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DEPARTMENT OF MATHEMATICS

MISCONCEPTIONS AND RESULTING ERRORS DISPLAYED BY 'O' LEVEL STUDENTS IN THE LEARNING OF ALGEBRA

BY

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APPROVAL FORM

The undersigned certify that they have supervised, read and recommended to Bindura University of Science Education for acceptance a research project entitled, **misconceptions and resulting errors displayed by 'o' level students in the learning of algebra**

Submitted by **Takawira Sharon Getrude** in partial fulfillment of the requirement of the Bachelor of Science Education Honours Degree in Mathematics

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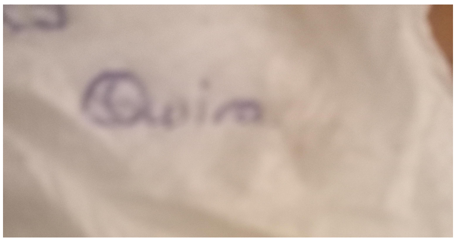

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(Signature of candidate)

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DEDICATION

This research project is dedicated to my sons, Tavonga and Kunashe and my father.

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I would like to express my appreciation to my supervisor DR L Mutambara for the support, guidance and dedication which helped me to enable this project to be a success. I also acknowledge all the prior researchers whose ideas have been used in this research study

ABSTRACT

This qualitative research study investigates the common misconceptions and resulting errors displayed by form 3 learners at Nyatso Secondary School in their learning of algebra. The study was conducted at Nyatso Secondary School, with a focus on a form 3 mathematics class of 40 students. From this population, a sample of 20 students was selected to participate in the research and the two mathematics teachers at the school. The sampling was done using a purposive sampling technique. The researcher identified students representing a range of performance levels in algebra - high-achieving, struggling and average. This ensured the sample was representative of the diversity within the class in terms of algebraic understanding. The final sample included an equal number of male and female students. Data was collected through pre- and post-tests, an interview for the mathematics teachers, classroom observations and analysis of student work. The study found that the most prevalent misconceptions included inability to properly translate word problems into algebraic expressions, confusion between variables and constants, difficulties with applying the correct order of operations and struggles with simplifying algebraic expressions involving negative signs and exponents. These misconceptions often led to systematic errors in the students' algebraic problem-solving strategies and final answers. The findings suggest that addressing these core misconceptions through targeted instructional approaches may help improve students' overall understanding and performance in algebra. The study provides insights that can inform teaching practices and curriculum development in secondary mathematics education.

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CHAPTER I

RATIONALE OF THE STUDY

1.0 Introduction

This research deals with exploring the misconceptions and resulting errors displayed by O level students in the learning of algebra. Nyatso Secondary School in Gokwe North district was used as the case study to explore these misconceptions and resulting errors and coming up with solutions. However this introductory chapter provides the background information about the research project, statement of the problem, research questions, research objectives, and justification of the research, assumptions, delimitations, limitations and the definition of terms.

1.1 Background of the study

This research deals with exploring learners' misconceptions and resulting errors in learning algebra at selected schools in Gokwe North district. Nyatso Secondary School was used as the case study to explore the misconceptions, resulting errors and coming up with solutions. The learning of algebra is a complex process that often poses difficulties for learners. Misconceptions and resulting errors are common in this domain and have been the subject of extensive research. Understanding the background of these misconceptions and errors can help educators develop effective teaching strategies to address them.

Algebra is a foundational subject in mathematics that serves as a gateway to advanced mathematical concepts. However, many students struggle with algebra and display persistent misconceptions and errors in their learning. Understanding the nature and sources of these misconceptions is crucial for developing effective teaching strategies to support student learning (Ndemo & Mtetwa, 2015). Algebra is a topic that must be studied by every student from primary school to university. Algebra is one of the major topics that require learners to adopt logical thinking, which in turn, challenges the learners' skills and allows them to focus on arithmetic operations while focusing on the use of symbols to represent equations and to establish the relationships in mathematical operations (Witzell et al, 2023).

Research has identified several common misconceptions that students exhibit in algebra, including lack of understanding of variables, that is students often view variables as arbitrary symbols rather than representing unknown quantities. They struggle to interpret expressions with variables and use them correctly in equations.

Difficulty with algebraic operations, this is whereby students have trouble with basic operations like combining like terms, factoring and applying the rules of exponents. They often rely on memorized procedures without conceptual understanding.

Students have difficulties isolating the variable and applying inverse operations to solve for the unknown. They may also struggle to interpret the meaning of the solution in the context of the problem. Students have trouble translating between different representations of algebraic concepts, such as verbal descriptions, tables, graphs, and symbolic expressions (Usiskin, 2018). Students lack problem-solving strategies and often approach algebra word problems with inadequate strategies, focusing on surface features rather than underlying mathematical structures.

Problem-solving performance can be determined based on five broad independent factors, namely knowledge acquisition and utilisation, control, beliefs, as well as social and cultural contexts (Lester, 2017). Problem-solving skills become important when applying mathematics in different situations. Learners need to integrate the skills and concepts of problem-solving to solve specific mathematical situations. One common misconception involves the transition from concrete to abstract thinking (Bell, 2017). Algebra introduces abstract symbols and variables that represent unknown quantities. Some learners struggle to grasp this shift and continue to rely on concrete representations, leading to errors in solving algebraic equations and understanding algebraic concepts. Algebra is frequently taught as though it is unrelated to any prior mathematics that students have experienced (Carraher & Schliemann, 2017).

Learners may misunderstand the concept of equality in algebra. They may perceive the equal sign (=) as an operational symbol and mistakenly treat it as an instruction to perform an action, such as adding or subtracting. Another typical sign whose use in arithmetic is inconsistent with its meaning in algebra is the equal sign. Beginning learners tend to see the equal sign as a procedural marking telling them “to do something”, or as a symbol that separates a problem from its answer, rather than a symbol of equivalence (Steinle, 2016). Equality is commonly misunderstood by beginning algebra learners (Knuth et al., 2018). Several researchers have noted that such a limited view of the equal sign exists among some learners in secondary schools (Herscovics & Linchevski, 2014) and also at college level.

This misconception can lead to errors in solving equations and simplifying expressions. Researchers found that even college students had trouble understanding and using the equal sign (Barcellos, 2015)

Negative numbers can be challenging for learners in algebra. Common errors include incorrectly applying the rules of operations with negative numbers, such as adding instead of subtracting, or multiplying instead of dividing. Misunderstandings related to negative numbers can affect various algebraic concepts, including solving equations and graphing linear functions.

Learners often struggle with understanding variables as placeholders for unknown quantities. They may treat variables as fixed values or confuse them with constants. One of the best known of the misconceptions is the "letter as object" misconception, described by Kuchemann (2014), in which the letter, rather than clearly being a placeholder for a number, is regarded as being an object. This misconception can hinder the ability to solve equations and work with algebraic expressions effectively.

Algebra requires procedural fluency, which involves the ability to perform algebraic operations accurately and efficiently. Learners may lack fluency in basic algebraic skills, such as factoring, simplifying expressions, or solving equations. Insufficient procedural fluency can result in errors and hinder the application of algebraic concepts. Kieran (2018) attributed many errors committed by students of algebra appear to be a result of their long-term inattention to structure of expressions and equations. Kieran (2018) documented the results of various studies that provided evidence of the inability and the difficulties of students to distinguish structural features of algebraic expressions and equations.

Understanding these common misconceptions and resulting errors is crucial for educators to design effective instructional strategies. By addressing these misconceptions explicitly, providing concrete examples and promoting conceptual understanding, educators can help learners overcome difficulties and develop a solid foundation in algebra.

1.2 Statement of the problem

These misconceptions and errors can have significant consequences for students' future mathematical learning and academic success. Unresolved issues in algebra can hinder the development of more advanced mathematical skills, making it difficult for students to progress in subjects like calculus, statistics, and other fields that rely heavily on algebraic foundations.

Furthermore, these misconceptions can lead to frustration, lack of confidence, and a negative attitude towards mathematics, potentially discouraging students from pursuing further STEM-related (Science, Technology, Engineering, and Mathematics) educational and career opportunities. The sources of these misconceptions are multifaceted, stemming from incomplete prior knowledge, reliance on flawed intuitive reasoning, instructional factors, and the inherent challenges posed by the abstract and symbolic nature of algebra.

Despite the extensive research on algebra misconceptions, there is a need for a comprehensive understanding of the specific misconceptions and errors exhibited by learners, as well as the underlying factors that contribute to their development. This knowledge is crucial for informing the design of targeted instructional strategies and interventions to address these issues and improve student learning outcomes in algebra.

The purpose of this study is to investigate the nature and prevalence of misconceptions and resulting errors displayed by learners in the learning of algebra, and to explore the potential sources of these misconceptions.

1.3 Research objectives

The expectations of this study are:

1. To determine difficulties learners have with understanding and working with variables, expressions and equations
2. To determine misconceptions and resulting errors learners have when learning algebra
3. To determine methods that can be used in order to minimize misconceptions and resulting errors in learning algebra

1.4 Research questions

The focus of this study lies in learners' difficulties, conceptions and attitudes towards learning algebra in the framework of conceptual change. In order to gain information on the relevance of these areas, the research questions that will guide this study are:

1. What difficulties do learners have with understanding and working with variables, expressions and equations
2. What misconceptions and resulting errors do learners have when learning algebra?
3. Which methods can be used in order to minimize misconceptions and resulting errors in learning algebra?

1.5 Importance of the study

1.5.1 Students

Students are the primary beneficiaries of this research, as identifying and addressing their misconceptions in algebra can lead to improved learning outcomes and a stronger foundation in mathematics. Understanding the common pitfalls and challenges faced by students can empower them to develop more effective learning strategies and self-assessment skills.

1.5.2 Parents and guardians

Parents and guardians play a crucial role in supporting their children's academic progress and are often concerned about their child's performance in mathematics, particularly in foundational subjects like algebra.

Insights from this research can help parents and guardians better understand the specific difficulties their children may face in learning algebra and provide them with guidance on how to support their child's learning.

1.5.3 The school

It is expected that after this study if acknowledged its results will be helpful to the school as this information can help educators better understand the underlying factors that contribute to these challenges as this can involve modifying teaching methods, providing more explicit explanations, or using alternate representations to help students develop a deeper and more accurate understanding of algebraic principles. This can prevent the development of more complex and deeply rooted misunderstandings. The insights gained from studying misconceptions can inform the development of more effective algebra curricula and assessments. Curriculum designers can incorporate strategies to address common misconceptions, while assessment tools can be designed to effectively identify and diagnose these issues. The study of misconceptions and errors in algebra learning can help bridge the gap between educational research and classroom practice. By identifying and addressing these challenges, educators can translate theoretical knowledge into practical, effective instructional approaches.

