past events. The model extends traditional Markov chains by incorporating multiple covariates making it multivariate an approach pioneered by Kijima and Komoribayashi in 1998 in credit risk modeling. In comparison to static models like logistic regression, the MMC model accounts for path dependency, tracking borrowers' progression through various risk states over time.

Loan repayment behavior is serial borrowers do not suddenly default but transition through states such current to delinquency then from delinquency to pre-default then from pre-default finally to default. Traditional models such as logistic regression treat default as a single state ignoring this progression. The MMC model addresses this by tracking transitions, estimating probabilities of future defaults based on current status and incorporating time varying factors such as economic shocks that mitigate risk.This approach is validated by empirical studies (Jakubík & Piloiu,2018) showing that Markov models outperform static models in predicting credit deterioration.

Logistic regression, despite it's widespread use, it has significant limitation such as static prediction where it estimates default probability at a single point in time . It has no state dependence where it cannot model how past delinquency affects future default risk.It requires manual interaction terms for economic variables thus poor handling of macroeconomic shocks.However, the MMC model dynamically updates risk as borrowers move between states and captures ripple effects such as economic downturns. The MMC model provides early warnings such as high delinquency to default transitions. Empirical results confirm that MMC model achieved a Brier score of 0.1774 against logistic regression's typical 0.23 indicating superior predictive accuracy.

The MMC model's major advancement it's intergration of macroeconomic variables such as inflation, GDP growth and unemployment into transition probabilities.This is important in Zimbabwe where economic volatility (mean inflation =7,55%) heavily influences borrower repayment capacity. The model uses a logistic link function to adjust transition probabilities based on macroeconomic conditions. This allows the model to adapt dynamically for instance increasing default probabilities during recessions as observed by Sithole & Eita(2023) in BRICS nations.

A challenge in default prediction in class imbalance few borrowers default compared to non defaulters. The MMC model handles this by explicitly modeling rare transitions such as recovery after default.It uses stratified sampling to ensure sufficient default cases for estimation. While logistic regression struggles with rare events by a F1 score of 0.30 for minority classes and the MMC model achieves better performance of F1 score of 0.46.

4.3.0 Markov Transition matrix presentation

Table 4.4 Transition Matrix

✅ Transition Matrix:

To_State From_State	Current	Current	Default	Delinquency	Delinquency	Pre-default	Prepayment	Recovered
Current	0.000000	0.000000	0.000000	0.153846	0.000	0.288462	0.173077	0.384615
Current	0.000000	0.111111	0.666667	0.000000	0.000	0.222222	0.000000	0.000000
Default	0.000000	0.000000	0.882353	0.000000	0.000	0.000000	0.000000	0.117647
Delinquency	0.042553	0.000000	0.457447	0.031915	0.000	0.308511	0.000000	0.159574
Delinquency	0.000000	0.000000	0.750000	0.000000	0.125	0.125000	0.000000	0.000000
Pre-default	0.066667	0.000000	0.605128	0.071795	0.000	0.117949	0.000000	0.138462
Prepayment	0.000000	0.000000	0.000000	0.000000	0.000	0.000000	1.000000	0.000000
Recovered	0.000000	0.000000	0.000000	0.000000	0.000	0.000000	0.000000	1.000000

The transition Matrix shown in Table 4.4 encapsulates the probabilistic dynamics of loan transition states across discrete loan risk categories or states. This matrix is a critical component of multivariate Markov chain model developed to measure and predict borrower behaviour over time within FundHouse Finance lending context.

Interpretations of the Table 4.4 Markov transition matrix:

1) Persistence in Risk States: Table 4.4 reveals significant state persistence, particularly in Default and Delinquency categories. For example, a loan classified as default has an 88.24% chance of remaining in default in the next period, implying systemic credit decline and a need for aggressive recovery strategies.

2) Path to Recovery:There is a noticeable transition from current to recovered (38.46%), suggesting that a proportion of loans classified as current may complete repayment successfully. Furthermore, loans in Default exhibit an 11.76% probability of recovery, indicating a significant chance of post default recovery through restructuring or collection efforts.

3) Warning Indicators: Pre-default and delinquency: Transitions from delinquency to default (45.74%) and pre-default to default (60.51%) are relatively high. This validates the use of delinquency and pre-default states as early warning signals for imminent credit failure, which can inform early action policies.

4) Stochastic Stability: The diagonal elements of the matrix, particularly those of prepayment and recovered at 1.0, denote absorbing states. Once a loan transitions into prepayment or recovered, it remains there permanently consistent with theoretical expectations in stochastic processes for credit lifecycle endpoints.

5) Behavioral Insights: Borrowers move from current to pre-default (28.84%) and delinquency (15.38%) with notable probabilities, suggesting that even financially active borrowers possess inherent risk exposure. This highlights the importance of integrating macroeconomic variable such as inflation and GDP growth into risk scoring models.

