BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE EDUCATION

INVESTIGATING ERRORS AND MISCONCEPTIONS DISPLAYED BY LEARNERS IN TRANSFORMATIONAL GEOMETRY AT ORDINARY LEVEL IN ZIMBABWE

SIBANDA TAFIREI

(REG NUMBER: B225868A)

YEAR 2024

APPROVAL FORM

The undersigned certify that they have read and recommended to Bindura University of Science Education for acceptance of a project entitled "An investigation into errors and misconceptions displayed by Ordinary Level learners in transformational geometry" by Sibanda Tafirei in partial fulfilment of requirements of Bachelor of Education Honors Degree in Mathematics

Mutambara

2/01/2025

SUPERVISOR DATE

CHAIRMAN DATE

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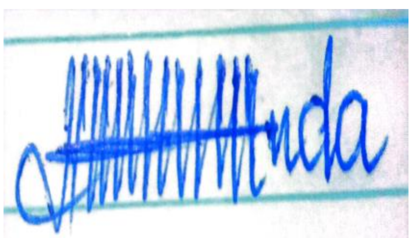
STUDENT DATE



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RELEASE FORM

Name: SIBANDA TAFIREI B225868A

Title of Project: Investigating errors and misconceptions displayed by learners in transformational geometry at ordinary level in Zimbabwe.

Program: Bachelor of Education Honors Degree in Mathematics

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SIGNED ………………………………………………

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DATE: …………………………………………….

DEDICATION

My research is dedicated to my wife Tore Molline as without her sacrifice in some financial obligations, prayers and support I could never have reached this far. Completing this program was going to be very tough without her support. I also remember my children; Vallerie Sibanda, Vannesa Sibanda and Nigel Sibanda as well as my parents for their support.

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ABSTRACT

The study aimed at identifying errors and misconceptions displayed by form 3 learners when learning transformational geometry at Kahobo Secondary School in Gokwe North District in the Midlands Province. To select the sample, the class of 90 learners was first divided into three subclasses (A, B, and C). Then, a random sample of learners was selected from each subclass, proportional to the size of each subclass. An action research approach was conducted with a sample of 50 Form 3 learners. Data was collected using tests while semi-structured interviews were used to complement results from the tests. Content analysis of participants to written tasks and interviews was done to determine errors thus resulting in the formulation of misconceptions about transformational geometry. The study highlights the need for teachers to adopt more effective teaching methodologies that address the specific areas where learners struggle. This may involve incorporating visual aids, hands-on activities, and technology-based tools to enhance understanding of abstract concepts. The findings suggest that the curriculum could be revised to provide more opportunities for students to practice applying transformational geometry concepts in real-world contexts. This could help to solidify their understanding and reduce misconceptions. Also, teachers should receive ongoing professional development to equip them with the knowledge and skills to teach transformational geometry effectively. This training could focus on strategies for identifying and addressing common misconceptions, as well as incorporating innovative teaching techniques. The study underscores the importance of using a variety of assessment methods to evaluate students' understanding of transformational geometry. This could include both traditional written tests and more formative assessments, such as projects, presentations, and discussions. Furthermore, collaboration between researchers, educators, and policymakers is essential to address the challenges identified in this study. This could involve developing evidence-based teaching materials, sharing best practices, and advocating for policy changes that support effective mathematics education. Longitudinal studies could track students' understanding of transformational geometry over time to identify patterns in their learning and to assess the impact of interventions designed to address misconceptions. By addressing the implications and recommendations outlined in this study, it is possible to improve the teaching and learning of transformational geometry in Zimbabwe and help students develop a deeper understanding of this important mathematical concept.

Contents

CHAPTER ONE: INTRODUCTION

1.0 Introduction

Transformational geometry, a fundamental concept in mathematics poses significant challenges to learners. Errors and misconception hinder students understanding and application of geometric transformations. This chapter gives the background of the study, statement of the problem, significance of the study, spells out research questions which seek answers to the purpose of the research and will give a demarcation to the study showing its limitations. Also, key terms will be defined in line with the research.

1.1 Background of Study

Transformational geometry, is thought to be one of the foundation of mathematical thinking and problem solving. It includes different types of transformations such as translations, rotations, reflection, enlargement, shear and stretch that help students in learning the spatial relationship between shapes. Nevertheless, students have difficulties when learning about and using geometric transformations (Battista, 2007). This gap in knowledge can interrupt the natural development of their mathematical skills resulting in an incomplete set of learned concepts which further suppresses their growth. Zimbabwe places a strong emphasis on mathematics as one of the key subjects in the education system yet poor performance still remains an area for concern (ZIMSEC, 2020). Many studies have confirmed students find this still difficult, as some of those include: Lack of complete foundational understanding and problem experiences is one potential factor. Clear strategies are needed to illustrate the application of geometry concepts in practical situations.

Recent Zimbabwean initiatives to strengthen mathematics education have included, among other efforts, curriculum reform and one-off, in-service teacher training programmes (MoPSE, 2015). It is hoped that these reforms will result in more effective teaching pedagogies, as teachers work towards integrating creative and innovative methods to assist students in their pursuit of a holistic notion of the underlying fabric of mathematics. Nonetheless, such initiatives notwithstanding, the problems remain to be there especially in transformation geometry. Grouped together, the factors that may lead to these challenges include but are not limited to: The current curriculum might not adequately cater to the various dimensions of transformational geometry, leading to gaps in understanding among students (Lamon, 2016). This is exacerbated by traditional teaching methods, which often focus on rote memorization rather than conceptual understanding (Hiebert & Lefevre, 2017). As a result, students may struggle to apply transformations in different contexts, even when they are familiar with the concept (Baturo & Niesche, 2017). Several schools lack sufficient resources for visualizing geometric transformations, such as models or technology (Geiger et al., 2015). This limitation can hinder students' ability to develop a deeper understanding of transformational geometry. Although training programs are available, some teachers may not feel comfortable or skilled enough to deliver transformational geometry instruction on their own (Pence & Ward, 2018). This can lead to inadequate instruction and a lack of confidence among teachers. Negative attitudes towards transformational geometry can arise from associated behaviours and challenges experienced by students over time, affecting their willingness to learn (Di Martino & Zan, 2015). These attitudes can be influenced by various factors, including prior experiences, teacher support, and parental involvement (Mujtaba & Reiss, 2017).

Transformational geometry is a fundamental branch of mathematics that explores the properties of geometric shapes under various transformations, such as reflections, rotations, translations, and dilations (Usiskin, 2017). These transformations involve moving shapes without altering their intrinsic properties, such as size, shape, and angle measures. Understanding these transformations is crucial for various fields, including engineering, architecture, and computer graphics. Despite the significance of transformational geometry, students often encounter challenges in mastering its concepts. Previous research has highlighted common misconceptions and difficulties students face in this area (e.g., Ada & Kurtuluş, 2010; Bansilal & Naidoo, 2012; Evbuomwan, 2013). More recent studies have delved deeper into specific challenges, such as spatial reasoning difficulties (Clements & Saram, 2014) and the impact of technology on teaching (Jones, 2017). This study aims to build upon this existing research by investigating the specific errors and misconceptions that learners at Ordinary Level in Zimbabwe exhibit in transformational geometry. By identifying these difficulties, we can develop targeted interventions to enhance their understanding and improve their overall performance in mathematics.

Kahobo Secondary has a history of academic challenges, particularly in the realm of mathematics. Over the past five years, the school has consistently faced the significant challenge of a zero percent pass rate in Ordinary Level Mathematics. This alarming statistic underscores the urgent need for innovative teaching strategies and targeted interventions. Transformational geometry, in particular, has proven to be a particularly challenging topic for students, with many struggling to grasp the abstract concepts and apply them to problem-solving.

Students often face the following challenges when learning transformational geometry:

- Conceptual Misunderstandings: Students may have difficulty grasping the underlying concepts of transformations, such as the difference between reflection and rotation (Hercovics, 1989).
- Incorrect Application of Rules: Many students struggle to apply the correct rules and properties of transformations to solve problems (Ada & Kurtuluş, 2010).
- Spatial Reasoning Difficulties: Visualizing and mentally manipulating shapes in different orientations can be challenging for some learners (Clements & Sarama, 2014).
- Lack of Practical Experience: Limited hands-on activities and real-world applications can hinder students' understanding of the practical significance of transformations (Bansilal & Naidoo, 2012).
- Limited Resources: Inadequate resources, including textbooks, stationery, and qualified mathematics teachers, may hinder effective teaching and learning.
- Large Class Sizes: Overcrowded classrooms can make it difficult for teachers to provide individual attention to students, especially those struggling with mathematics.
- Socioeconomic Factors: Socioeconomic disparities among students may impact their access to quality education and extra support.
- Lack of Parental Involvement: Limited parental involvement in students' education can negatively affect their academic performance.

By addressing these issues, we can empower students to develop a deeper understanding of transformational geometry and achieve better academic outcomes.

1.2 Statement of the problem

In Zimbabwe, transformational geometry constitutes an obligatory part of the ordinary level mathematics curriculum. Transformational geometry is a crucial component of the Ordinary Level mathematics curriculum, providing learners with a foundational understanding of spatial relationships and geometric transformations. However, despite its significance, research indicates that learners often encounter significant difficulties in grasping key concepts and applying them to problem-solving (Makhubele, 2021). These challenges are particularly evident in areas such as identifying and describing transformations, understanding the properties preserved under transformations, and accurately performing transformations on geometric figures. Studies have shown that learners frequently exhibit misconceptions regarding the nature of transformations, leading to errors in their calculations and interpretations (Makhubele, 2021). For instance, they may struggle to differentiate between reflections and rotations, or they may incorrectly apply rules for finding the images of points under transformations. These misconceptions can hinder learners' progress in mathematics and limit their ability to apply geometric concepts in real-world contexts.

Furthermore, research suggests that the teaching and learning approaches employed in classrooms may contribute to these challenges. Traditional teaching methods, often characterized by a focus on rote memorization and procedural skills, may not adequately address the conceptual understanding required for success in transformational geometry (Makhubele, 2021). To address these issues, it is essential to investigate the specific errors and misconceptions that learners exhibit in transformational geometry at the Ordinary Level. By identifying these areas of difficulty, educators can develop targeted interventions and instructional strategies to enhance learners' understanding and improve their performance in this critical area of mathematics.

1.3 Research objectives

By the end of the research project, the research will be able to:

1. Determine common errors and misconceptions in the learning of transformation among Zimbabwean Ordinary Level students.

2. To identify the possible causes of the errors and misconceptions displayed by learners in transformational geometry.

3. Develop suggestions for best practice instructional strategies.

1.4 Research Questions

The study seeks to answer the following question;

1. What types of errors and misconceptions do Zimbabwean Ordinary Level students commit in transformational geometry?

2. What are the causes of the errors and misconceptions in the teaching and learning of transformational geometry?

3. What can be done to help learners understand transformational geometry?

1.5 Assumptions of the Study

A number of important assumptions underpin this study into an investigation of errors and misconceptions displayed by learners in transformational geometry at the Ordinary Level in Zimbabwe.

It is assumed that the learners would be willing to participate in the research and reflect with sincerity on their responses with regards to understanding transformational geometry. By conducting this research, it is assumed that the teachers who participated in the research had a basic understanding in transformation geometry and could provide rich data on common mistakes and misconceptions their students make. This will give meaning to the interpretation of the data gathered. It is expected that the results of this research can be generalized to a wider applicability within the Zimbabwean education system. Even if specific schools or geographical locations are targeted for the study, the researcher hopes that the misconceptions and errors established would mirror some trends in other areas.

The study assumes that the mathematics curriculum, particularly about the content of transformational geometry, is offered in similar degrees within the various schools and classrooms. This consistency is crucial in ensuring that the misconceptions identified are not due to differing ways of instructional methods. It is assumed that while the influence of socio-economic factors on educational outcomes is there, it may not seriously bias this study's findings on errors and misconceptions in transformational geometry. The study assumes that the basic educational resources - textbooks and teaching aids - will be available to both teachers and learners despite resource constraints.

1.6. Significance of the Study

This study is important and has significance with respect to the type of errors displayed by Ordinary Level learners when dealing with Transformational Geometry across some crucial dimensions.

Identifying these pitfalls helps teachers to know their instructional strategies to mitigate the areas of toughest struggle for students. Such a focused treatment can result in better methods for teaching, thereby improving students' cognitive grasp and practice of geometric reasoning. In addition to that the results of this study may offer critical insights to curriculum developers in Zimbabwe. These findings can help guide the need to change mathematical practices or concepts that are mistaken or not well understood, appropriate for 3rd grade math coursework.

The study will also have greater implications on teacher preparation and professional development. Familiarity with the incorrect beliefs students have about this counter-intuitive stage should enable teacher education programs to communicate better how to teach transformational geometry systematically. This may mean setting up workshops that are designed around pedagogies in which the challenges in question can be addressed.

A better understanding of transformational geometry will give students the confidence to further explore complex mathematical problems, and help them perform better academically. More broadly, such a shift in mathematics can impact young people's academic and career prospects downstream. The study will add to the literature carried out on misconceptions in mathematics education, more especially those dealing with transformation geometry. The study will fill the gap in the literature by focusing on the Zimbabwean educational system and provide a foundation for other studies that could be conducted in similar contexts. The findings can thus help other researchers in studying other aspects of mathematics education. The results can also shape educational policy right from the school administration to the national education authorities. This study can also point out evidence of the existence of certain problems that learners experience and, thus, inform policy modifications toward the enhancement of teaching mathematics as well as budgeting for schools. In addition, the research may allow the community members including parents to appreciate some of the problems the students face in learning mathematics. Given an understanding of such issues, the members of the community can better support education programs and provide an environment that values and supports learning in mathematics.

1.7 Delimitations of the study

According to Creswell (2024), delimitations are the boundaries or restrictions that a researcher intentionally sets for their study. They are the choices made to define the scope and focus of the research, ensuring that the project remains manageable and achievable. This study will be delimited to learners enrolled in Ordinary Level mathematics at a selected school in Zimbabwe "Kahobo Secondary school" in Gokwe North. The focus will be on errors and misconceptions exhibited in the specific area of transformational geometry, excluding other topics within the Ordinary Level mathematics curriculum. Additionally, the investigation will be limited to the current academic year, providing a snapshot of learner understanding at this specific point in time.

1.8 Limitations of the study

This study's findings are limited to the specific sample of learners and the particular Ordinary Level subjects examined. Therefore, the results may not be generalizable to all Ordinary Level learners or all subjects within the curriculum. Additionally, the study focused on errors and misconceptions identified through written assessments, which may not capture all the complexities of learners' thinking and understanding. Further research is needed to explore the nature and prevalence of errors and misconceptions in a wider range of Ordinary Level subjects and using a variety of assessment methods.

