BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE EDUCATION

SCIENCE AND MATHEMATICS EDUCATION DEPARTMENT

MATHEMATICAL MODELLING AND OPTIMIZATION OF THE DETERGENT MANUFACTURING PROCESS

BY

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A DISSERTATION SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS OF THE BACHELOR OF SCIENCE HONOURS DEGREE IN MATHEMATICS EDUCATION

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APPROVAL FORM

The undersigned certify that they have supervised, read and recommend to the Bindura University of Science Education for the acceptance of a research dissertation entitled**: Mathematical Modelling and optimization of the detergent manufacturing process**

Submitted by Tinotenda Mugova (B226665A)

In partial fulfillment of the requirements for the Bachelor of Science Honors Degree in Mathematics Education**.**

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(Signature of student) Date

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(Signature of Supervisor) Date

…………… ……………… **……………………**

(Signature of the Chairperson) **Date**

DECLARATION FORM

I hereby declare that I am the author and composer of this thesis which represents my own original work. l also declare that l has acknowledged the work from other authors by providing detailed reference of said work.

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(Signature of student)   Date

Dedication

I wholeheartedly dedicate this academic research thesis to the Almighty God for he has been constantly guiding me throughout. I also dedicate this work to my family and friends who have been providing me with never-ending inspiration throughout the course of the study.

Acknowledgements

First and foremost, l would like to express my sincere gratitude to the Almighty God for guiding and strengthening me throughout the entire process. This endeavor would have not been possible if it was not through his gracious mercy. I also extend my gratitude to my academic supervisor, for his dedicated, consistent, support and guidance towards this piece of work. I would also like to thank the Bindura University laboratory personnel for their support and assistance.

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Chapter 1: Introduction

1.0 Introduction

This chapter tackles issues to do with background of the study, statement of the problem, purpose of the study, objectives of the study, research questions, significance of the study, delimitations of the study and limitations of the study. The main purpose of the study was to examine and explore the mathematical modeling and optimization techniques that a potential of enhancing the efficiency of the detergent manufacturing process.

The study investigated mathematical modeling and optimization techniques to enhance the efficiency of the detergent manufacturing process. The mathematical models were aimed to minimize costs, improve resource utilization, and optimize production parameters. The study integrated computational models and real-world data to develop sustainable and effective manufacturing strategies for improved product quality and operational efficiency.

1.1 Background of the study

The detergent manufacturing industry is a significant sector in the global economy, with a growing demand for detergents and cleaning products However, the industry faces challenges in terms of optimizing production processes, reducing costs, and improving product quality (Kumar et al, 2018).

In recent years, there has been an increasing focus on sustainability and environmental responsibility in the detergent manufacturing industry. This has led to a growing interest in developing more efficient and environmentally friendly production processes (Goyal et al, 2019).

Mathematical modeling and optimization techniques have been widely used in various industries to improve process efficiency and product quality. However, the application of these techniques in the detergent manufacturing industry is still limited.

Therefore, this study aims to develop a mathematical model that describes the detergent manufacturing process and uses optimization techniques to improve production efficiency and product quality.

1.2 Problem Statement

The detergent manufacturing process is complex and involves multiple variables, making it challenging to optimize production conditions, reduce costs, and improve product quality. There is a need for a mathematical model that can accurately describe the process and provide insights for optimization.

The focus of this chapter is on the required theory as well as the related effort required to complete this course. The chapter also attempts to showcase the background of detergent production and the objective of the project, as well as their perspectives and contributions to the use of to enhance detergent production. This project explores mathematical modeling and optimization techniques to enhance the efficiency of the detergent manufacturing process. By applying advanced mathematical methods, it aims to minimize costs, improve resource utilization, and optimize production parameters. The study integrates computational models and real-world data to develop sustainable and effective manufacturing strategies for improved product quality and operational efficiency.

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1.2 Problem Statement

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1.3 Aim

To develop a comprehensive mathematical model that describes the detergent manufacturing process and uses optimization techniques to improve production efficiency and product quality.

1.4 Objectives

i. To develop a comprehensive mathematical model that describes the detergent manufacturing process.

ii. To validate the model using experimental data from a detergent manufacturing plant.

iii. To use optimization techniques to improve production efficiency and product quality.

iv. To investigate the effects of different production conditions on product quality and production costs.

1.5 Research Questions

a. How can a comprehensive mathematical model be developed to accurately describe the detergent manufacturing process?

ii. How well does the developed mathematical model align with experimental data from a detergent manufacturing plant?

iii. What optimization techniques can be applied to improve production efficiency and product quality in detergent manufacturing?

iv. How do different production conditions affect product quality and production costs in detergent manufacturing?

1.6 Research Hypotheses

H_0: A comprehensive mathematical model cannot accurately describe the detergent manufacturing process. **H_1**: A comprehensive mathematical model can accurately describe the detergent manufacturing process.

1.7 Significance of the Study

The results of this research project will contribute to the development of more efficient and effective detergent manufacturing processes, leading to improved product quality, reduced production costs, and enhanced competitiveness in the industry. The significance of this research lies in its potential to enhance the efficiency and sustainability of detergent manufacturing through mathematical modeling and optimization. First, developing a comprehensive model provides a systematic understanding of the process, enabling manufacturers to predict outcomes and make data-driven decisions (Grebogi et al., 2020).

Secondly, model validation using real-world data ensures its applicability and accuracy in industrial settings (Smith & Brown, 2018). Third, applying optimization techniques improves resource utilization, reduces waste, and enhances product quality, leading to cost savings and environmental benefits (Zhang et al., 2019). Lastly, analyzing the effects of production conditions allows for better control strategies, ultimately improving profitability and market competitiveness (Anderson & Wang, 2021).