1.5.4 School administrators and curriculum developers

School administrators and curriculum developers are responsible for designing and implementing effective mathematics programs, including the delivery of algebra instruction.

The findings of this research can inform the development of curriculum materials, instructional strategies, and professional development opportunities for teachers to better address the identified misconceptions and errors in algebra.

1.5.5 Textbook and educational resource providers

Textbook and educational resource providers play a significant role in shaping the learning experiences of students in algebra. This research can provide valuable feedback to these stakeholders on the areas where existing instructional materials and resources may be falling short in addressing common misconceptions, enabling them to create more effective and targeted learning resources.

1.5.6 Policymakers and education researchers

Policymakers and education researchers are responsible for developing and evaluating policies, standards, and practices that impact mathematics education, including the teaching and learning of algebra. The insights from this research can inform policy decisions, curriculum standards, and future research directions aimed at improving student learning in algebra and mathematics more broadly.

By considering the interests and needs of these additional stakeholders, the justification for this research on algebra misconceptions and errors becomes even stronger, as the potential impact and applications of the findings extend beyond the immediate classroom setting.

1.6 Assumptions

This research was based on various assumptions. The first assumption related to the learners that they may have gaps or misconceptions in their foundational mathematical knowledge, such as arithmetic, fractions, or number sense. These gaps can lead to difficulties in grasping algebraic concepts. It was assumed that the use of variables, expressions and equations in algebra can be challenging for some learners, who may have difficulty translating between verbal, numerical and symbolic representations. It was assumed that the researcher did not have a prejudice against the participating learners during the implementation of the research and the interpretation of the data collected.

1.7 Delimitation of the study

This study is geographically restricted to Gokwe North District. Participants in this study will be form three learners within the year 2024. This study will be limited content area of form 3.

There are many methods in participatory teaching methods. This study will be limited to misconceptions and resulting errors in the learning of algebra.

1.8 Limitations of the study

Due to financial constraints coupled with time the researcher will be limited to one secondary school, Nyatso Secondary School in Gokwe North district.

1.9 Definition of key terms

Misconceptions is a view or opinion that is incorrect because it is based on faulty thinking or understanding. Misconceptions are false or inaccurate beliefs, understandings or ideas that people hold about a particular topic or concept. Misconceptions can arise from various sources, such as personal experiences, cultural influences or incomplete or incorrect information (Tanner and Allen 2005). It is important to identify and address misconceptions, as they can hinder learning and lead to the perpetuation of erroneous beliefs.

Resulting errors also known as propagated errors or compounded errors, refer to the errors that arise in the final result of a calculation or measurement due to the errors or uncertainties in the input variables or measurements. (Taylor, 2017) These errors can significantly impact the accuracy and reliability of the final result and it is important to understand and quantify them.

Learning is the process of acquiring new understanding, knowledge, behaviors, skills, values, attitudes and preferences. Learning is a process that leads to change, which occurs as a result of experience and increases the potential for improved performance and future learning. The change in the learner may happen at the level of knowledge, attitude or behavior (Ambrose et al, 2010). Algebra is a branch of mathematics in which arithmetical operations and formal manipulations are applied to abstract symbols rather than specific numbers.

Algebra is a branch of mathematics that deals with the study of mathematical expressions, equations, and the relationships between them. It is a fundamental discipline that underpins many areas of mathematics and science. (Rusczyk, 2007) Algebra is the branch of mathematics that deals with the use of letters and other symbols to represent unknown or changing quantities. (Sterling, 2020)

1.10 Summary

The researcher was motivated to research about the challenges and solutions of teaching and learning mathematics online in secondary education. The chapter gives an outline of the questions which will be answered towards the objectives and the end of research study. The chapter paves way for the following chapter. The next chapter dealt with related literature review.

CHAPTER II

LITERATURE REVIEW

2.0 Introduction

The primary purpose of this chapter of literature review is to unpack different views of scholars on misconceptions and resulting errors displayed by O level learners in the learning of algebra. The distinction developed in this aspect of the literature review raise important curricular questions on how to assist learners in developing an understanding of these various meanings of the symbol strings of algebra and in overcoming the learning difficulty associated with the language of mathematical symbols that is foreign to their previous experiences (Douglas, Headley, and Hadden, 2020). Moreover, learners need to learn and acquire an efficient way of representing properties of operations and relationships among them in the algebraic conventional system of notation which is different from the conventions used in arithmetic.

2.1 Difficulties learners have with understanding and working with variables, expressions and equations

Algebra deals with expressions with symbols and the extended numbers beyond whole numbers in order to solve equations, to analyse functional relations, and to determine the structure of the representational system which consist of expressions and relations (Lew, 2016).Teachers, mathematics educators, and mathematicians consider algebra to be one of the most important areas of school mathematics. Despite the importance placed on algebra in school mathematics curricula, many learners find it abstract and difficult to comprehend (Bora and Ahmed, 2019). Algebra concepts include unknowns and variables, expressions and equations, and the expansion of the meaning given to the equal and minus signs (Kieran, 2018).

Magezi and Igba (2018) points out that a difficulty is something that inhibits the learner in accomplishing correctly or in understanding quickly a given item. Difficulties may be due to several causes: related to the concept that is being learned, to the teaching method used by the teacher, to the learner's previous knowledge, or to his ability. Other possible explanations for the difficulties that learners experience with algebra involve the lack of the basic knowledge needed for a correct understanding of a given concept or procedure, the pace at which the algebra concepts is covered and also the formal approach often used in its presentation (Bracciali, Brogi and Canal, 2015).

According to Kieran (2018), learners' learning difficulties are centred on the meaning of letters, the change from arithmetical to algebraic conventions and the recognition and solving equations. Wagner and Parker (2023) agree with Kieran that most obstacles inherent to algebra stem from notational conventions or the complexity of concepts that arise with the use of letters as variables. Some of these problems are amplified by teaching approaches. The teaching methods used to convey content often exacerbate these algebra learning barriers, possibly becoming a unique barrier themselves (Leitzel, 2019). Teaching methods that focus on skill or procedural levels rather than relational understanding of abstract mathematical ideas, often requires a lengthy, iterative process are often insufficient for helping learners understand the abstract, structural concepts necessary for supporting the demonstrated procedural activities in algebra (Kieran, 2018; Rakes et al, 2020). As a result, many learners do not have the time to construct a good intuitive basis for the ideas of algebra or to connect these with the pre-algebraic ideas they have developed in primary school, they fail to construct meaning for the new symbolism and are reduced to performing meaningless operations on symbols they do not understand (Drijvers, Goddijn, and Kindt, 2021).

Various studies (Lithner, 2014; Maharaj, 2015; Mason, 2020) have focused on the teaching and learning of school mathematics. These studies have indicated some important sources of learners' difficulties in mathematics. Particular studies have focused on the equal sign (Carpenter et al, 2023), variables (Drijvers et al., 2021), graphing (Fennema and Carpenter, 2022), and equations and inequalities (Bazzini and Tsamir, 2021). These studies identify the complexities that learners experience but that the curriculum with its focus on algorithms or solution methods, does not address explicitly (Chazan & Yerushalmy, 2020).

Learners beginning the study of algebra face learning challenges that form a general foundational set of understandings necessary to negotiate through several complex topics with multiple sources of difficulty (Rakes et al., 2020). Algebra is often the first course in which learners are asked to engage in abstract reasoning that is, the ability to abstract common elements from situations, to conjecture, and to generalise and problem solving (Vogel, 2018). For the learners, all the previous mathematical problems had nothing that varied, they were presented with some numbers and operator, a plus, minus, multiplication or division sign and had to come up with the answer. With algebra, the situation is much more subtle. Instead of simply mathematical problems, learners are presented with expressions and equations. It seems to the learners as a switch from four arithmetic operations with numeric operands into terra incognita of some quantities that are both unknown and tend to change (Sinitsky & Ilany, 2019). Janvier, and Lepage (2022) note that "learners have difficulty acquiring and developing algebraic procedures in problem-solving" (p. 65); while Geary (2014) concludes, "Most learners find the solving of algebraic-word problems a cumbersome task” (p. 127). The obstacles are considered two-fold. Firstly “learners encounter major difficulties in representing word problems by equations” (Herscovics, 2019, p. 63). “Generating equations to represent the relationships found in typical word problems is well known to be an area of difficulty” (Kieran, 2018, p. 721) where learners are required to engage in abstract algebraic reasoning (Vogel, 2018). Similarly, Lochhead and Mestre (2018) write: “It is well known that word problems have traditionally been the nemesis of most mathematics learners. The translation process from words to algebra equation is perhaps the most difficult step in solving word problems” (p. 134).

Kieran (2018) attributed many errors committed by students of algebra appear to be a result of their long-term inattention to structure of expressions and equations. Kieran (2018) documented the results of various studies that provided evidence of the inability and the difficulties of students to distinguish structural features of algebraic expressions and equations. For example, students often fail to recognise the differences between expressions and equations. They also have difficulty conceptualising an equation as a single object rather than a collection of objects. The meaning of equality is often confused within the algebra contexts as well. These structural challenges often prevent students from recognising the utility of algebra for generalising numerical relationships. In her studies, Kieran (2018) showed that beginning students do not regard $x+7=4$ and $x=7 – 4$ as equivalent equations. In another study, Wagner, Rachlin and Jensen (2016) found that some high school students do not regard $7w+22=109$ and $7n+ 22=109$ as equivalent equations and do not perceive the structure of, for example, $4(2r+1)+7=35$ is the same as $4x+7 =35$. The findings of this study show that most algebra students have trouble and difficulty dealing with multi-term expressions as a single unit including ones in which the unknown occurs on both sides.

2.2 Misconceptions learners have when learning algebra

Researchers agree that learners enter classrooms with different conceptions due to different life experiences or prior learning. An important task for teachers is to identify learners’ preconceptions and misconceptions in order to help learners learn mathematics effectively and efficiently. Ignoring learners’ misconceptions may have negative effects on learners’ new learning and may also result in the original misconceptions being reinforced. Algebra uses its own standardized set of signs, symbols and rules about how something can be written (Drijvers et al., 2021). Algebra seems to have its own grammar and syntax and this makes it possible to formulate algebraic ideas unequivocally and compactly. In this symbolic language, “variables are simply signs or symbols that can be manipulated with well-established rules, and that do not refer to a specific, context-bound meaning" (Drijvers et al., 2021, p. 17).

There is a great deal of empirical evidence showing that rational number reasoning is very difficult for learners at all levels of instruction and in particular when new information about rational numbers comes in contrast with prior natural number knowledge (Ni and Zhou, 2015) in their previous learning experiences. Learners have difficulties interpreting and dealing with rational number notation, in particular when it comes to fractions (Gelman, 2021). They do not realise that it is possible for different symbols (decimals and fractions) to represent the same number and thus they treat different symbolic representations as if they were different numbers (Vamvakoussi and Vosniadou, 2017).

