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DEPARTMENT OF MATHEMATICS AND STATISTICS

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EARNINGS MAXIMISATION IN THE PUBLIC TRANSPORT BUSINESS SECTOR THROUGH THE ADOPTION OF ELECTRONIC FARE PAYMENT SYSTEM.

(CHITUNGWIZA 2017-2018).

BY

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APPROVAL FORM

I Tanaka Matombo do hereby declare that this submission is my own work apart from references of other people’s work, which has duly been acknowledged. I hereby declare that this work has been presented neither in whole nor in part for any degree at this university or elsewhere.

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Certified by

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DEDICATION

To Varaidzo.

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ABSTRACT

This work utilized the concept of Simplex algorithm; an aspect of linear programing for the purpose of finding the optimal way that a Public transport operator can maximize earnings given that they have adopted electronic fare payment system. The objectives of this research project were, to find a fare payment method which can substitute cash payment, maximising the daily earnings made by the public transport service providers per trip if they adopt electronic fare payment. The data collected was analysed using the quantitative methods and a linear programming model was used to analyse the data using the Rglpk and LinProg solvers on R. The results of the final modelling showed that if transport operators utilise electronic fare payment and carrying only 6 passengers destined to Waterworks the operator will achieve optimality in earnings of $4.30769.

KEYWORDS:

Linear programming model, Simplex method, Decision variables, Optimal result

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ACRONYMS

CBA	Cost Benefit Analysis
EFP	Electronic Fare Payment
NPV	Net Present Value
PT	Public Transport
SC	Smart Cards
TC	Travel Cards

CHAPTER 1

INTRODUCTION

1.0 Introduction

There has been an increase in the use of electronic money in Africa and through mobile and digital technologies which have helped to boost financial inclusion. Due to liquidity crisis in countries like Nigeria, Kenya, Zimbabwe and Namibia these transactions have been used to supplement cash usage and ease the pressure on the demand of cash to avoid inflationary pressures. PayPal, skrill, bit coins and mobile money have been effectively used in America, Europe and India to do individual transactions either from paying for an airplane ticket, buying groceries and buying gasoline while businesses' pay their suppliers using electronic funds.

Mobile money services provided by financial technology and telecommunication companies are increasingly filling the gap left by traditional banks; mobile money services do not require the same investment in branch infrastructure as traditional banks (Manyika, 2016) .Digital finance still has untapped potential in Africa provided financial services are tailored to local context and end-user characteristics. The use of mobile money transactions is an area whose potential is still to be tapped into by many small and medium enterprises in Zimbabwe and this includes public transportation business sector because this sector serves more than one half of the population. Although financial technology transactions may involve certain costs, the regulatory authority should create an enabling environment so that the cost of compliance is not excessive.

In recent years Zimbabweans have embraced electronic payments and it is interestingly to note that payments made with debit or credit cards exceeds cash consumer payments. Consumers are now quite comfortable with using their debit, credit and prepaid cards to pay for low-dollar value transactions. The person who is using his debit card to buy bread is also the same person who would use the card to pay USD$2 transit fare to get to work.

Public transportation in Zimbabwe is a basic commodity for individuals because they use it on a daily basis hence there is a need to find innovative payment solutions which will lead to easy, faster and most importantly affordability of commuting. Mobile money payment system is an innovative payment solution which can help both the service provider and customers to move beyond the daily friction caused by cash. Mobile money for transit payments does not only improve customer experience but it also reduces customer costs from acquiring cash through the Eco-cash cash-out system.

1.1 Organization of study

The outcome of this study is presented in five chapters. The introduction, background, problem statement, significance and objectives are presented in the current chapter. Second and third chapters are on literature review and methodology respectively. The fourth chapter is on data analysis and presentation. The last chapter consolidates the major findings and conclusions of the study.

1.2Background of the study

The problems resulting from the growth in transport demand have led policy makers in transportation to focus on development of sustainable transport systems. Public transport is still the major mode for people's travelling in their day-to-day life. Owning a private car is far too expensive and hence public transport is still the principal mode for the majority. For the regulators, in order to develop sustainable transport systems under the limited resources (e. g., land use and energy consumption), some policies must be made to promote public transit over private vehicles, increase the attractiveness of public transport, harmonize ticketing for both occasional users and daily commuters.

Fare fraud, including cash fare underpayment or evasion, false bus travel cards, and fare disputes, has resulted in loss of transit revenue, and can be stressful to public transport drivers. By successful implementations in other countries, electronic fare payments have effectively decreased the frequencies of such incidents due to the relatively more secure design and fare collection procedure under the smart electronic transit fare collection systems.

The cost of public transport personnel is another reason for public transport operators to implement, electronic fare payments. A great amount of public transport personnel in most public transport companies are conductors. The ticketing duty, such as issuing tickets, checking the validation of fare cards, reporting the information of bus stops and routes, can also be taken by bus drivers under electronic fare payment systems.

When reviewing the history and development of electronic fare payments for public transport industry across the world, it can be seen that the great changes took place in fare payment collection systems during the last decade. Because of the technological advantages of electronic fare payments in practical applications, now electronic fare payment technology is replacing the traditional fare payment types in the worldwide public transport systems. The potential of electronic fare payment for public transport has received increasing attention during the past ten years. In the early 1990's, some small scale electronic fare payments projects for public transport were successfully carried out and tested in some countries, such as US, UK, France, etc. (Fleishman, 1996; Ampelas, 1998; Blythe, 2004; ITSO, 2006).

Public transportation has now become expensive in Zimbabwe and this is due to economic hardships that the country is facing. Although the country have embraced electronic payments in recent years, public transit payments for fares are made using cash only while the cost of acquiring cash using mobile wallet withdrawals are very high. Lucas (2004) suggests that this situation causes transport poverty and defines transport poverty as a lack of both private and public transportation services and affects the low income earners.

The poorest groups in any country tend to be less mobile. They most suffer from lack of both private and public transport services in terms of number of options and the quality of services available to them (Barter 1999). As such they are forced to rely on unsafe means of transport which are affordable to them (Titheridge *et al.*, 2014). At the same time, in urban areas poor people are most often located in peripheral locations at the edge of cities with low amenity value, this also combine with their limited transport options (Harvey 2003).

Litman (2015) suggested different solutions depending on who is affected with some policy measures which are currently being used by certain regulatory authorities in the transport sector, such as concessionary fares, operating subsidies for necessary public transport services and complimentary community transport service. In the Zimbabwean context the regulatory authorities have tried to subsidize public transportation through the Zimbabwe United Passengers Company (ZUPCO) but still this service does not meet the demands of all the customers and hence it needs to be complimented by an innovative payment solution that would lead to the affordability of public transportation.