Furthermore, the study relies on a limited number of schools and learners, which may not be representative of the entire Ordinary Level population. This could potentially introduce sampling bias and limit the generalizability of the findings. Additionally, the study focused on errors and misconceptions identified in a single academic year, which may not capture the long-term development of learners' understanding. Longitudinal studies are needed to track the evolution of errors and misconceptions over time and to identify potential interventions that can address these issues.

1.9 Definition of terms

1. Transformational Geometry

"Transformational geometry is a branch of mathematics that deals with the study of geometric transformations, such as translations, reflections, rotations, and dilations, and their effects on the properties and relationships of geometric figures" Smith et al, (2015). Transformation geometry is that aspect of mathematics dealing with the properties of geometric figures by means of transformations. Among these are translation (slides), rotation (turns), reflection (flips), enlargement (size changes), shear (size changes) and stretch (size changes). Their understanding allows students to investigate properties of shapes and their relationships in space.

2. Misconceptions

"Misconceptions are persistent, deeply rooted beliefs or ideas that are inconsistent with the accepted understanding of a particular concept or topic" (Mutambara, 2021, p. 62). In mathematics education, misconceptions are defined as incorrect beliefs or understanding of learners about mathematical concepts. These may lead to persistent errors in problem-solving and inhibit students from applying the principles correctly in mathematics.

3. Ordinary Level

"Ordinary level refers to the secondary education stage, typically comprising grades 9 to 11, where learners are expected to develop a basic understanding and proficiency in various mathematical concepts, including transformational geometry" (Mutambara, 2021, p. 24). Ordinary Level can also be defined as a stage at which a student goes through secondary school in Zimbabwe and is usually completed by a student at an age of about 16 years. The attainment by students in various subjects, such as mathematics, is normally tested through O-Level examinations that are administered through the Zimbabwe School Examination Council.

4. Errors

"Errors refer to specific mistakes or inaccuracies made by learners in their understanding, application, or performance of transformational geometry concepts" Smith et al, 1993. Math errors can be defined as any mistakes that are made while solving problems. Their source may range from calculation mistakes, misunderstanding of concepts to the incorrect application of procedures. Errors have to be identified to diagnose misconceptions.

5. Geometry

Geometry is the branch of mathematics that deals with points, lines, angles, surfaces, and solids with their properties and relationships. The latter comprises the study of the shape of objects, their size, and the spatial relationship that exists between different objects.

1.10. Chapter summary

The whole chapter discussed the foundational facts of the research into investigating errors and misconceptions displayed by learners in transformational geometry at Ordinary Level in Zimbabwe. The chapter began with an introduction that reasserted the importance of transformational geometry in the mathematics curriculum, stated the purpose of the study, and gave an overview of what was to be covered in the chapter. This was followed by the background of the study, which identified the prevailing problems existing among learners in understanding transformational geometry and requirements for focused research. The problem statement, therefore, identified specific issues relating to misconceptions and errors that influence students' performance in mathematics. These objectives were clearly pointed out by emphasizing the identification and analysis of common misconceptions together with their implications on academic achievement. Research questions accompanied the objectives in order to guide the investigation and ascertain the causes that presently occur to be the roots of misconceptions. A discussion of the significance of the study was done, focusing on how it would be able to inform education practices, curriculum development, and policy recommendations. Assumptions of the study were made by creating a platform for an understanding of the context within which the research will be done. Delimitations and limitations were also addressed to put into perspective what the research encompasses, as well as the constraints that come with it. The definitions of key terms were included so that clarity and consistency could be conveyed and maintained throughout this study. To this end, the current chapter should be a general overview of the study to be developed further in subsequent chapters covering a related literature review, methodology, results, and recommendations regarding misconceptions on transformational geometry.

CHAPTER TWO: LITERATURE REVIEW

2.0 Introduction

This chapter provides a theoretical framework and literature review that underpin the study of errors and misconceptions displayed by ordinary level learners. The theoretical framework draws on cognitive theories of learning, specifically schema theory and cognitive load theory, to explain how learners construct and process information, which can lead to errors and misconceptions. The literature review explores existing research on common errors and misconceptions in various subjects at the ordinary level, highlighting patterns and trends. Additionally, the literature review examines the factors that contribute to the development of errors and misconceptions, such as instructional practices, learner characteristics, and the nature of the subject matter. By understanding these theoretical underpinnings and the empirical evidence, the study aims to identify specific errors and misconceptions prevalent among ordinary level learners and to explore potential interventions to address these challenges.

2.1 Literature Review of Errors and Misconceptions in Transformational Geometry

The chapter reviews the literature on errors and misconceptions in transformational geometry in both Zimbabwean and international contexts. Geometry, which deals with the study of geometric figures through movements such as translations, rotations, reflections, and non-rigid transformations like shear, stretch, and enlargement, poses unique challenges to learners. Transformations have proven to be complex for many students due to the difficulty in distinguishing between rigid and non-rigid transformations. Research indicates that learners frequently conflate these concepts, leading to incorrect conclusions about the properties preserved after transformations (Ada & Kurtuluş, 2010). Furthermore, visualizing the effects of transformations, especially in the absence of dynamic geometry software, presents a significant challenge (Clements & Sarama, 2014). This issue is particularly pronounced in Zimbabwean classrooms, where access to such technology is limited. Misconceptions about the properties of geometrical figures lead to an incorrect application of rules of transformation. For example, students often mistakenly believe that a reflection alters the dimensions of a figure, rather than preserving its size and shape. This common misconception, observed both in Zimbabwe and internationally, highlights the need for effective instructional strategies to address these misunderstandings (Hercovics, 2014).

Furthermore, students' mastery of transformational geometry depends on the knowledge and preparation of teachers. Many teachers in Zimbabwe may not be adequately prepared to teach geometry, inadvertently perpetuating misconceptions among students. However, teachers who have undergone relevant professional development are better equipped to address these issues and ensure accurate instruction (Ball, Thames, & Phelps, 2008). In addition to that, cultural perceptions of mathematics can also affect students' attitude towards transformational geometry. In many Zimbabwean communities, geometry is often regarded as a purely theoretical subject with limited real-world applications. This perception can lower student interest and motivation, hindering conceptual understanding. Less attention has been given internationally to some of the strategies for addressing misconceptions in transformation geometry. However, research suggests that technology, such as dynamic geometry software, can enhance understanding by providing opportunities for visualization and exploration (Jones, 2017). Collaborative learning approaches, where students work together to discuss and solve problems, can also be effective in promoting deeper understanding and addressing misconceptions (Vygotsky, 2016).

In short, while errors and misconceptions in transformational geometry are prevalent both in Zimbabwe and internationally, addressing these challenges requires a multifaceted approach. Improving teacher preparation, integrating technology into the classroom, and fostering a positive attitude toward mathematics are crucial steps in enhancing students' understanding of this complex topic. By implementing these strategies, educators can empower students to overcome common misconceptions, develop strong spatial reasoning skills, and achieve success in transformational geometry.

2.2 Theoretical Framework

Cognitive Load Theory (CLT) (Sweller, 2016) posits that the limited capacity of working memory significantly influences learning outcomes. In the context of transformational geometry, CLT can help explain why learners often struggle with this topic (Kalyuga, Sweller, & Kirschner, 2015). The inherent complexity of understanding and applying multiple transformations, such as translations, rotations, reflections, and dilations, can exceed the cognitive capacity of many students, leading to errors and misconceptions (Kalyuga et al., 2015). Intrinsic load refers to the inherent difficulty of the learned material (Sweller, 2015). For example, in transformational geometry, understanding several transformations e.g., translations, rotations, reflections, and dilations—may pose a high intrinsic load, particularly for students who still conceptualize basic geometric concepts (Ainsworth, 2014). On the other hand, extraneous load refers to the added cognitive load placed on the learner as a result of how the information is presented (Kalyuga et al., 2015). Poor instructional design, such as unclear explanations or complex visual aids, can increase extraneous load (Sweller, 2015). In many Zimbabwean classrooms, resource constraints may lead teachers to rely on traditional teaching methods that inadvertently increase this load (Mkhatshwa & Moolla, 2017). Germane load, on the other hand, refers to the mental effort dedicated to processing and comprehending the material (Sweller, 2015). This type of load is essential for learning but can be hindered by high intrinsic or extraneous loads. In transformational geometry, germane load is fostered through engaging problem-solving activities and exploration of concepts (Kieran, 2017). However, when learners are burdened by extraneous demands, such as disorganized information or limited use of visual aids, their ability to process and internalize geometric concepts is compromised (Ainsworth, 2014). It is also possible for educators to ease the cognitive load by using simplified instructional materials and teaching complex tasks one step at a time (Sweller, 2015). For example, presenting transformations sequentially, rather than simultaneously, can facilitate students' comprehension (Kieran, 2017).

Than simultaneously, can help learners develop a deeper understanding with less strain on working memory (Kalyuga, Sweller, & Kirschner, 2015). Additionally, the use of visual aids and dynamic geometry software can concretize abstract concepts, reducing cognitive load. By visualizing the immediate effects of transformations, students can more easily grasp and apply these concepts (Clements & Sarama, 2014). In general, germane load can be enhanced by engaging students in structured, scaffold practice. For instance, predicting the outcomes of transformations before performing them can foster deeper cognitive processing and reduce error likelihood. Collaborative learning, such as working in pairs or small groups, can also be beneficial. Students can share cognitive resources, discuss thought processes, and help each other identify and correct misconceptions (Vygotsky, 2016).

Recent research suggests that incorporating technology, such as dynamic geometry software, can further enhance learning by providing interactive and visual representations of transformations. This can help students visualize the effects of transformations and develop a deeper understanding of the underlying concepts (Jones, 2017). According to Posner et al. (2014), Conceptual Change Theory suggests that prior knowledge and misconceptions significantly influence learning. Learners must confront and revise their existing beliefs to accommodate new information. In the context of transformational geometry, students' prior knowledge of geometric concepts can either facilitate or hinder their understanding of transformations. Misconceptions, such as believing that reflections change the size of a shape or confusing different types of transformations, can impede learning (Ada & Kurtuluş, 2010). To address these misconceptions, teachers can employ various instructional strategies. One effective approach is to use concrete manipulative and dynamic geometry software to help students visualize and experiment with transformations. By actively engaging with these tools, students can develop a deeper understanding of the underlying concepts and identify and correct their own misconceptions. Additionally, collaborative learning activities, such as group discussions and peer teaching, can foster critical thinking and problem-solving skills, leading to a more robust understanding of transformational geometry.

To develop conceptual understanding, students have to be made aware of the inadequacy of their prior knowledge and conceptions. This awareness has been said to be a motivating factor in leading students to learn the new ideas. Posner et al. (2014) proposed that for conceptual change to occur there are four necessary conditions, including: Dissatisfaction: The learner must become dissatisfied with his or her existing understanding. Also there is intelligibility: New ideas should be understandable and logically fit the schema of the learner's previous knowledge. Apart from that there is plausibility: New concepts should come off as believable and sensible. In addition to that fruitfulness: The new insight should allow possibilities of continued learning and use. To effectively address misconceptions in transformational geometry, teachers must first diagnose these errors through assessments, class discussions, and reflective activities. Once identified, instructional strategies can be implemented to highlight the discrepancies between students' existing conceptions and correct geometric principles. Manipulative activities, such as using physical objects or virtual simulations, can help learners visualize and experiment with transformations, challenging misconceptions and fostering a deeper understanding. Additionally, the use of dynamic geometry software allows students to explore and manipulate geometric shapes, providing immediate feedback and facilitating the development of accurate mental models (Clements & Sarama, 2014). Helping them to reflect on the thought processes and changes in understanding of students, journaling can be done to instill a growth mindset in them; discussion in small groups can be carried out, or presentation. Also, full guidance in structured support may help students navigate through the complexities of transformational geometry and manage their cognitive load to facilitate conceptual change. This may involve guided practice, peer teaching, and problem-solving tasks in gradual increments.

2.3. Errors and Misconceptions in Transformational Geometry

According to Battista (2007), the Inability to visualize the transformation of figures is leading to errors and misconceptions. In his 2007 work, Battista looks at the students' inability to visualize geometric transformations and brings forth the role of spatial reasoning in comprehending these concepts. The inability to visualize really hampers the learner's attainment of transformation geometry and leading to errors and misconceptions. Transformational geometry covers knowledge about various types of transformations- translations, rotations, reflections, shear, stretch and enlargement that can change the position, orientation, or even the size of a shape. Each has its rules and properties that must be learned for successful application. According to Battista (2007), because these forms of transformations have an abstract nature, students' abilities to visualize and perform mental manipulations of geometric figures are complicated.

Spatial reasoning can be described as the capability of a learner to perceive and transform objects in three-dimensional space. Battista has indicated that all students have problems as far as spatial reasoning is concerned, which is crucial for understanding how different transformations change geometric figures. Most learners have difficulties rotating or reflecting shapes mentally; thus, they often make incorrect predictions of the transformation outcome. Therefore, this hampers the learning process. Also, the prior experiences with geometry play a significant role in students' visualization of transformation capabilities. Students who have fewer concrete, manipulative geometric activities will find it more difficult to establish an accurate mental image of transformed figures. Battista goes on to say misconceptions of basic properties can further result in complication; students will apply incorrect assumptions while visualizing transformation. To cope with such challenges in visualization as outlined by Battista, the following are some of the strategies that a facilitator could use: The use of physical models and manipulative allows students to visualize the transformation of objects more concretely. The learners manipulate the physical shapes and can see directly the immediate outcome of transformations made on the shapes. Furthermore, the use of technology like dynamic geometry software allows students to work in an active environment where they can change figures and see transformations for themselves. This type of hands-on activity will hopefully improve their spatial reasoning and visualization skills. Also, clear annotated diagrams for each step of any transformation will provide a clearer view of the processes involved. Visual aids can serve as references for students while learning various transformations.

Battista reinforces the use of metacognitive strategies-teaching students to think about their thinking. Encouraging learners to describe their thought processes when visualizing transformations helps them notice gaps in their understanding and develop better visualization skills. Battista's research certainly underlines the high level of difficulty in visual transformation in geometry. Emphasizing spatial reasoning skills and applying appropriate instructional strategies will provide students with an opportunity to overcome such difficulties and thus improve their performance in transformational geometry and reduce errors and misconceptions. Inadequate understanding of geometric properties_ (Clements & Battista, 2014). The issue of the inadequacy of understanding geometric properties, from the work carried out by Clements and Battista (2014), shall be elaborated upon in the following paragraphs: In the seminal study undertaken in 2014, Clements and Battista examined how students' poor conception of geometric properties interferes with their learning and application of concepts in geometry. This study shows how misconceptions and superficiality in learning could later become stumbling blocks to the proper understanding of higher geometric concepts, including that of transformational geometry. Clements and Battista found out that students generally have basic misconceptions in the geometric properties when they enter the geometry classes. For instance, students would probably not be well aware of the congruence and similarity that form the basis for more insightful thought about transformation. Not having any sound knowledge about these properties, students would find it hard to understand how the transformation will maintain or alter these properties. For them, students have incomplete or partial mental models of geometric properties. It is this incoherence in the framing that prevents students from making connections between different concepts in geometry. For example, students who view transformations as mechanical processes are not likely to understand the underlying geometry involving properties like invariance under certain transformations.