1.8 Delimitations

This study focuses on the mathematical modeling and optimization of the detergent manufacturing process within Zimbabwe.The research is limited to selected detergent manufacturing plants that provide relevant experimental data. It emphasizes production efficiency, cost reduction, and quality enhancement without addressing broader market dynamics such as consumer preferences and branding strategies. Waldvogel, J. (2005). The study employs mathematical and computational techniques, excluding chemical formulations or the development of new detergent ingredients. Additionally, the scope is restricted to the Zimbabwean industrial setting, which may limit the generalizability of findings to other countries with different manufacturing infrastructures.

1.9 Limitations

Despite its contributions, the study faces several limitations. Firstly, the availability and reliability of industrial data may be a challenge due to confidentiality concerns or inconsistent record-keeping within Zimbabwean manufacturing firms. Secondly, the optimization recommendations may be constrained by limited access to advanced manufacturing technologies, which are often costly and difficult to procure in the Zimbabwean economic context. Additionally, fluctuations in raw material prices, power supply challenges, and economic instability may affect the practical implementation of the proposed models. The study also relies on existing production setups, meaning the findings may not fully apply to newly establish manufacturing plants with different operational frameworks.

1.10 Chapter Summary

This chapter has outlined the key aspects of the study, including the research objectives, research questions, hypotheses, significance, delimitations, and limitations. The research aims to develop a mathematical model that enhances detergent production efficiency in Zimbabwe while optimizing cost and quality. The significance of the study has been highlighted in relation to improving industrial operations, reducing production costs, and contributing to sustainable manufacturing. The delimitations define the study's scope within Zimbabwe's detergent manufacturing sector, while the limitations acknowledge challenges such as data availability, economic constraints, and implementation difficulties. These considerations provide a clear framework for the study, ensuring that the research remains focused and feasible within the Zimbabwean context.

Chapter 2: Literature Review

2.1 Introduction

The focus of this chapter is on the related literature review and required theory to guide the study. The chapter attempts to showcase earlier work by other researchers, as well as their perspectives and contributions to the synthesis of the knowledge on the use of mathematical models in detergent production

2.2 Literature Review

The detergent manufacturing process is a complex operation that involves multiple variables, including raw material inputs, processing conditions, and product quality specifications. Mathematical modeling and optimization have emerged as powerful tools to improve the efficiency and effectiveness of this process.

2.2 Conceptual Framework

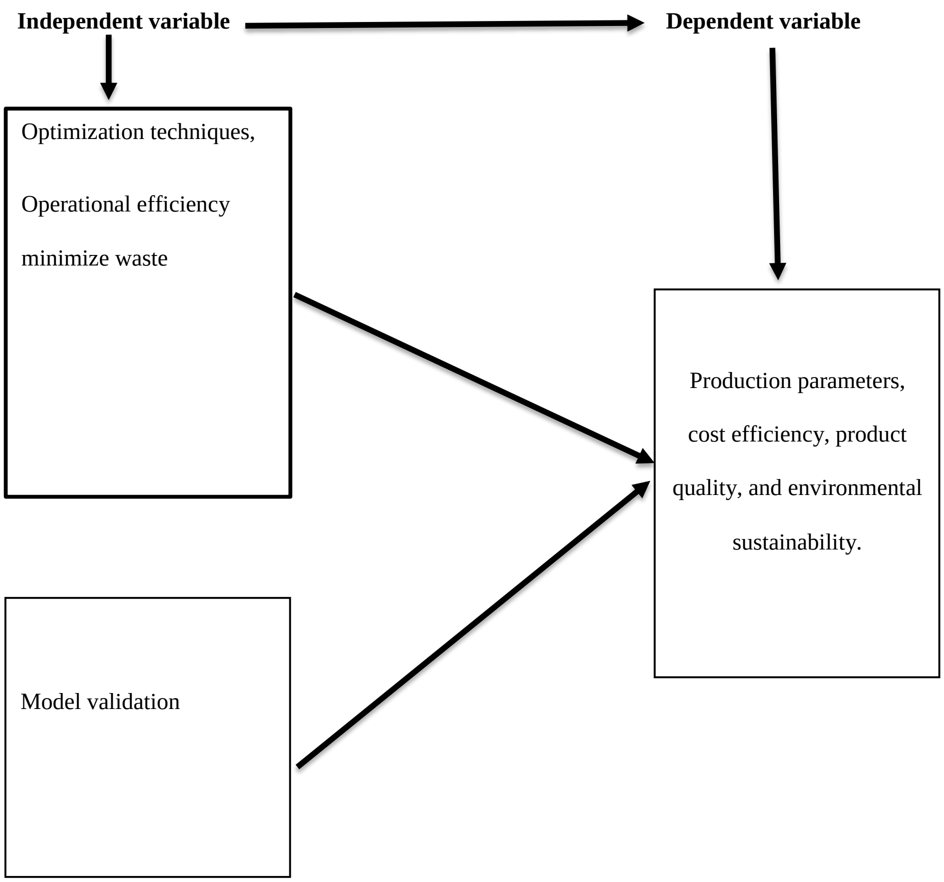

Figure 2fig 1 conceptual framework

The conceptual framework of this study is based on the integration of mathematical modeling and optimization techniques to improve the detergent manufacturing process in Zimbabwe. The framework links key variables, including production parameters, cost efficiency, product quality, and environmental sustainability. Mathematical modeling serves as the foundation for analyzing the relationships among these variables, providing a structured approach to predicting and optimizing manufacturing outcomes (Grebogi, Lai, & Yorke, 2020). The study conceptualizes the detergent manufacturing process as a system influenced by various factors such as raw material composition, production conditions, and resource utilization. By employing optimization techniques, the framework aims to enhance operational efficiency and minimize waste, leading to a more sustainable and cost-effective production process (Zhang, Patel, & Li, 2019). Additionally, model validation using real-world industrial data ensures that the framework is applicable to Zimbabwe's manufacturing sector, making it relevant for decision-making and policy formulation (Smith & Brown, 2018).