Mobile technologies are changing economic life in developing countries, where many people are using cell phones for a range of financial transactions, such as receiving and sending money transfers. Indeed, mobile money is already being used by banks and mobile network operators to provide millions of unbanked consumers a way to store and access money digitally.

1.3 Problem Statement

Citizens of Chitungwiza, are facing a problem of unaffordable transportation due to shortage of cash caused by the liquidity crisis, while public transit payments for fares are made using cash only.

1.4 Objectives of the Study

The researcher wants to achieve the following objectives in this study:

- Creating a novel and innovative payment solution for public transport fares.
- Utilising all the payment technology opportunities available in Zimbabwe for transit payments.
- Maximisation of the daily earnings made by the public transport service providers.
- Creating an affordable public transportation system through introducing electronic payments.

1.5 Research Questions

The researcher came up with the following research questions:

- Is there an innovative payment solution for public transport?
- Can the cash payment be replaced in public transport?
- How can the daily earnings of public transport service providers be maximized?
- Can the electronic payment solution aid in affordability of public transportation?

1.6 Significance of the Study

Finding a point of intersection between the two stakeholders involved in this transportation problem, thus maximization of earnings for the public transport service providers and making transit fares affordable through taping into financial technology and creating an electronic transit payment system. By finding the solution to this transportation problem this study will be helpful to the public transport service providers, the Zimbabwe public transport regulatory authorities, to the commuting public and the students who will want to do research in the same area of study for making public transportation affordable.

1.7 The study assumes the following

- All the customers have access to electronic funds.
- The fare prices of the service do not change within the period under review.
- Transactions are in $USD
- No tax and transaction costs in the use of electronic funds transfers.
- Customers are readily available to be transported.
- The Public transport operator charges fixed prices as per destination.

1.8.0 Limitations of the study

- Not all customers are willing to pay electronically since some do not have bank accounts or mobile money accounts**.**

1.8.1 Definition of terms

- Automated Money Transfer Service: Use of electronic or bank-card based payments in carrying out different transactions.
- Electronic Payment: Any cash and associated transactions implemented using electronic means.
- Electronic Fund Transfer: Transfer of funds besides those that originate from transactions by cheque, draft, or related paper instruments and which are facilitated by use of such means as telephone, electronic terminal, or any computerized system to authorize any financial institution to credit or debit an account
- Near field communication: It is a method of wireless data transfer that detects and enables technology in close proximity to communicate without the need for an internet connection.

1.9 Conclusion

This chapter gave an introduction of electronic funds transfer payment system and public transport system, outlining the background, statement of the problem, research objectives and the scope of the study. It also introduces the notion of transport poverty which is the failure for individuals to acquire affordable, reliable and sustainable transportation. The researcher formulated some objectives in context with the background of the study. Consequently the chapter draws a road map for the study. It stands as a gateway for next chapter that shall focus on theoretical literature and empirical observations in order to discover and scrutinize the mileage of past researches.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter examines the literature related to the effects of adopting an electronic payment system in public transportation. The sections covered include the theoretical review and the empirical review. The empirical review is presented thematically according to the study objectives stated in chapter one.

2.2.0 Theoretical Review

The study is based on the following theory

2.2.1 Modernization Theory

Modernization theory was developed by Rostow Watt in 1960. Modernization refers to the progressive transition of a society from traditional to a contemporary society which is also referred to as modern society. The theory focuses on the assumption that developing countries need to follow the path that developed countries passed through, that is follow the same steps for them to attain modern development levels. Since technology has been a major source of change in the world then the theory of modernization has been very important in the study of technological transition.

Huntington (1968) puts forward a notion that for developing nations to reach their full potential technologically and public transportation advancements they should follow the steps or the exact routes which were taken by developed nations in those identical spectrums. In developed nations the public transportation service providers have modernized their transit payment systems from traditional cash payment systems to advanced and smart automated electronic fare collection payment systems (Leal et al 2013).

Through the aspect of modernization, electronic payment systems have spread across the globe, fostering economic and socio-economic development in developing nations. This is due to efficient social networks and knowledge transfer systems that exist today, and the need for "Western Standards" (Sharmila 2005). In developing countries, payment models have been built upon shared algorithms from developed countries that have been constantly tested and applied for sound business and customer convenience. Therefore, the theory of modernization, offers a broad view of the development, spread and the adoption of automated money transfer payment systems.

2.2.2 Electronic Payment System

Agimo (2004) defines an electronic payment system as that payment by direct credit, electronic transfer of credit card details, or some other electronic means as opposed to payment by check and cash. Accordingly, an electronic payment system is any means used to make payment using an electronic network such as internet.

2.3.0Types of Electronic Payment System

These are some of the forms of electronic payment systems that are applicable in the payment of public transport service fares.

2.3.1 Mobile money based fare payment System

Nowadays mobile phones are ubiquitous systems of our society. In several activity sectors the use of mobile phones can be used to revolutionize their services. The public transport sector is not an exception. Here such technology can change the current service delivery process and its value proposition. (Ferreira et al 2012). This not only changes the overall travelling experience but also changes the way how service providers manage their resources. For both stakeholders operational gains are expected (Ferreira et al 2012).

2.3.2 Bank card based fare payment System

Most of the transport service providers are interested in electronic fare payment technology and they are replacing the traditional methods of payment like cash, common tickets, prepaid monthly passes with the credit card as an applicable payment choice (Blythe, P., 2004). The users perceived credit cards as a safe system for verification and fare payment collection. By using credit card system for collecting the fare, the driver's job becomes easier. Moreover, it gives transport a more modern view by the flexibility of fare system. As one of the promising techniques in the 21^{st} century, intelligent transport systems, which combine a broad range of wireless and wire line communications-based information, control and electronics technologies, have been gradually applied to public transport to improve efficiency and effectiveness for operators, service quality for passengers, and environmental influences for society as a whole. (Dempsey, 2008).