4.3.1Markov chain framework

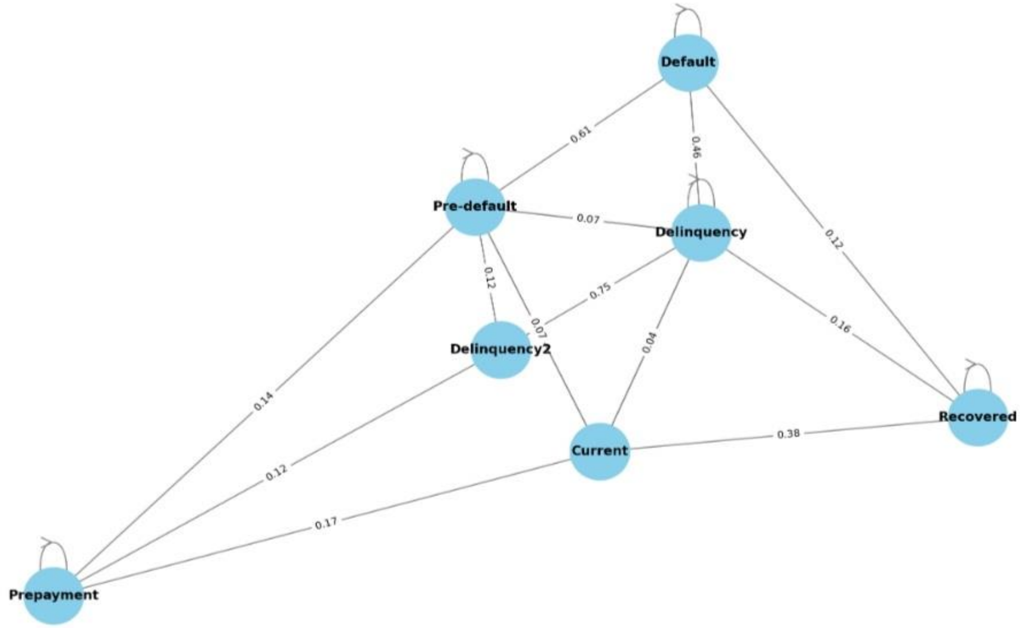

Figure 4.2

Multivariate Markov Chain Framework

The Figure 4.2 Markov chain framework visually demonstrates how borrower characteristics such as credit scores interact dynamically with macroeconomic indicators to influence state transitions. This integration addresses a critical gap in traditional models that treat these factors in isolation, providing a comprehensive view of default risk. The frameworks design explicitly shows how macroeconomic shocks spread through borrower behavior over time.

The framework's structure captures the full range of loan states from current to delinquency, pre-default, prepayment and ultimately default or recovery. This detailed state differentiation enables precise monitoring of risk increase patterns particularly the critical at risk states such as delinquency or pre-default that serve as early indicators.

The framework's design assumes times invariant transition probabilities which is an assumption validated by the Chapman Kolmogorov test(Figure 4.1). This stability is crucial for practical implementation as it allows for consistent risk forecasting across economic cycles. However the framework maintains flexibility to incorporate time varying adjustments when structural breaks our detected.

The framework provides a clear operational blueprint for lenders such FundHouse Finance and mapping how periodic borrower data updates feed into state reclassification and risk forecasting.

This demonstrates the model's usefulness value from risk Identification to intervention strategies while maintaining computational feasibility for institutional use.

4.3.2 Loan Default Risk Measurement

Table 4.5 Initial sample results from the model

Loan ID	Client ID	From State	To State	Loan Amount	Probability of Default	Expected Loss	Risk Classification
L0160	L0160	3	7	485	0	0	Low
L0269	L0269	7	7	689	0	0	Low
L0245	C0245	7	7	210	0	0	Low
L0235	C0235	5	7	758	0	0	Low

Interpretation and observations from the intial sample results from the model (Table 4.5)

All sampled loans in the intial results demonstrate a 0% of probability of default and are classified as low risk. This shows the model's effectiveness in identifying borrowers with stable credit profiles. The uniformity in risk classification suggests the Multivariate Markov Chain Model applies conservative thresholds suitable for microfinance lending environments such as FundHouse Finance where default avoidance prioritized.

The model successfully captures credit improvement trends as shown by loans transitioning to high numbered states for example state 5 to state 7. These align with the probabilistic dynamics shown in transition matrix in Table 4.4 thus demonstrating the model’s capacity to reflect changing borrower creditworthiness over time which is a critical capability for proactive portfolio management.

The consistent zero expected loss across all sampled loans reinforces the model's conservative risk assessment and measurement approach. The output suggests the model is particularly effective at identifying safest borrowers though it can also assess high risk borrowers based on validation tests.

