BINDURA UNIVERSITY OF SCIENCE EDUCATION

ENGINEERING AND PHYSICS DEPARTMENT

PH 208: SOLID STATE PHYSICS

JAN 2025

DURATION: THREE HOURS

Answer **ALL** parts of Section A and any THREE questions from Section B. Section A carries 40 marks and Section B carries 60 marks.

SECTION A

1 (a) (i) Inspite of their long range order in the arrangement of particles, why are the crystals usually not perfect? [2]

(ii) Which of the following statements are true about semiconductors? [4]
(a) Silicon doped with electron rich impurity is a p -type semiconductor
(b) Silicon doped with an electron rich impurity is an n-type semiconductor
(c) Delocalised electrons increase the conductivity of doped silicon
(d) An electron vacancy increases the conductivity of n-type semiconductor

(b) (i) Under which situations can an amorphous substance change to crystalline form? [3]

(ii) Match the items given in Column I with the items given in Column II. [5]

Column I	Column II
A. Mg in solid state	1. P-type semiconductor
B. MgCl2 in molten state	2. N-type semiconductor
C. Silicon with phosphorus	3. Electrolytic conductors
D. Germanium with Boron	4. Electronic com

(c) Find the kinetic energy of an electron with a wavelength of 0.5 nm [4]

(d) Determine the energy and radius of the electron orbit of s hydrogen atom for n=1. [10]

(e) Using Figure 1.1, determine the dipole moment associated with the ionic model of the water molecule. NB: the length of the O-H bond is 0.097 nm and the angle between the bonds is 104.5°. [6]

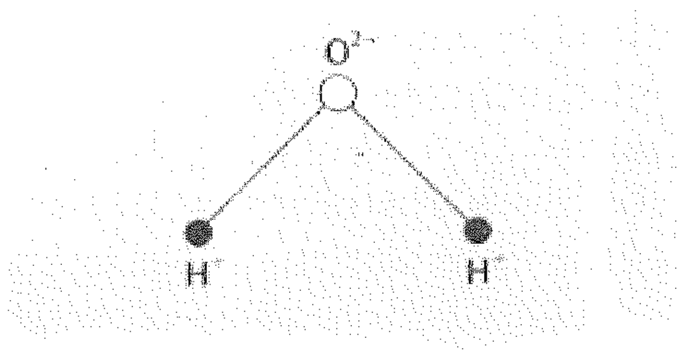

Figure 1.1 Ionic model

(f) Estimate the stress required to open a surface crack in a glass rod given that the crack has a depth of 2.0 μm and a radius of curvature at the tip of 1.0 nm. [4]

(g) Estimate the typical conductivity of a metal at 295 K assuming that the mean free path is about 1 nm and that the number of valency electrons is about 10^{29} m^{-3}. [3]

Section B

2 (a) [8]

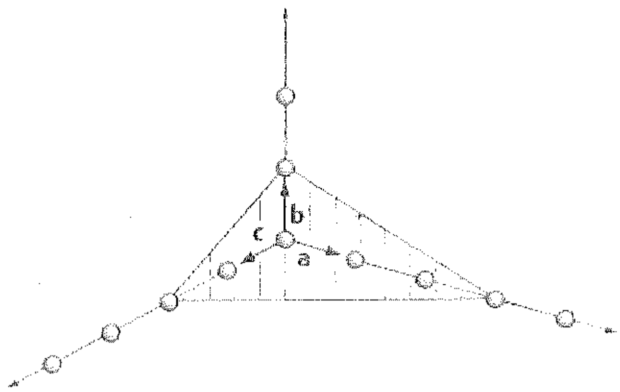

Figure 2

Determine the Miller indices for the plane shown in Fig. 2

(b) Determine the actual volume occupied by the spheres in the simple cubic structure as a percentage of the total volume. [12]

3 (a) The energy-wavevector (E – k) dispersion relation for a particle in two dimensions is E = Ck, where C is a constant. If its density of states D (E) is proportional to E^p. What is the value of p? [4]

(b) The energy dispersion for electrons in one dimensional lattice with lattice parameter a is given by $E(k) = \varepsilon_0 - \frac{1}{2} W \cos ka$, where W and E_0 are constants. What is the effective mass of the electron near the bottom of the band? [4]

(c) Discuss the advantages and disadvantages of using electron microscopes in comparison to X-ray diffraction for determining the structure of a crystal. [6]

(d) A lift cable of diameter 2 cm has a yield stress of 2 x $10^8 Nm^{-2}$. Assuming that the empty lift has a mass of 150 kg and that the average mass of a person is 70 kg, determine the maximum number of people that can be carried in the lift given that the safe operating limit is20%ofthe elastic limit. [6]

4 (a) Explain why the conductivity of germanium crystals increases on doping with gallium? [10]

(b) How does the doping increase the conductivity of semiconductors? [10]

5 (a) (i) n-type semiconductors. [10]
Use Bohr's model to determine the radius of the donor electron orbit for a phosphorus atom in a silicon crystal.

(ii) Estimate the number of silicon atoms in a sphere of this radius. (The nearest neighbour separation in silicon is 0.234 nm.)