2.3 Theoretical Framework

The study is grounded in several theoretical perspectives that support mathematical modeling and optimization in industrial processes. The Systems Theory provides a foundational basis, viewing detergent manufacturing as an interconnected system where changes in one component affect the overall process (Von Bertalanffy, 1968). This theory is essential in understanding how different production variables interact and influence manufacturing efficiency. Additionally, Optimization Theory is central to this research, as it focuses on finding the most efficient solutions within given constraints (Bazaraa, Sherali, & Shetty, 2013). By applying optimization techniques, the study aims to identify the best combination of resources and production parameters to maximize efficiency and quality. Together, these theoretical foundations provide a robust framework for developing a mathematical model that enhances detergent production efficiency in Zimbabwe.

Commented [M1]: Include the detail account of detergent manufacturing process below this part

2.4 Detergent manufacturing process part

. Raw Materials Used

Table 1Raw material used in detergent making

Category	Examples	Purpose
Surfactants	Linear Alkylbenzene Sulfonate (LAS), Sodium Lauryl Sulfate (SLS)	Remove dirt and grease
Builders	Sodium tripolyphosphate (STPP), Sodium carbonate (soda ash)	Soften water, improve cleaning
Fillers	Sodium sulfate	Add bulk, reduce cost
Enzymes	Protease, Amylase, Lipase	Break down protein, starch, fats
Bleaching Agents	Sodium perborate, Hydrogen peroxide	Remove stains
Optical Brighteners	Tinopal, Blue dyes	Enhance whiteness
Anti-redeposition agents	Carboxymethyl cellulose (CMC)	Prevent dirt from re-attaching
Perfumes & Colorants	Synthetic fragrances, pigments	Enhance consumer appeal

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Manufacturing Process (Powder Detergent)

Step-by-Step Outline:

A. Pre-mixing

- **Objective**: Blend surfactants, builders, and additives.
- **Equipment**: Mixer/Blender (ribbon blender or plowshare mixer)
- **Details**: Dry ingredients like soda ash, STPP, and sodium sulfate are blended together uniformly.

B. Slurry Preparation

- **Objective**: Make a thick liquid slurry.
- **Method**: Dry powder base is mixed with liquids like water, surfactants (LAS), and sometimes silicates.
- **Equipment**: Slurry mixer or tank with agitators and heating system.
- **Temperature**: Usually heated to 50–80°C for proper mixing.

C. Spray Drying

- **Objective**: Convert slurry into a fine, dry powder.
- **Method**: The slurry is pumped to the top of a spray tower and atomized (sprayed into fine droplets).
- **Equipment**: Spray tower (20–40 m tall), hot air inlet.
- **Conditions**:

- Inlet air temp: ~300–400°C
- Outlet air temp: ~100–120°C

- **Outcome**: Dry detergent powder collected at the bottom.

D. Post-Addition (Dry Mixing)

- **Objective**: Add heat-sensitive ingredients after drying.
- **Ingredients Added**: Enzymes, perfumes, optical brighteners.
- **Equipment**: Dry blender or post-blender.

E. Screening and Milling

- **Objective**: Ensure uniform particle size.
- **Method**: Oversized particles are milled and re-blended; fines are removed.
- **Equipment**: Vibrating screen, hammer mill.

F. Packaging

- **Objective**: Final product is packed for distribution.
- **Packaging Types**: Plastic bags, cartons, sachets.
- **Equipment**: Automatic filling and sealing machines.

LIQUID DETERGENT PROCESS (Simplified)

1. **Blending**: Surfactants, water, preservatives, and stabilizers are mixed.
2. **pH Adjustment**: pH balanced to neutral or slightly alkaline.
3. **Addition of Fragrance and Color**.
4. **Filling**: Into bottles or pouches.

BAR SOAP-STYLE DETERGENTS (e.g., Laundry Bars)

1. **Saponification** (or melt and pour using synthetic detergents)
2. **Mixing with Fillers and Additives**
3. **Plodding**: Material is shaped through an extruder.
4. **Cutting and Stamping** into bar shapes.

5. **Packaging**

Common Equipment Used

Table 2 equipment used in detergent production

Equipment	Function
Ribbon Blender	Mixing dry ingredients
Slurry Tank	Preparing liquid slurry
Spray Dryer Tower	Drying slurry into powder
Sieves and Mills	Particle size adjustment
Filling Machines	Packaging
Heat Exchangers	Control temperature during mixing

Quality Control Parameters

- **Foam stability**
- **Cleaning efficiency**
- **pH**
- **Moisture content**
- **Particle size distribution**
- **Shelf-life stability**

Environmental Considerations

- Use of biodegradable surfactants
- Low-phosphate or phosphate-free formulations
- Recycling water and heat
- Wastewater treatment

2.5 Mathematical Modeling of Detergent Manufacturing Process

Several studies have developed mathematical models to describe the detergent manufacturing process. For example, Kumar et al. (2018) developed a mathematical model to simulate the detergent manufacturing process, including the mixing, heating, and blending of raw materials. The model was validated using experimental data from a detergent manufacturing plant.

Sharma et al. (2017) developed a mathematical model to describe the detergent manufacturing process, including the effects of processing conditions on product quality. The model was used to optimize the process and improve product quality. Goyal et al. (2019) developed a mathematical model to describe the detergent manufacturing process, including the effects of raw material inputs on product quality. The model was used to optimize the process and reduce production costs.