These difficulties are exacerbated to the study of algebra in which students are introduced to the principled ways in which letters are used to represent numbers and numerical relationships – in expression of generality and as unknowns – and to the corresponding activities involved with these uses of letters … simplifying, factorising, substituting, and solving, resulting in a series of misconceptions with the use of notation – a tool to represent numbers and quantities with literal symbols but also to calculate with these symbols (Kieran, 2018). With regard to learners' possible misconceptions of the meaning of variable, often contexts call for multiple usages and interpretations of variables. Learners must switch from one interpretation to another in the course of solving a problem which makes it difficult for an observer and for the individual himself to disentangle the real meaning being used (Mulungye, 2016). The meaning of variable is variable, using the term differently in different contexts can make it hard for learners to understand (Schoenfeld & Arcavi, 2019), hence giving rise to many misconceptions learners have in understanding the different uses of literal symbols in different contexts. Previous studies have demonstrated a series of misconceptions which learners have in relation to the use of literal symbols in algebra. A common naïve conception about variables is that different letters have different values. Alternatively, when students think of literal symbols as numbers they usually believe that they stand for specific numbers only (Knuth et al., 2015). This misconception is illustrated by students' responses of "never" to the following question: "When is the following true – L+M+N=L+P+N; always, never or sometime?" Kuchemann (2021) reported in the CSMS project that 51 percent of learners answered "never" and Booth (2014) reported in SESM project that 14 out of 35, that is, about 41 percent of 13 – 15 year-old learners responded likewise. Kieran (2018) in her study found out that learners do not understand that multiple occurrences of the same letter represent the same number. Even learners who have been told and are quick to say that any letter can be used as an unknown may, nonetheless, believe that changing the unknown can change the solution to an equation.

One of the best known of the misconceptions is the “letter as object” misconception, described by Kuchemann (2021), in which the letter, rather than clearly being a placeholder for a number, is regarded as being an object. For example, learners often view literal symbols as labels for objects, that is, they think that ‘D’ stands for David, ‘h’ for height, or they believe that ’y’ - in the task “add 3 to 5y -“refers to anything with a ‘y’ like a yacht. The term “fruit-salad algebra” (MacGregor & Stacey, 2019) is sometimes used for this misconception, infamously presented in examples such as “a for apples and b for bananas, and so 3a+ 2b is like 3 apples and 2 bananas, and since you can’t add apples and bananas we just write it as 3a+ 2b.” One difficulty is that 3a in algebra does not represent 3 apples, but three times an unknown number. The second difficulty here concerns the mathematical idea of closure: in saying we cannot add apples and bananas we contradict the fact that 3a+2b is the sum. According to Chick (2019), the letter as object misconception may be reinforced by formulas like A=lb, where A = area.

Another typical sign whose use in arithmetic is inconsistent with its meaning in algebra is the equal sign. Beginning learners tend to see the equal sign as a procedural marking telling them “to do something”, or as a symbol that separates a problem from its answer, rather than a symbol of equivalence (Steinle, 2016). Equality is commonly misunderstood by beginning algebra learners (Knuth et al., 2018). Several researchers have noted that such a limited view of the equal sign exists among some learners in secondary schools (Herscovics & Linchevski, 2014) and also at college level.

2.3 Methods that can be used in order to minimize misconceptions, difficulties in learning algebra

Studies carried out by Zuya (2014) indicated that merely being aware of learner misconceptions was not sufficient, since even when teachers explained fully correct methods based on the knowledge of learner’s misconceptions, the misconceptions in most cases remained unchanged. Davis (2020, p. 109) states that, “before children will change their beliefs, they must be persuaded that their ideas are no longer effective or that another alternative is preferable.” This is undoubtedly why misconceptions are so resistant to change. Roberts (2021) suggests that if students were to overcome these misconceptions, the teaching must expose and discuss them and assist the student to achieve a resolution which is consistent with the claim made by Davis (2020) when he wrote that learning involves progressive consideration of alternative perspectives and the resolution of anomalies. This presents a need for ‘conflict’ in teaching. Research shows that teaching is more effective when it assesses and uses prior learning to adapt to the needs of students (Pierce and Stacey, 2018). This section describes the diagnostic teaching model designed to diagnose and modify students' conceptions through exposure and rejection.

Diagnostic teaching, also known as conflict teaching (Welder, 2021), lays great importance on learners' wrong answers and the role of cognitive conflict in the process of teaching and learning. This type of teaching is based largely on the theories of Piaget in that the method involves initially ascertaining the existing cognitive structures, as related to the topic or task being taught possessed by the learners, and on the basis of these structures attempting to both expand and refine them by presenting tasks which lead to cognitive conflict (Perso, 2021). Creating tension and cognitive conflict may then be resolved through discussion (Bell, 2023). The notion that a pre-existing misconception cannot be replaced or corrected unless it is first recognised as being incorrect and cognitive conflict facilitates exposure of misconceptions was succinctly described by Bell et al. (2015) when they wrote that persistent, well retained bodies of knowledge and skill are those which are richly inter-connected and that fresh ideas are often rejected until they become so strong that they force a reorganisation of the existing material into a new system, holding together the new idea and the transformed old ones.

The diagnostic teaching approach is based on identifying key conceptual points and misconceptions. Teaching is then designed to focus on these points, giving learners substantial open challenges, provoking cognitive conflict by exposing misconceptions, and resolving through intensive discussion (Bell, 2023). This approach involves "appropriate exploratory work in the topic first followed by conflict/discussion lessons where learners are given carefully designed tasks together with encouragement to voice their ideas and question each other's assumptions with the teacher's support" (Kennewell, 2014, p. 3). A key feature of the method is that learners are placed in situations in which they are required to make their ideas and beliefs explicit, exposing them to challenge in a supportive situation. Their ideas may be carefully challenged by the teacher through prompts and questions which will help them to adapt their thinking, or through argument with their peers who may be equally, but differently, wrong (Kennewell, 2014).

When learners come to realise that there is something wrong with their existing interpretation of the situation, cognitive conflict occurs, and creating such conflict in the learner is one strategy that has been recommended for situations where learners need to move from one way of thinking to another (Light & Glachan, 2015, Bell, 2023). New ideas are constructed through reflective discussion. Opportunities are provided for learners to consolidate what has been learned through the application of newly constructed concepts. The role of the teacher in contrast to traditional roles of being an expositor or a desk-to-desk imparter of knowledge is required to be a facilitator of the diagnostic teaching process (Perso, 2021). In essence, the teaching methodology involved carefully chosen task or problem to be resolved through a process of discussion with peers, shared methods, articulation of conflicting points of view and whole-class discussion. Through such an approach the conflict is resolved and new learning is consolidated.

The aim of diagnostic teaching is to help learners to adopt more active approaches towards learning (Swan, 2016). Diagnostic teaching lesson places emphasis on learners' interaction and participation, particularly in the whole class elements of each lesson, and encourages discussion and co-operation between learners in group/paired work (Swan, 2016, Bell, 2023). This method allows learners to engage in discussing and explaining ideas, challenging and teaching one another, creating and solving each other's questions and working collaboratively to share methods and results (Swan, 2016). The model of teaching emphasises the interconnected nature of the subject and it confronts common conceptual difficulties through discussion and also allowing learners opportunities to tackle problems before offering them guidance and support. This encourages the learners to reapply pre-existing knowledge and allows the teacher to assess and then help them build the knowledge. This approach has a thorough empirically tested research base (Swan, 2016). Implementing a cognitive conflict approach has been reported in studies on a variety of topics, such as division (Tirosh and Graeber, 2021), sampling and chance statistics (Watson, 2022), decimal domain (Liu, Huang, and Chang, 2017, Bell, 2023), fractions (Tanner and Jones, 2020), literal symbols (Fujii, 2018) or directed numbers (Gallardo, 2022). However, Swan (2016) warns that this approach is challenging but research shows that it develops connected, long-term learning.

Earlier research into provoking cognitive conflict (Bell, 2023) suggests that the benefit to long-term learning is greater when learners encountered misconceptions through their own work than when teachers choose to draw attention to potential errors/misconceptions in their introduction to topics. Using a teaching methodology called diagnostic teaching, the Diagnostic Teaching Project based at Nottingham University Shell Centre reported long-term retention of mathematical skills and improvement in achievement when using teaching packages that were designed to elicit and address learners' misconceptions. Developing the work of the Diagnostic

Teaching Project, Swan (2021) believes that mistakes and misconceptions should be welcomed, made explicit, discussed and modified if long-term learning is to take place. Askew and William (2015) commented that we have to accept that learners will make some generalisations that are not correct and many of these misconceptions remain hidden unless the teacher makes specific efforts to uncover them. Swan (2016) argued that if learners' errors and misconceptions are more effectively addressed through being encouraged to examine their own ideas and confront inconsistencies and compare their own interpretations with those of other learners and with accepted conventions, then adequate time would have to be given for reflection and discussion. He reiterated that when learners feel less pressurised to give quick response which is correct, creating more time for such reflection and dialogue would clearly lead to less mathematical content being taught but, perhaps, more long-term mathematical learning taking place.

Despite its promise, this teaching method has not been adopted as normal teaching practice, at least partly because of the difficulty of reliably generating usefully wrong answers (Stacey et al., 2017). As mentioned earlier in this review, diagnostic teaching is challenging and is not easy to implement (Swan, 2015). It is more time consuming than traditional or rote teaching methods. It needs an environment, a community of enquiry, which is missing from many classrooms at present (Kennewell, 2014; Swan, 2015). Furthermore, it requires that teacher possesses well developed facilitation skills and a thorough understanding of the topic or phenomenon in question in terms of the planning of the tasks and intervention during learning activities to provide scaffolding and challenge (Noss, 2016). Or else, peer discussion in potential conflict situations may merely reinforce naïve conceptions, and the teacher may be too ready to provide the right answer in a way which will not influence the students' intuitive thinking (Kennewell, 2014).

The research dealing with cognitive conflict in science and mathematics education is divided, with some research finding cognitive effective at advancing students' cognitive structure, and other research finding that cognitive conflict did not lead to conceptual change. The findings demonstrate that cognitive conflict can have constructive, destructive or meaningless potential (Fraser, 2017). A considerable number of researchers (Kang et al 2020; Lee and Kwon, 2021; Liu, Huang and Chang, 2017; Treagust and Duit, 2018) found cognitive conflict an effective method of changing students' existing conceptions (if incorrect, misconceptions). Other studies (Dreyfus et al., 2020; Tirosh, Stavey and Cohen, 2018) found cognitive conflict was not consistently effective at modifying students' existing conceptions. A possible disagreement among the researchers as to the effectiveness of cognitive conflict is that there are different types of cognitive conflict and there may be a significant difference in the level of conflict required to properly resolve these different types of conflict (Fraser, 2017). Lee and Kwon (2021) describe sub-types of cognitive conflict researchers have categorised, for example, conflict between two internal concepts; conflict between two external sources of information; or conflict between and internal concept and an external source of information. Kwon, Lee and Beeth (2020) and Lee and Kwon (2021) found that the different types of cognitive conflict demand different levels of conflict to resolve. There were however, key details in the methods of the two groups that attest that the strategy used to manage cognitive conflict is extremely important. The common features of these methods were an introduction of the relationships and the context of the concept, presentation of a problem that will induce cognitive conflict and after having generated a conflict it is essential to provide an environment that will facilitate the proper resolution of the conflict.