The lack of proper understanding of properties of geometric shapes significantly influences the performance of students in transformation geometry. Students wrongly generalize that all transformations operate on shape properties similarly, which then leads to incorrect reasoning. For instance, they would mistake that rotation changes the size of a figure while it does not but results in retention of the size and shape. In their research, Clements and Battista insist on new learning approaches to determine the geometrical properties more profoundly. The learning of concepts should be taught rather than the memorization of it. Different exploratory and discovery activities would help learners to construct a more robust conceptual knowledge of geometric properties. Another approach to teaching geometric properties involves the use of charts, graphs, and manipulative. One can explain the geometric properties of shapes to students and allow them to independently manipulate these shapes to observe those properties in question in action.

The following discusses the theme of insufficient practice and reinforcement in mathematics education, based on the work of Kulm: Kulm examines the vital place of practice and reinforcement that is meant to form part of learning mathematics in his 1994 research. He identifies how inadequate practice hinders the concept-learning process of students, which includes ideas on transformational geometry. This research stresses the need for regular and meaningful practice in developing competence in mathematics. Important points garnered from Kulm's work are enumerated below: Kulm argues that practice is crucially important in the process of learning mathematics. When problems are presented repeatedly, there is consolidation of acquired knowledge and the eventual development of procedural fluency. In transformation geometry, for example, several transformations that are applied to shapes require students, upon constant practice, to get a feel for the rules and properties governing these transformations. In addition to that the nature of practice is critical. Kulm delineates the following types of practice: Drill Practice while rote practice may result in mechanical skill, it often does not foster deep understanding. In transformational geometry, for example, students may commit the steps for carrying out a transformation to memory without understanding any of the conceptual ideas underlying the transformation. Engage students in a problem-solving activity in which they continuously apply the transformations within different contexts to make connections, deepening their understanding. For instance, there is meaningful learning when applications and visualizations are brought into reality.

Kulm makes a point that students, without continuous opportunities for application and reflection, may forget the main points or not firm up their understanding. In transformation geometry, practice repeatedly with feedback so that students can correct misconceptions and consolidate how transformations affect geometric properties. It is believed that ineffective Practice results in superficial understanding, with a lack of confidence in the application of mathematics concepts. In the case of transformation geometry, due to a lack of practice, the students face difficulty while visualizing and carrying out the transformation correctly, leading to errors like misapplication of rules for the transformation and failure to recognize invariant properties. Following are some of the strategies Kulm advocated for improving the practice and reinforcement in mathematics: Differentiated Practice Opportunities: More frequent and varied problem sets involving divergent application requirements for different transformations build deeper understandings. As an example, tasks that call for rigid and non-rigid transformation can be challenging for the students to think more deeply about what properties are preserved in each. Timely, constructive feedback on the work of the students will help to realize areas they need to improve, while others are for consolidation of knowledge. This kind of feedback loop is crucial in avoiding misconceptions in transformational geometry. By allowing students to work together in solving practice problems, there arises an environment of discussion and further analysis. Peer teaching can also foster reinforcement while students are explaining to their peers.

In Kulm's research, practices and reinforcement play a very important role. In transformation geometry, ample practice opportunities and reinforcement make students solidify their concepts through feedback mechanisms or peer group discussions that help in conceptual purification and reduce fallacies and misconception. Students often struggle with differentiating between rigid and non-rigid transformations, leading to incorrect predictions and solutions (Ada & Kurtuluş, 2010). They may also misapply rules for transformations, particularly when dealing with combinations of transformations. Spatial reasoning difficulties, such as visualizing and mentally manipulating shapes, can hinder their ability to accurately predict the outcomes of transformations (Clements & Sarama, 2014). Additionally, misconceptions about the properties of shapes, such as assuming that a shape's orientation affects its size or shape, can lead to errors in identifying congruent and similar figures after transformations. Yanik (2014) noted that some students have the misconception that circular figures cannot be translated, as there are no corners to act as "handles." Other students have the misconception that non-circular figures cannot be rotated because rotation changes the figure (a square is changed to a diamond, for example). Along those lines, students may believe that similar (or even congruent) figures that are oriented differently (such as a square and a diamond, which is the square rotated 45∘) are not the same figure (Seago, Nikula, Matassa, & Jacobs, 2012). In our proposed learning progression, we have included a section at each level that lists misconceptions or errors that may indicate that a student should be placed at that level.

Addressing Misconceptions and Improving Learning

To address these challenges, educators can implement a variety of strategies. Dynamic geometry software can provide students with opportunities to visualize and experiment with transformations, helping them develop a deeper understanding of the underlying concepts. Hands-on activities, such as paper folding, cutting, and drawing, can help students develop spatial reasoning skills and concretely experience the effects of transformations. Collaborative learning can promote critical thinking, problem-solving, and the sharing of ideas, allowing students to identify and correct misconceptions. Effective questioning can probe students' thinking, identify misconceptions, and guide their learning. Connecting transformations to real-world contexts, such as art, architecture, and design, can increase student engagement and motivation. By employing these strategies, teachers can create a supportive learning environment that fosters conceptual understanding and reduces the prevalence of errors and misconceptions in transformational geometry.

2.4 Factors Contributing to Errors and Misconceptions

Several factors contribute to the development of these misconceptions. A weak foundation in basic geometric concepts can hinder students' ability to understand and apply transformations. Limited exposure to real-world applications can limit students' understanding and motivation. Ineffective instructional practices, such as those that rely heavily on rote memorization and procedural learning, may not adequately address the conceptual underpinnings of transformations. Furthermore, the complexity of transformational geometry can overwhelm students' cognitive capacity, leading to errors and misconceptions (Kalyuga, Sweller, & Kirschner, 2015). Understanding the sources of the errors and misconceptions in transformational geometry will provide a clear insight into effective teaching and learning. There are a number of interrelated factors that may influence students' geometric conceptual developments, thus leading to errors and misconceptions. The following subsections outline some of the key factors, including cognitive, instructional, and finally contextual elements. Cognitive factors such as working memory limitations and difficulties in spatial reasoning can significantly impact students' understanding of transformational geometry. Students may struggle to process and retain information about multiple transformations simultaneously, leading to confusion and errors (Kalyuga, Sweller, & Kirschner, 2015). Additionally, difficulties in visualizing and mentally manipulating shapes in different orientations can hinder their ability to accurately predict the outcomes of transformations.

The theory of Cognitive Load is based on the very important aspect that the students have only limited working memory. When dealing with complex transformations, such as carrying out many steps in a sequence of execution, students may not be in a position to hold all relevant information in mind; that can lead to mistakes. The inability to visualize often causes students some problem in mentally manipulating shapes. Poor development of spatial reasoning skills makes it hard to comprehend how a transformation will change the position and orientation of a figure; hence, misconceptions arise. Also, students enter learning environments with previously acquired knowledge and some misconceptions concerning geometric properties. If not dealt with, such misconceptions interfere with the proper grasp of new knowledge, resulting in sustained mistakes. Quality of Instruction: Indeed, the quality of instruction is one of the main factors affecting students' comprehension. Poor explanations or a lack of clarity or presentation in too abstract a way burdens the student's cognition and creates bewilderment. According to Kulm (1994), poor practice hinders the internalization of ideas in the minds of students. Reinforcements on properties and transformations are greatly needed in geometry; without such reinforcements, application remains hard. In addition, Inappropriate or scarce use of dynamic geometry software and visual aids hinders students in building imagination about transformations and their effects. Where students are not able to see the transformations in action, they may build misconceptions about their properties.

More recent studies have highlighted the importance of effective instructional practices in teaching transformational geometry. For example, Jones (2017) emphasizes the role of technology, such as dynamic geometry software, in enhancing student understanding. By providing interactive and visual representations of transformations, technology can help students visualize the effects of transformations and develop a deeper understanding of the underlying concepts. Additionally, research by Clements and Sarama (2014) has shown the importance of hands-on activities and manipulative in promoting spatial reasoning and understanding of geometric concepts. Curriculum Design: How the mathematics curriculum is structured and what it covers can either facilitate or hinder student learning. A curriculum that fails to knit together conceptual understanding with procedural skills may result in students picking up pieces of knowledge and misconceptions. Generally speaking, cultural attitudes toward mathematics may have a negative or positive effect on the motivation of learners toward learning activities. In such contexts where cultural perceptions about mathematics are not positive, this may result in neglecting to put effort into understanding geometric concepts and hence committing errors. The extent of training a teacher has about the geometric concepts is a significant determinant of the learning of the students. A teacher who has poor training about transformational geometry cannot easily identify and discard pupils' misconceptions.

It follows that mathematics anxiety will impinge on performance and perhaps even the willingness of students to tackle more complex ideas. Lack of confidence in one's ability to perform certain mathematical activities might lead an individual to shy away from challenging tasks, which would be conducive to the accumulation of mistakes and misunderstandings. Students who find mathematics boring, uninteresting, or lack motivation might not put in the effort required to comprehend transformational geometry. A lack of engagement here can, in fact, contribute to superficial learning and persistent misconceptions. Errors and misconceptions in transformational geometry arise from a complex interaction of cognitive, instructional, contextual, and emotional influences. Awareness of such contributing factors provides points for teachers to create specific interventions toward developing deeper understanding and fewer errors for an improved learning environment for the students.

2.5. Zimbabwean Context

Students' understanding and experiences in teaching and learning transformational geometry in Zimbabwe depend on several factors (Mkhatshwa & Moolla, 2017). This section discusses the unique context of Zimbabwean education, bringing to light the curriculum, resources, teacher preparedness, and cultural attitude toward mathematics. The mathematics curriculum in Zimbabwe is structured to emphasize various mathematical concepts at different educational levels; geometry inclusive (ZIMSEC, 2016). However, depth and breadth of coverage can vary across different educational levels (Makonye, 2017). While transformation geometry is included, greater emphasis might be placed on procedural skills at the expense of conceptual understanding, leading to misunderstandings (Kieran, 2017). The majority of schools in Zimbabwe face resource constraints regarding textbooks, media and technological tools (Mkhatshwa & Moolla, 2017). Without access to dynamic geometry software, for example, students may be limited in their ability to apply transformations fundamental in promoting insight into geometric concepts (Geiger et al., 2015). Zimbabwe teacher training programs may not be strongly enough focused on transformational geometry and spatial reasoning (Makonye, 2017). Most teachers are not confident about their understanding of geometric concepts, which can be transmitted to students in the form of misconceptions (Kalyuga et al., 2015). Some Zimbabweans consider mathematics to be abstract and complex, hence difficult (Mkhatshwa & Moolla, 2017). Such considerations may engender anxiety in students' minds, such that they do not want to engage with transformational geometry (Di Martino & Zan, 2015). Too much weight given to standardize testing lets teaching focus on memorization and procedural skills at the expense of conceptual understanding (Kieran, 2017). Therefore, students may become hung up on passing rather than deeply understanding transformation geometry, leading to misconceptions. Limited opportunities for formative assessment and feedback are likely to limit students' abilities in highlighting misunderstandings and correcting those (Black & Wiliam, 2017). Regular feedback would be greatly supportive in revision. The Zimbabwean context provides a challenge and an opportunity in teaching transformational geometry (Mkhatshwa & Moolla, 2017). Addressing challenges of curriculum design, resource availability, teacher preparedness, and cultural attitudes may go a long way toward creating a more supportive learning environment for improving learners' geometric conceptual understanding and minimizing errors and misconceptions.

Several factors contribute to the development of these misconceptions. A weak foundation in basic geometric concepts can impede students' ability to understand and apply transformations. Limited exposure to real-world applications can diminish students' interest and motivation. Ineffective instructional practices, such as those that rely heavily on rote memorization and procedural learning, may not adequately address the conceptual underpinnings of transformations. Furthermore, the complexity of transformational geometry can overwhelm students' cognitive capacity, leading to errors and misconceptions (Kalyuga, Sweller, & Kirschner, 2015). Cognitive factors such as working memory limitations and difficulties in spatial reasoning can significantly impact students' understanding of transformational geometry. Students may struggle to process and retain information about multiple transformations simultaneously, leading to confusion and errors (Kalyuga, Sweller, & Kirschner, 2015). Additionally, difficulties in visualizing and mentally manipulating shapes in different orientations can hinder their ability to accurately predict the outcomes of transformations. The quality of instruction plays a crucial role in students' understanding of transformational geometry. Poor explanations, lack of clarity, and abstract presentations can burden students' cognition and create confusion. As Kulm (2014) argues, ineffective instructional practices hinder the internalization of ideas. Reinforcements on properties and transformations are essential for application. Additionally, the appropriate use of dynamic geometry software and visual aids can help students visualize and understand transformations.

The design of the mathematics curriculum can significantly impact students' learning of transformational geometry. A curriculum that fails to connect conceptual understanding with procedural skills may lead to fragmented knowledge and misconceptions. Cultural attitudes toward mathematics can also influence students' motivation and engagement. In contexts where mathematics is not valued, students may be less likely to invest effort in understanding geometric concepts. Furthermore, the quality of teacher training can significantly impact students' learning. Teachers who lack a strong understanding of transformational geometry may struggle to identify and address students' misconceptions effectively.

2.6 Research Gaps, Gaps identified:

1. Limited research on the type of errors committed by Zimbabwean students

2. Few studies done on effective instructional strategies

2.7 Chapter Summary

The chapter's objective was to find out errors and misconceptions learners face in transformational geometry as revealed by other authorities. Some aspects that were left hanging have also been discussed. A rationale for the inclusion of transformation geometry in the Zimbabwean Curriculum has been given. This literature review, therefore, forms a basis of the methodology to be used in this study in an effort to establish the errors and misconceptions in transformational geometry among the Zimbabwean Ordinary Level students. This literature review has explored existing research on student difficulties in learning transformational geometry, with a focus on identifying common errors and misconceptions. Studies have consistently highlighted a range of challenges faced by students, including: Difficulties in grasping core concepts such as the properties of transformations (translations, rotations, reflections and enlargements), the concept of invariance, and the relationship between transformations and geometric properties. This review highlights the significance of addressing the identified errors and misconceptions to improve student learning and achievement in transformational geometry. The findings from this review will inform the design and implementation of the current research study, which aims to investigate the specific errors and misconceptions displayed by learners in transformational geometry at the Ordinary Level in Zimbabwe. This summary provides a concise overview of the key findings and implications from the literature review, laying the groundwork for your subsequent research on errors and misconceptions in transformational geometry among Ordinary Level learners in Zimbabwe.

CHAPTER THREE: RESEARCH METHODOLOGY

3.0 Introduction

This chapter outlines the methodology used in the exploration of errors and misconceptions in transformational geometry among Zimbabwean Ordinary Level students. In essence, this means giving a clear understanding of the research design, population, sampling methods, and data collection and analysis procedures employed in this study.