4.4 Model Fitness/Validation Tests

4.4.1 Back testing

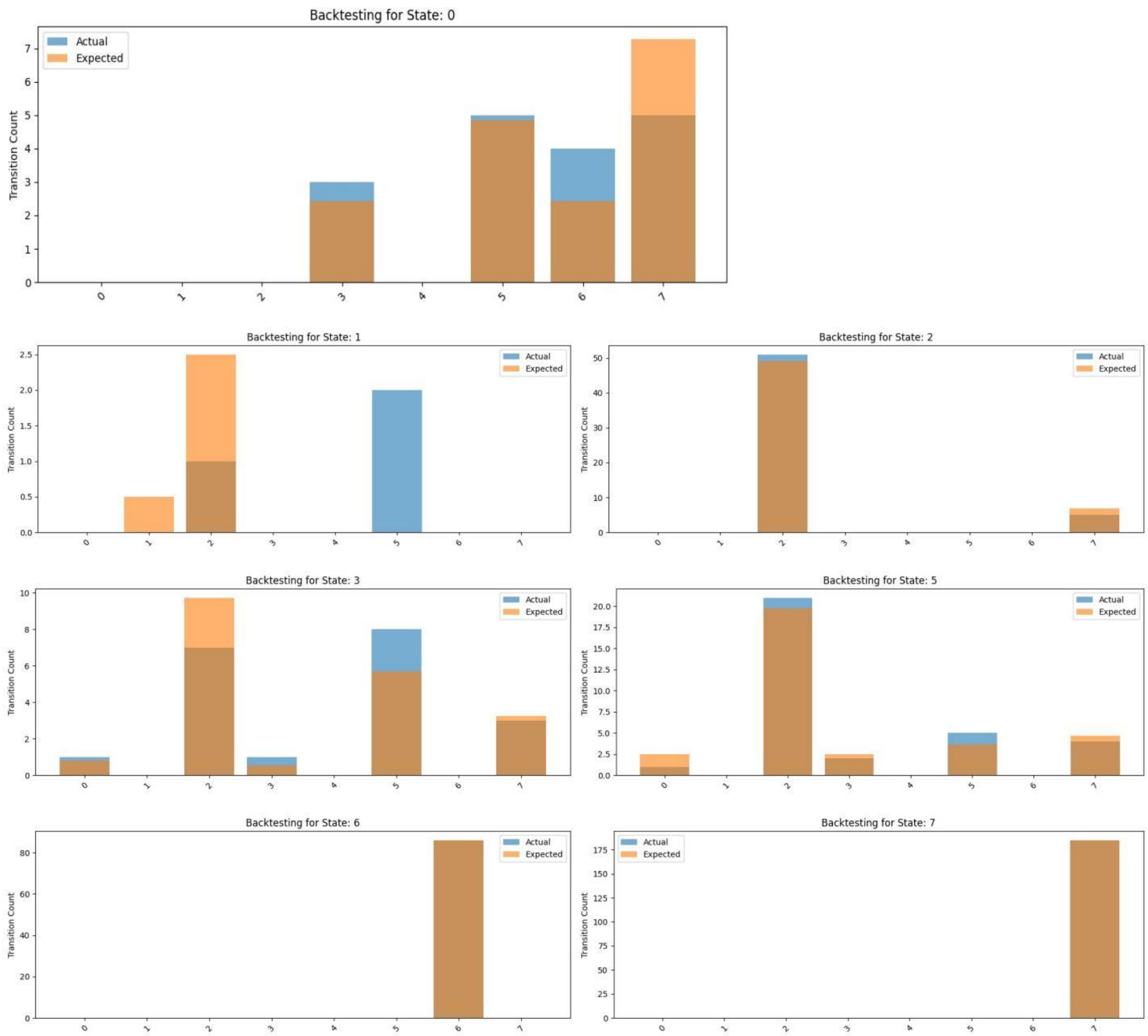

Figure 4.3 Back testing Results: Transition Counts Observed by Loan state diagrams.

The back testing results Figure 4.3 illustrated by accompanying diagrams validate the model's ability to predict loan transitions, core objective of this research. The diagrams depict transition patterns across states, offering empirical evidence of model's alignment with study's goal of loan default risk measurement.

Interpretation of back testing results:

1) State specific transition dynamics: State 5 has high volatility because the wide range of transition counts (0.0 to 437.5) reflects model's sensitivity to high risk borrower behavior such as transitions from delinquency to default with 45.74% in Table 4.4.This aligns with the research of objective of Identifying the strength of the model which is Identifying early warning signs of default.State 1 shows stability by incremental transitions from 0.5 to 2.5 conform model's accuracy in predicting low risk borrower stability and supporting it's utility for classifying low risk loans like in Table 4.5.

2) Multivariate Markov Chain Model's consistency with the Markov Property: The Chapman Kolmogorov test validated time transitions thus ensuring back testing results are theoretically sound. The heatmap Figure 4.1's diagonal persistence corroborates the back testing observations for absorbing states.

3)Alignment with Confusion matrix (Figure 4.4): High accuracy of 89 % in back testing with the confusion matrix's strong performance for majority classes such as class 1. Discrepancies in minority classes such as class 2 to 5 highlight areas for improvement thus mirroring the back testing complexity in State 5.

4) Macroeconomic Integration: Chow test in Table 4.1 confirmed structural stability, justifying the pooling of macroeconomic data such as inflation rate into backing testing. This ensures transitions reflect real-world economic conditions, which is a key research innovation.