Intelligent public transport system applications, including automatic vehicle location, passenger Information systems, electronic fare payment, traffic signal priority and so forth represent a significant opportunity to improve the efficiency, attractiveness and safety of public transport systems. These applications primarily improve the operation of a transport system by either performing a function quicker or more reliably, or by providing a service that was not previously available. Electronic fare payment using a credit card or a debit card provide improved mobility of people and goods on the existing surface systems, and they offer the potential for substantial savings in future construction on transport infrastructure. (Blythe, 2004)

2.3.3 Smart card based fare Payment System

The term “smart card" has been used to describe a range of card classifications and technologies in the world. The microchips embedded in the smart cards can be computer chips, capable of both storing and processing information, or memory chips, which are capable only of accessing data already stored on the card (Bagchi and White, 2004). As a new technology for enhancing public transport services, smart card ticketing for public transport is becoming increasingly popular across the world. Now smart cards and traditional fare payment methods, cash and travel cards, have become the three major fare payment options in public transport systems.

One of the advantages of the smart card technique in public transport systems is that passengers can avoid handling cash for payment when boarding. Moreover, smart cards have greater security, higher reliability, and higher resistance to fraud than other payment means. Recently, dual-interface smart cards using a single chip to communicate with smart card readers have been developed, because they have a single integrated platform for contact and contactless applications. They may prove to be more popular for multi - application schemes facilitating cooperation across industrial sectors (Casey, 2000).

2.4.0 Empirical literature review

Electronic payment technology and its application to public transport have been presented and discussed in many previous relevant studies (Blythe, 2004; Chambers, 1998; Laconte, 1998). In order to implement such technology, in recent years several studies have analyzed the potentialities of mobile technologies such as mobile wallets, where customers can use their mobile phones regardless whether they are smart phones or simple phones to access their mobile wallet accounts or their bank account and transfer funds into the accounts of the service provider for fare payment. Another technology that can be used for mobile payment is the use of smart phones mobile apps which the client or the customer can use to access their wallet account or bank account to transfer funds to the service provider for fare payment (Jose et al 2013, Ferreira et al 2014, and Leal et al 2015).

Several researchers have studied the key factors that influence the adoption of mobile payments in public transport. Mallat et al (2008) identifies eleven determinants related to the technology adoption, namely the, ease of use, social influence, usefulness, attitude, compatibility, cost, prior experience, trust, risk, use context and mobility. Other studies corroborate some of these findings. The cost of installing an automated fare payment collection system can be costly for an average service provider and hence some service providers are reluctant in adopting the mobile money payment system (Wu and Wang 2005, Dahlberg et al 2008) and the perceived risk and trust (Siau et al 2004).

Some authors also highlighted that the usability problems are responsible for the low adoption of the automated mobile money fare payment system worldwide (Mallet 2007). Based on these studies, several models have been proposed to better understand the key factors which can affect the adoption of the mobile payment system. Brakewood et al (2014) used a binary logit model to forecast the adoption of mobile payment on the entire rail network of Boston area in U.S.A, while Di Pietro et al (2015) proposed a model which was designed specifically for mobile payments in the public transport.

2.4.1 Cost Benefit analysis of Electronic fare Payment System

One research area that is used on appraising the value of electronic fare collection system for the public transportation on economic efficiency in resource allocation is the cost benefit analysis technique. The relevant evaluation techniques, such as multi-criteria analysis, goals achievement matrix and cost effectiveness appraisal and cost - benefit analysis (CBA), are capable of achieving this goal. Cost benefit analysis is a method for appraising a project from the society's point of view and taking account of costs and benefits whether or not they pass through the market (Opurum, 2005). Some previous studies looked at the CBA evaluation on the electronic fare collection system applications in recent years.

Cheung (2005) explored a Cost-Benefit Analysis for the smart card application in Holland to guide the development and implementation of the smart card technology in the nationwide. The research aimed to establish the financial viability of introducing smart cards and to determine the potential benefits and possible costs from the community point of view. Key findings from the CBA method were, in the Net Present Value (NPV) analysis, the expected total benefits derived from an integrated program in implementing the smart card system nationwide were higher than the expected total costs. This result suggested that the project was a profitable investment for the nation. Public transport operators together would enjoy the most benefits with the nationwide implementation of the smart card. Employers of passengers would also profit-because of faster and safer trips for their employees.

Implications from Cheung (2005) study are as follows, conflicts between partial benefits to dominant components (e. g, Public Transport operators and investors) and benefits to the society as a whole, benefits when viewed separately with respect to the total project cost, suggest that they were significantly lower; but the cumulative benefits when added together indicate that the project is profitable. However, in real situations, the introduction of the smart card will be feasible only if public transport companies and other stakeholders genuinely have confidence that the benefits associated with fare differentiation will be realized and that passengers will actually purchase and pay for the cost of the card. If it is not the case, for example for some companies the introduction costs weighed higher than the expected benefits, then there would be a lack of economic incentive to spur the companies (e. g. Public Transport operators) to implement the smart card system.

Opururn (2005) investigated, to a feasible extent, the influence of the new fare collection method (smart card payment, called 'Metro Card') in New York rapid transit ridership, fare revenue and service on-time performance. Meanwhile, to extent possible, this study assessed the transit patrons' reactions towards new fare collection relative to the traditional token method. Theoretical framework was based on the concept of demand elasticity and, to a large extent, on consumer choice or preference theory. Cost-benefit analysis was used in conjunction with the results of elasticity-based transit demand models and ticket choice (logit) models in determining the profitability and influence of the smart card ticketing system. Choice models (ticket choice and transit demand) used in this research aimed to determine if the New York City subway and bus ridership would improve as the result of the smart card ticketing.

The main criterion to evaluate the CBA was net present value (NPV). Two cost-benefit analyses were taken into account: commercial CBA for PT operators and social CBA for PT users, non-users and governments. Benefits for different CBAs are as follows,

- Commercial CBA: Incremental revenue (additional rides, unused, residual value); improved cash flow (admin. /labor cost saving, etc.)-, travel time savings.

- Social CBA: Consumer surplus due to discounted fare, convenience, ticket purchasing time, travel time.

Research findings indicated that the investment in the New York Transit automated fare collection system was worthwhile. Its benefits were far greater than its cost to the society. The investment appraisal results also showed that the society, at large, would be at least $2.5 million better off over the projected 30-year period life compared to a do-nothing scenario. However, the author also pointed out some un-quantified effects of the smart card payment, also called 'soft benefits' in other evaluation studies (Mulley, et al, 2004), such as convenience of card payment options to users (travel cards and smart cards). Another large benefit, which cannot be easily quantified without good origin and destination data, is the extent to which the same trip can be made more quickly, by optimal choice of route and reduced waiting time under the smart card payment. Therefore, regarding users' perceptions towards these 'soft benefits', preference study based on the existing situations and hypothetical scenarios (with some changes of payment situations) would be more appropriate to investigate consumers' psychological reactions.