3.1 Research Design

The study employs a mixed-methods approach, integrating both quantitative and qualitative data to provide a comprehensive understanding of student errors and misconceptions in learning. This design is particularly effective in educational research, as it combines the strengths of both methodologies to offer richer insights.

Quantitative Component

The quantitative aspect involves the collection of numerical data through structured assessments, such as standardized tests and quizzes. These assessments are designed to identify common errors and misconceptions across a broad sample of students. Statistical analysis, including descriptive statistics and inferential tests, will be utilized to quantify the prevalence of specific errors and to explore correlations between student demographics and performance. This data will enable the identification of patterns in student misunderstandings, allowing for the development of targeted interventions. It focuses on measuring and quantifying variables to identify patterns, test hypotheses, and make generalizations about a population (Creswell, 2014). In the context of the research on transformational geometry errors and misconceptions among Ordinary Level learners in Zimbabwe, a quantitative approach could provide valuable insights into the prevalence, patterns, and potential predictors of these challenges. Quantitative research typically involves formulating testable hypotheses or research questions that can be answered through statistical analysis.

Quantitative data analysis typically involves statistical methods to analyze numerical data and test hypotheses. Common techniques includes: Descriptive Statistics to summarize and describe the data, such as calculating means, standard deviations, and frequencies (Gravetter & Wallnau, 2013). By carefully considering these components and adhering to rigorous research practices, you can effectively utilize quantitative research to gain valuable insights into the factors that contribute to student learning and performance in transformational geometry.

Qualitative Component

Complementing the quantitative data, the qualitative component includes interviews, focus groups, and open-ended survey questions. These methods will provide deeper insights into the thought processes and reasoning behind student errors. By engaging students in discussions about their learning experiences, the research will uncover the underlying cognitive and emotional factors contributing to misconceptions. Thematic analysis will be employed to identify recurring themes and patterns in the qualitative data, enriching the overall understanding of the issues at hand. It refers to the aspects of a study that focus on exploring and understanding phenomena in a more nuanced and in-depth way, often through non-numerical data. It delves into the subjective experiences, perspectives, and meanings that individuals or groups ascribe to their realities (Creswell, 2014). Qualitative research often employs open-ended, exploratory research questions that aim to understand complex issues or phenomena from the participants' perspectives. These questions typically begin with "how," "why," or "what" (Merriam & Tisdell, 2016). In-depth conversations with individuals to gather detailed information about their experiences, beliefs, and perspectives (Kvale & Brinkmann, 2009). Also Group discussions that allow researchers to explore a topic of interest from the perspectives of multiple participants (Krueger & Casey, 2018). Qualitative data analysis involves a systematic process of identifying themes, patterns, and meanings within the collected data. Techniques such as thematic analysis, grounded theory, and narrative analysis are commonly used (Braun & Clarke, 2006; Charmaz, 2014; Riessman, 2008). While qualitative research does not rely on statistical methods, it emphasizes the importance of rigor and trustworthiness. This includes ensuring data credibility, dependability, conformability, and transferability of findings (Lincoln & Guba, 2016). By carefully considering these components and adhering to rigorous research practices, researchers can effectively utilize qualitative research to gain valuable insights into complex social phenomenon.

Integration of Data

The integration of quantitative and qualitative data will occur at multiple stages of the research process. Initial quantitative findings will inform the development of qualitative instruments, ensuring that the interviews and focus groups address specific areas of concern identified through statistical analysis. Conversely, qualitative insights will contextualize quantitative results, providing a more nuanced understanding of the factors influencing student performance. This mixed-methods design not only enhances the reliability and validity of the findings but also facilitates a holistic view of the educational challenges faced by students. By bridging the gap between numerical data and personal experiences, the study aims to contribute to a more effective educational framework that addresses both the cognitive and emotional dimensions of learning.

3.2 Population of the study

In research, the population refers to the entire group of individuals, objects, or events that share a common characteristic and are of interest to the researcher. It is the group from which a sample is drawn for the study (Creswell, 2014). For example, if one is studying errors and misconceptions displayed by learners in transformational geometry at ordinary level in Zimbabwe, the population would be all Ordinary level students who are doing transformational geometry in Zimbabwe. The population for this study will comprise all Ordinary Level (O-Level) mathematics students enrolled in government and independent secondary schools in Midlands Province, Gokwe North District during the academic year, 2024. This population includes both urban and rural schools, catering to a diverse range of socioeconomic backgrounds and educational resources. By focusing on this broad population, the study aims to capture a comprehensive understanding of the errors and misconceptions prevalent among O-Level mathematics learners in transformational geometry across various learning contexts. This approach aligns with the recommendations of researchers like Darawsheh (2014), who emphasizes the importance of considering diverse learning environments to gain a nuanced understanding of students' epistemological obstacles in geometry.

3.3 Sampling

In research, a sample is a subset of individuals or objects selected from a larger population. Researchers use samples to make inferences about the characteristics of the entire population (Creswell, 2014). This is often done because studying the entire population can be time-consuming, costly, or even impossible. According to Yin (2018), sampling in case study research involves selecting cases that are relevant to the research questions and can provide rich, in-depth data. The goal is to choose cases that are informative and representative of the phenomenon being studied. Yin emphasizes the importance of purposive sampling, where cases are selected based on their potential to illuminate the research questions. From this study, a stratified random sampling will be employed to ensure that the sample cut across different strata, including; Geographic Location: Urban verses rural schools. Secondly, type of School: Public and private institutions. Thirdly, Performance Academics: High achievement versus low achievement. The sample will be limited to 50 students to maintain equilibrium between quantitative data with statistical analysis and qualitative insights.

For this study, a stratified random sampling technique was employed to ensure a representative sample of Ordinary Level students across diverse educational contexts in Zimbabwe. The target population encompassed all Ordinary Level students enrolled in mathematics classes within Gokwe North Midlands. To ensure a balanced representation, the population was stratified into three categories namely: Geographic Location (Stratified into urban and rural schools), School Type (Stratified into public and private schools) and Academic Performance Stratified into high-achieving and low-achieving schools based on historical performance in national examinations as indicated by Zimbabwe School Examinations Council (ZIMSEC) examination results (ZIMSEC, 2024). The research was conducted at a school with a total enrollment of 432 students. The study focused on a specific class of 90 learners, which was divided into three subclasses: A, B, and C. A sample of 50 learners was randomly selected from the class of 90 learners using stratified random sampling. This sampling method ensured that the selected learners were representative of the larger class population. To select the sample, the class of 90 learners was first divided into three subclasses (A, B, and C). Then, a random sample of learners was selected from each subclass, proportional to the size of each subclass. This resulted in a total sample of 50 learners. The sample size of 50 learners was deemed sufficient to provide reliable and generalizable results, while also being mindful of the potential limitations and constraints of the research study.

3.4 Research instruments

For data gathering, the researcher used interviews, questionnaires and diagnostic tests. To investigate errors and misconceptions in transformational geometry at the ordinary level, a comprehensive research design involving multiple research instruments is essential. These instruments will help the researcher to collect both quantitative and qualitative data, providing a deeper understanding of the learners' challenges.

Diagnostic test

The researcher developed a well-structured test consisting of a variety of questions covering different aspects of transformational geometry, such as translations, reflections, rotations, and enlargements. This included both multiple-choice and open-ended questions to assess both conceptual understanding and procedural skills. The researcher administered the test to a representative sample of learners at the ordinary level and analysed the test results to identify common errors and misconceptions. A cornerstone of my research was the development and administration of a comprehensive diagnostic test to accurately assess learners' understanding of transformational geometry at the Ordinary Level in Zimbabwe. This meticulously designed instrument played a pivotal role in identifying specific errors and misconceptions displayed by student. Firstly, the diagnostic test was meticulously designed to cover all key transformation types, including translations, rotations, reflections, and enlargements (NCTM, 2000). To ensure a comprehensive assessment, the test incorporated a diverse range of question formats: Multiple-choice questions: To assess basic knowledge, identify common misconceptions, and measure recall (Haladyna, 2004). Short-answer questions: To evaluate procedural knowledge and problem-solving skills, requiring students to demonstrate their understanding of transformation rules and perform basic calculations. Long-answer questions: To assess higher-order thinking skills, such as analysing complex transformations, justifying solutions, and explaining their reasoning (Bloom, 2016). All test items were meticulously aligned with the Zimbabwean Ordinary Level mathematics curriculum and underwent rigorous review by experienced mathematics educators to ensure content validity, clarity, and appropriate difficulty levels. Also, prior to full-scale administration, a pilot study was conducted with a small group of learners to assess the clarity, difficulty, and discriminatory power of the test items (Airasian, 2010). Item analysis was also conducted to identify any problematic items and to refine the test for subsequent administrations. This iterative process ensured the reliability and validity of the diagnostic test, maximizing its effectiveness in identifying student strengths and weaknesses. Apart from that, the analysis of student responses on the diagnostic test was a crucial phase of the research. Descriptive statistics, such as frequencies, percentages, and mean scores, were calculated to summarize student performance across different transformation types and overall. In-depth error analysis was conducted to identify specific types of errors made by learners, such as conceptual misunderstandings, procedural errors, and calculation errors. This analysis provided valuable insights into the nature and origin of student difficulties.

The findings from the diagnostic test were meticulously triangulated with data from student questionnaires, teacher interviews, and classroom observations (Creswell, 2014). This triangulation provided a more comprehensive and nuanced understanding of the factors contributing to learners' errors and misconceptions. The researcher conducted semi-structured interviews with a selected group of learners to gain insights into their thinking processes and reasoning behind their errors. The researcher used open-ended questions to encourage detailed explanations and to probe deeper into their understanding. The researcher also focused on specific questions from the diagnostic test where learners made mistakes, allowing them to explain their thought processes.

Interviews

The researcher utilized both teacher and student interviews as crucial qualitative data sources to gain a multifaceted understanding of learners' errors and misconceptions in transformational geometry at the

Ordinary Level in Zimbabwe. The interviews covered topics such as: Common student errors and misconceptions observed in transformational geometry, teaching approaches and strategies used for teaching transformational geometry, challenges and successes in teaching transformational geometry, curriculum implementation and its impact on student learning and factors that may influence student learning beyond the classroom. Semi-structured interviews were conducted with a sample of mathematics teachers. The interviews covered topics such as: Students' perspectives on their own learning experiences in transformational geometry, identification of specific difficulties and challenges encountered, explanation of their reasoning and thought processes when solving problems, utilization of learning strategies and their perceived effectiveness and attitudes towards mathematics and transformational geometry in particular. Individual or small group interviews were conducted with a sample of students. Both teacher and student interviews were audio-recorded and transcribed. Thematic analysis was employed to identify key themes, patterns, and insights from the interview transcripts (Braun & Clarke, 2006). The findings from both teacher and student interviews were triangulated with data from the diagnostic test and student questionnaires to provide a more comprehensive and nuanced understanding of the factors contributing to learners' errors and misconceptions in transformational geometry (Creswell, 2014). The combined insights from teacher and student interviews informed the development of targeted instructional strategies and interventions that addressed the specific needs and challenges of learners in transformational geometry. By incorporating both teacher and student perspectives, the research provided a richer and more holistic understanding of the factors influencing student

3.5 Data Collection

1. Questionnaires and Surveys: A structured questionnaire will be designed to assess students' understanding of transformational geometry concepts. The questionnaire will contain multiple-choice format, true/false questions, and open response questions.

2. Diagnostic Tests: A test concentrated on transformation geometry will be given to the students to diagnose prevalent errors and misconceptions. It shall include some of the important concepts about translations, rotations, reflections, shear, stretch and enlargement.

3. Interviews: Semi-structured interviews will be conducted among subsamples of 30 students to delve deeper into the thought processes and reasoning behind the errors and misconceptions. This will yield more qualitative insights.

Data Collection Process

Data collection for this study involved a mixed-methods approach, combining quantitative and qualitative methods to gain a comprehensive understanding of learners' errors and misconceptions in transformational geometry at the Ordinary Level in Zimbabwe. The diagnostic test was administered to the selected sample of students in their respective classrooms under controlled conditions. Clear and concise instructions were provided to ensure all students understood the requirements of each task. The test was administered within a designated time frame, allowing sufficient time for students to complete the assessment. Following the diagnostic test, students were administered a self-report questionnaire. The questionnaire, designed to gather information on students' learning styles, study habits, attitudes towards mathematics, and metacognitive awareness, was distributed and collected in a confidential manner (Neuman, 2014). Semi-structured interviews were conducted with mathematics teachers from the selected schools. The interview guide focused on eliciting information on common student errors and misconceptions observed in transformational geometry, teaching approaches, curriculum implementation, and factors influencing student learning (Kvale & Brinkmann, 2009).

Individual and small group interviews were conducted with a subset of students from the sample. These interviews aimed to gain deeper insights into students' perspectives on their learning experiences in transformational geometry. Students were encouraged to explain their reasoning, identify specific challenges, and describe their learning strategies (Kvale & Brinkmann, 2009). At Kahobo Secondary School, a more in-depth case study was conducted. This involved: Observing mathematics lessons focusing on transformational geometry to gain insights into teaching practices and student engagement. Document Analysis: Examining relevant documents such as lesson plans, student work samples, and school records to gain a broader understanding of the school context. Conducting follow-up interviews with a subset of students and teachers at Kahobo Secondary School to further explore specific issues and gain deeper insights into the learning environment. All data collected, including diagnostic test scores, questionnaire responses, interview transcripts, and field notes from classroom observations, were meticulously recorded and organized using appropriate data management software. This ensured the accuracy, reliability, and accessibility of the data for subsequent analysis. This section provides a detailed description of the data collection process, outlining the specific steps undertaken and the rationale behind each method. It also emphasizes the importance of ethical considerations, such as ensuring confidentiality and obtaining informed consent.

3.6 Data Analysis

The current study will employ descriptive statistics in summarizing data, while t-tests and ANOVA will be employed for testing differences between groups on the basis of demographic variables. The mistakes which were commonly found in the diagnostic tests would be grouped and analysed for the frequency and kinds of misconceptions held by students. The interviews shall be transcribed and thematic analysis conducted to identify any recurring themes or patterns of insight into misconceptions and reasoning processes among students. Also, initial codes will be developed based on the data, followed by thematic categorization that will help in drawing connections between student responses and identified misconceptions.

3.7 Ethical Considerations

Free and Informed Consent: Informed consent in writing will be obtained from both students and their guardians, ensuring that they understand the purpose of the study and their right to withdraw at any time. Also, all reports and publications will not show the identity of the participants. Data will be stored securely and accessed only by the research te

3.8 Limitations of the study

As the sample is specific, and the context is, too, one may not be able to generalize to all Zimbabwean students. In addition, reliance on self-reported data may introduce bias; hence, triangulation with objective measures will be employed. Also, cultural considerations of geometric ideas may not be represented as such, hence impacting the true meaning of misconceptions.