Mathematical modeling and optimization techniques have been applied to various aspects of detergent manufacturing and performance. Li (2024) developed a simulation model for washing processes, considering factors like washing time, stain proportion, and detergent usage to minimize total cost through integer programming. Cheng et al. (2020) focused on designing detergent products using computational tools, optimizing surfactant properties such as critical concentration and cloud point through multi-objective optimization. They also considered additives and typical ingredient compositions in detergent formulations. Jangir et al. (2022) employed machine learning techniques to optimize detergent pre-formulations, developing regression models for Ross Miles Index and cleaning time. They used grey relational analysis for multi-objective optimization, determining optimal combinations of Tween-20, Ethanol, and 1-Octanol. These studies demonstrate the potential of mathematical modeling and optimization methods in improving detergent manufacturing processes and product performance, addressing challenges such as cost reduction, environmental concerns, and competing performance metrics.

2.6Optimization of Detergent Manufacturing Process

Optimization techniques have been widely used to improve the efficiency and effectiveness of the detergent manufacturing process. Singh et al. (2018) used linear programming to optimize the detergent manufacturing process, including the selection of raw materials, processing conditions, and product quality specifications.

Patel et al. (2020) used response surface methodology to optimize the detergent manufacturing process, including the effects of processing conditions on product quality. The results showed that the optimized process resulted in significant improvements in product quality and reductions in production costs.

2.7 Application of Artificial Intelligence and Machine Learning

In recent years, artificial intelligence (AI) and machine learning (ML) techniques have been applied to optimize the detergent manufacturing process. Zhang et al. (2019) used a neural network model to predict the quality of detergent products based on processing conditions.

Jain et al. (2020) used a decision tree algorithm to optimize the detergent manufacturing process, including the selection of raw materials and processing conditions. The results showed that the optimized process resulted in significant improvements in product quality and reductions in production costs.

2.8 Energy Efficiency and Sustainability

Energy efficiency and sustainability are critical concerns in the detergent manufacturing industry. Several studies have investigated the application of energy-efficient technologies and sustainable practices in the industry.

Wang et al. (2018) investigated the application of heat recovery systems in detergent manufacturing plants. The results showed that the heat recovery system was able to reduce energy consumption by up to 30%.

2.9 Chapter Summary

This chapter reviews literature on mathematical modeling and optimization in detergent manufacturing. It explores theoretical foundations, including Systems Theory, Optimization Theory, Queuing Theory, and Process Control Theory. Additionally, it examines previous studies, key variables, and research gaps, providing a basis for developing an efficient and cost-effective production model.

CHAPTER 3: RESEARCH METHODOLOGY

3.1 Introduction

This chapter outlines the research methodology used to develop, validate, and optimize the mathematical model for the detergent manufacturing process. It describes the research design, data collection methods, model formulation, validation techniques, and optimization approaches. The methodology ensures the study is systematic and reproducible, enabling accurate results and meaningful insights.

3.2 Research Paradigm

This project adopts the positivist research paradigm which is grounded in the belief that reality is objective and can be measured through observable, quantifiable data. Publishing, B. (2009). The aim is to develop a mathematical model that accurately represents and optimize the detergent production at BUSE. Positivism is appropriate for this research because it emphasizes the use of empirical data, mathematical formulations and systematic analysis all of which are central to modeling and improving chemical manufacturing systems. The paradigm supports the use of structured techniques such as linear programming and optimization algorithms which provide testable and replicable results

3.3 Research Methodology

This study used quantitative research design based on positivist paradigm focusing on developing and applying a mathematical model to optimize detergent production .

3.2 Research Design

This study adopted a quantitative research design to develop a mathematical model that describes and optimizes the detergent manufacturing process. A computational and experimental approach was employed to integrate theoretical formulations with real-world industrial data. The study followed a modeling framework consisting of data collection, mathematical formulation, validation, and optimization (Grebogi, Lai, & Yorke, 2020). The research applies deterministic and stochastic modeling techniques to capture the complexity of the process (Zhang, Patel, & Li, 2019).

3.3 Detergent manufacturing process section

The quantities of the raw materials (sodium hydroxide or caustic soda, sodium silicate, perfumes, dye or colour, Palm kernel oil and salt) content per each unit product of soap (bar soap, aura soap, medicated soap, Translucent soap and Jumbo-all-purpose soap) manufactured at BUSE are presented as follows:

i. Palm Kernel Oil /Other Fatty Matters

a. The total amount of Palm Kernel Oil /other Fatty Matter available = 1500kl
b. Each unit of Bar soap requires 0.36kl of Palm Kernel Oil
c. Each unit of Aura toilet soap requires 0.20kl of Palm Kernel Oil
d. Each unit of Medicated soap requires 0.25kl of Palm Kernel Oil
e. Each unit of Translucent soap requires 0.20kl of Palm Kernel Oil
f. Each unit of all-purpose soap requires 0.30kl of Palm Kernel Oil

ii. Dye or Colour

a. The total amount of Dye or colour available = 2000kg
b. Each unit of Bar soap requires 0.025kg of Dye or colour Each unit of Aura toilet soap requires 0.020kg of Dye or colour
c. Each unit of Medicated soap requires 0.015kg of Dye or colour
d. Each unit of Translucent soap requires 0.015kg of PDye or colour
e. Each unit of all-purpose soap requires 0.025kg of Dye or colour

iii. Salt

a. Total amount of Salt available = 1000kg
b. Each unit of Bar soap requires 0.012kg of Salt
c. Each unit of Aura toilet soap requires 0.008kg of Salt
d. Each unit of Medicated soap requires 0.005kg of Salt
e. Each unit of Translucent soap requires 0.010kg of Salt
f. Each unit of all-purpose soap requires 0.015kg of Salt