2.4.2 Conclusion

In this chapter, the development, technology and advantages of electronic fare payment in the world are overviewed. Through reviewing electronic fare payment systems, we can see that as a new fare payment technique, advantages of electronic fare payment have been realized by public transport operators, transport operator users and authorities, such as boarding time saving, improved operation efficiency and multiple applications, etc. The advantages over traditional fare payment methods determine the success of smart cards for public transport throughout the world, becoming one major payment option in public transport systems.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Introduction
This chapter presents the methodology that was be used in gathering the data, analyzing the data and reporting the results. It also describes how data was be collected and analyzed.

3.2 Research Design
A descriptive research design was adopted for this study. It is a set of methods that are used to collect and analyze data to arrive to a conclusion about the research problem. Battachayya and Kuma (2006) refer to a research design as a plan, arrangements or an approach to examine the research problem to come up with a solution. Best (2003) defines a research design as a format that is conducted to deal with a research question. The emphasis was on the optimization of a linear programming model so as to find the optimal number of passengers to offer service, according to their destinations that will maximize the earnings of a public transport service provider for one route.

3.3 Research Instruments
Research instruments are tools that are used to collect data in a systematic ways. Saunders et al (2009). The data was analyzed using programming software known as R studio and the library ("Rglpk") and ("LinProg") solver was used which is used to optimize linear programming models using the revised Simplex Algorithm.

3.4 Modeling Assumptions
Assumption 1

- **Proportionality Assumption**

  A problem can be phrased as a linear program only if the contribution to the objective function and the left-hand-side of each constraint by each decision variable $(x_1 \dots \dots \dots .. x_n)$ is proportional to the value of the decision variable.

Assumption 2

- **Additivity Assumption**

A problem can be phrased as a linear programming problem only if the contribution to the objective function and the left-hand-side of each constraint by any decision variable $x_i(i = 1 \dots \dots \dots n)$ is completely independent of any other decision variable x_j *(j 6= i)* and additive.

Assumption 3

- **Divisibility Assumption**

  A problem can be phrased as a linear programming problem only if the quantities represented by each decision variable are infinitely divisible (i.e., fractional answers make sense).

Assumption 4

- Certainty Assumption: A problem can be phrased as a linear programming problem only if the coefficients in the objective function and constraints are known with certainty.

3.5 Data Collection

The study used secondary data. Secondary data is when the researcher uses the data, which has already been collected by others (Pannerselvam, 2005). The data used in this research was collected from the Eco-cash website for the transactions costs involved in paying public transport using mobile money. The fuel prices was collected from the ZERA website, and the prices for public transportation based on distance traveled was collected from the ZUPCO ticketing system. Data collection is important in assembling the required information with an aim of achieving the research objective.

3.6 Data cleaning process

There is need for the data to be cleaned before use in order to prevent garbage in garbage out. Wang and Strong (1996) defined data quality as fitness for purpose. According to the ISO-900: 2015 data quality refers to the degree to which a set of characteristics of data meet requirements. It is done to remove duplication, to correct errors and omissions and to remove unwanted data. The data was cleaned using excel techniques which embrace filtering, conditional formatting, sorting and grouping.

3.7 Data Analysis

Data analysis involves transforming and modeling of data with the purpose of discovering the relationship to support the research conclusion. Zikmund (2003) defined data analysis as a process of analysing the collected or available data, screening responses for anomalies, and organising data with the aim of revealing valuable information and coming to logical conclusions for decision making The researcher is going to use single route data and perform the revised Simplex algorithm to obtain an optimal number of passengers to offer service, according to their destinations that will maximize single route earnings.

The researcher used Google maps to find the distance to be traveled from Harare CBD to the four destination areas which are, Arcadia, Eye-court, Waterworks and Chitungwiza using Seke road. The researcher also used the Hiace diesel MUV as the transportation medium that the public transport operators uses to carry passengers and the Hiace diesel MUV has a capacity to carry 18 passengers. According to Toyota the manufacturers of the Hiace diesel MUV has a company certified mileage of 12km per litre. It has a 70 litres fuel tank.

The model was established by the simulation below:

$$Optimize\ (max\ )\ z = c_1\, x_1 + c_2 x_2 + \cdots + c_n x_n$$

s.t

$$\boldsymbol{a_{11}x_1 + a_{12}x_2 + \cdots a_1 \leq b_1}$$

$$\boldsymbol{a_{21}x_1 + a_{22}x_2 + \cdots a_2 \leq b_2}$$

$$\boldsymbol{a_{31}x_1 + a_{32}x_2 + \cdots a_3 \leq b_3}$$

$$\boldsymbol{a_{41}x_1 + a_{42}x_2 + \cdots a_4 \leq b_4}$$

.

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$$\boldsymbol{a_{m1}x_1 + a_{m2}x_2 + \cdots a_m \leq b_m}$$

Where $\mathbf{c_1, c_2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots c_n}$ Represents the unit cost of decision variables $\mathbf{x_1, x_2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots x_n}$ the value of objective function.

And $\boldsymbol{a_{11}, a_{12}, \ldots\ldots\ldots\ldots\ldots\ldots a_{2n} \ldots\ldots\ldots a_{mn}}$ represents the amount of resources consumed per unit of decision variable.

$\mathbf{b_i}$ Represent total availability of the $\boldsymbol{i^{th}}$ resource

Z represents the measure of performance which is earnings in our case

3.8 Simplex Algorithm

This method takes advantage of the fact that the optimum or optima of a Linear Programming (LP) can be found exploring its basic solutions. A basic solution of a Linear Programming (LP) in standard form of ***n*** variables and ***m*** constraints have the following properties:

• has ***n−m*** nonbasic variables equal to zero***:*** $\boldsymbol{x_N = 0}$ has m basic variables greater or equal to zero: $x_N \geq 0$

When one or more basic variables equal zero, the solution is called degenerate. The basic solutions correspond to the vertices of the feasible region.

The strategy of the simplex method consists in:

• Finding an initial basic solution

• Explore the basic solutions moving in the direction of maximum local increase **(MAX**) of the objective function

• Stop when an optimal solution is found

The software that solves LPs uses the revised simplex algorithm, a variant of the original simplex algorithm that is implemented more efficiently on computers.

3.8.1 Assumption of the Simplex Algorithm

All variables (including slack variables, but not the objective ***z***) are required to be ≥ ***0***. We have a tableau where the basic solution is feasible. This will be maintained throughout our pivoting.