3.9 Summary

This chapter has described the methodology that shall be followed in investigating errors and misconceptions in transformational geometry among Zimbabwean Ordinary Level students. The mixed-method approach allows for a profound investigation into the topic at hand, since quantitative data may easily be combined with qualitative insights. The next chapter will present findings from this methodology to contribute to a deeper insight into the problems of student learning in transformational geometry.

CHAPTER FOUR: DATA ANALYSIS AND RESULTS

4.0 Introduction

This chapter analyses and shows the results from the study that investigated errors and misconceptions in transformational geometry among Zimbabwean Ordinary Level students. This domain is very important in mathematics, and it gives a firm foundation in understanding spatial reasoning and problem-solving; it covers a number of topics, including translations, rotations, reflections, enlargement, shear and stretch. The research will ascertain common errors made by students in the process of learning, mostly attributed to a lack of comprehension of the concept of geometry or an ineffective teaching approach. These types of error analysis can provide insight into the nature of conceptual misunderstandings that result in poor performance in geometry. The study adopted a combined qualitative and quantitative approach wherein data were collected through assessments, interviews, and classroom observations across schools in Zimbabwe. This chapter outlines the methodology applied, states the findings, and discusses implications for teaching practices in mathematics education. The findings are organized into themes that highlight specific areas where pupils struggle to apply transformations, understand geometric properties, and visualize and manipulate shapes. Such conceptions, if addressed, can help the educator take steps toward providing better support to students in building a solid foundation of transformation geometry concepts, hence improving their overall mathematical competence.

4.1 Quantitative Data Analysis

Diagnostic Test

A diagnostic test is a tool used to assess learners' existing knowledge and understanding of a particular subject or topic before beginning a more in-depth study. In the context of research on errors and misconceptions in transformational geometry, a diagnostic test aims to identify specific areas where learners may have misconceptions or lack clarity. It helps pinpoint gaps in their knowledge, enabling educators or researchers to tailor their teaching strategies or interventions to address these issues effectively, ensuring a more targeted and supportive learning experience. As such, a diagnostic test consisting of 6 multiple-choice questions and 5 open-ended questions was administered to assess students' understanding of transformation geometry. The test covered various aspects of transformation geometry, including visualization, geometrical properties, and transformation techniques. 74% of the students (37 out of 50) reported difficulty in visualizing transformations for example if given shapes that have undergone transformation, the students could not figure the type of transformation, indicating a possible lack of spatial reasoning ability. Also, 62% of the respondents (31 out of 50) had problems understanding geometrical properties, for example distinguishing the isomers and non-isomers, which would hinder their ability to apply transformation techniques. The test consisted of 6 multiple-choice questions and 5 open-ended questions, with a total score of 11. The results from the diagnostic test showed that the average score was 5 out of the 11 questions attempted. While 74% of the students scored below 6 out of 11. Only 26% of the students scored above 7 out of 11. The diagnostic test results indicate significant difficulties among students in understanding transformation geometry, particularly in visualization and geometrical properties. These findings suggest a need for targeted interventions to address these challenges and improve students' understanding of transformation geometry concepts.

These findings were further corroborated by the qualitative data obtained from student interviews and classroom observations. Many students expressed confusion about the concepts of reflection, rotation, translation, and enlargement. They often struggled to distinguish between these transformations and apply the correct rules for finding image points. Furthermore, students frequently made errors in calculating the coordinates of image points and determining the scale factor of enlargement. The types of errors that the students manifested are discussed below.

Error Analysis

An error analysis was carried out in as much detail as possible to identify the most frequent problem that the students had with the transformational geometry:

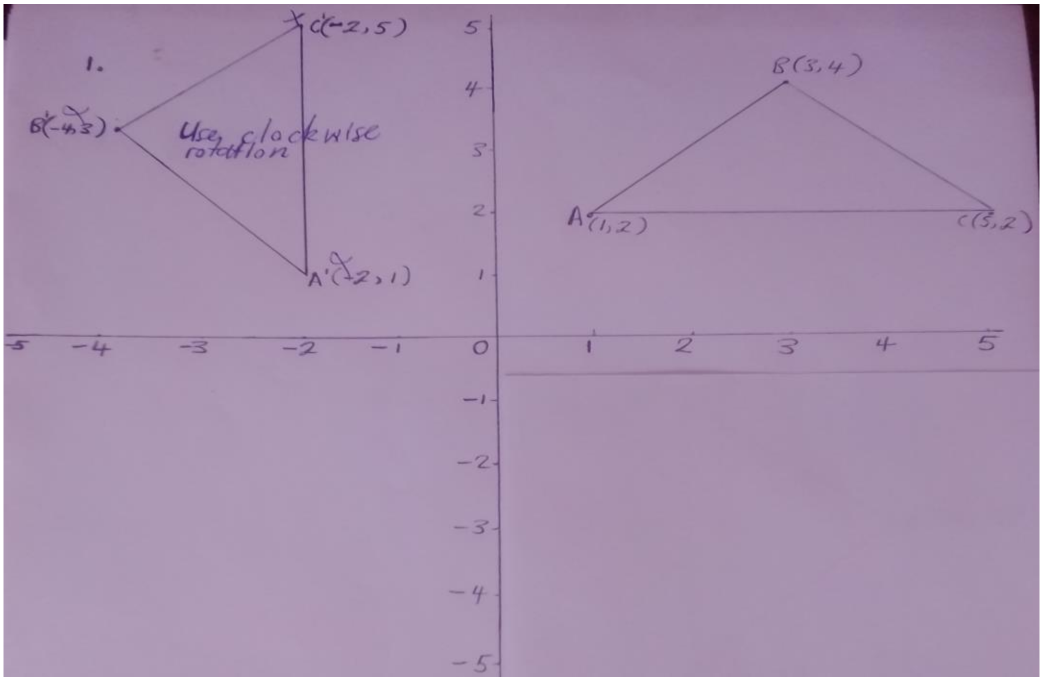
1. Common Errors made by O level students at Kahobo secondary: (APPENDIX (I) B)

I gave Form 3 students the following question:

A triangle with vertices at (A (1, 2), (B (3, 4), and (C (5, 2). The task was to rotate this triangle 90 degrees counterclockwise about the origin.

Incorrect Application:

Most students mistakenly apply the transformation rule for a 90-degree clockwise rotation instead (which is ((x', y') = (y, -x))), thus why they got this wrong result:



1. For point (A (1, 2)):

A' = (2, -1)

2. for point (B (3, 4)):

B' = (4, -3)

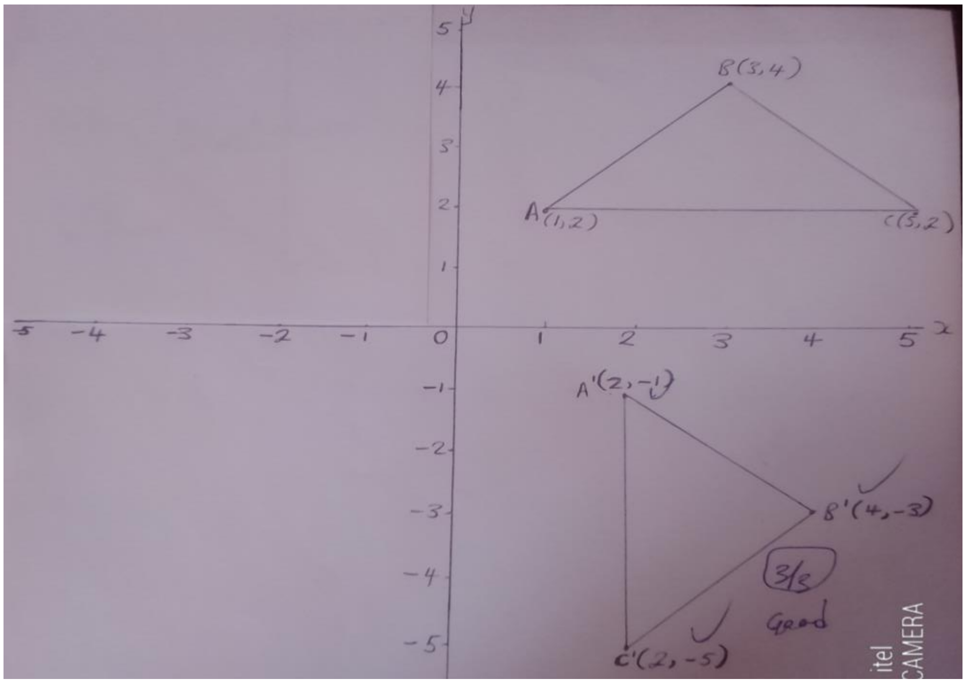
3. for point (C (5, 2)):

C' = (2, -5)

This was incorrect application of rules of transformation in mathematics especially, rotation. In some cases learners lacked the idea that counterclockwise and clockwise rotations share some similarities, though they differ significantly in terms of direction, mathematical representation, and impact on shape and position. For example, a counterclockwise direction, a shape is rotated in a direction opposite to the clockwise direction. Imagine a clock face: a counterclockwise rotation would move the hour hand from 12 to 11 to 10, and so on. While for a clockwise direction, a shape is rotated in the same direction as the clock hands. Using the same clock face example: a clockwise rotation would move the hour hand from 12 to 1 to 2, and so on. To correctly apply the 90-degree counterclockwise rotation, the transformation rule is as follows:

-For a point ((x, y)), the new coordinates after a 90-degree rotation counterclockwise are given by:

(X', y') = (-y, x)



Applying the Correct Rule will give us this result:

1. For point (A (1, 2)):

A' = (-2, 1)

2. for point (B (3, 4)):

B' = (-4, 3)

3. for point (C (5, 2)):

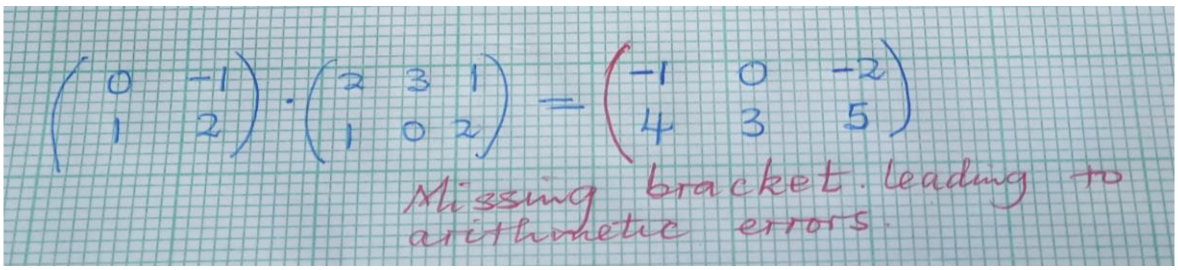
C' = (-2, 5)

Thus, the new vertices after the correct transformation are (A'(-2, 1)), (B'(-4, 3)), and (C'(-2, 5)).

I gave form 3 students the following task to test their ability in using brackets when multiplying a 2×2 matrix by a 2×3 matrix.

A triangle with vertices at A(2,1), B(3,0) and C(1,2). The task was to multiply it by a matrix below: $\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$

Missing brackets:



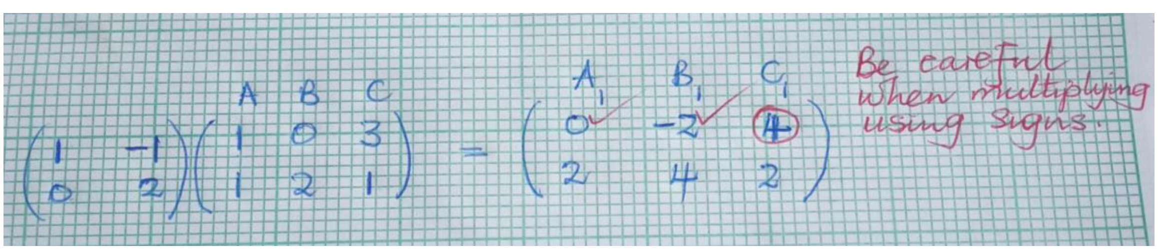
Some learners who showed this error were skipping the parenthesis. This led some learners for example learner S1 whose work has been shown, to take the equal signs as meaning negative signs hence, leading to wrong results.

I gave form 3 students the following question just to test their accuracy when multiplying matrices.

Multiply the matrix below:

$$\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 3 \\ 1 & 2 & 1 \end{pmatrix}$$

Omission of signs:



The work above suggests that the learner S2 carelessly omitted the negative sign in the multiplication of (1x3) + (-1x1), and got 4 instead of 2. This is because for the other coordinates, she got them correct. That gave a wrong image for the transformation.

I also gave form 3 students the following task:

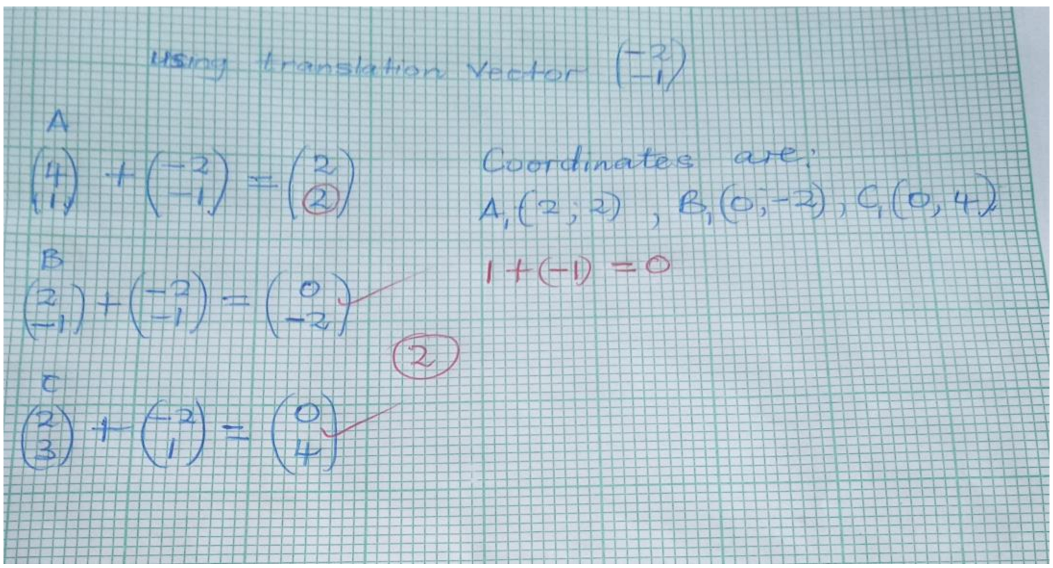
Add the following vectors using the translation vector (-2,-1) to find the images.

a) A (4, 1)

b) B (2, -1)

c) C) (2, 3)

Errors in solving basic operations:



Some leaners failed to use basic operations. This has been due to the poor mastery of directed numbers. The above illustration shows work done by S1 who failed some basic operations. The student did not master the idea that 1+ (-) 1=0 which means that 4+ (+) 3=7 and so on.