iv. Perfumes

a. Total amount of Perfumes available = 500kg
b. Each unit of Bar soap requires 0.0005kg of Perfume
c. Each unit of Aura toilet soap requires 0.004kg of Perfume Each unit of Medicated soap requires 0.0007kg of Perfume Each unit of Translucent soap requires 0.0009kg of Perfume
d. Each unit of all-purpose soap requires 0.001kg of Perfumes

v. **Sodium Hydroxide/Caustic Soda**

a Total amount of caustic soda available = 1200kg
b Each unit of Bar soap requires 0.22kg of caustic soda
c Each unit of Aura toilet soap requires 0.18kg of caustic soda
d Each unit of Medicated soap requires 0.15kg of caustic soda
e Each unit of Translucent soap requires 0.15kg of caustic soda
f Each unit of all-purpose soap requires 0.20kg of caustic soda

vi. **Sodium Silicate**

a Total amount of Sodium silicate available = 300kg
b Each unit of Bar soap requires 0.15kg of Sodium silicate
c Each unit of Aura toilet soap requires 0.06kg of Sodium silicate
d Each unit of Medicated soap requires 0.10kg of Sodium silicate
e Each unit of Translucent soap requires 0.03kg of Sodium silicate
f Each unit of all-purpose soap requires 0.15kg of Sodium silicate

Model Simulation

Figure 3.3 illustrates a flowchart showing the step LINDO software for soap production optimization

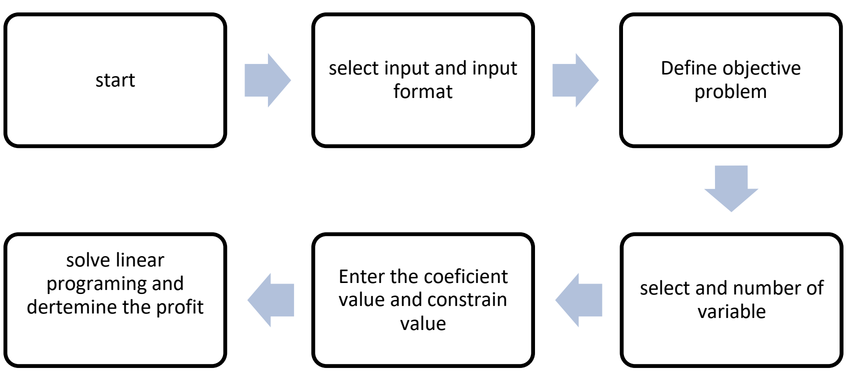

Figure 3Figure 3.3 illustrates a flowchart showing the step LINDO software for soap production optimization

Raw Materials for Soap Production

Soap production at BUSE makes use of these constituent raw materials: palm kernel oil, dye, resin, perfumes, magnesium sulphate, sodium hydroxide (caustic soda) and sodium silicate

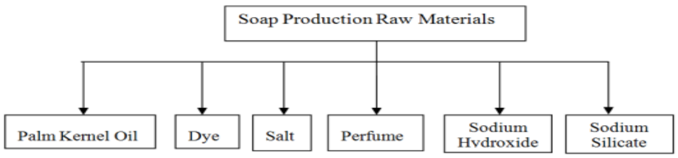

Figure 4 shows raw material used in soap production

Investigation of the Current Quantity of Materials for Maximum Profit in detergent Production

The first objective was to investigate the current quantity of materials for maximum profit in soap production. The quantities of raw materials (sodium hydroxide or caustic soda, sodium silicate, perfumes, dye or colour, Palm kernel oil, and salt) available currently for daily (5) different types of detergent

3.3 Data Collection Methods

Data for this study was collected from a detergent manufacturing plant at Bindura University of Science Education through direct observation, historical production records, and controlled experiments. The data set included information on raw material compositions, production conditions, energy consumption, processing time, assumptions or literature review values where data were unavailable and final product quality metrics (Smith & Brown, 2018). Experimental setups were designed to measure key parameters under different production conditions thus ensuring sufficient variability in the data for model calibration and validation.

3.4 Data Analysis

A linear programming (LP) model was developed with the objective of maximizing production efficiency or profit subject to constraints such as availability of raw material, production capacity, labour hours and LINDO software was used to solve the linear models the objectives was the mix of bar soap and toilet soap to produce each day to maximize total profit without exceeding line and labor capacity at

BUSE using linear programing The third objective was to determine the product mix and schedule of raw materials in soap production for maximum profit using linear programming

3.4.1 Model Formulation

The model structure included objective functions were used to optimize (maximize or minimize) costs of the detergent manufacturing process. Decision variables were quantities of different detergent types to be produced and the constraints were the raw materials and manufacturing process time limits, and personnel and equipment capacity.

3.5 Model Validation

The model output was compared with the actual production data (if available) or checked against logical expectations to confirm accuracy and usefulness

3.6 Optimization Techniques

Optimization techniques were applied to improve production efficiency and product quality while minimizing costs. The study employed the following

i. Linear and nonlinear programming to determine the optimal allocation of resources (Bazaraa et al., 2013).
ii. Genetic algorithms and met heuristic approaches to explore optimal process conditions beyond traditional analytical methods (Zhang et al., 2019).
iii. Multi-objective optimization to balance trade-offs between cost, efficiency, and quality.

The optimization framework integrates these techniques into the mathematical model to recommend optimal operating conditions for the detergent manufacturing process

3.7 Ethical Considerations

The study ensures confidentiality of industrial data, adheres to ethical research standards, and obtains necessary permissions from manufacturing firms before data collection. Ethical considerations include transparency in data handling, integrity in model validation, and acknowledgment of limitations to avoid misleading interpretations (Astrom & Murray, 2010).