The literature discussed in the above leads to a conclusion that teaching to avoid learners developing misconceptions appears to be unhelpful and could result in misconceptions being hidden from the teacher and from the learners themselves. This implies that a shift in the mind-set is needed for the teachers to move from planning mathematical lessons to avoid errors and misconceptions from occurring, to actively planning lessons which will confront students with carefully chosen examples that will allow for accommodation and conceptual change. It appears that effective teaching of mathematics involves planning to expose and discuss errors and misconceptions in such a way that students are challenged to think, encouraged to ask questions and listen to explanations, and helped to reflect upon these experiences. Diagnostic teaching methodology provides for such an opportunity for students to go through the process. This suggests that the more aware the teachers are of the common errors and possible misconceptions associated with a topic, the more effective will be the planning to address and deals with children’s potential difficulties. The role of questioning, dialogue and discussion is significant (Swan, 2016) if students are to shift their perspectives on only contributing if they think they have a correct answer they believe is wanted by their teacher.

The reason to make mistakes in simplifying algebraic expressions is that mathematics teachers do not care about the origin of these mistakes (Guler & Celik, 2016). Mathematics teachers not only need to have arithmetical skills, but also must have the ability to think, select aimful training strategies, and curriculum algebraic ideas to help learners move from arithmetic to algebra (Anne Hayata, 2022). In most classrooms, traditional methods dominate, and these traditional approaches have failed to teach algebra. (Norton & Irvin, 2017). Studies repeatedly highlight the role of teaching as an important variable, which affects learners' performance in mathematics as well as an effective factor in causing student mistakes (Mulungye, 2016). It is reasonable to say that traditional methods do not provide meaningful educational options for addressing learners' mistakes in mathematics, and particular algebra (Owusu, 2015). Creating a new educational method that improves students learning (Doerr, 2014).

2.4 Summary

This chapter gave an account of other studies which have relevance to the present research, discussions of past research on resulting errors and misconceptions displayed by students in the learning of algebra.

CHAPTER III

METHODOLOGY

3.0 Introduction

This chapter focused on research methodology showing the research design which the researcher used so as to be able to gather valid and reliable data for the study. Tools of data collection, data collection procedure, and data analysis and research integrity were also discussed in this chapter.

3.1 Research design

Before the researcher starts on the process of collecting information from the participants of the study, the first thing that one is supposed to do is determine the overall approach that one is going to take for the success of the study. This involves identifying the relevant instruments and procedures for the study while outlining the whole study framework systematically and in a fashionable manner (Flick, 2018). Research design can, therefore, be said to be the techniques and framework of the research methods that the investigator selects to complete the project (Creswell & Creswell, 2017). According to Blanche et al (2017), a design refers to "plans that guide the arrangement of conditions for collection and analysis of data in a manner that aims to combine relevance to the research purpose with economy in procedure" and that it "serves as a bridge between research questions and the execution or implementation of the research" (p.34). For the current study, the researcher chose to conduct a qualitative approach. In order to satisfy the objectives of the dissertation, a qualitative research was held. The main characteristic of qualitative research is that it is mostly appropriate for small samples, while its outcomes are not measurable and quantifiable (Flick, 2018). Its basic advantage, which also constitutes its basic difference with quantitative research, is that it offers a complete description and analysis of a research subject, without limiting the scope of the research and the nature of participant's responses.

3.2 Target Population

Target population in any research refers to the whole set of units for which the research information is to be used for making the final conclusions (Taherdoost, 2018). It defines the units which the investigator intends to make generalizations on from the findings of the group of individual units from which the researcher selects a sample (Faugeir & Sargeant, 2015). The definition of a target population should be as specific and precise as possible because it acts as a check for the eligibility of the entire research. Asiamah et al. (2017) states that the temporal and geographic features of a target population should be outlined, in addition to the types of units which the researcher intends to include. Sometimes, the target population in a study is limited to exclude those members whose access is impossible or difficult to avoid inconveniencing the researcher.

The target population for this research were the form three learners and two mathematics teachers at Nyatso secondary school in Gokwe North district. The research was conducted at Nyatso secondary school which is a satellite school in Gokwe North district. The school comprise of one hundred and seventy five learners, sixty learners in form one, forty in form two, forty in form three and thirty five in form four. Twenty learners and two mathematics teachers were selected for the research. The sampling was done using a purposive sampling technique. The researcher identified students representing a range of performance levels in algebra - high-achieving, struggling and average. This ensured the sample was representative of the diversity within the class in terms of algebraic understanding. The final sample included an equal number of male and female students. The district had approximately three thousand learners who were doing mathematics. Learners at Nyatso Secondary school were selected to participate in the study for easy access and convenience sake.

3.3 Sampling Technique

Since the researcher could not work with the entire population of the individuals, it was necessary to select part of the group to work with, and this is referred to as the sampling process. In Touvila's (2020) article at Investopedia, sampling process has to do with predetermining the number of units selected from the entire population. Therefore, a sample is defined as a subset of the entire targeted group representing the needed population characteristics (Acharya, 2016). It could either be selected using probability sampling or non-probability sampling. The current research used non-probability sampling approach to select the working sample. In non-probability sampling, the researcher decides who ought to be included to take part in the study according to their relevance to help in answering the research questions. The sampling approach used for the current research was convenience type of non-probability sampling. Here, the researcher draws a sample from a group of people who are easy to contact. Two mathematics teachers and twenty learners were selected for the research, these are form three learners at Nyatso secondary school.

3.4 Research Instruments

Data was collected through pre- and post-tests, an interview for teachers, classroom observations and analysis of student work. The tools used by the researcher to collect data to find solutions to the problem under investigation, included an interview, pre-tests and post-tests. Research instruments were clearly described so as to bring out their strengths and weaknesses as a way of justifying their selection and suitability to the research (Terry et al., 2017). The developments of these instruments were based on the research questions and objectives. Measures used to control the identified weaknesses were spelt out so as to ensure the validity and reliability of the data to be collected.

For the purposes of this research, interview guide was used. Interview guide are personal and unstructured interviews, whose aim is to identify participant's emotions, feelings, and opinions regarding a particular research subject. The main advantage of personal interviews is that they involve personal and direct contact between interviewers and interviewees, as well as eliminate non-response rates, but interviewers need to have developed the necessary skills to successfully carry an interview (Fisher, 2016, Wilson, 2018). The interview on the mathematics teachers can provide rich, qualitative data that complements and contextualizes the qualitative findings typically associated with research on misconceptions and errors in algebra. The teacher's expertise and first-hand knowledge can offer invaluable insights to inform both research and instructional practices.

For the purposes of this research, pre-tests and post-tests were used. Pretests and post-tests are research instruments commonly used to assess the knowledge, misconceptions, and learning gains of learners before and after an intervention or instructional program (Bell, 2018). In the context of studying misconceptions and errors displayed by learners in the learning of algebra, pretests and post-tests can be valuable tools to measure the effectiveness of educational interventions in addressing those misconceptions. Pretests and post-tests are powerful research instruments that provide valuable insights into the effectiveness of educational interventions in addressing misconceptions and errors in algebra learning. They allow researchers to assess the impact of interventions and identify areas for improvement in instructional practices, curriculum development and teacher training.

3.4.1 Pre-tests and post-tests

Pretests and post-tests are research instruments commonly used to assess the knowledge, misconceptions and learning gains of learners before and after an intervention or instructional program (Bell, 2018). In the context of studying misconceptions and errors displayed by learners in the learning of algebra, pretests and post-tests are valuable tools to measure the effectiveness of educational interventions in addressing those misconceptions.

A pretest is administered to participants before they receive any instruction related to the topic of interest, in this case, algebra. The purpose of the pretest is to assess the participants' baseline knowledge, identify their misconceptions and understand the common errors they may display. The pretest typically consists of a series of questions or problems that cover various concepts and topics within algebra. These questions are designed to target specific misconceptions or errors that are commonly observed among learners.

After the pretest, participants receive intervention via the diagnostic teaching aimed at addressing the identified misconceptions and errors. These instructional strategies should address the specific challenges learners face in algebra.

Following the intervention, the diagnostic teaching, a post-test is administered to the participants. The post-test is similar to the pretest and assesses the participants' knowledge and understanding of algebra at the end of the diagnostic teaching sessions. By comparing the results of the pretest and post-test, researchers can determine the effectiveness of diagnostic teaching in correcting misconceptions and reducing errors displayed by learners.

The comparison between pretest and post-test scores allows researchers to quantitatively measure the learning gains made by participants. It provides valuable data on the extent to which the diagnostic teaching has successfully addressed the misconceptions and errors identified in the pretest. Additionally, researchers can analyze the specific questions or problem types in which learners showed the most improvement, helping to identify the areas that require further attention or alternative instructional strategies.

Pretests and post-tests are powerful research instruments that provide valuable insights into the effectiveness of educational interventions in addressing misconceptions and errors in algebra learning. They allow researchers to assess the impact of interventions and identify areas for improvement in instructional practices, curriculum development and teacher training.

3.4.2 Interview guide for teachers

This is an interview with an experienced mathematics teacher focused on the topic of common misconceptions and resulting errors displayed by learners in the learning of algebra. By interviewing a practicing teacher, you can tap into their first-hand experiences and insights into the specific misconceptions and errors they've observed among their students. This provides valuable, on-the-ground knowledge that may not be easily captured through other research methods. The teacher can provide deeper explanations for why certain misconceptions arise and how they manifest in the types of errors students make. This can give important clues about the underlying cognitive and conceptual barriers students face when learning algebra. The teacher can share the strategies and techniques they've found effective in addressing these misconceptions and helping students overcome their errors. This can provide guidance on best practices for teaching algebra and designing effective interventions. The interview allows the teacher to candidly discuss the challenges they face in helping students master algebraic concepts, as well as highlight areas where students demonstrate greater understanding or potential for growth. Incorporating the teacher's real-world experiences and classroom-based observations can help ground the research in the practical realities of teaching and learning algebra, rather than relying solely on theoretical models or standardized test data.