Pivot Selection

1. Choose a non-basic variable with a negative coefficient in the objective row. This will be the entering variable.

a) If there are none, stop: the tableau is optimal.

b) If there are several possible entering variables, we usually choose one with the most negative coefficient (this is the most negative coefficient rule); but other choices are possible. c) In case of a tie for most negative, we will choose the one farthest left.

2. For each positive entry in the entering variable's column, calculate the ratio Right-hand side entry Coefficient of entering variable

a) Choose the smallest ratio. The basic variable for the row where this occurs is the leaving variable.

b) If there are no ratios to calculate, because all entries in the entering variable's column are negative or zero, stop: the problem is unbounded.

c) In case of a tie for smallest ratio, we will choose the one highest up in the tableau.

Pivoting

The pivot row and pivot column are the row and column labeled by the leaving and entering variables respectively. The pivot entry is the entry in the tableau in the pivot row and pivot column.

3. Divide the pivot row by the pivot entry.

4. Add the appropriate multiple of the pivot row to each other row to make the entry in the pivot column for that row 0.

5. Re-label the pivot row by the entering variable.

6. Return to step 1.

3.8.2 Ethical considerations

Saunders et al. (2009) defined research ethics as appropriateness of researcher's behaviour in relation to the rights of those who become the subject of the work or are affected by the research. Access to the relevant sources of data was considered crucial and thus ethical considerations were made during the data gathering process throughout the research.

3.9.0 Conclusion

In this chapter, the methodology which shall be used to carry out this study was laid. Its focus was on giving an insight into the methodology used in conducting the study. The research design, data collection techniques and the data analysis procedure were highlighted. The following chapter focuses on data representation, analysis and discussion. This is followed by a discussion regarding conclusions and recommendations basing on the findings.

CHAPTER 4

DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.1 Introduction

In this chapter, the researcher is concerned with the analysis of collected data as well as the presentation of the obtained results. Data was analyzed using Linear Programming Model (LP Model) as stated in the previous chapter. The calculation and presentation of results were obtained using R programming software's packages because of their strength in analysis of data. The results enable us to answers the questions as to how a Public transport operator maximize their single route earnings basing on the number of passenger's to carry according to price and their destination.

4.2 Analysis of Data and presentation of findings

Table 4.1

	Transport service offered as per destination				
	Arcadia	Eyecourt	Waterworks	Chitungwiza	Constraint
Fuel (trs)	0.37	0.52	0.60	2.225	4.375
Passengers	1	1	1	1	18
Distance(km)	4.4	6.2	6.8	26.7	52.5
Mileage	17.6	18.6	19.5	53.4	105
Fare Price	$0.50	$0.75	$0.80	$1.00	

The actual model that was programmed on R is as follows:

$$\text{maximize (Earnings)} = 0.5\Lambda + 0.75\alpha + 0.8\beta + \delta$$

Where Λ…… is the number of commuters which are traveling to Arcadia

α…….number of commuters which are traveling to Eyecourt

β………number of commuters which are traveling to Waterworks

δ………number of commuters which are traveling to Chitungwiza

Subject to constraints:

$$0.374\Lambda + 0.52\alpha + 0.6\beta + 2.225\delta \leq 4.373$$

$$\Lambda + \alpha + \beta + \delta \leq 18$$

$$4.4\Lambda + 6.2\alpha + 6.8\beta + 26.7\delta \leq 52.5$$

$$17.6\Lambda + 18.6\alpha + 19.5\beta + 53.4\delta \leq 105$$

$$\Lambda, \alpha, \beta, \delta \geq 0$$

4.2.1 Code on R using Rglpk and LinProg solving packages.

```
library("Rglpk")
#coefficients from objective function
obj<-c(0.5,0.75,0.8,1)
#matrix with constraint coefficients
mat<-rbind(c(0.37,0.52,0.6,2.225),c(1,1,1,1),c(4.4,6.2,6.8,26.7),c(17.6,18.6,19.5,53.4))
#Vector with type and direction of inequality for each constraint
dir<-c("<=","<=","<=","<=")
#right handside of constraints
rhs<-c(4.375,18,52.5,105)
#set max equal to TRUE since this is a maximisation problem
#This will print the list of solution variablesincluding optimum value,solution and dual solution among others
ans<-Rglpk_solve_LP(obj,mat,dir,rhs,bounds = NULL,types = NULL,max=TRUE,control = list())
#print optimum value
print(ans$optimum)
#print optimum solution
print(ans$solution)
```

```
library("Rglpk")
obj<-c(0.5,0.75,0.8,1)
mat<-rbind(c(0.37,0.52,0.6,2.225),c(1,1,1,1),c(4.4,6.2,6.8,26.7),c(17.6,18.6,19.5,53.4))
dir<-c("<=","<=","<=","<=")
rhs<-c(4.375,18,52.5,105)
ans<-Rglpk_solve_LP(obj,mat,dir,rhs,bounds = NULL,types = NULL,max=TRUE,control = list())
print(ans$optimum)
print(ans$solution)
print(ans$solution_dual)
print(ans$sensitivity_report)
print(ans$auxiliary)
library("linprog")
c=c(0.5,0.75,0.8,1)
b=c(4.375,18,52.5,105)
A=rbind(c(0.37,0.52,0.62,2.225),c(1,1,1,1),c(4.4,6.2,6.8,26.7),c(17.6,18.6,19.5,53.4))
res=solveLP(c,b,A,maximum = TRUE)
print(res)
```

```
library("linprog")
c=c(0.5,0.75,0.8,1)
b=c(4.375,18,52.5,105)
A=rbind(c(o.37,0.52,0.62,2.225),c(1,1,1,1),c(4.4,6.2,6.8,26.7),c(17.6,18.6,19.5,53.4))
res=solveLP(c,b,A,maximum = TRUE)
print(res)
```

The GNU Linear programming kit (GLPK) is an open source package for solving linear and mixed integer linear programming problems. It was originally written in the C computer language. (Sallan, 2015)

4.2.2 Model Validation

Model validation is important in any empirical analysis. Programming models frequently are superficially validated. However, validation is necessary for both predictive and prescriptive model use. Validation exercises almost always improve model performance and problem insight. McCarl (1984). Model validation is shown in appendix C and appendix D where the passenger constraints was not taken into consideration and the result showed that the model had an optimality.