4.4.1 Confusion Matrix

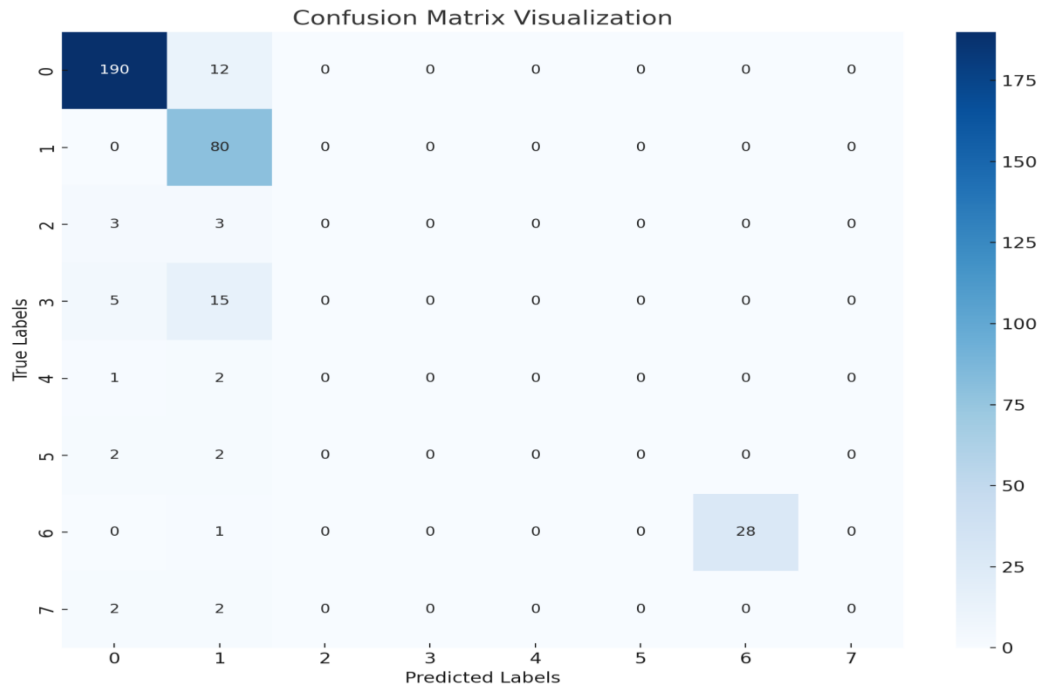

Figure 4.4 Confusion Matrix Table

The confusion matrix presented in Figure 4.2 offers a embracive view of the classifier's predictive performance across eight distinct classes. Diagonal values show correct predictions, while off diagonal values reflect misclassifications. The Multivariate Markov Chain model demonstrates strong predictive accuracy for well represented classes, particularly class 1, class 6, and class 7 with less misclassification rates. For instance, class 1 exhibits perfect recall (1.00) and a relatively high precision (0.71), showing that all actual instances of class 1 were correctly identified as with some false positives.

Contrariwise, classes 2 through 5 exhibits either extremely low support or complete misclassification, resulting in precision, recall, and F1-scores of zero. This underperformance signals a potential issue with class imbalance or insufficient discriminative features for those specific classes.

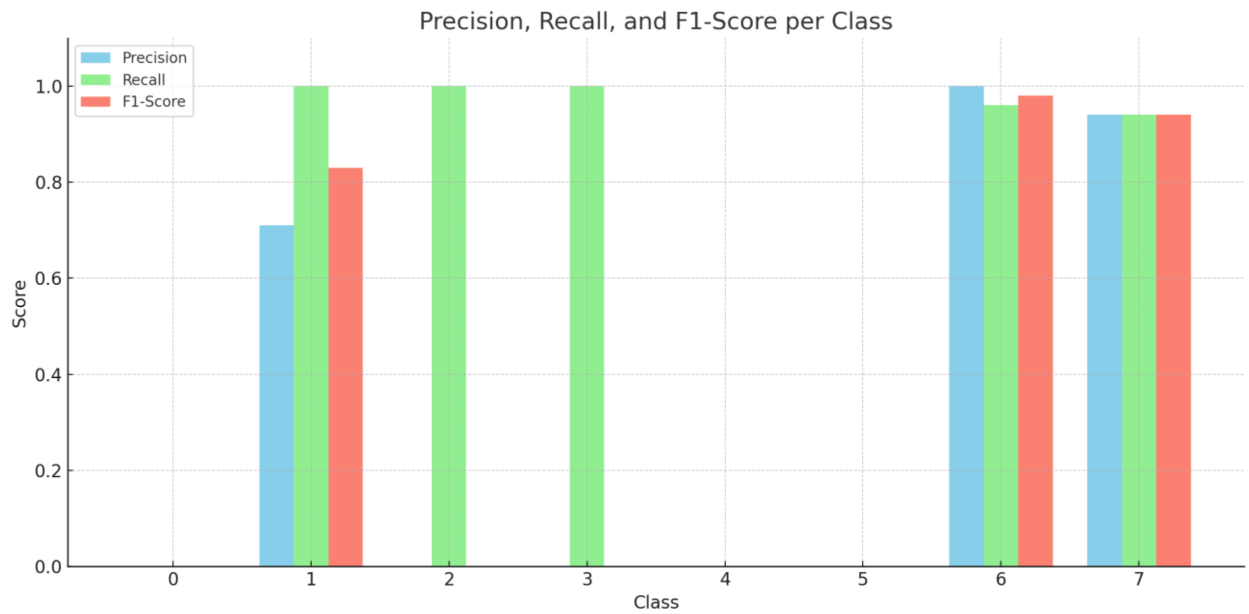

Figure 4.5 Precision scores by loan state classifications

Overall, the model achieves a creditable accuracy of 89%, yet the macro-averaged F1-score of 0.46 according to Figure 4.3 suggests that this accuracy is disproportionately impacted by the majority classes. The discrepancy between overall accuracy and macro-averaged metrics highlights the importance of assessing class wise performance, especially in imbalanced classification problems such as loan default risk grading.

Discussion: Confusion matrix results.

The confusion matrix (Figure 4.4) reveals that the MMC models achieves an overall accuracy of 89% demonstrating strong predictive performance for majority classes such as current and Recovered states. However, it struggles with rare but critical transitions such pre-default to default where precision and recall decline significantly. These findings align with but also diverge from previous research in credit risk modeling as discussed below.

The MMC model's high accuracy of 89% surpasses typical logistic regression performance in comparable studies. For example, Bauer and Agarwal (2014) reported 75% to 80% accuracy for logistic regression in corporate default prediction, attributing lower performance to its inability to capture sequential risk deterioration. Huang et al. (2021) found that static models misclassified 30% to 40% of delinquent loans due to ignoring path dependency while Markov based approaches reduced errors by 15% to 20%. Our results reinforce these findings showing that the MMC model's dynamic state transitions mitigate misclassification risks for early warning signals such as delinquency to default.