4.3.3 Clinical interviews

Clinical interviews allow researchers to deeply probe learners' thought processes and underlying understanding of algebraic concepts. By asking probing questions and having learners verbalize their reasoning, researchers can uncover specific misconceptions that contribute to errors, such as confusing variables with constants, misinterpreting algebraic expressions, or applying inappropriate procedures. Clinical interviews go beyond just examining learners' procedural fluency in algebra. They allow researchers to delve into the depth of learners' conceptual understanding of algebraic principles and relationships (Aydin-Guc & Aygun 2021). By probing learners' explanations, justifications, and representations of algebraic concepts, researchers can better understand the root causes of learners' errors and misconceptions. By employing clinical interviews in the study of algebra learning, researchers can gain a rich and nuanced understanding of the cognitive and metacognitive processes underlying learners' difficulties, which can inform the design of more effective instructional approaches and support the development of mathematical proficiency in algebra.

After writing the pre- and post-tests, the researcher conducted the clinical interviews on the pupils who gave wrong answers so as to have a deeper understanding on the root of the errors and misconceptions. During the clinical interviews, the students were asked to solve the questions they got wrong and to express their thoughts in detail during the solution process. No guidance was given to the students in the clinical interview. To enable them to express their thoughts clearly, the following questions were directed: “What is the reason for giving this answer?” “Why do you think this solution is correct?”

3.5 Data collection procedure

Cohen and Swerdlik, (2018) define data collection as the procedures that are followed during data gathering and measurement of data on issues of curiosity. They further explain that the process takes place in a way that is pre-established and fashioned systematically in line with specific pacts and laws. A survey instrument is used to gather data from learners. The researcher gave a pre-test to the students, and marked the test and recorded the marks. The intervention process took place, and the pre-test was administered. The researcher was particularly interested in student participation during class and student perceptions of the diagnostic teaching unit taught throughout the teaching of the unit and the testing procedure. The researcher did clinical interviews to dig deeper into the misconceptions and resulting errors the students had. The researcher interviewed the two mathematics teachers at the school.

3.6 Data analysis

The data collected was presented in tables so that the researcher can analyze the data. The data was analyzed making use of discussions. Data was obtained using pre-test, post-test and an interview guide of the learners and teachers respectively. All of the qualitative data were then analysed as a means of answering the research questions.

3.6 Ethical Considerations

In an effort to stand by ethics, the researcher assured participants confidentiality and participants kept anonymity. The research also ensured consent and self-determination. Ethics are defined by Bell (2018) as acceptable standards governing research conduct and influence the welfare of human being.

3.7 Summary

In this chapter the research design employed by the researcher was explained together with the sampling procedure taken and its justification. In addition, an account of how data was collected was spelt out. Lastly data analysis and presentation procedure were laid out. The next chapter will deal with data presentation, analysis and discussion.

CHAPTER IV

DATA ANALYSIS AND PRESENTATION

4.0 Introduction

This chapter presented the findings of misconceptions and resulting errors displayed by learners in the learning of algebra. The findings are based on the interpretation and analysis of the data obtained through questionnaires and pre-tests and post-tests. Survey statistics results in relation to the objectives of the research are laid out in this chapter.

4.1 Biographical of the learners

Table 4.1.1 Gender of participants

Male	11
Female	11

A sample of 20 learners took part in the study by writing a pre-test and a post-test of which 10 were male and 10 were females. Two mathematics teachers also took part on the research by answering interview questions to make a total of twenty two participants.

Table 4.1.2 Age of participants

Age	Frequency
15	8
16	7
17	5
Above 17	2
Total	22

Table 4.1.2, shows the ages of the participants. The research examined misconceptions and resulting errors among form three learners of different ages, 15 years old: 8 learners, 16 years old: 7 learners and 17 years old: 5 learners. The above 17 years old is for the two teachers who participated in the interview. The researcher also explored how misconceptions and resulting

errors in algebra learning may vary across adolescent age range as students progress through their secondary school mathematics education. Studying multiple age groups allows the researchers to potentially identify developmental trends in the types of algebra-related misconceptions made by learners of different ages.

4.2 Difficulties learners have with understanding and working with variables, expressions and equations

From the tests, students viewed variables as representing specific objects or values, rather than understanding them as representing unknown or varying quantities. This led to errors like thinking x + x = x instead of 2x, 2r-r=2 instead of r. The following was observed; treating variables as objects or specific values rather than as representing unknown or varying quantities, confusing the role of variables in different contexts (e.g., independent vs. dependent variables) and struggling to understand that a variable can represent multiple possible values.

Learners had trouble identifying which terms were "like" and can be combined, leading to errors such as combining 2x and 3y as 5xy. This was due to over-reliance on memorized procedures and algorithms, difficulties in adapting problem-solving strategies when encountering unfamiliar or more complex algebraic problems and lack of metacognitive skills to monitor their own understanding and identify areas of confusion.

Results from the diagnostic test and lesson observations revealed that students interpret the '+' and '-' signs in terms of the action to be performed and tend to conjoin open algebraic expressions. The results from the pre-test showed that an alarming large proportion of students writing responses on the question x+9x+2. About 35% of that is 7 out of 20 students were found to have difficulty accepting an unclosed expression such as ′10x+2′ as a legitimate answer. To these students 10x+2 does not look like an answer. The presence of the operator symbol, +, makes the answer appear unfinished. In short, students see the plus symbol as invitations to do something, and if something is still to be done, then they ought to do it. The need for 'closure' is a major obstacle. This kind of misconception greatly distorts perception of the function of the operator symbols.

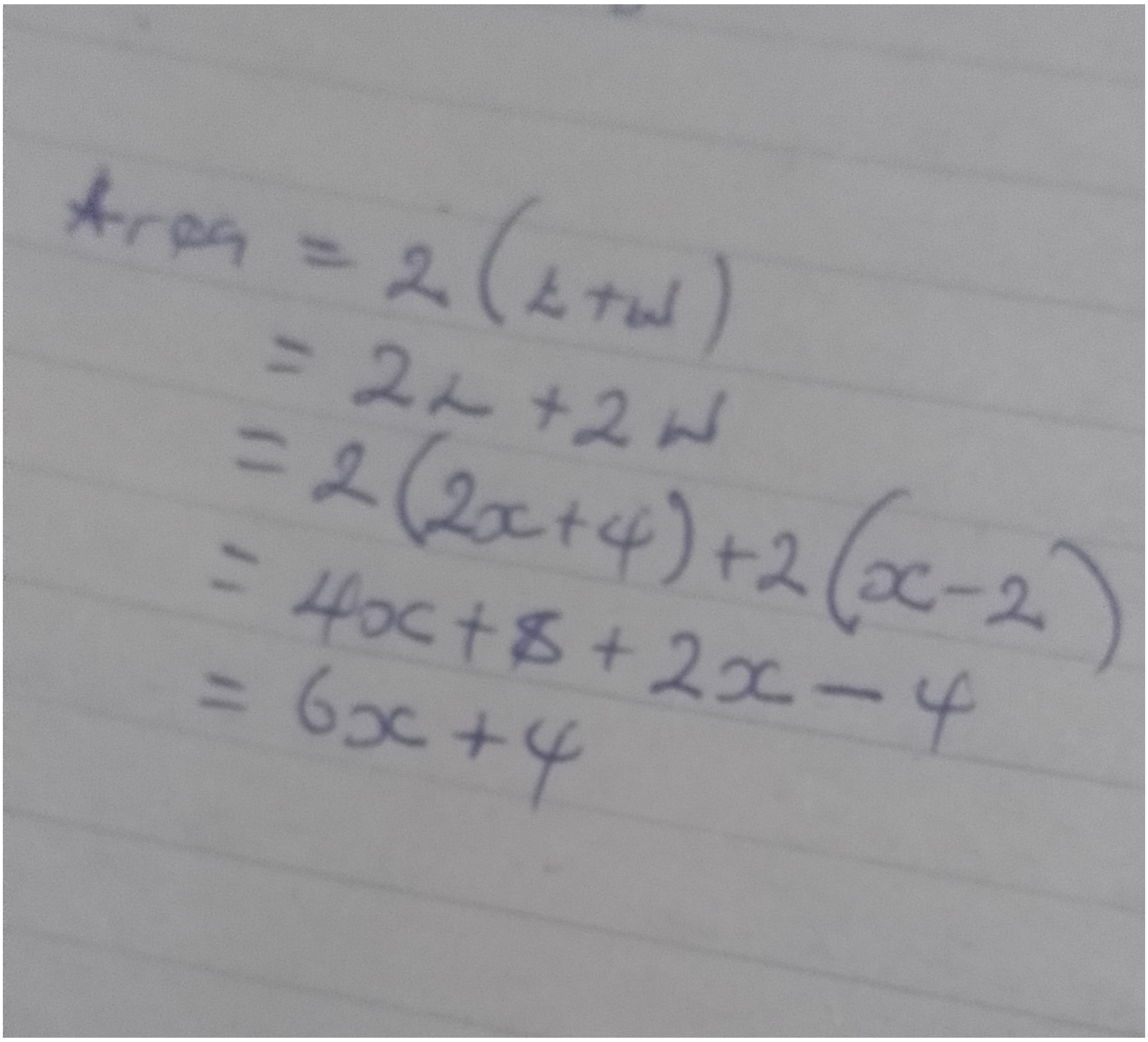
Students had misconceptions about the rules for operations with negative numbers, resulting in errors like subtracting a negative number by adding it or incorrectly simplifying expressions with negative exponents. Learners struggled with the rules for raising a power to a power or multiplying expressions with the same base, leading to mistakes like simplifying x^2 * x^3 as x^6 instead of x^5.

Students had difficulty isolating the variable or applying the correct inverse operations, resulting in errors like solving 2x + 3 = 7 as x = 5 instead of x = 2. From the tests the following was observed; difficulties isolating the variable and applying inverse operations correctly, confusion about the meaning of the equal sign (interpreting it as "the answer is" rather than a relationship), struggles with solving multi-step equations, especially those involving fractions or negative numbers and trouble with solving equations with variables on both sides

Figure 4.2.1 Difficulty in word problems

Question: A rectangle has a length of (2x+4)cm and width of (x-2)cm. What is the area of the rectangle in terms of x?

Solution from one of the students:



From figure 4.3.1 above, these are some of the wrong responses from students on the word problem, which required learners to calculate the area of a rectangle in terms of x. The correct answer was 2(x-2)(x+2). Some students failed to interpret that it was area which was to be calculated which led to the wrong answer. Students had difficulty translating verbal descriptions of algebraic situations into appropriate mathematical expressions or equations, resulting in errors in the problem-solving process. Learners may rely too heavily on memorized procedures and struggle to adapt their approach when encountering unfamiliar or more complex algebraic problems.