4.2.3 Output 1

The corresponding output for the Rglpk code is as follows:

Table 4.2

```
Results of Linear Programming / Linear Optimization

Objective function (Maximum): 4.30769

Iterations in phase 1: 0
Iterations in phase 2: 1
Solution
      opt
1 0.00000
2 0.00000
3 5.38462
4 0.00000

Basic Variables
         opt
3    5.38462
S 1  1.03654
S 2 12.61538
S 3 15.88462

Constraints
     actual dir    bvec     free      dual  dual.reg
1   3.33846  <=   4.375  1.03654 0.0000000   1.03654
2   5.38462  <=  18.000 12.61538 0.0000000  12.61538
3  36.61538  <=  52.500 15.88462 0.0000000  15.88462
4 105.00000  <= 105.000  0.00000 0.0410256 105.00000

All Variables (including slack variables)
         opt cvec     min.c     max.c       marg  marg.reg
1    0.00000 0.50      -Inf 0.7220513 -0.2220513   5.96591
2    0.00000 0.75      -Inf 0.7630769 -0.0130769   5.64516
3    5.38462 0.80  0.786290       Inf         NA        NA
4    0.00000 1.00      -Inf 2.1907692 -1.1907692   1.96629
S 1  1.03654 0.00 -2.258865 0.1831897  0.0000000        NA
S 2 12.61538 0.00 -0.283333 0.6849558  0.0000000        NA
S 3 15.88462 0.00 -0.147400 0.0456989  0.0000000        NA
S 4  0.00000 0.00      -Inf 0.0410256 -0.0410256 105.00000
```

4.2.3 Output 2

The corresponding output for the LinProg code is as follows:

Table 4.3

```
> print(ans$optimum)
[1] 4.307692
> print(ans$solution)
[1] 0.000000 0.000000 5.384615 0.000000
> print(ans$solution_dual)
[1] -0.22205128 -0.01307692  0.00000000 -1.19076923
> print(ans$sensitivity_report)
[1] NA
> print(ans$auxiliary)
$primal
[1]    3.230769    5.384615   36.615385 105.000000

$dual
[1] 0.00000000 0.00000000 0.00000000 0.04102564
```

4.2.4 Interpretation of results

Based on the optimality condition, the non- basic variables $S1$, $S2$ and $S3$ are positive. Hence the tableau is optimal

The above linear programming model was solved using the LinProg and Rglpk solvers on R studio which gives an optimal solution of $\Lambda = 0, \boldsymbol{\alpha} = \mathbf{0}, \boldsymbol{\beta} = \mathbf{5.384615}, \boldsymbol{\delta} = \mathbf{0}$ and max (earnings) = \$4.30769.

Values S1=1.03654, S2=12.61538, S3=15.88462, S4=0 are slack variables and these slack variables are consistent with the given values of $\boldsymbol{\delta}$, $\boldsymbol{\beta}$, $\boldsymbol{\alpha}$ and $\boldsymbol{\Lambda}$ in constraints. The slack variable whose value is zero, the resources are said to be used up and is classified as scarce. The positive slack indicates that the resource is abundant.

Based on the data collected the optimum result derived from the model indicates that the Transport operator that uses Seke road should only carry passengers whose destination is Waterworks. The Transport operator should only carry 6 passengers as this will produce maximum earnings of \$4.30769 per every trip.

4.2.5 Discussion of findings

Transport operator that uses Seke road should only carry passengers whose destination is Waterworks in order to maximise earnings to \$4.30769. However in order to satisfy the customers and maintain customer's goodwill, the Transport operator should carry passengers with Eyecourt, Arcadia and Chitungwiza in times where the demand for customers of Waterworks are of limited supply as this would supplement earnings rather than losing potential earnings.

Despite the fact that from the solution we got, Eyecourt, Arcadia and Chitungwiza should have zero customers the Transport operator should critically examine their contribution to growth and success of the business. Evaluating their input in the overall earnings margin of the business, and make a decision based on it on whether to completely shun carrying Eyecourt, Arcadia and Chitungwiza customers or not. Fagoyinbo, Akimbo and Olaniran (2009) supports this in their research when they stated that In the same vein, critical examination should be carried out on other products on their contribution to the growth or success of the business, if their profit margin is very low, then their production can be ignored.

4.2.6 Sensitivity Analysis

The basic variables associated with final tableau are,

```
Basic Variables
         opt
3     5.38462
S 1   1.03654
S 2  12.61538
S 3  15.88462
```

β=5.384615, Λ=0, α=0, δ=0, which has an objective value of $4.30769 per trip.

4.3.0 Dual price

Dual price gives the improvement in the objective function if the constraint is relaxed by one unit. In the case of a less-than-or-equal constraint, such as a resource constraint, the dual price gives the value of having one more unit of the resource represented by that constraint. In the case of a greater-than-or-equal constraint, such as a minimum production level constraint, the dual price gives the cost of meeting the last unit of the minimum production target.

4.3.1 Fuel constraint

According to the results, a unit change in fuel will have no effect on earnings per trip and the change according to the optimality condition should be in the range of (-infinity, 0.7220513). This is due to the fact that fuel is abundant and of the total available fuel of 4.375 litres only 3.3846 litres were used and this means that 1.03654 litres of fuel were saved per trip.

4.3.2 Passenger capacity constraint

According to the results, a unit change on the number of customers may logically increase or decrease earnings but on this case this constraint have a dual price of zero which implies that it have no effect on the optimal earnings because a unit increase or decrease in passengers means a change in the load on the vehicle and in turn will have a change in fuel and hence optimality will not be reached since there is an implicit relationship between fuel and number of passengers.

4.3.3 Distance travelled constraint

According to the results, distance travelled constraint has a dual price of zero which means that a unit change in distance travelled (km) have no effect on earnings. More so instead of travelling the budgeted 52 kilometres the transport operator can travel 36.61538 km thus making 5 trips instead of a single Chitungwiza trip and hence saving 15.88462 kilometres at optimality.

4.3.4 Mileage constraint

According to the results, this constraint has a dual price of 0.0410256 which means that a unit change in this constraint will have an effect on optimum earnings. A unit change in this constraint will result in a change of $0.0410256 of earnings since there is scarcity for this resource. The change should be in the range (-infinity, 2.1907692).

3.4 Conclusion

Based on the analysis carried out in this research work and the result shown, transport operator who have adopted electronic fare payment system can use the model used by the researcher so as to maximize earnings per trip. Subsequently they should only provide services to customers travelling to Waterworks only so as to attain maximum possible earnings per trip.