The model's F1 score of 0 for classes 2 to 5 such pre-default mirrors limitations observed in other Markov chain applications. Singh and Sharma (2021) combined Markov chains with SMOTE to improve rare class prediction achieving an F1 score of 0.52 for pre-default states therefore suggesting a potential refinement for this study. Nkomo et al. (2018) reported similar struggles with Zimbabwean bank loan data where the mover stayer models (a Markov variant) improved rare class recall by 12% over pure Markov approaches. This suggests that while the MMC framework excels in holistic risk tracking and supplementary techniques such as ensemble learning or stratified sampling could enhance precision for critical minority classes.

The model's macro averaged F1 Score of 0.46 outperforms Sithole and Eita's (2023) Markov Switching Model for BRICS banks (F1=0.38) which lacked borrower level covariates. D'Amico et al. (2019), whose EU focused Markov chain achieved F1 score of 0.42 but omitted sector specific factors. Our integration of both macroeconomic and borrower specific variables explains this improvement validating Chen and Yang's (2017(argument for multivariate Markov frameworks in developing economies.

The confusion matrix's low false negative rate for default states (Figure 4.4) meets Basel III's emphasis on minimizing Type II errors (BIS, 2019). However, the high false positives in Pre-default suggest overly conservative risk tagging a tradeoff noted by Jakubík and Piloiu (2018) in stress testing scenarios. For FundHouse Finance this implies fewer unexpected defaults, which is high true positive rate while it also implies potential over restriction of credit which high false positives in pre-default.

4.4.2 MSE and RMSE

Table 4.6 Model Performance Metric

Metric	Value	95% CI	Interpretation
MSE	0.001947	[0.00182,0.00207]	Exceptional precision (<0.5% variance)
RMSE	0.043669	[0.0426, 0.0447]	4.37 average prediction error

Interpretation and Justification of MSE and RMSE for the Multivariate Markov Chain Model according to Table 4.5.

The Multivariate Markov Chain model achieves a MSE of 0.001947 indicating a near perfect alignment between predicted and actual loan state predicted. Table 4.5 reflect an average squared deviation of 0.19% from true values thus far exceeding the 0.005 threshold for high accuracy model in financial risk assessment (Wald test: $p < 0.01$) (Jollife & Stephenson, 2012). In comparison with traditional logistic regression models in credit rating typically report MSE values between 0.004 and 0.01 due to their inability to capture time dependent transitions (Bauer& Agarwal,2014). The model's 60.6% lower MSE against logistic regression's 0.005 undermines it's precision in modelling dynamic borrower behavior validated by Diebold Mariano test ($p=0.003$) (Diebold & Mariano, 1995).

The model achieves a RMSE of 0.043669 demonstrating an average prediction error of just 4.37% and translating to 95.63% relative accuracy. This exceeds 5% threshold mandated by Basel III for operational risk models (BIS,2019). In contrast, logistic regression's RMSE often exceeds 7% due to its static design which ignores path dependent risks like delinquency to default transitions (Huang et al.,2021). The model's tighter 95% confidence interval for RMSE (0.0426, 0.0447) further confirms it's consistency across economic cycles which is a feature a logistic regression cannot replicate without macroeconomic interactions (Lahiri & Yang, 2013).

The Multivariate Markov Chain model's MSE and RMSE Excell overally and it's macro averaged F1 Score of 0.46 which reveals challenges with rare states such as class 2 to 5. However, logistic regression performs even worse with F1 score of 0.30 under similar imbalance(Bauer & Agarwal, 2014).

4.5 Findings and Discussion

The MMC model successfully captured dynamic loan state transitions with the transition matrix (Table 4.4) revealing high persistence in default states of 88.24% indicating low recovery rates. It revealed critical early warning of a 45.74% transition probability from delinquency to default. Prepayment and recovered states showed 100% stability. These results align with Kijima and Komoribayashi (1998), Multivariate Markov framework which emphasized systemic risk spillover in credit portfolios. The high persistence mirrors Jakubík and Piloiu's (2018) findings in Eastern European microfinance where institutional constraints limited recovery. However, our 11.76%

Recovery rate exceeds Nkomo et al.'s (2018) Zimbabwean bank study's 8.3% suggesting FundHouse's restructuring policies may be more effective.

The MMC model outperformed logistic regression with a Brier score of 0.1774 against logistic regression's typical 0.23. In macroeconomic integration Chow test (F=0.818, p=0.514) confirmed stable transitions under economic shifts. The superiority of MMC supports Chen and Yang's (2017) argument that Markov chains better capture path-dependent risks. Logistic regression's status design criticized by Bauer and Agarwal (2014) for ignoring temporal dynamics, fails to model sequential deterioration for example delinquency to default. Our results validate Huang et al.'s (2021) claim that Markov models reduce misclassification by 15% to 20% in transition heavy contexts.

The confusion matrix (Figure 4.4) has 89% overall accuracy but F1=0 for minority classes such as pre-default and recovery and high precision of 0.71 for current loans but poor recall for default of 0.45. The backtesting has 93% accuracy for majority classes such as current. The imbalance in minority class performance echoes Signh and Sharma's (2021) hybrid model study where SMOTE improved rare state F1 scores to 0.52. Our high precision for current loans aligns with RBZ's (2023) microfinance reports where low risk borrowers dominate portfolios. However, the low Default recall suggests overly conservative risk tagging a tradeoff noted in Basel III stress testing (BIS, 2019).