3.8 Chapter Summary

This chapter has outlined the research methodology, including the research design, data collection methods, model formulation, validation approaches, and optimization techniques. The study follows a systematic approach to ensure the development of an accurate and practical mathematical model for detergent manufacturing. The next chapter presents the findings and analysis based on the implemented model and optimization results.

CHAPTER 4

4.1 INTRODUCTION

This chapter focuses on the results produced during the study that examined how mathematical modelling and optimization can be used in detergent manufacturing process to produce the required product. The results produced will be presented in different ways (tables, diagrams etc.) in a step by step as the experiment progressed

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4.2 Liquid soap manufacturing

Figure 5 shows products at Buse detergent plant

Liquid soap is manufactured with the help of mathematical models from drafting of formulation to quantity of raw material used in which linear programing (LINDO) and ratio are mathematical modes used

Linear Interactive and Discrete Optimizer (LINDO)

Linear Interactive and Discrete Optimizer (LINDO) program is an algorithm, that is, a mathematical set of programs (mathematical software). It is an optimization system in the area of operations research. The Linear Interactive and Discrete Optimizer (LINDO) computer program provides linear programming solutions that will be useful and available to the management of production industries in reducing wastages of their production system resources and set-up times which consequently lead to reduced expenses (LINDO Systems Inc., 2003). The study will also assist manufacturing industries to determine an optimal product mix for their production process flow line that will produce the most profit after sales.

Quantities of Raw Materials for liquid Soap

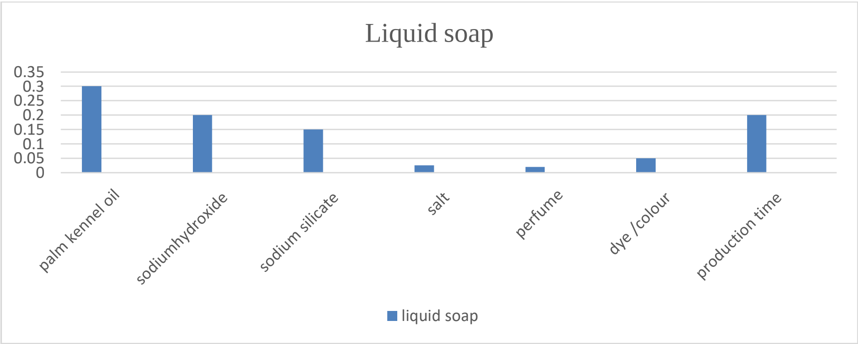

Figure 6 illustrates the quantities of raw materials for the production of liquid

Figure 3.5 illustrates the quantities of raw materials for the production of liquid soap at BUSE Soap industry. The illustration revealed that the liquid soap produced by the company used 0.30kl of Palm Kernel Oil /other Fatty Matter, 0.20kg of Sodium hydroxide or caustic soda, 0.15kg of Sodium silicate, 0.015kg of salt, 0.001kg of perfumes, and 0.025kg of dye daily. This result is in good agreement with results obtained in studies reviewed in the literature (Okolie et al., 2014; Balogun, et al., 2013).

3.2 Understudying the Time Taken to Produce Different Variables of Soap

The second objective was to understudy the time taken to produce each of the different variables (types) of soap. The time taken to produce each of the different variables (types) of soap (bar soap, aura soap, medicated soap, Translucent soap and jumbo-all-purpose soap) was understudied and analyzed. The total amount of production time available for manufacturing daily was 20 hours.

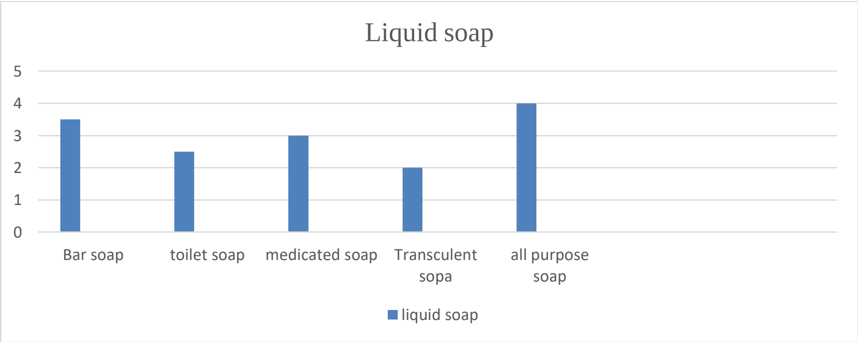

Figure 7illustrates the time taken to produce the different variables of soap (Bar Soap, Aura Soap, Medicated Soap, Translucent Soap and all purpose soap

Figure 7 illustrates the time taken to produce the different variables of soap (Bar Soap, Aura Soap, Medicated Soap, Translucent Soap and all purpose soap

illustration showed that the time taken to produce the Bar soap daily is 3.4hrs i.e. 17% of the total available time for production daily, and the time taken to produce the Aura soap daily is 2.6hrs i.e. 13% of the total available time for production daily, the time taken to produce the Medicated soap daily is 3.0hrs i.e. 15% of the total available time for production daily, the time taken to produce the Translucent soap daily is 2.4hrs i.e. 12% of the total available time for production daily, and the time taken to produce the Jumbo-all purpose soap daily is 3.8hrs i.e. 19% of the total available time for production daily. The result showed that all-purpose soap used the greatest amount of time for production while Translucent soap used the least amount of time for production

3.3 To determine the mix of bar soap and toilet soap to produce each day to maximize total profit without exceeding line and labor capacity at BUSE using linear programing

The third objective was to determine the product mix and schedule of raw materials in soap production for maximum profit using linear programming