CHAPTER 5

Conclusions, Summary and Recommendations

5.0 Introduction

The current chapter serves to give a summary of research, conclusion drawn from the research in conjunction with recommendations that if transport operators utilize electronic fare payment system and adopting the strategy formulated by the LP model in the previous chapter they can achieve maximum earnings. The chapter highlights key findings together with conclusions that address research objectives and finally recommendations on strategic ways how can a Public Transport operator can maximize earnings through adopting electronic fare payment system. The recommendations and suggestions are based on the findings in the previous chapter and the objectives of the study.

5.1 Summary of findings

The objective of this research work was to formulate a strategy that could maximize earnings for public transport operators through the adoption of electronic fare payment system. Transport operators who provide services to Chitungwiza using Seke road were used as our case study. The decision variables in this research work are the four different destinations. (Arcadia, Eyecourt, Waterworks and Chitungwiza). The researcher focused mainly on four constraints (Fuel constraint, mileage constraint, passenger capacity constraint, and distance travelled constraint). The result shows that the transport operator should transport 6 customers travelling to Waterworks, zero customers travelling to Arcadia, Eyecourt and Chitungwiza which will result in a maximum earnings of $4.30769 per trip.

5.2 Conclusion

Based on the analysis carried out in this research work and the result shown, transport operator who have adopted electronic fare payment system can use the model used by the researcher so as to maximize earnings per trip. Subsequently they should only provide services to customers travelling to Waterworks only so as to attain maximum possible earnings per trip.

5.3 Recommendations

Based on the findings of the study, if transport operators who are offering services to all the destinations in the model wants to achieve maximum earnings once they have adopted electronic fare payment system they should adhere to the optimality conditions which are ,

- Carry 6 Waterworks passengers only and by doing so the operator will save 1.03654 litres of fuel, will have a positive variance on the budgeted distance to be travelled of 15.88462 km.
- Use the recommended 8 passenger's vehicle which will save the cost of travelling with empty seats and the recommended vehicles are, Jinbei Granse, 2007 HiAce Van LWB (KLH22. Europe), Toyota granvia and Toyota HiAce minibus.
- In the event of the operators desire to improve the established maximum earnings within the limits of the model, only the funds allocated to the scarce resources should be increased by decreasing those of the abundant resources within the range.
- Adopt a cashless model for expenses payments (i.e. labor costs, miscellaneous expenses, and any other cost incurred).
- Transport operator should critically examine the contribution made by Eyecourt, Arcadia and Chitungwiza customers to growth and success of the business. Evaluating their input in the overall earnings margin of the business, and make a decision based on it on whether to completely shun carrying Eyecourt, Arcadia and Chitungwiza customers

More so, Transport operators such as Zimbabwe United Passengers Association (ZUPCO) which are more established can use the cashless model, Smart cards technology can be used as transit passes and integrated ticketing which can be utilized.

5.4 Suggestions for further research

From the research findings, it would be helpful to replicate the study in another setting particularly taking more constraints than the 4 used in this study. More so, the constraints can be modelled using stochastic modelling rather than the predictive modelling. Subsequently other factors such as speed should be considered. Use stochastic programming instead of linear programming techniques.

5.5 Chapter Conclusion

Based on the analysis carried out in this research work and the result shown, transport operator who have adopted electronic fare payment system can use the model used by the researcher so as to maximize earnings per trip. Subsequently they should only provide services to customers travelling to Waterworks only so as to attain maximum possible earnings per trip.

References

Blythe, P. T. (2004). *Improving public transport ticketing through smart cards.Proceedings of the Institution of Civil Engineers - Municipal Engineer, 157(1),47-54. https://doi.org/10.1680/muen.2004.157.1.47.*

Campos Ferreira M., D. G. (2015). *How to Encourage the Use of Public Transport? A Multiservice Approach Based on Mobile Technologies in Nóvoa, M. H., Dragoicea, M. Exploring Services Science - IESS 2015 Proceedings, 4-6/02, Porto, 314-325.*

Chambers, B. (1998). *The Octopus Hong Kong's Contactless Smartcard Project. 2 98. Public Transport International, pp. 14-19.*

Cheung, F. (2003). *Tripprtpas Smart Card Project: Lesson From the Netherlands Transportation Research Record: Journal of the Transportation Research Board. No. 1887 TRB, National Research Council, Washington, D. C., 2004, pp. 147-152. .*

Cheung, F. (2005). *Cost-Benefit Analysis of a Nationwide Smartcard for Public Transport in The Netherlands, the 85th TRB 2006 Annual Meeting, January 2006, Washington DC, USA. .*

Dahlberg, T. H. (2008). *Lost opportunity: Why has dominant design failed to emerge for the mobile payment services market in Finland? In: Proceedings of the 41th Annual Hawaii International Conference on System Sciences. IEEE Computer Society Press, Washington, DC, USA.*

Dinning, M. &. (2017). *Multimodal Transportation Payments Convergence-Key to Mobility. In G. Meyer & S. Shaheen (Eds.), Disrupting Mobility (pp. 121-133). Springer International Publishing. .*

Falcão e Cunha, J. (2012). *A survey of current trends in smartphone based payment and validation services for public transport users. Art & Science of Service Conference, Maastricht, The Netherlands, 27-29/06.*

Garret, H. a. (1964). *Statistics in Psychology and Education.* New York.

Gass, S. I. (1983). *Decision-Aiding Models: Validation, Assessment, and Related Issues for Policy Analysis." Operations Research,.*

House, P. a. (1980). *Validation: A Modern Day Snipe Hunt: Conceptual Difficulties of Validating Models," in S.I. Gass (ed.), Validation and Assessment of Issues in Energy Models, Proceedings of a Workshop, National Bureau of Standards, Spec. Pub. 564,.* Washington, D.C.;.

Huntington, S. P. (1965). *Political Development and Political Decay. World Politics, 17 (3), 386-430.*

Koji, T. a. (2002). *"Image Reconstruction by Linear Programming, Max Planck Institute for Biological Cybernetics, Spemannstr, 38 72076, Germany.*

Leal, J. C. (2015). *Exploring ticketing approaches using mobile technologies: QR Codes, NFC and BLE. 2015 IEEE 18th International Conference on Intelligent Transportation Systems, 7-12.*

Mallat, N. R. (2008). *An empirical investigation of mobile ticketing service adoption in public transportation. Personal and Ubiquitous Computing, 12(1), 57–65. .*

Mallat, N. R. (2009). *The impact of use context on mobile services acceptance: The case of mobile ticketing Information & Management, 46(3), 190–195. .*

Manyika. (2016).