4.3 Misconceptions and resulting errors learners have when learning algebra

Table 4.3.1 Misconceptions, frequency and percentages in the pretest

Misconceptions	Frequency	Percentages (%)
The minus at the beginning of the algebraic expression has no meaning	8	40
The operation on one side of the equation should be applied to the other side so that equality is not broken	5	25
When writing sentences as algebraic expressions, we must consider the order of precedence of integers	7	35
Everything before the parentheses is distributed into the parentheses	6	30

When Table 4.3.1 is examined, it is seen that students have four different misconceptions about algebraic expressions and operations noted from the pre-test. It is noteworthy that the most common misconception is that the minus at the beginning of the algebraic expression has no meaning and that eight of the students have this misconception. The number of students who have the misconceptions, the operation on one side of the equation should also be applied to the other side so that the equality is not broken, we should take into account the order of operation priority in integers while writing the sentences as algebraic expressions and everything before the parenthesis is distributed into the parentheses are observed to be very close to each other.

Table 4.3.2 Misconceptions, frequency and percentages in the posttest

Misconceptions	Frequency	Percentages (%)
The minus at the beginning of the algebraic expression has no meaning	1	5
The operation on one side of the equation should be applied to the other side so that equality is not broken	2	10
When writing sentences as algebraic expressions, we must consider the order of precedence of integers	1	5
Everything before the parentheses is distributed into the parentheses	3	15

From table 4.3.2, there was a significant improvement in the post-test. Overall, the data indicates that the instructional or learning process between the two tests was effective for some students, resulting in significant improvements. However, the fact that there are still some students with misconceptions on the post-test suggests that there were still a subset of students who continued to struggle with certain algebraic concepts or errors even after the intervention. This data aligns well with the common challenge of addressing deeply rooted misconceptions in mathematics. While the overall improvement is encouraging, the persistence of lower scores highlights the need for continued support and targeted instruction to fully remediate these types of foundational algebraic misunderstandings.

The analysis of this pre-test and post-test data provides valuable insights into the effectiveness of the research intervention and the areas that may require further attention and refinement in the teaching of algebra. This type of assessment can help guide future instructional approaches and curriculum development in this domain.

Students are required to apply a succession of transformation rules in their manipulation of letters, symbols or variables involved in the process of simplifying algebraic expression or solving an algebraic equation. In the process, students frequently fail to realise that formulas in mathematical symbol systems have an intrinsic structure where expressions are structured explicitly by the use of parentheses, and implicitly by assuming conventions for order in which we perform arithmetic operations. Students commit errors as a result of their inattention to the structure of expressions and equations. Among the misconceptions identified were order of operation is unimportant, parentheses don't mean anything and over-generalisation of some mathematical rules while ignoring others.

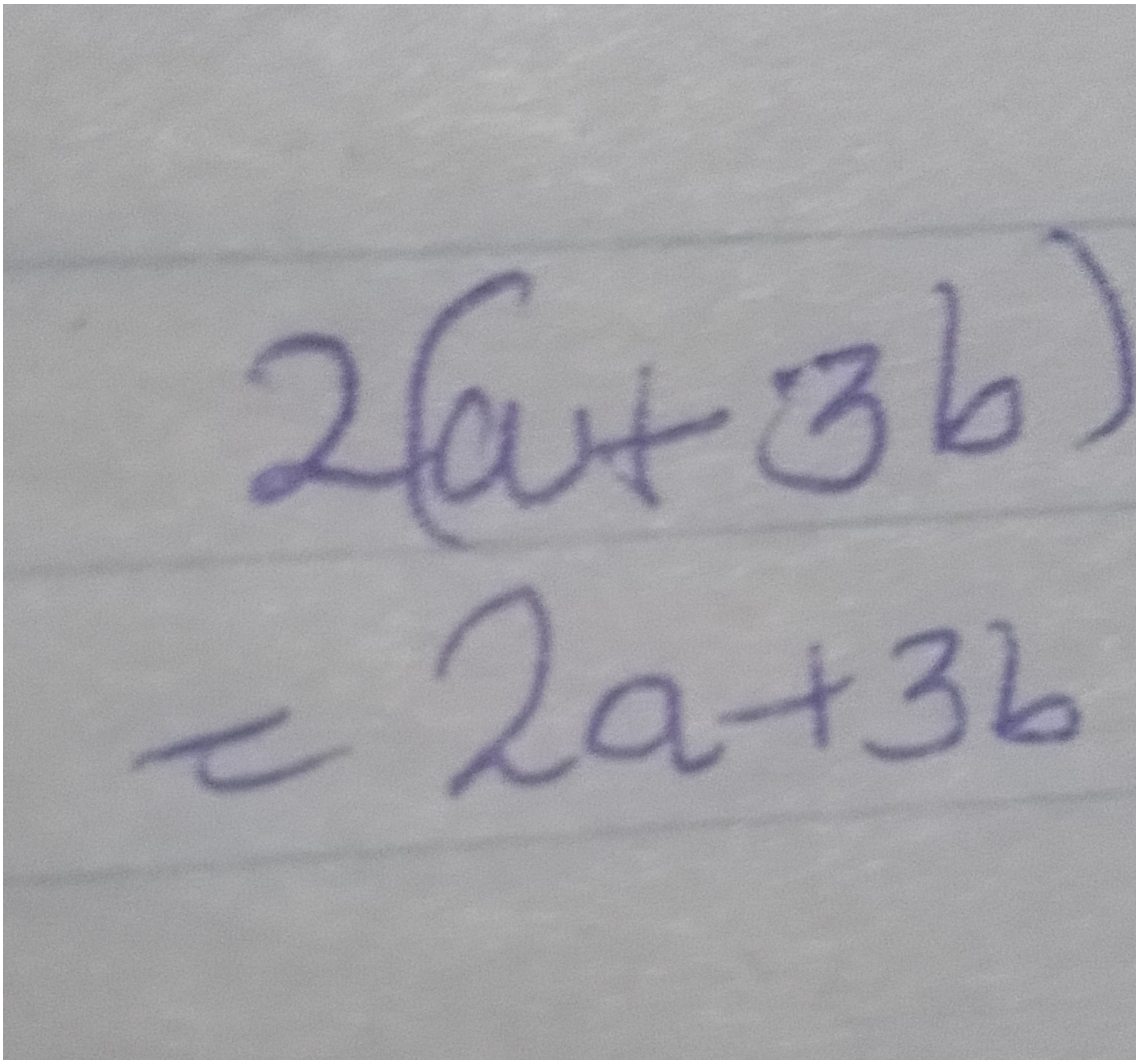
The order of operation is very important when simplifying expressions and equations. The failure to use the order of operation can result in a wrong answer to a problem. For example, to find the value of $3 + y \times 2$, without the order of operation one might decide to simplify the problem working left-to-right. The resulting answer would be $6y$. In contrast, the correct answer is $3+2y$. Alarmingly, there were 7 students (out of 20) in form three choosing to ignore the need for a rule in this item. These students simply did not see a need for rules presented within the order of operation. Instead, they attempted to perform multiple operations in any order which appears possible – they will attempt any part of an expression which they think they can do, and in any order. Hence, solving the expression based on how the items are listed, in a left-to-right fashion, consistent with their cultural tradition of reading and writing.

Figure 4.3.3 Errors in removing brackets

Students are found neglecting to use brackets when needed. Brackets are an essential element of mathematical notation in both arithmetic and algebra. Algebra, however, requires students to have a much more flexible understanding of brackets. In arithmetic, brackets are generally used as a static signal telling students which operation to perform first (or to specify the order of operation). Although symbolising the grouping of two terms (in an additive situation) is one important usage, students need to understand that brackets can also be used as a multiplicative operator (Linchevski, 2015). Although this dual usage is vital in understanding algebraic equivalence, Gallardo (2015) found that students were unable to correctly apply both usages of brackets to disapprove the number sentence: 20 – (7 – 8) = (20 – 7) – 8. Many students, however, accept the need for brackets in arithmetic but the rule: ‘do the the brackets first’ often is impossible in algebra. Therefore, students simply ignore it. To some students, parentheses don’t mean anything in algebra. This is related to the misconception that working is always from left to right; the way we read. For example, simplify the expression 2(a+3b). Responses such as 2a+3b as shown in figure 4.3.3 below and 5ab are not uncommon instead of the correct answer 2a+6b. The results clearly indicated that there were approximately 20% of the students who wrote the response such as 2a+3b and roughly 10% of them writing the response such as 5ab, indicating that they were either compelled to either ‘close’ their answer or to ignore the parentheses and work from left to right.

Question: Simplify the expression 2(a+3b)

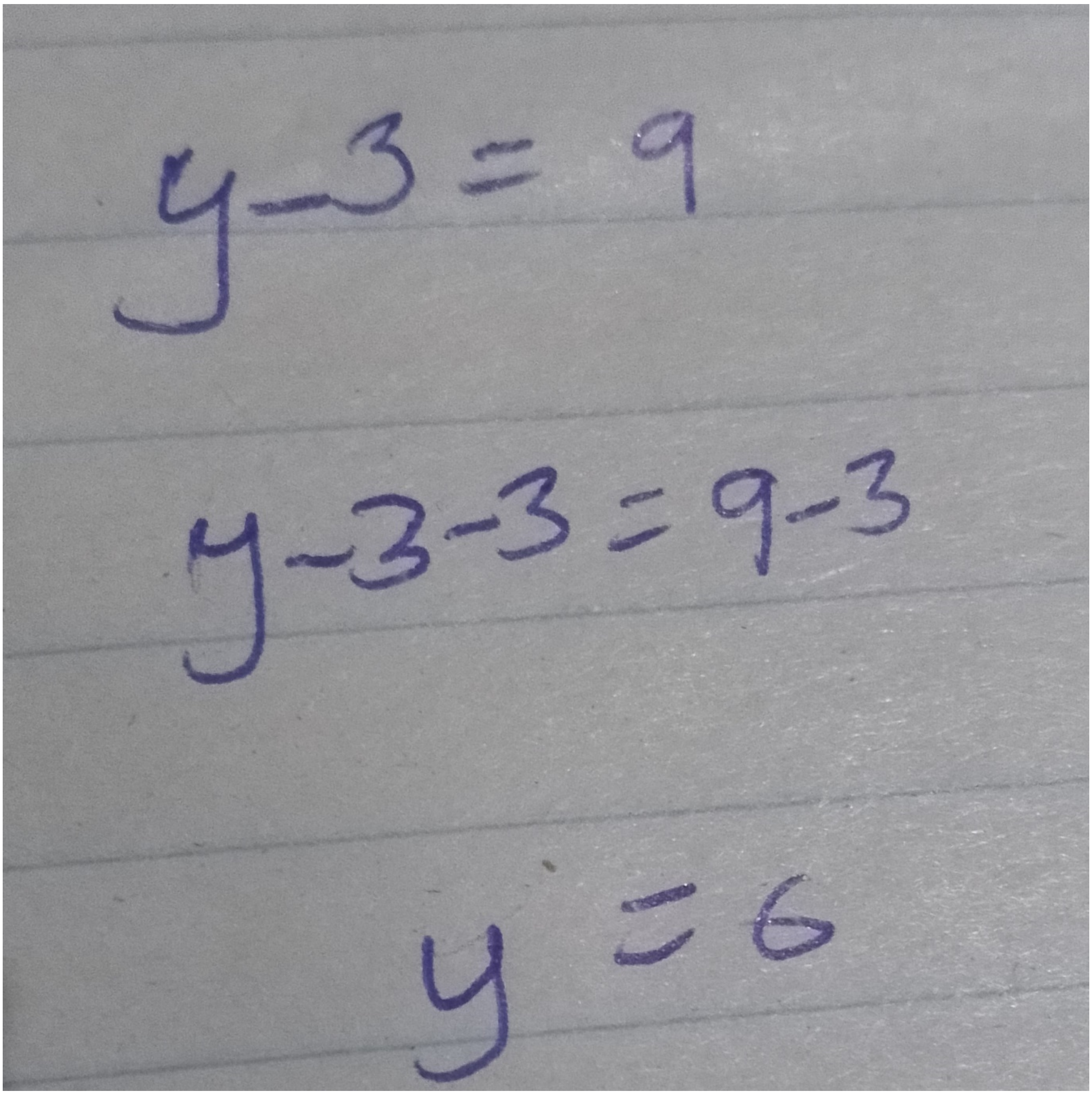
The figure below shows a written response by one of the students:



Chua and Wood (2015) argued that among the common misconceptions students have in algebra, the most significant were errors due to over-generalisation of concepts and rules. These authors attribute these errors to misconceptions that students have actively constructed when they use their existing schema to interpret data which are grounded in faulty understanding. Another student's misconceptions of over-generalisation and inability to generalise mathematical rules discussed are to remove a term from the equation, students subtract it from both sides of the equation and dividing the larger number by the smaller rather than respecting the order of inversing.

Question: Solve the equation y-3=9

The figure below shows a written response by one of the students:



A common strategy students have for solving equations is that if they want to remove a term from the equation, they subtract it from both sides of the equation irrespective of the adjoining operator symbol. This works just fine for removing 3 from the equation y+3=9. However, when they encounter equations like y-3=9, many students still try to subtract 3 from both sides to solve the problem as observed in one of the questions. Booth and Koedinger (2018) attribute this misconception to students' inability to process the fact that the negative sign modifies the 3 and is a necessary part of the 'term' they are trying to remove. Students were found to often ignoring it to the detriment of their goal of solving the problem.

The misconception of dividing the larger number by the smaller rather than following the order of inversing is related to students' inability to generalise because of a lack of understanding of arithmetic operations. It also assumes that frequently students do not know which operation will undo another operation. For example, results concerning this misconception as 5=9m six students answered this question wrongly by solving m=9/5 instead of 5/9. Here, it is obvious that students perceived the need to isolate the variable, but were unsure which operation was necessary to inverse the one given. Thirty three percent of the students writing 9/5 appeared to be aware of the need to perform the operation of division but exhibit a lack of understanding of the arithmetic operator. They did not consider that the inverse operation for multiplying by 9 was dividing by 9. It was simply seen that it is easier to divide a smaller number into a larger one, and so this was done. Generalisation from early experiences that you can't divide smaller numbers by larger ones is thought to be the cause of this misconception.

Students frequently have difficulty that some signs are attached to numbers (or variables) but not others. Students believed that negative signs represent only the subtraction operation and do not modify terms (Vlassis, 2017). For example, -3 mean that the '-' is part of the 3 whereas in 'x 3' or '÷ 3' the signs are detached. This confusion may lead to students totally ignoring signs, particularly negative signs, in an algebraic expression or equation. It is difficult to understand how a variable can have a negative. This misconception is also related to the fact that many students are unwilling to work with fractions and negative numbers and will consequently ignore any ties which appear to exist between numbers, signs and variables. Eight students out of twenty students (40%) seemed to ignore the negative sign and just multiply -3a and 7 to get 21a. This answer is consistent with the misconception that negatives can enter and exit phrases without consequence and that their locations (and connections to numbers or variables in the problem) are detached or not significant. Failing to tie the negative sign to the term it modifies or to understand how changing or moving a negative sign impacts the equation resulting in incorrect strategies for solving algebraic equations. Not only does the failure to understand the negative sign but also the failure to apply the correct rules of algebraic manipulation will result in incorrect strategies for solving algebraic equations.

4.4 Methods that can be used in order to minimize misconceptions and resulting errors in learning algebra

The first question for the interview was, how long have you been serving as a qualified teacher of Mathematics? The first response was, I have been a qualified teacher of Mathematics for the past ten years. During this time, I have had the opportunity to teach students from secondary school through high school. The second response was, as a qualified mathematics teacher, I have been serving in this role for the past fifteen years. During this time, I have had the opportunity to work with students of diverse backgrounds and skill levels, which has greatly informed my teaching practice. From the teachers' responses, the researcher noted that experienced teachers have observed and analyzed a wide range of student thinking patterns over many years. They can identify common stumbling blocks, misconceptions, and points of confusion that students tend to exhibit. This deep understanding of student cognition allows them to anticipate potential problem areas and proactively address them.

According to Draper and Lott, (2020), experienced teachers have had the opportunity to try various instructional approaches and techniques in their classrooms. They have developed a rich repertoire of strategies that they have found to be effective in helping students overcome specific misconceptions and errors. This allows them to draw from a toolbox of proven methods to tailor their instruction to the unique needs of their students.

Teachers with experience have honed their ability to quickly identify the root causes of students' misconceptions and errors. They can accurately diagnose the underlying conceptual gaps or procedural weaknesses that lead to these issues (Gürel & Okur, 2017). This diagnostic expertise enables them to design targeted interventions and provide personalized support to address the specific challenges faced by each student. Experienced teachers have developed the ability to adapt their teaching approaches based on student feedback and performance. They can recognize when their initial instructional strategies are not effectively addressing students' misconceptions and adjust their methods accordingly. This flexibility allows them to continuously refine and improve their practices to better support student learning.

Experienced teachers have a comprehensive grasp of the entire algebra curriculum and how concepts build upon one another. They can identify the interdependencies between different algebraic topics and understand how misconceptions in one area can impact a student's understanding in other areas. This holistic perspective enables them to design instructional sequences and scaffolding that address misconceptions in a coherent and effective manner. By leveraging their deep experience and accumulated expertise, experienced mathematics teachers are well-positioned to develop and implement effective strategies to minimize the occurrence of misconceptions and errors in the learning of algebra (Larino, 2018). Their insights and practical knowledge can be invaluable in informing both classroom practices and educational research in this domain.

The second question on the interview was do you agree that students should master some basic skills before moving to new topics? If so, what basic skills do you think a student has to master before dealing with quadratic equations? The first respondent gave the following response: Yes, I strongly believe that students should master some basic skills before moving on to new, more complex mathematical concepts. For quadratic equations specifically, I believe students should have a solid understanding of the following foundational skills, simplifying algebraic expressions, solving linear equations, factorizing expressions and graphing linear functions. Mastering these basic skills will provide students with the necessary background knowledge and problem-solving abilities to successfully navigate the complexities of quadratic equations.

The second respondent gave the following response: Yes, I strongly believe that students should have a solid foundation of basic skills before tackling more advanced mathematical concepts like quadratic equations. Some of the key foundational skills I consider essential include, proficiency in operations with integers, fractions, and exponents, ability to simplify algebraic expressions, fluency in solving linear equations and inequalities, understanding of function types and their properties, competence in graphing linear and simple nonlinear functions and familiarity with the properties of equality and manipulating equations. Ensuring students have mastered these fundamental skills helps provide the necessary building blocks for a successful transition to more complex topics, such as quadratic equations.

The researcher agreed with the perspective on the importance of mastering foundational skills before moving on to more advanced mathematical concepts like quadratic equations. Developing a strong grasp of the prerequisite skills outlined - simplifying algebraic expressions, solving linear equations, factoring polynomials and graphing linear and parabolic functions - is crucial for students to be able to fully understand and work with quadratic equations. By solidifying these basic competencies, students build a robust mathematical framework that allows them to, manipulate the symbolic representations in quadratic expressions with confidence, leverage their ability to solve linear equations to inform strategies for solving quadratic equations, factor quadratic expressions, which is a key technique for finding solutions and interpret the graphical behavior of quadratic functions, which provides valuable geometric insight. According to Tooher & Johnson, (2020) mastering these foundational skills enables students to approach quadratic equations with the requisite background knowledge and problem-solving tools. This lays the groundwork for deeper conceptual understanding and the ability to tackle more advanced algebraic concepts. Ensuring students have this solid foundation before moving forward is an effective teaching strategy that can significantly improve their chances of success in learning mathematics.

Another question on the interview was, what strategies can you use to assist students with problems of solving quadratic equations? The first respondent’s response was; there are several strategies that can be employed to help students with solving quadratic equations. These are providing step-by-step examples and guided practice, encouraging students to graphically represent quadratic functions to visualize the solutions, incorporating real-world applications and contextualized problems to make the concepts more relevant, utilizing various solving methods, such as factoring, completing the square, and using the quadratic formula, encouraging students to explain their thinking and problem-solving process and offering targeted feedback and opportunities for students to learn from their mistakes.

The second respondent’s response was; to help students with solving quadratic equations, there are a variety of strategies to be employed including, scaffolding instruction by starting with simple, concrete examples and gradually increasing complexity, incorporating visual aids like graphs, tables, and diagrams to help students conceptualize the relationships, encouraging students to explore multiple solution methods (factoring, completing the square, quadratic formula) and choose the most appropriate approach, providing opportunities for students to work collaboratively, share their thought processes, and learn from their peers, incorporating technology like graphing calculators or computer software to assist with graphing and numerical computations and offering differentiated support, such as one-on-one coaching or small group targeted instruction, for students who are struggling.

The researcher noted that, rather than focusing solely on memorizing formulas and procedures, put emphasis on developing a deep conceptual understanding of algebraic principles. This helps students make connections and apply concepts more flexibly. Introduce new algebraic concepts using physical manipulatives, visual models, or real-world examples that students can relate to. This grounds the abstract ideas in tangible experiences (Pedone, 2018). Break down complex algebraic problems into smaller, more manageable steps. Use guiding questions and step-by-step examples to support students as they develop their problem-solving skills. While conceptual understanding is crucial, also provide opportunities for students to practice algebraic procedures and develop fluency with skills like simplifying expressions and solving equations.