Conclusion:

This example illustrates how a misunderstanding of transformation rules can lead to significant errors in geometric transformations. The errors observed in students may stem from such incorrect applications, highlighting the need for practice and a solid grasp of transformation concepts. This could be interpreted to mean that the stronger the students' foundation in geometry, the fewer the mistakes they will have. In this respect, it becomes quite important to reinforce the basics before higher-order transformations are introduced.

4.2 Qualitative Data Analysis

4.2.1 Interviews with Teachers:

(APPENDIX (1) C)

a) General Questions

i) Core Concepts: What do you consider to be the most challenging concepts in transformational geometry for students to grasp?

ii) Common Errors: What are the most common errors or misconceptions you've observed in students' understanding of transformations (translations, rotations, reflections, dilations)?

iii) Student Difficulties: What specific difficulties do students encounter when applying transformations to geometric shapes?

IV. Misconceptions: Is there any misconception that students have on the properties of transformations?

Specific Transformation Questions

b) Translations:

i) How do students often misinterpret the direction or magnitude of a translation?

ii) What challenges do students face when translating shapes on a coordinate plane?

c) Rotations:

i) What common errors do students make when determining the center of rotation or the angle of rotation?

ii) How do students struggle with the concept of rotational symmetry?

d) Reflections:

i) What misconceptions do students have about the line of reflection and the image of a reflected shape?

ii) How do students confuse reflections with other transformations?

e) (Enlargement):

i) What difficulties do students face in understanding the concept of scale factor?

ii) How do students misapply the scale factor to find the coordinates of dilated points?

f) Instructional Strategies and Assessment

i) Effective Teaching Strategies: What teaching strategies or activities have you found effective in addressing common misconceptions and improving students' understanding of transformations?

ii) Assessment Challenges: What challenges do you encounter when assessing students' understanding of transformational geometry?

iii) Technology Integration: How do you use technology (e.g., dynamic geometry software) to enhance students' learning of transformations?

Interview insights:

(a) General questions

i). the most challenging concepts in transformational geometry for students to grasp include understanding the properties of transformations (e.g., preservation of shape and size), visualizing and describing transformations, and applying transformations to complex shapes.

ii) Common errors or misconceptions include confusing translation with rotation or reflection, incorrectly applying transformation rules, and struggling with coordinate geometry.

iii) Students encounter difficulties when applying transformations to geometric shapes, particularly with visualizing and describing transformations, using coordinate geometry, and understanding the relationships between different transformations.

iv) Persistence of misconceptions include believing that transformations change the shape or size of an object, confusing the order of transformations, and struggling with the concept of rotational symmetry.

b) Translation

i) Students often misinterpret the direction or magnitude of a translation by confusing the x- and y-coordinates or incorrectly applying the translation vector.

ii) Students face challenges when translating shapes on a coordinate plane, particularly with visualizing and describing the translation, using coordinate geometry, and understanding the relationships between different translations.

c) Rotations

i) Common errors include incorrectly identifying the center of rotation, confusing the angle of rotation with the direction of rotation, or struggling with visualizing and describing the rotation.

ii) Students struggle with the concept of rotational symmetry, particularly with understanding the relationships between different rotations, visualizing and describing rotational symmetry, and applying rotational symmetry to solve problems.

d) Reflections

i) Misconceptions include incorrectly identifying the line of reflection, confusing the image of a reflected shape with the original shape, or struggling with visualizing and describing the reflection.

ii) Students confuse reflections with other transformations, such as rotations or translations, particularly when working with complex shapes or coordinate geometry.

e) Enlargement

i) Difficulties include understanding the concept of scale factor, applying the scale factor to find the coordinates of dilated points, and visualizing and describing the dilation.

ii) Students misapply the scale factor by incorrectly multiplying or dividing the coordinates, or struggling with understanding the relationships between different dilations.

f) Instructional Strategies and Assessment

i) Effective teaching strategies include using visual aids, real-world examples, and technology to enhance students' understanding of transformations, as well as providing opportunities for students to practice and apply transformations.

ii) Assessment challenges include evaluating students' understanding of abstract concepts, assessing students' ability to apply transformations to complex shapes, and using technology to enhance assessment.

iii) Technology can be used to enhance students' learning of transformations by providing interactive visual aids, real-world examples, and opportunities for students to practice and apply transformations.

4.2.2 Student Interviews:

<u>(APPENDIX (1) D)</u>

Basic Concepts

a) What is a transformation? Can you give examples of different types of transformations?

b) What is the difference between a translation, rotation, reflection, and dilation?

c) How does a transformation affect the size and shape of a figure?

d) What is the line of symmetry?

e) What is the centre of rotation?

f) How can you translate a figure on a coordinate plane?

g) What happens to the coordinates of a point when it is reflected across the x-axis or y-axis?

h) How can you rotate a figure 90, 180, or 270 degrees about a point?

i) What is the effect of a dilation on the size of a figure?

j) Can you explain how to find the image of a figure after a sequence of transformations?

Problem-Solving

k) Given a figure and a transformation, how can you draw the image of the figure?

l) How can you determine the coordinates of the image of a point after a transformation?

a) S5: A transformation is a change in the position, size, or shape of a figure. Some examples include (moving the figure), reflection (flipping the figure), also enlargement, shear and stretch (shrinking the figure).

Error/misconception; A common misconception is that enlargement, shear and stretch always make a figure smaller. However, they can either enlarge or shrink a figure, depending on a scale factor.

b) S1: Translation moves the figure, rotation turns it, while enlargement, shear and stretch shrink and enlargement it.

Error: Reflection only changes orientation, not position.

c) S8: Translation and rotation preserve size and shape. Reflection changes orientation, and enlargement, shear and stretch changes size.

Error: reflection does not change shape, only orientation.

d) S7: The line of symmetry is a line that divides a figure into two identical parts.

Error: not all shapes have a line of symmetry.

e) S6: The center of rotation is the point at center of a figure around which a figure rotates.

Error: The centre of rotation can be anywhere not just in the figure' centre.

f) S2: To translate a figure on a coordinate plane, add the translation vector to the coordinate of a point.

Error: Translation vector is added to all points of a figure.

g) S4; when reflecting across the x-axis, the y-coordinate and x-coordinate changes sign.

Error: Only one coordinate changes sign.

h) S26: To rotate a figure 90°, swap the x and y coordinates and change the sign of the new x-coordinate. For 180°, change the signs of both coordinates. For 270°, swap the coordinates and change the sign of the new y-coordinate.

Error: For 270, the sign of new y-coordinate changes.

i) S15: An enlargement enlarges or reduces a figure by a scale factor, changing its size but preserving its shape.

j) S4: To find the image after a sequence of transformations, apply each transformation in order, using the result of each step as the input for the next step.

Error: some learners apply transformations in the wrong order.

k) S23: To draw the image of a figure, apply the given transformation to each point of the original figure.

Error: Some forget to apply the transformation to every point.

l) S8: To determine the coordinates of the image of a point, apply the transformation to the original coordinates.

Error: Some assume transformations automatically give the new coordinates.

Error/Misconception analysis

From the above responses students often made key errors in transformation geometry. They confused different types of transformations (translations, rotations, reflections, enlargement, shear and stretch) and misunderstanding their properties. They incorrectly calculated new coordinates after a transformation, struggle with identifying the centre of rotation or reflection, and misapply symmetry concepts, such as line of symmetry. Additionally, Students did not know that transformations like reflections and rotations preserve properties such distance and angle, leading to incorrect conclusions about congruence. Misunderstanding scale factors in enlargements, shear and stretch or confusing them with other transformations is another common issue, as is difficulty visualizing changes in shapes or orientations. These conceptions can be addressed by reinforcing the concepts by practising step-by step practice.

4.2.3 Classroom Observations

(APPENDIX (1) E)

Classroom observation revealed that limited use of visual aids and technology was being made in the classroom. Seldom did teachers use any tools, like geometric software or an interactive whiteboard, to enable learners to visualize transformations more clearly. In addition to that, instruction was teacher-centered; students were rarely involved in discussion or hands-on activities. This made many students passive, hence part of the problem in their difficulties in understanding transformational concepts. The general feeling observed in the classroom setting was rigid, with no room for exploration or experimentation. Encouraging a more student-centered approach may help create a better learning environment. There were no appropriate mechanisms observed to give effective feedback. Often, teachers were not in a position to go through the student's work in a timely manner and provide constructive feedback that would help the students learn from their mistakes and improve.

4.2.4 Interview-Interviewee Sessions and Common Responses

(APPENDIX (1) F)

Translation

Interviewer: Here is your first question: A point is at (2, 3). We translate it 4 units to the right. What are the new coordinates?

Interviewee (S2): We add 4 to the y-coordinate. The new point is (2, 7).

Interviewer: Not quite. Translation affects both x and y coordinates. Moving right affects the x-coordinate. Since we are moving 4 units to the right, we add 4 to the x coordinate, so 2 + 4 = 6. The y-coordinate stays the same. So the new point is (6, 3).

Interviewer: Last one. A shape is translated by the vector (-3, -1). This means?

Interviewee (S6) It moves 3 to the right and 1 up.

Interviewer: Remember, a negative x-value in the vector means movement to the left, and a negative y-value means movement down. So, (-3, -1) means 3 to the left and 1 down.

Errors

Learners confuse which coordinate is affected by horizontal/vertical movement. Adding instead of subtracting (or vice versa) when moving left/down.

Misinterpreting translation vectors, especially with negative values.

Reflections

Interviewer: Let's try reflecting a line segment. The endpoints are at (1, 4) and (5, 1). We reflect it across the y-axis. What are the new endpoints?

Interviewee (S9): If we reflect across the y-axis, we change the y-coordinate. So the new endpoints are (1, -4) and (5, -1).

Interviewer: Almost. Reflecting across the y-axis changes the x-coordinates, making them their opposite, while the y-coordinates stay the same. The new endpoints should be (-1, 4) and (-5, 1).
Interviewer: Okay, let's try reflecting across the line y = -x. We have the point (3, -2). What’s its reflection?

Interviewee (S11): Reflecting across y = -x is the same as reflecting across the x-axis and then the y-axis. So, first, reflecting across the x-axis gives (3, 2), and then reflecting across the y-axis gives (-3, 2).

Interviewer: That's also correct. Reflecting across y=-x is equivalent to swapping the x and y coordinates and then changing the signs of both. So, from (3, -2), swapping gives (-2, 3), and then changing signs gives (2, -3). Your way is correct too.

Interviewer: Last one. A point is at (0, 4). We reflect it across the line x = 2. What are the new coordinates?
Interviewee (S14): If the point is at x=0 and the reflection line is at x=2, then we just move it 2 more spaces to the right. So the new point is (4, 4).

Interviewer: It's close, and the x-coordinate is correct. The point (0, 4) is 2 units to the left of the line x=2. Its reflection will be 2 units to the right of the line x=2. So the x-coordinate will be 2 + 2 = 4. The y-coordinate does indeed stay the same. So the correct coordinates are (4, 4).

Errors

Confusing reflections across the x-axis and y-axis (changing the wrong coordinate). Not correctly applying the rules for reflections across y = x or y = -x. Difficulty reflecting across vertical or horizontal lines that are not the axes (like x = 2).

3. Rotations

Interviewer: First question: A point is at (1, 0). We rotate it 90 degrees clockwise around the origin (0, 0). What are the new coordinates?

Interviewee (S16): 90 degrees clockwise… that means it moves to (0, 1).

Interviewer: A line segment has endpoints at (0, 2) and (1, 2). We rotate it 180 degrees around the origin. What are the new endpoints?

Interviewee (S17): A 180-degree rotation just flips the signs of both coordinates. So the new endpoints are (0,-2) and (-1, -2).

Interviewer: That's correct. A 180-degree rotation around the origin changes the sign of both the x and y coordinates.

Interviewer: Here's a slightly more complex one. A point is at (3, 4). We rotate it 90 degrees clockwise around the point (1, 1). What are the new coordinates?

Interviewee (S19): If it was around the origin, it would be (4, -3). But since it is around (1, 1), we have to add 1 to the x and y. So, the new coordinates are (5, -2).

Interviewer: That's where the common error is. You can't just add 1 to the result of the rotation around the origin. You need to first translate the point so the center of rotation becomes the origin. Then perform the rotation. Then translate the point back. Translate the point (3, 4) so that (1, 1) becomes the origin. We subtract (1, 1) from (3, 4) which gives us (2, 3).

Rotate (2, 3) 90 degrees clockwise around the origin. This gives us (3, -2).Translate the result back by adding (1, 1) to (3, -2). The new coordinates are (4, -1).

Errors

Confusing clockwise and counterclockwise directions. Incorrectly applying the rules for 90-degree rotations. Not properly handling rotations around centers other than the origin. Not recognizing the equivalence of certain rotations (e.g., 270° clockwise = 90° counterclockwise).
4. Enlargement

Interviewer: First question: A point is at (2, 3). We enlarge it by a scale factor of 2, using the origin (0, 0) as the center of enlargement. What are the new coordinates?

Interviewee (S22): We multiply both coordinates by 2, so the new point is (4, 6).

Interviewer: That's correct! When the center of enlargement is the origin, you simply multiply the coordinates by the scale factor.

Interviewer: Now, let's change things. A point is at (2, 1). We enlarge it by a scale factor of 2, but this time the center of enlargement is (1, 0). What are the new coordinates?

Interviewee (S25): Okay, so we multiply the original coordinates by 2 to get (4, 2).

Interviewer: That's the common mistake. When the center of enlargement isn't the origin, you can't just multiply the coordinates. You need to consider the vectors from the center of enlargement to the point.

Find the vector from the center of enlargement (1, 0) to the point (2, 1): This is (2-1, 1-0) = (1, 1). Multiply this vector by the scale factor 2: (12, 12) = (2, 2). Add this new vector to the center of enlargement (1, 0): (1+2, 0+2) = (3, 2). So the new coordinates are (3, 2).

Interviewer: Let's try another one with a different center. A triangle has vertices at (2, 2), (4, 2) and (3, 4). We enlarge it by a scale factor of 0.5, with the center of enlargement at (1, 1). **Interviewee (S3)**: With a scale factor of 0.5, it gets smaller. So, we divide the coordinates by 2 to get (1, 1), (2, 1) and (1.5, 2).

Interviewer: You're right about it getting smaller, but again, you can't simply divide. You must use vectors. Let's take the point (2, 2): Vector from center of enlargement (1, 1) to point (2, 2) is (2-1, 2-1) = (1, 1) multiply the vector by 0.5: (0.5, 0.5). Add to the center of enlargement (1, 1): (1.5, 1.5). Using the same method, the other points become (2.5, 1.5) and (2, 2.5).