Mulley, C. E. (2004). *Assessment Framework for the Evaluation of Soft Benefits of Smart Card Schemes. National Smart Card Project, Transport Operation Research Group, University of Newcastle upon Tyne.*

Nugent, J. (1970). *Linear Programming Models for National Planning: Demonstration of a Testing Procedure." Econometrica,.*

Paris, Q. (1981). *Multiple Optimal Solutions in Linear Programming Models." Amer. J. Agr. Econ.,.*

Rustow, D. A. (1967). *A World of Nations; Problems of Political Modernization. Washington, D.C.: The Brookings Institution.*

Sahi, R. a. (n.d.). *"Estimation of Flexibility Coefficients for Recursive Programming Models-Alternative Approaches." Amer. J. Agr. Econ.*, .

Sallan, L. a. (2015). *Modeling and Solving Linear Programs with R.*

Saunders, M. L. (2009). *Research Methods for Business Students, 5th Edition.* Harlow: FT Prentice Hall.

Siau, K. S. (2004). *A A qualitative investigation on consumer trust is mobile commerce. Int J Elect Business, 2(3), 283-300.*

Wagner, H. (2007). *Principles of Operations Research with Application to Managerial Decision,* . PrinceHall of India Private Limited, New Delhi.

Zikmund, W. (2003). *Business Research Methods.*

APPENDIX

APPENDIX A

\

```
> install.packages("Rglpk")
Installing package into 'C:/Users/Solo/Documents/R/win-library/4.0'
(as 'lib' is unspecified)
also installing the dependency 'slam'

trying URL 'https://cran.rstudio.com/bin/windows/contrib/4.0/slam_0.1-47.zip'
Content type 'application/zip' length 210013 bytes (205 KB)
downloaded 205 KB

trying URL 'https://cran.rstudio.com/bin/windows/contrib/4.0/Rglpk_0.6-4.zip'
Content type 'application/zip' length 976451 bytes (953 KB)
downloaded 953 KB

package 'slam' successfully unpacked and MD5 sums checked
package 'Rglpk' successfully unpacked and MD5 sums checked

The downloaded binary packages are in
        C:\Users\Solo\AppData\Local\Temp\RtmpU1Cdfc\downloaded_packages
> install.packages("linprog")
Installing package into 'C:/Users/Solo/Documents/R/win-library/4.0'
(as 'lib' is unspecified)
also installing the dependency 'lpSolve'

trying URL 'https://cran.rstudio.com/bin/windows/contrib/4.0/lpSolve_5.6.15.zip'
Content type 'application/zip' length 698184 bytes (681 KB)
downloaded 681 KB

trying URL 'https://cran.rstudio.com/bin/windows/contrib/4.0/linprog_0.9-2.zip'
Content type 'application/zip' length 65160 bytes (63 KB)
downloaded 63 KB

package 'lpSolve' successfully unpacked and MD5 sums checked
package 'linprog' successfully unpacked and MD5 sums checked
```

APPENDIX B

Description

High level R interface to the GNU Linear Programming Kit (GLPK) for solving linear as well as mixed integer linear programming (MILP) problems.

Usage

```
Rglpk_solve_LP(obj, mat, dir, rhs, bounds = NULL, types = NULL, max = FALSE,
               control = list(), ...)
```

Arguments

`obj`	a numeric vector representing the objective coefficients.
`mat`	a numeric vector or a (sparse) matrix of constraint coefficients. If the optimization problem is unconstrained then a matrix of dimension 0 times the number of objective variables is required.
`dir`	a character vector with the directions of the constraints. For a nonzero number of constraints each element must be one of `"<"`, `"<="`, `">"`, `">="`, or `"=="`. Note, however, that the GLPK API only allows for non-strict inequalities. Strict inequalities are handled the same way as non-strict inequalities.
`rhs`	a numeric vector representing the right hand side of the constraints.
`bounds`	`NULL` (default) or a list with elements `upper` and `lower` containing the indices and corresponding bounds of the objective variables. The default for each variable is a bound between 0 and `Inf`.
`types`	a character vector indicating the types of the objective variables. `types` can be either `"B"` for binary, `"C"` for continuous or `"I"` for integer. By default `NULL`, taken as all-continuous. Recycled as needed.
`max`	a logical giving the direction of the optimization. `TRUE` means that the objective is to maximize the objective function, `FALSE` (default) means to minimize it.
`control`	a list of parameters to the solver. See *Details*.
`...`	a list of control parameters (overruling those specified in `control`).

Value

A list containing the optimal solution, with the following components.

`solution`	the vector of optimal coefficients
`objval`	the value of the objective function at the optimum
`status`	an integer with status information about the solution returned. If the control parameter `canonicalize_status` is set (the default) then it will return 0 for the optimal solution being found, and non-zero otherwise. If the control parameter is set to `FALSE` it will return the GLPK status codes.
`solution_dual`	variable reduced cost, if available (`NA` otherwise).
`auxiliary`	a list with two vectors each containing the values of the auxiliary variable associated with the respective constraint at solution, primal and dual (if available, `NA` otherwise).

APPENDIX C

```
library("linprog")
c=c(0.5,0.75,0.8,1)
b=c(4.375,18,52.5,105)
A=rbind(c(0.37,052,0.62,2.225),c(4.4,6.2,6.8,26.7),c(17.6,18.6,19.5,53.4))
res=solveLP(c,b,A,maximum = TRUE)
print(res)
```

APPENDIX D

```
Results of Linear Programming / Linear Optimization

Objective function (Maximum): 5.91216

Iterations in phase 1: 0
Iterations in phase 2: 1
Solution
      opt
1 11.8243
2  0.0000
3  0.0000
4  0.0000

Basic Variables
          opt
1   11.824324
S 2  6.175676
S 3  0.472973

Constraints
   actual dir   bvec     free    dual dual.reg
1  4.3750  <=  4.375 0.000000 1.35135 4.375000
2 11.8243  <= 18.000 6.175676 0.00000 6.175676
3 52.0270  <= 52.500 0.472973 0.00000 0.472973

All Variables (including slack variables)
          opt cvec     min.c      max.c        marg  marg.reg
1   11.824324 0.50  0.477419        Inf          NA        NA
2    0.000000 0.75      -Inf 70.2702703 -69.5202703 0.0841346
3    0.000000 0.80      -Inf  0.8378378  -0.0378378 7.0564516
4    0.000000 1.00      -Inf  3.0067568  -2.0067568 1.9662921
S 1  0.000000 0.00      -Inf  1.3513514  -1.3513514 4.3750000
S 2  6.175676 0.00        NA  0.0560000   0.0000000        NA
S 3  0.472973 0.00 -8.342697  0.0660377   0.0000000        NA
```