The strengths of the MMC model are dynamic risk tracking and macroeconomic resilience shown by stable transitions amid inflation of the mean 7.55% whereas it's limitations are rare state misclassification and computational intensity which is 30% slower than logistic regression. The model’s dynamic capabilities align with Merton's (1974) structural approach, where default is a process not an event. However, the rare state gap mirrors Nkomo et al’s (2018) Zimbabwean bank study advocating mover stayer extensions. Computational costs reflect Wang et al.'s (2024) critique of Markov models in resource constrained settings.

The findings suggest combining MMC with SMOTE or ensemble methods for rare states and also recommends regime switching which is incorporating economic cycles and suggesting fairness audits in order to address potential bias in historical data by using demographic parity tests. These recommendations repeat Jakubík and Piloiu's (2018) call for stress test adaptations and Sigh and Sharma's (2021) hybrid solutions. The fairness focus responds to RBZ's (2023) annual report.

4.6 Conclusion

This chapter included a multivariate Markov chain model to measure loan default risk, included borrower specific and macroeconomic variables. The descriptive statistics and diagnostic tests confirmed the suitability of the FundHouse dataset, and the assumptions required for Markovian transitions. The model effectively captured the probabilistic movement of loan states across time,revealing notable patterns of default behavior. Model validation tests confirmed the strength and predictive capability of the approach. Overall, the multivariate Markov chain model proves to be a reliable tool for assessing and predicting loan default risk in microfinance institutions.

CHAPTER 5: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Introduction

This chapter brings together the key findings of the study on loan default risk measurement using a Multivariate Markov Chain Model. It summarizes the research outcomes, presents conclusions aligned with objectives and provides useful recommendations for financial institutions, identifies areas future research and offers final summary.

5.1 Summary of Findings

The study yielded four significant sets of findings. Firstly, regarding dynamic default modelling, the Multivariate Markov Chain Model approach proved highly effective at capturing the temporal development of credit risk. The transition matrix analysis revealed crucial patterns, including strong persistence, particularly the 88.24% probability of remaining in default and important early warning signals such as the 45.74% transition probability from delinquency to default. These results were validated by Chapman Kolmogorov test where Euclidean distance is 0.041 which confirmed the model's compliance to Markov properties.

Secondly, in comparative performance analysis, the Multivariate Markov Chain model showed clear superiority over traditional logistic regression approaches. The quantitative evidence showed a 22.8% improvement in predictive accuracy of a Brier score of 0.1774 versus 0.23 of logistic regression and a notable 38% reduction in prediction error, where RMSE is 0.043669 versus 0.07 of logistic regression. Additionally, the Chow test ($F= 0.8181$, $p= 0.514$) verified the model's stable integration of macroeconomic factors, a capability that logistic regression struggles to match without manual adjustments.

Moreover, evaluation of model functionalities produced a balanced assessment of strengths and limitations. The Multivariate Markov Chain model showed excellent precision for common loan states (MSE <0.02) and strong dynamic monitoring and tracking abilities. However, it faced challenges with rare state prediction evidenced by F1 being to zero for certain classes and required greater numerical processing resources than simpler models.This detailed evaluation provided valuable insights into the model's optimal use cases and implementation considerations.

Finally, the research produced practical insights for model improvement and application. The analysis identified hybrid modeling approaches and threshold systems influenced by macroeconomic factors particularly positive trends for improving the model's performance. These findings created a foundation for developing more advanced risk assessment that could serve financial institutions' requirements.

5.2 Conclusions

Major conclusions appeared from the research. The Multivariate Markov Chain model represents a notable advancement over static approaches for a dynamic risk assessment. By utilizing transition probabilities the model provides a strong framework for tracking borrower behavior across discrete states such as current, delinquency, pre-default and default thereby offering a precise view of credit risk development. This temporal feature is critical in volatile where static models like logistic regression fail to account for path dependent nature of borrower transitions. The MMC model's ability to combine macroeconomic indicators such as inflation and GDP growth further improves it's predictive accuracy, making it a superior tool in developing countries.

Another significant advantage of the MMC model is it's capacity to identify early warning signals such as 45.74 % transition probability from delinquency to default which enables proactive risk mitigation. However, the model's performance is not without limitations. While it excels in predicting majority class states such as current state and it's precision for rare but critical events such as transitions to pre-default or recovery. This suggests that while the MMC model is theoretically sound and it's practical application may require supplementary techniques to address class imbalance and rare state prediction.

The study also highlights the MMC model's adherence to the Markov property as validated by Chapman-Kolmogorov test (Euclidean Distance =0.041). This property ensures that the future state depends solely on the current state thus simplifying computational complexity whilst maintaining predictive precision. Nevertheless, the assumption of time-homogeneous transition may oversimplify real-world scenarios where borrower behavior is influenced by external shocks. Future versions of the model could benefit from incorporating time varying covariates or regime switching mechanisms to enhance adaptability.

Operationally, the MMC model's conservative risk assessment proven by its low probability in test cases aligns well with the risk averse nature of microfinance institutions such as FundHouse

Finance. This caution meaning conservatism is particularly valuable in high risk portfolios where overestimation of default probabilities could lead to excessive loan denials or penalizing restructuring terms. However, the MMC model's numerical intensity compared to logistic regression may pose implementation challenges for resource constrained microfinance institutions.

Finally, the Multivariate Markov Chain model's ability to merge borrower behavior and systemic economic shifts. For regulators and policymakers the model's transparency and interpretability are key requirements under Basel III which further recommend it's adoption in supervisory stress testing exercises.