Interviewer: Last one. A point is at (1, 2). We enlarge it by a scale factor of -1, with the center of enlargement at the origin. What are the new coordinates?

Interviewee (S7): A negative scale factor means it goes to the opposite side. So it becomes (-1, -2). **Interviewer:** That is correct. A scale factor of -1 is equivalent to a 180-degree rotation about the center of enlargement.

Errors

Forgetting to use vectors when the center of enlargement is not the origin. Incorrectly applying fractional scale factors. Not fully understanding the effect of negative scale factors (combining enlargement with a 180-degree rotation).

5. Shear

Interviewee (S8): A horizontal shear means the x-coordinate changes. Since the shear factor is 2, we multiply the x-coordinate by 2. So the new point is (4, 3).

Interviewer: Not quite. The y-coordinate stays the same. The new x coordinate is found by adding the product of the shear factor and the y-coordinate to the original x coordinate. So, -1 + (-1 * 2) = -1 - 2 = -3. The new coordinates are (-3, 2).

Interviewer: Okay, let's switch to a vertical shear. A point is at (4, 1). We apply a vertical shear with a shear factor of 3 and the y-axis as the invariant line. What are the new coordinates?
Interviewee (S28): For a vertical shear, the y coordinates change, so we multiply the y-coordinate by 3. The new point is (4, 3).

Interviewer: Not quite. In a vertical shear with the y-axis as the invariant line, the x-coordinate stays the same. The y-coordinate changes by adding the product of the shear factor and the x-coordinate. So, the new y-coordinate is 1 + (3 * 4) = 1 + 12 = 13. The new coordinates are (4, 13).
Interviewer: Last one. After a shear, a square becomes a parallelogram. What stays the same?

Errors

Confusing horizontal and vertical shears (affecting the wrong coordinate). Incorrectly applying the shear factor (not adding the product of the shear factor and the relevant coordinate). Not understanding the concept of an invariant line. Thinking that the area changes after a shear.

6. Stretches

Interviewer: Let's start with a horizontal stretch. A point is at (2, 3). We apply a horizontal stretch with a scale factor of 2 and the y-axis as the invariant line. What are the new coordinates?
Interviewee (S14): A horizontal stretch affects the x-coordinate. Since the scale factor is 2, we just multiply the x-coordinate by 2. So the new point is (4, 3).

.**Intervie we**r: Correct! When the invariant line is an axis, it's a simple multiplication of the relevant coordinate by the scale factor

Errors

forgetting to use the distance between the point and the invariant line when the invariant line is not an axis. Incorrectly applying the scale factor (often adding it directly to the coordinate instead of multiplying the distance from the invariant line). Confusion between horizontal and vertical stretches.

Learners' Errors and Misconceptions in Transformational Geometry

The following table and diagrams summarize the findings of the leaners' errors and misconceptions observed during the interview process on transformational geometry. The learners demonstrated challenges in understanding various geometric transformations, such as translations, reflections, rotations, enlargements, shears and stretches. Key issues included confusion between which coordinate is affected by horizontal and vertical movements, misinterpretation of translation vectors and difficulties in reflecting across lines other than the axes. Rotational transformations were particularly challenging, with learners struggling to correctly apply rotations around center other than the origin and learners often failed to use the correct vector- based approach. Shearing transformations led to confusion about the effect on the right coordinate, and learners misapplied shear factors. The table and diagrams visually represent the frequency and types of errors and misconceptions, providing insight into areas requiring further focus in teaching transformational geometry concepts.

Table. 4.2

Transformation	Relative Misconception / Error Contribution (%)
Translation	8
Reflection	16
Rotation	24
Enlargement	32
Shear	10
Stretch	10

Table 4.2 Shows results from interview er-interview ee (session)

The information above was gathered through structured interviews with learners, where each was asked to solve problems involving geometric transformations such as translations, reflections, rotations enlargements, shears and stretches. Learners' responses were analysed to identify errors or misconceptions, which were then categorised by transformation type. The frequency of each error was calculated and expressed as a percentage. This data was organised into a table to provide a clearer and more comprehensive overview of the learner' struggles with the different transformations. The table includes two columns: 'Transformation' and 'Relative Misconception/Error Contribution (%)'. The 'Transformation' column lists the six transformation types under consideration: Translation, Reflection, Rotation, Enlargement, Shear, and Stretch. The 'Relative Misconception/Error Contribution (%)' column displays estimated percentages representing the proportion of errors observed for each transformation relative to the total errors observed across all transformations. The analysis reveals a clear trend: Enlargement is identified as the most challenging transformation, exhibiting the highest error contribution (32%), likely due to the complexities of scale factors and centers of enlargement. Rotation also presents significant difficulty (24%), often involving errors in direction and center of rotation. Reflection (16%) and Shear (10%) show moderate error rates, while Translation (8%) and Stretch (10%) are associated with the lowest error contributions, suggesting a stronger grasp of these concepts among students. These findings underscore the need for targeted interventions and pedagogical strategies to address specific misconceptions and improve student understanding of transformational geometry.

4.3 Thematic Analysis

Analysis of the qualitative data highlighted a few key themes which center on the type of challenges experienced both by students and teachers in the context of transformational geometry. Students find it difficult to visualize various transformations, including translation, rotation, and reflection. Such cognitive barriers lead students to have misconceptions and commit errors in problem-solving. Abstract Thinking: One major obstacle is the abstractness of geometric ideas. Students usually have a hard time connecting them with real-life examples; hence, poor comprehension and retention. Misconceptions of Concepts: Misconceptions regarding basic concepts such as symmetry and congruence are common among students, as evidenced through the interviews, and once more hamper the learning process of transformational geometry.

One of the strongest emerging themes concerned a lack of sufficient teaching resources including visual, manipulative, and technology resources. This scarcity restricts teachers in employing appropriate instructional strategies. Need for Teacher Training: The qualitative data showed the dire need for professional development, as indicated by a demand for training in methodologies. "The course on innovative pedagogies would be very valued, because these are highly interactive and will help to keep students engaged and enhance active learning." Curriculum Matters: The respondents also said that the curriculum is not always flexible, making it difficult, at times, to try and adapt lessons to meet the needs of learners with varied abilities. This can limit teaching and learning effectively. In this analysis, the relation of the prior knowledge that the students had about geometry and their present understanding of the transformation concept were outstanding; thus, students who had a good foundation did better and would have fewer misconceptions. Practical Application Experience: Students who reported experiences of accessing geometry in contextualized settings-for example, projects or real-life applications-appeared to evidence deeper understanding and greater confidence using transformational geometry. Some students reported that their socio-economic status affects the experiences and opportunities they get from mathematics outside of school, which then affects the overall learning of the subjects. The thematic analysis underlines the complex interplay of cognitive, instructional, and experience-based factors that influence students' grasping of transformation geometry. These themes can be addressed for useful insights that might help in improving teaching practices and learning outcomes for students in geometry.

4.4 Chapter summary

This chapter focused on the fundamental process of data collection, interpretation, and analysis, with particular emphasis on identifying and understanding the significant errors and misconceptions that emerges during these stages. Through careful examination of visual presentations, the chapter underscores common pitfalls and misinterpretations that often hinder accurate data analysis. Additionally, it explores the root causes of these errors, as articulated by learners themselves, offering valuable insights into their thought processes. In response to these challenges, the chapter proposes a range of practical strategies and preventive measures aimed at addressing and mitigating these issues, ultimately enhancing the overall quality and accuracy of data interpretation and analysis. Chapter five is going to deal with summary, conclusions and recommendations.

CHAPTER FIVE: CONCLUSION AND RECOMMENDATION

5.0 Introduction

This chapter summarizes the findings of the study, interprets the implications of these findings for mathematics education in Zimbabwe, and makes actionable recommendations for improving the teaching and learning of transformational geometry among Ordinary Level students. It is important, in this regard, that addressed challenges are mitigated for better student understanding and performance in this important area of mathematics.

5.1 Summary of Findings

There were cognitive difficulties that involved visualizing and abstracting the concept of
Transformation. The students lacked the ability to apply the principles of geometry through
Transformation by reason of concept visualizations and abstractions. Also, the dominance of teacher-centered instruction was compounded by a lack of resources and hence limited the students' opportunities to explore and practice. This led to a lack of practical experiences in learning, which fostered the sustenance of misconceptions. Utilization of prior knowledge and experiences from real life significantly influenced how students conceptualized transformational geometry throughout the study. Students who had more understandings of the basic concepts of geometry were more likely to perform better. These deductions from the results imply that educators need to focus on embedding visual aids and technology into the lesson, give specific scaffolding for weak learners, and increase teacher training and resources. If educators do something regarding these issues, they can offer a far more effective learning environment; this develops further conceptual depth and engagement in mathematics. The implication of this study, therefore, is that the teaching and learning of transformational geometry need a multi-faceted approach. Indeed, besides the expected improvement in performance as a result of the recommendations cited, there is the possibility to improve students' attitude towards mathematics through such moves, thus aiding their skills in later years. The several important messages about students' difficulties regarding transformational geometry that came out from the study were: Significant Errors and Misconceptions: A large percentage of students constantly showed developing errors and misconceptions in visualizing geometric transformation and applying the rules of transformation correctly. Cognitive, Instructional, and Resource-Related Factors: The findings highlighted those cognitive challenges such as abstract thinking combined with instructional limitations and inadequate

resources, significantly contributed to students' difficulties in mastering transformational geometry. Prior Knowledge and Experience Influencing Learning: It may be deduced from the analysis that the students' prior knowledge concerning geometry and experience of its practical application have directly influenced their comprehension and performance. Students with a good grounding usually made fewer mistakes.

5.2 Consequences

The study's findings carry several implications for the educational landscape: Mathematics Education Policy and Curriculum Development: The need for a curriculum that specifically emphasizes transformational geometry is critical. Policymakers should consider integrating these concepts more thoroughly into the national curriculum to better prepare students. Teacher Training and Professional Development: Developing teacher training programs with special workshops about transformational geometry will conveniently prepare the teachers to acquire the required skills and methods for teaching such content. Instructional Strategies and Resource Use: Schools shall adopt instructional strategies that include media and technology so as to ensure adequate resources are available for teaching and learning.

5.3 Recommendations

Curriculum Review and Revision; the curriculum needs to be reviewed in order to give due emphasis on transformation geometry, stating full learning objectives and attainment standards/criteria. Develop National Standards for Mathematics Education: This can help develop a framework for schools to ensure uniformity and excellence in mathematics education.

Teacher Training and Professional Development

Regular Workshops: Regular workshops in innovative methods of teaching transformational geometry, allow sharing of best practices and resources by teachers. Encourage Buddy Mentorship and Learning: Allow mentorship programs where experienced tutors can help other colleagues as a source of stimulating continuous professional development.

Instructional Strategies

Incorporate Visual Aids, Technology, and Real-World Applications: Teachers should use a range of visual aids and technology to facilitate the visualization of transformations by students and to illustrate links with real-life applications of geometry. Encourage Problem-solving and Critical Thinking: Provide instructional methods that are problem-solving and critical thinking-oriented; students would engage deeply in their course material.

Resource Allocation

Mathematics Education Infrastructure Resource Allocation: This will provide for the required infrastructure necessary to improve the learning and teaching environment, including technology-equipped classrooms. Make Available Accessible Digital Resources for Students: Guarantee that students have been provided with digital resources, such as online tutorials and interactive software, to facilitate independent learning.

5.4 Implication for Teaching and Learning

The findings realized in this study have great implications for improving the teaching and learning of transformation geometry among ordinary level students in Zimbabwe. These issues being addressed, there will be more effective teaching and learning. Some of these recommendations go this way:

1. Use of Visual Aids and Technology

Visual Tools Integration: An extension of the techniques applied by teachers in varied visual tools should be made, like diagrams and geometric models, to provide insights into the transformations. For instance, this can be furthered with the use of dynamic geometry software, such as Geo Gebra, to facilitate explorations that give a profound insight into geometric concepts.

- Multimedia Resources: The use of multimedia resources such as videos and interactive simulations will attract the attention and interest of the students in understanding such complex transformations in an easier manner. This will bridge the gap between abstract and concrete learning.

- Classroom Environment: The classroom environment will be adjusted to allow students to explore using visual aids. This would enhance active learning by allowing students to work in groups to manipulate shapes and transformations, hence enhancing collaboration and learning among peers.

2. Provide Targeted Support for Struggling Students

- Differentiated Instruction: Such differentiated instructional strategies on the part of the teacher will help him or her present the lesson to them, considering different needs that may be represented in the classroom. Noticing those students who face difficulties in concepts and then providing them with individualized interventions will bring about tremendous understanding and improvement.

- More Practice Opportunities: Providing extra practice sessions for students through tutoring or after-school programs to work on foundational skills will give them additional reinforcement regarding the concepts that are being addressed. Regular feedback will also help the students detect and set right the misconceptions.

• Peer Tutoring Programs: The peer tutoring programs allow students to work with one another, whereby the concepts that come more easily to some will be coached through by their peers. This will serve as a confidence builder and enable learning via collaboration.

3. Enhance Teacher Training and Resources

• Professional Development: Investments in continuous professional development for teachers are to be made. Training and workshops in innovative teaching methods, use of technology, and best practices in assessment will upgrade their level of expertise, which shall lead to enhancement of student learning.

• Resource Availability: Schools shall give priority toward acquiring teaching resources such as textbooks, manipulative, and technology. If good-quality materials are made available to the teachers, they can certainly impart more interesting and valuable lessons.

Collaborative Teaching Strategies: It would also facilitate collaboration amongst the teachers to share best practices and experiences. In this direction, the professional learning community in a school would enable the teachers to support each other in implementing effective teaching-learning strategies.

5.5 Recommendations for Stakeholders

Ministry of Education

Develop National Mathematics Education Policy: Formulate comprehensive policy and focus on mathematics education. Prioritization shall be given to transformational geometry. Providing Resources for the Training of Teachers: Ensure funds and materials are available to carry out teacher training programs.

Teachers

Attend Workshops and Peer Mentoring: Actively participate in professional development opportunities to enhance their teaching skills. Include Visuals and Technology: Use various teaching aids and technology in the classroom to facilitate better understanding of geometric concepts. Students Engage in Mathematics Learning Materials: Take responsibility and make use of other learning materials that may be available online or extra textbooks. Seek Assistance from Teachers and Classmates: Encourage students to ask questions and seek help whenever they do not understand a concept. Appendices Appendix A: Survey Questionnaire Appendix B: Mathematics Achievement Test Appendix C: Interview Protocols

5.6 Conclusion

This has, therefore, contributed immensely to the understanding of errors and misconceptions in transformational geometry among Ordinary Level students in Zimbabwe. The findings have pointed out some areas that are critical for improvement in terms of teaching practices and resource allocation. These recommendations are made in an effort toward improving students' learning and informing mathematics education policy and practice for a more robust classroom understanding of geometry.