The bar soap unit produces $100 profit per day and each toilet soap unit produces $150 profit per day

At Buse the liquid soap line produces 100 bars of soap per day

The toilet soap unit produces 120 liters of toilet soap

There is limited supply of daily labor at Buse in total there is 8 hours available each

Liquid soap require 1 hour of labor and toilet soap require 2 hours of labor

Results from LINDO

```
LINDO/WIN64 21.0.38 (22 Apr 2025), LINDO API 15.0.6099.233

Licensee info: Eval Use Only
License expires:  1 NOV 2025

Global optimal solution found.
Objective value:                              28000.00
Infeasibilities:                              0.000000
Total solver iterations:                             0
Elapsed runtime seconds:                          0.12

Model Class:                                        LP

Total variables:                     2
Nonlinear variables:                 0
Integer variables:                   0

Total constraints:                   3
Nonlinear constraints:               0

Total nonzeros:                      4
Nonlinear nonzeros:                  0


                              Variable           Value        Reduced Cost
                         LIQUID soap          100.0000            0.000000
                          TOILET Soap         120.0000            0.000000

                                   Row    Slack or Surplus      Dual Price
                                     1        28000.00            1.000000
                                     2        0.000000            100.0000
                                     3        0.000000            150.0000
```

Figure 8 show lindo results of operation to determine the product mix

The results above was obtained from LINDO and after doing a operation to determine the product mix and schedule of raw materials in soap production for maximum profit using linear programming from 100 liquid soap and 120 toilets soap and the objective value was $ 28 000 from the above operations there is evidence where mathematical operations can be used to optimize operations in detergent plants

Chapter 5

5.1 introduction

This chapter focuses on the results produced during the study ,sumary of key results conclusion and recommendations on mathematical modelling and optimization in detergent manufacturing process to produce the required product. This chapter presents the results obtained from applying mathematical modeling techniques to optimize the detergent manufacturing process. The primary objective was to evaluate how quantitative models, particularly linear and non-linear programming, could improve the allocation of raw materials, reduce production costs, and enhance detergent quality parameters such as foam stability and cleaning efficiency.The chapter begins by outlining the experimental setup and simulation inputs used to model the production process. This includes the definition of decision variables, objective functions, and constraints based on ingredient costs, performance indices, and product formulation standards. Results from both linear programming (LP) and non-linear programming (NLP) models are presented, analyzed, and compared.Furthermore, the impact of mathematical modeling on operational efficiency and resource management within the detergent formulation process is critically examined.The discussion integrates findings with existing literature, highlighting how mathematical tools can support evidence-based decisions in chemical manufacturing. The chapter also outlines limitations of the models used and suggests areas for practical implementation and future research

5.2 Summary of key results

The project objective was to develop a comprehensive mathematical model that describes and optimizes the production of liquid detergent therefore the model that was developed was linear programming .The model simulates how formulation parameters and process conditions influence final product quality and it also predicts the mixing time .This model is developed based on production constrains such as resource availability ,labor and machine time .The model is solved using LINDO this model enables resource utilization ,sensitivity analysis .The model developed is user friendly interface easy to input models using algebraic notation ,its ideal for students and researchers new optimization .The model solves complex models handles large scale linear ,integer ,nonlinear and stochastic models efficiently .LINDO is fast and reliable solvers built in solvers use advanced simplex and interior point methods. It delivers quick solution even with multiple variable and constraints. The model is a sensitivity analysis it provide post solution reports on shadow price allowable ranges and resource utilization Schrage,L.(2009) .It is widely used in academics for teaching operations research ,economics and engineering optimization .It is used in production planning ,logistics ,finance and scheduling .However LINDO has its limitation whereby it is a limited free version the free /student version has restrictions on problem size eg number of variables and constrains . The model is less intuitive for non-technical users; while it has a user interface, it's not as visually intuitive as tools like Excel Solver for beginners. There is no built-in graphical output. LINDO lacks rich graphical tools; external software is needed for plotting results. The model is primarily text- based;odels using algebraic syntax,,, which can be error-one if not done carefully. LINDO has limited community support compared to open-source tools (like Python with Pulp or R Lindo has a smaller user base and fewer tutorials The full version requires commercial licensing, the paid licenses,, which may be costly for individuals or small institutions. The developed mathematical model alignss with experimental data from a detergent manufacturing plant at B.U.S.E., wherebyby the barsoap unit produces $100 profit per day andeach toilet soap unit produces$150 profit per day.. At Buse,, the liquid soap line produces 100 batcheses of soap per day. The toilet soap unit produces 120 liters of toilet soap.. There is a limiteded supply of daily labor at Buse. In total,, there aree 8 hours available.

Each liquidd soap requiress 1 hour of labor,, and toilet soap requiress 2 hours of labor.The results from LINDO and after doing an operationn to determine the product mix and schedule of raw materials in soap production for maximum profit using linear programming from 100 liquid soapss and 120 toilet soaps,e objective value was $28,000. From the operations,, there is evidence that a mathematical model can determine the means of operation in a detergent manufacturing plant.The model is less intuitive for non Technical users while it has a user interface its not as visually intuitive as tools like excel solver for beginners .It is no built in graphical output LINDO lacks rich graphical tools ,external software is needed for plotting results .The model is primarily a text based modeling users must define models using algebraic syntax which can be error –prone if not careful done .LINDO has limited community support compared to open source tools (like python with pulp or R).Lindo has a smaller user base and fewer tutorials .The full version requires commercial licensing the paid licenses which may be costly for individuals or small institutions. The developed mathematical model align with experimental data from a detergent manufacturing plant at B.U.S.E were by the bar soap unit produces $100 profit per day and each toilet soap unit produces $150 profit per day At Buse the liquid soap line produces 100 batch of soap per day The toilet soap unit produces 120 liters of toilet soap There is limited supply of daily labor at Buse in total there is 8 hours available each Liquid soap require 1 hour of labor and toilet soap require 2 hours of labor . The results from LINDO and after doing a operation to determine the product mix and schedule of raw materials in soap production for maximum profit using linear programming from 100 liquid soap and 120 toilets soap and the objective value was $ 28 000 from the operations there is evidence mathematical model can determine the means of operation on detergent manufacturing plant .