The last question on the interview guide was, some teachers are also said to be a source of the problems that the students face when doing Mathematics. What strategies can teachers employ to counteract students from making errors? The first respondent's answer was; as teachers, it is important that we are mindful of the potential ways in which our own actions and instructional practices can contribute to student errors. Some strategies to be employed to counteract this include, emphasizing conceptual understanding over rote memorization of procedures, regularly checking for misconceptions and addressing them proactively, providing clear, step-by-step modeling of problem-solving techniques, encouraging students to review their work and identify potential sources of errors, offering opportunities for self-assessment and reflection on their learning process and fostering a growth mindset, where students understand that mistakes are a natural part of the learning journey.

The second respondent's answer was; as teachers, it is important that we watch our own practices and how they may contribute to student errors. Some strategies to minimize this include, providing clear, concise, and well-structured explanations of mathematical concepts, actively checking for student understanding throughout the lesson, encouraging students to ask questions and clarify any confusion, offering timely feedback and personalized support to address individual learning needs, regularly reviewing and assessing students' progress to identify and address any areas of difficulty and continuously reflecting on my own teaching methods and adjusting them as needed to better support student learning. By being attuned to potential pitfalls and employing these strategies, the aim is to create a supportive and engaging learning environment that empowers students to develop a deeper, more robust understanding of mathematics.

Mathematics requires learners to think in terms of symbolic representation or abstract conceptualisation. Pedagogically, the teacher facilitates discovery of mathematics principles, patterns and relationships by students through inductive discovery and deductive discovery teaching approaches to mathematics teaching and learning. Identifying what students may learn in algebra is of paramount importance (Kaput, J & Lins, 2014). Effective algebraic thinking sometimes involves reversibility. It is the ability to undo mathematical processes as well as do them. It is the capacity not only to use a process to get to a solution, but also to understand a process well enough to work backward from the answer to the starting point. Students should have the capacity for abstracting from computations. This is the ability to think about computations independently of particular numbers used, thus generalising arithmetic. One key characteristics of algebra has always been abstractness. Abstracting system regularities from computation is when thinking algebraically involves being able to think about computations freed from the particular numbers in arithmetic (Ndemo & Ndemo, 2018).

Help students become aware of their own thought processes and learning strategies. Teach them to monitor their understanding, identify areas of confusion, and adjust their approach accordingly. Use frequent, low-stakes assessments to gather information about students' progress and misconceptions. Use this data to adjust instruction and provide targeted support.

It is observed that students make some mistakes while performing operations with algebraic expressions. In this study, it was aimed to reveal the student understanding underlying these mistakes. In this context, while trying to identify misconceptions, instead of increasing the number of identical questions to reveal the systematisty of student concepts, clinical interviews were conducted with all students who gave wrong answers. Clinical interviews have shown that although written responses point to existing errors in the literature, insights underlying the errors may differ (Attorps, 2015). This reveals, once again, the role of clinical interviews in revealing the deep understanding underlying student insights. In contrast, it has been observed that questions designed for different purposes allow to reveal different misconceptions; this shows that further studies are needed to determine students' misconceptions. It reveals that different students in different sociocultural settings may have different misconceptions. Knowing the different misconceptions that students may have provides information about which conceptions teachers can encounter with students. Furthermore, the test developed in the study can be used by teachers to measure their students' pre-understanding. Therefore, it permits to include activities that will not allow for the misconceptions put forward in the design of learning environments or that will eliminate the existing misconceptions. The most important limitation of the study is that the clinical interviews were conducted only on wrong answers. This situation prevented the disclosure of misconceptions that directed students to the correct answer.

Emphasize that algebra is a learned skill, and that with effort and perseverance, all students can improve their algebraic abilities. Praise students' hard work and progress, not just their natural abilities. Provide opportunities for students to work together, discuss their thinking, and learn from their peers. Peer interactions can help identify and resolve misconceptions (Bragg, 2017).

4.5 Summary

This chapter focused on the presentation of data through the use of tables, graphs and discussions. In this chapter research questions were answered and the research findings were discussed in relation to literature. The next chapter will focus on the summary and conclusion of the study. Also, the chapter will look at recommendations and areas of further research.

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Introduction

This chapter provided a summary of the research findings and also offers some research solutions to reduce the misconceptions and resulting errors displayed by learners in the learning of algebra. The chapter also offers a scope for future studies.

5.1 Summary

Algebra is a foundational subject that is crucial for success in higher-level mathematics and many scientific fields. However, students often struggle with algebra and develop persistent misconceptions that lead to common errors. Understanding the nature of these misconceptions and errors is important for improving algebra instruction and student learning.

Students may view variables as representing specific numbers rather than unknown quantities. They may have difficulty manipulating and operating with variables. This can lead to errors in simplifying expressions, solving equations, and working with functions.

Students may struggle with the meaning and use of mathematical symbols, such as the equal sign, parentheses, and exponents. They may have trouble translating between verbal and symbolic representations of algebra. This can result in mistakes in simplifying expressions, evaluating formulas, and interpreting the meaning of algebraic statements.

Students may have trouble understanding the properties of operations, such as the distributive property, when working with variables. They may apply operations incorrectly, such as adding or multiplying variables without considering the variables' coefficients. This can lead to errors in simplifying expressions, solving equations, and working with algebraic inequalities.

Students may struggle to translate real-world situations into algebraic expressions or equations. They may have trouble identifying the relevant variables and relationships in word problems. This can result in errors in setting up and solving algebra-based problems, including those involving linear, quadratic and systems of equations.

Identifying and addressing these common misconceptions and errors can help improve algebra instruction and student learning. Strategies such as emphasizing the conceptual understanding of variables, focusing on the meaning of algebraic notation and providing ample practice with diverse problem-solving contexts can be beneficial. Incorporating formative assessments and targeted feedback can also help students overcome their misconceptions and develop a stronger foundation in algebra.

Investigating the nature and prevalence of misconceptions and errors in algebra learning is crucial for enhancing student understanding and success in this fundamental mathematics subject (Bragg, 2017). Ongoing research and the implementation of effective instructional practices can help address these challenges and promote better learning outcomes for students.

5.2 Conclusion

The research on misconceptions and errors in the learning of algebra has provided valuable insights into the persistent challenges that students face in this foundational mathematical subject. The studies have identified several key areas where students commonly develop misconceptions, including the understanding of variables, the interpretation of algebraic notation, the application of algebraic operations and the translation of real-world problems into algebraic models.

These misconceptions can lead to a wide range of errors, such as the mishandling of variables, the incorrect simplification of expressions, the inability to solve equations accurately, and the failure to set up and solve word problems effectively. These errors not only hinder students' immediate performance in algebra but can also have long-lasting consequences, as the mastery of algebra is crucial for success in higher-level mathematics and many scientific fields.

Addressing these misconceptions and errors is essential for improving algebra instruction and student learning. Effective instructional approaches emphasize the development of conceptual understanding, the explicit targeting of common misconceptions, the facilitation of productive struggle, the encouragement of multiple representations and connections, the incorporation of formative assessments and feedback and the fostering of a supportive learning environment.

By implementing these strategies, educators can help students overcome their misconceptions, develop a deeper and more flexible understanding of algebraic concepts and ultimately enhance their problem-solving skills and overall performance in algebra. Continued research in this area, coupled with the widespread adoption of these evidence-based instructional practices, can contribute to the improvement of algebra education and the long-term success of students in this critical mathematical domain.

5.3 Recommendations

While the current research has identified several key areas of misconceptions, such as variables, notation and operations, there may be other emerging or less-studied misconceptions that warrant investigation. Researchers should explore a broader range of algebraic concepts and skills to uncover a more comprehensive understanding of the challenges students face and the misconceptions they have.

Most of the existing research has focused on misconceptions at the secondary level, but it is important to understand how these misconceptions develop and evolve from the primary level to the post-secondary levels. Longitudinal studies tracking the progression of misconceptions across grade levels could provide valuable insights into the developmental aspects of algebra learning.

Further research is needed to evaluate the effectiveness of specific instructional strategies in addressing misconceptions and improving student learning in algebra. Comparative studies that contrast different teaching methods, such as the use of concrete models, technology-based interventions and problem-based learning, can help identify the most impactful approaches.

Since algebra is a foundational subject, it is important to understand how the persistence of misconceptions in algebra can affect students' performance and learning in more advanced mathematical domains. Investigating the cascading effects of algebra misconceptions on the learning of calculus, linear algebra and other higher-level mathematics could provide valuable insights.

Combine quantitative and qualitative research methods, such as large-scale assessments, detailed case studies and classroom observations, to gain a more comprehensive understanding of the nature and causes of misconceptions. Incorporate student interviews, think-aloud protocols, and analysis of student work to uncover the underlying cognitive processes and reasoning behind the observed misconceptions.

Strengthen the partnership between researchers and classroom teachers to ensure that the research agenda aligns with the practical needs and challenges faced by educators. Engage teachers in the research process, from the identification of misconceptions to the design and implementation of interventions, to enhance the relevance and applicability of the findings.

Effectively communicate the research findings to teachers, curriculum developers and policymakers to facilitate the widespread adoption of evidence-based instructional practices that address misconceptions in algebra learning. Provide professional development opportunities and support systems to help educators implement the recommended strategies in their classrooms.

By pursuing these research directions, the academic community can deepen our understanding of the complex nature of misconceptions in algebra learning, develop more effective interventions, and ultimately improve the teaching and learning of this foundational mathematical subject.

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APPENDIX A: Form three test

1. Simplify

   a) $2a-9a$

   b) $8d-3-7d$

   c) $r-3s-3t-4r+10s+8t$

   d) $x+x$

   e) $4c+c$

   f) $2r-r$

   g) $7q-7q$

   h) $x+2+9x$

   i) $3x+11-2$

   j) $7y-3x+4y$

2. Simplify

   a) $3+y \times 2$

   b) $x-2\times 3$

   c) $15\div 5+6y$

   d) $6x\times 4-2\times 7x$

1. Simplify

   a) $-5\times 2y$

   b) $-6\times -4x$

   c) $-3a\times 7$

   d) $-\frac{1}{3} of 36x$

4 Solve

a) $4=\frac{x}{9}$

b) $5=9m$

c) $8x-24=0$

3. Remove brackets

   a) $2(a+3b)$

   b) $-2(3m+4)$

   c) $\frac{1}{2}(2u-8)$

4. Expand

   a) $(b+2)(b-3)$

   b) $(3x+4)(x-2)$

5. A rectangle has a length of (2x+4)cm and width of (x-2)cm. what is the area of the rectangle in terms of x?

APPENDIX B: Interview guide for teachers.

1. How long have you been serving as a qualified teacher of Mathematics?
2. Do you agree that students should master some basic skills before moving to new topics? If so, what basic skills do you think a student has to master before dealing with quadratic equations?
3. What strategies can you use to assist students with problems of solving quadratic equations?

Some teachers are also said to be a source of the problems that the students face when doing Mathematics. What strategies can teachers employ to counteract students from making errors?