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APPENDICES

APPENDIX: 1

Questionnaire on Transformational Geometry Understanding

A. DIAGNOSTIC TEST

Section 1: Multiple-Choice Questions (MCQs)

1. Which of the following describes a reflection transformation?

A) A shape is turned about a point.

B) A shape is mirrored over a line.

C) A shape is enlarged or reduced in size.

D) A shape is moved along a straight path.

2. If a point P (3, 4) is translated by the vector (2,-3), what are the new coordinates of point?

A) (5, 1)

B) (1, 7)

C) (2, 7)

D) (5, 7)

3. Which of the following statements is true for a rotation of 90° clockwise about the origin?

A) The new coordinates of any point become (7, -3).

B) The new coordinates of any point become (-1, 4).

C) The new coordinates of any point become (3, 4).

D) The new coordinates of any point remain the same.

4. An enlargement with a scale factor of will: A) Double the size of the figure, while maintaining its shape.

B) Reduce the size of the figure by half.

C) Keep the size of the figure the same.

D) Flip the figure over a line.

5. Observe the diagram of a point on the coordinate plane. After a reflection over the x-axis, what will be the new coordinates of point?

A) The new coordinates of any point (x, y) becomes (y,-x)

B) The new coordinates of any point (x, y) becomes (-y, x)

C) The new coordinates of any point (x, y) becomes (-x, y)

D) The new coordinates of any point (x, y) remains the same

6. Given the triangle with vertices A (1, 1), B (3, 1) and C (2, 4) what are the coordinates of the triangle after a 90° counterclockwise rotation about the origin?

A) A (-1, 1), B (-1, 3), C (-4, 2)

B) A (-1, -1), B (-3, -1), C (-4, -2)

C) A (-1, 1), B (-1, -3), C (-4, -2)

D) A (-1, -1), B (-1, 3), C (-4, 2)

Section 2: Open-ended questions

5. A triangle with vertices at (A (1, 2), (B (3, 4), and (C (5, 2). The task was to rotate this triangle 90 degrees counterclockwise about the origin

6. Translate the point P (3. 5) by the vector (4, -2). What are the coordinates of the image?

6. Reflect the point (2,-3) over the x-axis. What are the coordinates of the image?

7. A rectangle has vertices at A (1, 2), B (4, 2), C (4, 5) and D (1, 5). Perform a 90° counterclockwise rotation about the origin and find the coordinates of the image of point.

8. An enlargement of a shape is made with a scale factor of ½. If the original coordinates of a point are (6, 8). What are the new coordinates of the point after the enlargement?

9. Describe the process and the rule used to reflect a quadrilateral with vertices A (1, 2), B (4, 2), C (4, 6) and D (1, 6) over the y-axis. Provide the coordinates of the reflected quadrilateral.

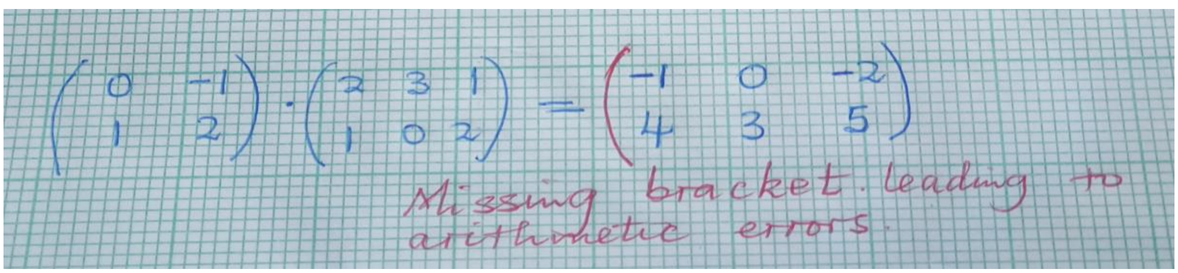
B. COMMON ERRORS MADE BY O' LEVEL STUDENTS

<u>Conceptual Errors Made by O level Leaners:</u>

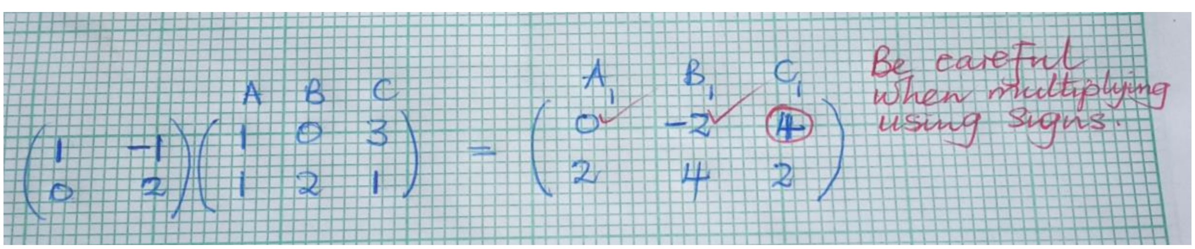
A triangle with vertices at (A (1, 2), (B (3, 4), and (C (5, 2). The task was to rotate this triangle 90 degrees counterclockwise about the origin.

I gave form 3 students the following task to test their ability in using brackets when multiplying a 2×2 matrix by a 2×3 matrix.

Incorrect use of brackets:



Omission of signs:



Errors in solving basic operations:

Using translation vector $\begin{pmatrix}-2\\-1\end{pmatrix}$

A

$\begin{pmatrix}4\\1\end{pmatrix} + \begin{pmatrix}-2\\-1\end{pmatrix} = \begin{pmatrix}2\\2\end{pmatrix}$

Coordinates are:

$A_1(2;2)$, $B_1(0;-2)$, $C_1(0,4)$

$1+(-1)=0$

B

$\begin{pmatrix}2\\-1\end{pmatrix} + \begin{pmatrix}-2\\-1\end{pmatrix} = \begin{pmatrix}0\\-2\end{pmatrix}$

2

C

$\begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}-2\\1\end{pmatrix} = \begin{pmatrix}0\\4\end{pmatrix}$

C. INTERVIEWS WITH TEACHERS

i) Core Concepts: What do you consider to be the most challenging concepts in transformational geometry for students to grasp?

ii) Common Errors: What are the most common errors or misconceptions you've observed in students' understanding of transformations (translations, rotations, reflections, dilations)?

iii) Student Difficulties: What specific difficulties do students encounter when applying transformations to geometric shapes?

iv. Misconceptions: Are there any persistent misconceptions that students hold about the properties of transformations?

Specific Transformation Questions

b) Translations:

i) How do students often misinterpret the direction or magnitude of a translation?

ii) What challenges do students face when translating shapes on a coordinate plane?

c) Rotations:

i) What common errors do students make when determining the center of rotation or the angle of rotation?

ii) How do students struggle with the concept of rotational symmetry?

d) Reflections:

i) What misconceptions do students have about the line of reflection and the image of a reflected shape?

ii) How do students confuse reflections with other transformations?

e) (Enlargement):

i) What difficulties do students face in understanding the concept of scale factor?

ii) How do students misapply the scale factor to find the coordinates of dilated points?

f) Instructional Strategies and Assessment

i) Effective Teaching Strategies: What teaching strategies or activities have you found effective in addressing common misconceptions and improving students' understanding of transformations?

ii) Assessment Challenges: What challenges do you encounter when assessing students' understanding of transformational geometry?

iii) Technology Integration: How do you use technology (e.g., dynamic geometry software) to enhance students' learning of transformations?

D. INTERVIEWS FOR LEARNERS

a) What is a transformation? Can you give examples of different types of transformations?

b) What is the difference between a translation, rotation, reflection, and dilation?

c) How does a transformation affect the size and shape of a figure?

d) What is the line of symmetry?

e) What is the center of rotation?

f) Applying Transformations

g) How can you translate a figure on a coordinate plane?

h) What happens to the coordinates of a point when it is reflected across the x-axis or y-axis?

i) How can you rotate a figure 90, 180, or 270 degrees about a point?

J) What is the effect of a dilation on the size of a figure?

k) Can you explain how to find the image of a figure after a sequence of transformations?

Problem-Solving

l) Given a figure and a transformation, how can you draw the image of the figure?

m) How can you determine the coordinates of the image of a point after a transformation?

n) How can you use transformations to create tessellations?

o) Can you identify the transformation(s) that map one figure onto another?

p) How can you use transformations to solve real-world problems (e.g., designing patterns, creating art)?

E. CLASSROOM OBSERVATIONS CHECKLIST

i. Did the teacher encourage student participation and engagement?

ii. Were students given opportunities to ask questions and seek help?

iii. Did students demonstrate a clear understanding of transformational geometry concepts (e.g., translations, rotations, reflections)?

iv. Were students able to apply these concepts to solve problems?

v. Did students demonstrate fluency in performing transformational geometry procedures (e.g., graphing **transformations)?**

F. INTERVIEW-INTERVIEWEE (session)

The researcher interview learners on the following subtopics on Transformational geometry:

i) Translation

ii) Reflection

iii) Rotation

iv. Enlargement

v) Shear

vi. Stretch

APPENDIX: 2

CONSERT FORMS

A. Parent consent form

Dear Parent/Guardian,

We invite your child to participate in a research study titled "Investigating errors and misconceptions displayed by learners in transformational geometry at ordinary level". This study aims to identify and understand common errors and misconceptions that learners encounter in transformational geometry.

Your child's participation will involve completing a diagnostic test in transformational geometry and possibly participating in an interview to discuss their understanding of the subject. The information gathered from this study will contribute to a better understanding of the challenges learners face in transformational geometry and inform the development of targeted instructional strategies.

Your child's participation is voluntary, and they may withdraw at any time. Please indicate your consent for your child to participate by signing and dating below.

Signature: ____

Date: ____

B. Learner Consent form

Dear Learner,

We invite you to participate in a research study titled "Investigating errors and misconceptions displayed by learners in transformational geometry at ordinary level". This study aims to identify and understand common errors and misconceptions that learners encounter in transformational geometry.

Your participation will involve completing a diagnostic test in transformational geometry and possibly participating in an interview to discuss your understanding of the subject. The information gathered from this study will contribute to a better understanding of the challenges learners face in transformational geometry and inform the development of targeted instructional strategies.

Your participation is voluntary, and you may withdraw at any time. Please indicate your consent to participate by signing and dating below.

Signature: ____

Date: ____

C. Teacher consent form

Dear Teacher,

We invite you to participate in a research study titled "Investigating errors and misconceptions displayed by learners in transformational geometry at ordinary level". This study aims to identify and understand common errors and misconceptions that learners encounter in transformational geometry.

Your participation will involve completing a questionnaire about your teaching practices and experiences with transformational geometry, as well as participating in an interview to discuss your perspectives on teaching and learning transformational geometry. Additionally, the researcher may observe your mathematics lessons to gain a better understanding of how transformational geometry is taught in the classroom.

The information gathered from this study will contribute to a better understanding of the challenges learners face in transformational geometry and inform the development of targeted instructional strategies.

Your participation is voluntary, and you may withdraw at any time. Please indicate your consent to participate by signing and dating below.

Signature: ____

Date: ____

D. Demographic Information

1. Age: __________

2. Genre:

[ ] Male

[ ] Female

[ ] Other

3. School Type:

[ ] Urban Public

[ ] Private Urban

[ ] Rural Public

[ ] Rural Private

4. Current Grade Level:

[ ] Form 3

[ ] Form 4

APPENDIX 3 LETTERS OF PERMISSION TO CONTACT A RESEARCH

SAMED

P Bag 1020
BINDURA
ZIMBABWE

Tel: 0271 - 7531 ext 1038
Fax: 263 - 71 - 7616

BINDURA UNIVERSITY OF SCIENCE EDUCATION

Date: 12/12/2024

TO WHOM IT MAY CONCERN

NAME: SIBANDA TAFIREI REGISTRATION NUMBER: B225868A

PROGRAMME: HBScEdMt PART: 2-2

This memo serves to confirm that the above is a bona fide student at Bindura University of Science Education in the Faculty of Science Education.

The student has to undertake research and thereafter present a Research Project in partial fulfillment of the HBScEdMt programme. The research topic is:

INVESTIGATING ERRORS AND MISCONCEPTIONS DISPLAYED LEARNERS IN TRANSFORMATIONAL GEOMETRY AT ORDINARY LEVEL IN ZIMBABWE

In this regard, the department kindly requests your permission to allow the student to carry out his/her research in your institutions.

Your co-operation and assistance is greatly appreciated.

Thank you

BINDURA UNIVERSITY OF SCIENCE EDUCATION
DEPARTMENT OF EDUCATIONAL FOUNDATIONS
OCT 2024
P. BAG 1020
BINDURA

Z. Ndemo (Dr.)
CHAIRPERSON - SAMED

Bindura University of Science Education
P. Bag 1020
Bindura

06 January 2025
Ministry of Primary and Secondary Education
P. Bag 7701
Causeway
Harare

Dear Sir/Madam

RE: APPLICATION FOR PERMISSION TO CONDUCT AN ACADEMIC RESEARCH AT KAHOBO SECONDARY SCHOOL IN GOKWE NORTH DISTRICT, MIDLANDS PROVINCE

I am requesting for permission to carry out an academic research at Kahobo Primary School in Gokwe North, Midlands province. The topic is, "Investigating errors and misconceptions displayed by learners in transformational geometry at ordinary level in Zimbabwe.

Questionnaires and interviews will be used to gather information.

I will be grateful if you take my application into consideration.

Yours faithfully

SIBANDA TAFIREI

CELL NUMBER: 0778 532 132 / 0783 120 528

All communications should be addressed to the
Secretary for Primary and
Secondary Education
Telephone: +263 242 794 509
Toll Free: 317

ZIMBABWE

Ministry of Primary and Secondary Education
88 Kwame Nkrumah Avenue
Queen Lozikeyi House
P O Box 121
Causeway, Harare

Reference: C/426/3

7 January 2025

Sibanda Tafirel
Bindura University of Science Education
P.O. Box 1020
Bindura

RE: PERMISSION TO CARRY OUT A RESEARCH IN MIDLANDS PROVINCE: GOKWE NORTH DISTRICT: KAHOSO SECONDARY SCHOOL.

Reference is made to your application to carry a research from the above mentioned district schools on the research title:

INVERSTIGATING ERRORS AND MISCONCEPTIONS DISPLAYED BY LEARNERS IN TRANSFORMATIONAL GEOMETRY AT ORDINARY LEVEL IN ZIMBABWE

Permission is hereby granted. However, you must liaise with the Provincial Education Director for Midlands Province, who is responsible for the schools which you want to involve in your research. You should ensure that your research work does not disrupt the normal operations of the schools. Where students are involved, parental consent is required.

You are also required to provide a copy of your final report to the Secretary for Primary and Secondary Education.

Musingarimi H
Acting Deputy Director Innovation and Development
for **SECRETARY FOR PRIMARY AND SECONDARY EDUCATION**

MIN. OF PRY & SEC. EDUCATION
POLICY PLANNING, RESEARCH
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7 JAN 2025
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