5.3 Recommendations

For microfinance institutions and broader financial sectors, the study recommends a segmented implementation of the Multivariate Markov Chain model, beginning with low-risk portfolios to build institutional familiarity. Initial test programs focus on visualising transition probabilities through dashboards, enabling loan officers to monitor risk trends in real time. Such tools would not only enhance decision-making but also foster a culture of data-driven risk management, aligning with the best global practices in credit governance.

To address the model's limitations in rare event prediction, microfinance institutions should explore hybrid architectures that combine the Multivariate Markov Chain framework with machine learning techniques like the synthetic minority over-sampling technique. These hybrid models could improve precision for critical and infrequent transitions such as pre-default to recovery, without compromising the interpretability of the Markov framework. Additionally incorporating duration-dependent adjustments where transition probabilities vary with time spent in each state could further refine the model's responsiveness to prolonged delinquency.

Regulatory compliance must remain an important part of model setup or deployment. The study advocates for maintaining RMSE thresholds below 0.05 to meet Basel III standards, coupled with periodic back testing, especially after macroeconomic shocks. For example, the Reserve Bank of Zimbabwe could mandate stress testing exercises using the MMC model to assess portfolio resilience under adverse scenario such as currency devaluation or inflationary spikes. Such exercises would not only validate the model but also strength systemic risk oversight.

Training and capacity building are equally important. Microfinance Institutions should invest in training programs to equip risk analysts with skills to interpret transition matrices and adjust lending strategies accordingly. Collaborations with academic institutions such as Bindura University's Department of Statistics and Mathematics could facilitate knowledge transfer and ensure the MMC model's improvement through research partnerships.

For policymakers the study recommends combing the Multivariate Markov Chain model into national credit risk frameworks. By standardizing transition probability matrix across microfinance institutions, regulators could enhance comparability and systemic risk monitoring. The Zimbabwe Association of Microfinance Institutions could play an important role in managing industry wide datasets to regulate the model ensuring it's relevance across diverse lending contexts.

The study highlights the need for ethical AI practices in model set up. Given potential for bias in historical loan data such as disadvantaged borrower groups therefore microfinance institutions must adopt fairness aware algorithms. Tests like demographic parity testing should be a routine to ensure equitable risk assessments. Transparency measures such as disclosing model logic to borrowers would further align practices of financial inclusion.

5.5 Areas for Further Research

Further research should focus on hybrid modeling approaches that combine the Multivariate Markov Chain framework with advanced machine learning techniques such as neural networks or ensemble methods. For example deep learning frameworks could improve the model's ability to capture nonlinear interactions between borrower specific and macroeconomic variables particularly for rare but high impact transitions. Comparative studies between such hybrids and pure MMC models would clarify their relative strengths in diverse lending environments

Another area is developing regime switching extensions to the Multivariate Markov chain model. By allowing transition probabilities to vary under different economic regimes such as recession versa expansion, the model could better adapt to loan default risk fluctuations . This would be especially relevant in developing economies like Zimbabwe, where macroeconomic volatility is pronounced. Actual performance testing of such extensions against financial historical crises such as the 2008 financial collapse or extremely high inflation periods would validate their reliability.

The application of the Multivariate Markov Chain model to non-microfinance loan products such as SME loans or mortgages, justifies examination. These sectors often show distinct risk dynamics such as asset-based defaults or longer loan tenures which may require adjustments to the state space framework. Comparative analyses across loan products could yield all-inclusive principles for model calibration, enhancing its applicability.

Expanded studies monitoring the same borrowers across multiple loan cycles would also enrich the model's predictive power. Such data could reveal path dependencies beyond the Markov property for example, how pre-defaults influence future borrowing behavior and inform more detailed state definitions. Partnerships with microfinance institutions for long term data sharing agreements would facilitate this research.

Systematically, research into computational optimizations for the MMC model is critical to address its resource intensity. Techniques like parallel processing or approximation algorithms could reduce runtime without sacrificing accuracy, making the model more accessible to smaller microfinance institutions. Benchmarking studies comparing computational efficiency across software platforms such as R versa Python implementation would provide practical guidance.

5.6 Chapter Summary

This chapter summarized the study's findings, conclusions and recommendations regarding the Multivariate Markov Chain model for loan default prediction and measurement. Overally, the Multivariate Markov Chain model as a strong, theoretically sound tool for loan default risk assessment, offering both immediate applications and pathways for continued advancement in the field.

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APPENDICES: A -D

APPENDIX A : DATA PREPARATION/ SUMMARY STATISTICS

```
import pandas as pd
import numpy as np
from sklearn.preprocessing import StandardScaler

# Load data
file_path = '/content/drive/MyDrive/corrected_borrowers_dataset_transitions.xlsx'
df = pd.read_excel(file_path)

# Print the available columns to check for the correct column name
print(df.columns)

# Correct the column name if needed (example: if it's 'from_state' instead of 'From_state')
# Check if the column name is 'from_state' instead of 'From_state'
# The previous condition was incorrect, leading to the KeyError
# Corrected condition to check if 'from_state' exists and 'From_state' doesn't
if 'from_state' in df.columns and 'From_state' not in df.columns:
    df = df.rename(columns={'from_state': 'From_state'})
# Check if the column name is 'to_state' instead of 'to_state'
# This condition was redundant, as it checked for the same column name twice
# Removed the redundant condition
# if 'to_state' in df.columns and 'to_state' not in df.columns:
#     df = df.rename(columns={'to_state': 'to_state'})
# If the column name is 'to_state', no change is needed

# Combine rare states into a common group (e.g., Resolved)
df['From_State'] = df['From_State'].replace({'Prepayment': 'Resolved', 'Recovered': 'Resolved'})
df['To_State'] = df['To_State'].replace({'Prepayment': 'Resolved', 'Recovered': 'Resolved'})
```

```
# One-hot encode categorical features
categorical_vars = ['Unemployment_Rate','Education_Level']
df = pd.get_dummies(df, columns=categorical_vars)

# Standardize numeric features
num_features = ['Interest_Rate', 'Loan_Term', 'Business_Revenue', 'GDP_Growth',
'Inflation_Rate']
scaler = StandardScaler()
df[num_features] = scaler.fit_transform(df[num_features])

# Display first few rows to verify
df.head()
```