5.3 Conclusion

The mathematical model LINDO proved to be an effective method and this shows that mathematical models can be effective in detergent manufacturing process . The results obtained from LINDO and after doing a operation to determine the product mix and schedule of raw materials in soap production for maximum profit using linear programming from 100 liquid soap and 120 toilets soap and the objective value was $ 28 000 from the above operations this is evidence where mathematical operations can be used to optimize operations in detergent plants . . The illustration revealed that the liquid soap produced by the company used 0.30kl of Palm Kernel Oil /other Fatty Matter, 0.20kg of Sodium hydroxide or caustic soda, 0.15kg of Sodium silicate, 0.015kg of salt, 0.001kg of perfumes, and 0.025kg of dye daily. This result is in good agreement with results obtained in studies reviewed in the literature (Okolie et al., 2014; Balogun, et al., 2013). Mathematical modelling is a powerful and versatile tool that enables us to understand, analyze, and predict the behavior of complex systems. by following a structured process and considering the challenges and ethical considerations, mathematical models can provide valuable insights and guide decision-making across various domains. The continuous advancements in computational tools, data-driven approaches, and integration with artificial intelligence are expanding the possibilities of mathematical modelling, making it an indispensable tool for addressing the challenges of the modern world.Therefore a comprehensive mathematical model can accurately describe the detergent manufacturing process.

5.4 Recommendations

I recommend the use of LINDO in conjunction with other models such as machine learning algorithms e.g. regression neutral networks or statistical models .The research recommend the use of linear and nonlinear programming technique to balance the proportion of ingredients based on performance metrics and constrains .I recommend the simulation of surfactant behavior in detergent production where creation of a model to simulate the behavior of surfactants in different washing conditions e.g. temperature ,water hardness ,soil type .I recommend creation of a model to assess the environmental impact of detergent production and use focusing on factors like biodegradability ,toxicity and energy consumption. The use of life cycle assessment techniques to quantify environmental impacts and identify areas for improvement should be done .There is need for development of a model that Scales up detergent production process from laboratory to industrial scale while maintaining product quality and consistency e.g. the use of dimensional analysis or computational fluid dynamics (CFD) to study the effects of scaling on mixing ,heat transfer and mass transfer in detergent production .Development of a Supply chain optimization model for detergent production should be done and ensuring of timely delivery of raw materials and finished products must be considered

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Appendix 1

Figure 9 appendix of ant plagiarism report

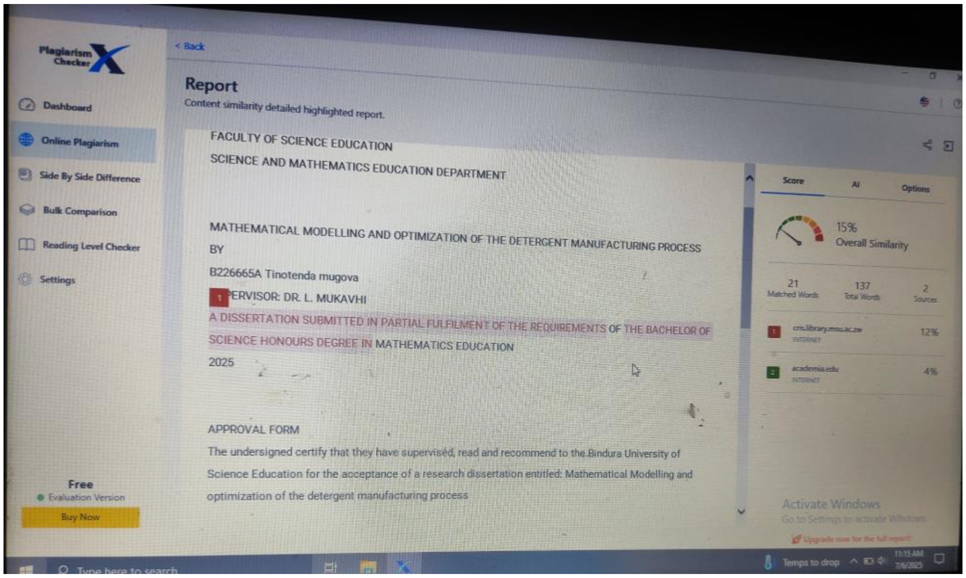

Figure 10 Apendix 2 Lindo results

Results from LINDO

```
LINDO/WIN64 21.0.38 (22 Apr 2025), LINDO API 15.0.6099.233

Licensee info: Eval Use Only
License expires:  1 NOV 2025

Global optimal solution found.
Objective value:                              28000.00
Infeasibilities:                              0.000000
Total solver iterations:                             0
Elapsed runtime seconds:                          0.12

Model Class:                                        LP

Total variables:                      2
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                    3
Nonlinear constraints:                0

Total nonzeros:                       4
Nonlinear nonzeros:                   0


                             Variable           Value        Reduced Cost
                          LIQUID soap        100.0000            0.000000
                          TOILET Soap        120.0000            0.000000

                                  Row    Slack or Surplus      Dual Price
                                    1        28000.00            1.000000
                                    2        0.000000            100.0000
                                    3        0.000000            150.0000
```