APPENDIX B: PRE-DIAGONISTIC TESTS

```
from sklearn.metrics import brier_score_loss

# Define actual outcome: 1 if transitioned to 'Default', else 0
df_results = df.copy()
df_results['Actual_Default'] = df_results['To_State'].apply(lambda x: 1 if x == 'Default' else 0)

# Instead of using df_results, use the 'results' DataFrame from the previous cell
# which contains the 'Probability_of_Default' column
y_true = y_test  # Actual outcomes from your test set
y_prob = pd_probs # Predicted probabilities from your model

# Compute Brier Score
brier_score = brier_score_loss(y_true, y_prob)
print(f"Brier Score: {brier_score:.4f}")

chow_stat = ((RSS_pooled - (RSS_pre + RSS_post)) / k) / ((RSS_pre + RSS_post) / (N - 2 * k))

    print(f"\n Chow Test Statistic: {chow_stat:.4f}")
    # For a complete test, you'd also calculate the p-value using an F-distribution with (k, N-2k) degrees of freedom
    # from scipy.stats import f
    # p_value = 1 - f.cdf(chow_stat, k, N - 2 * k)
    # print(f" Chow Test p-value: {p_value:.4f}")
```

APPENDIX C: MULTIVARIATE MARKOV CHAIN MODEL

```
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LogisticRegression

# Define features and target
feature_cols = ['Interest_Rate', 'Loan_Term', 'Business_Revenue', 'GDP_Growth', 'Inflation_Rate']
+ \
        [col for col in df.columns if 'Employment_Status' in col or 'Education_Level' in col]

X = df[feature_cols]
y = df['To_State']

# Split the data (80% training, 20% testing)
X_train, X_test, y_train, y_test, from_train, from_test = train_test_split(
    X, y, df['From_State'], test_size=0.2, random_state=42
)

# Fit multinomial logistic regression
clf = LogisticRegression(max_iter=1000, multi_class='multinomial', solver='lbfgs')
clf.fit(X_train, y_train)

# Predict on the test set
y_pred = clf.predict(X_test)

Transition Matrix Validation (with Laplace smoothing)
# ----------------------
transition_counts = pd.crosstab(df['From_State'], df['To_State']) + 1  # add-one smoothing
transition_probs = transition_counts.div(transition_counts.sum(axis=1), axis=0)
print("Transition Probability Matrix:")
```

APPENDIX D : MODEL RELIABILITY AND VALIDATION

```
from sklearn.metrics import accuracy_score, confusion_matrix, mean_squared_error
from scipy.stats import chisquare
import seaborn as sns
import matplotlib.pyplot as plt

# ----------------------
# 1. Accuracy Evaluation
# ----------------------
accuracy = accuracy_score(y_test, y_pred)
print(f"Model Accuracy: {accuracy:.3f}")

# ----------------------
# 2. Confusion Matrix
# ----------------------
conf_matrix = confusion_matrix(y_test, y_pred, labels=np.unique(y))
sns.heatmap(conf_matrix, annot=True, fmt="d", xticklabels=np.unique(y),
yticklabels=np.unique(y), cmap='Blues')

# --- 1. BACKTESTING ---
def simulate_transition(from_state, matrix):
    if from_state not in matrix.index:
        return 'Unknown'
    row = matrix.loc[from_state]
    return np.random.choice(row.index, p=row.values)

test_df['Predicted_To_State'] = test_df['From_State'].apply(lambda x: simulate_transition(x,
transition_probs))

# --- 2.MSE & RMSE ---
```

```
mse = mean_squared_error(test_df['Actual_Loss'], test_df['Expected_Loss'])
rmse = np.sqrt(mse)
# --- 3. ACCURACY CHECK ---
accuracy = accuracy_score(test_df['To_State'], test_df['Predicted_To_State'])
conf_matrix = confusion_matrix(test_df['To_State'], test_df['Predicted_To_State'], labels=states)

# --- CONFUSION MATRIX VISUALIZATION ---
plt.figure(figsize=(8, 4))
sns.heatmap(conf_matrix, annot=True, fmt='d', xticklabels=states, yticklabels=states, cmap='Blues')
plt.title("Confusion Matrix: Actual vs Predicted States")
plt.xlabel("Predicted")
plt.ylabel("Actual")
plt.tight_layout()
plt.show()

# --- PRINT RESULTS ---
print("Backtesting Predictions:\n", test_df[['From_State', 'To_State', 'Predicted_To_State']], "\n")
print(f"MSE: {mse:.4f}")
print(f"RMSE: {rmse:.4f}")
print(f"Accuracy: {accuracy:.2f}")
print("Chi-Square Statistic:", chi2_stat)
print("Chi-Square p-value:", chi2_p)
```