(b) By making use of the Bohr's model of the hydrogen atom estimate the energy of the donor electron in Silicon. [4]

(c) Given that the grains in a polycrystalline metal are typically 50 μm across and that metal ions have radius of 0.5 nm, estimate the average number of ions in a grain and the proportion of these ions which are adjacent to a grain boundary. [6]

6 (a) By considering the group velocity v_g of an electron in a crystal, show that the effective mass of the particle is given by $m_e^* = \hbar^2 \left(\frac{d^2E}{dk^2}\right)^{-1}$ [14]

(b) Explain why silicon is opaque, whereas diamond is transparent.
(Hint: consider the minimum frequency of light that is absorbed in each case.) [6]

Formula Sheet

A.1 Physical Constants

The following table contains the values in SI units for some physical constants that are particularly important in solid-state physics.

Name	Symbol	Value
speed of light		
in vacuum	c	$299\,792\,458\,\mathrm{m\,s^{-1}}$
magnetic constant		
(permeability of free space)	μ_0	$4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}$
electric constant		
(permittivity of free space)	$\epsilon_0 = 1/\mu_0 c^2$	$8.854\,188 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
elementary charge	e	$1.602\,176 \times 10^{-19}\,\mathrm{C}$
Planck constant	h	$6.626\,069 \times 10^{-34}\,\mathrm{J\,s}$
in eV	$h/\{e\}$	$4.135\,667 \times 10^{-15}\,\mathrm{eV\,s}$
reduced Planck constant	$\hbar = h/2\pi$	$1.054\,572 \times 10^{-34}\,\mathrm{J\,s}$
in eV	$\hbar/\{e\}$	$6.582\,119 \times 10^{-16}\,\mathrm{eV\,s}$
fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	$7.297\,353 \times 10^{-3}$
inverse of α	α^{-1}	$137.035\,999$
magnetic flux quantum	$\Phi_0 = h/2e$	$2.067\,834 \times 10^{-15}\,\mathrm{Wb}$
conductance quantum	$G_0 = 2e^2/h$	$7.748\,092 \times 10^{-5}\,\mathrm{S}$
inverse of G_0	$G_0^{-1} = h/2e^2$	$12\,906.404\,\Omega$
Josephson constant	$K_J = 2e/h$	$483\,597.9 \times 10^{9}\,\mathrm{Hz\,V^{-1}}$
von Klitzing constant	$R_K = h/e^2$	$25\,812.807\,\Omega$
Boltzmann constant	k_B	$1.380\,650 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Avogadro constant	N_A	$6.022\,142 \times 10^{23}\,\mathrm{mol^{-1}}$
molar gas constant	$R = N_A k_B$	$8.314\,472\,\mathrm{J\,mol^{-1}\,K^{-1}}$
Bohr magneton	$\mu_B = e\hbar/2m_e$	$9.274\,009 \times 10^{-24}\,\mathrm{J\,T^{-1}}$
nuclear magneton	$\mu_N = e\hbar/2m_p$	$5.050\,783 \times 10^{-27}\,\mathrm{J\,T^{-1}}$
Bohr radius	$a_0 = 4\pi\epsilon_0\hbar^2/m_e e^2$	$0.529\,177 \times 10^{-10}\,\mathrm{m}$
electron mass	m_e	$9.109\,382 \times 10^{-31}\,\mathrm{kg}$
electron magnetic	μ_e	$-9.284\,764 \times 10^{-24}\,\mathrm{J\,T^{-1}}$
moment		$-1.001\,160\,\mu_B$
electron g-factor	$g_e = 2\mu_e/\mu_B$	$-2.002\,319$
electron gyromagnetic	$\gamma_e = 2\lvert\mu_e\rvert/\hbar$	$1.760\,860 \times 10^{11}\,\mathrm{s^{-1}\,T^{-1}}$
ratio	$\gamma_e/2\pi$	$28\,024.9532\,\mathrm{MHz\,T^{-1}}$
neutron mass	m_n	$1.674\,927 \times 10^{-27}\,\mathrm{kg}$
neutron magnetic	μ_n	$-0.966\,236 \times 10^{-26}\,\mathrm{J\,T^{-1}}$
moment		$-1.913\,043\,\mu_N$
neutron g-factor	$g_n = 2\mu_n/\mu_N$	$-3.826\,085$
proton mass	m_p	$1.672\,622 \times 10^{-27}\,\mathrm{kg}$
proton electron		
mass ratio	m_p/m_e	$1836.152\,667$
electron proton		
mass ratio	m_e/m_p	$5.446\,170\,10^{-4}$
proton magnetic	μ_p	$1.410\,607 \times 10^{-26}\,\mathrm{J\,T^{-1}}$
moment		$2.792\,847\,\mu_N$
proton g-factor	$g_p = 2\mu_p/\mu_N$	$5.585\,695$
proton gyromagnetic	$\gamma_p = 2\mu_p/\hbar$	$2.675\,222 \times 10^{8}\,\mathrm{s^{-1}\,T^{-1}}$
ratio	$\gamma_p/2\pi$	$42.577\,481\,\mathrm{MHz\,T^{-1}}$
Rydberg constant	$R_\infty = \alpha^2 m_e c/2h$	$10\,973\,731.569\,\mathrm{m^{-1}}$
Rydberg energy	$\mathrm{Ry} = R_\infty hc$	$2.179\,872 \times 10^{-18}\,\mathrm{J}$
in electronvolts		$13.605\,692\,\mathrm{eV}$
Hartree energy	$E_h = e^2/4\pi\epsilon_0 a_0$	$4.359\,744 \times 10^{-18}\,\mathrm{J}$
in electronvolts		$27.211\,384\,\mathrm{eV}$

Quantum theory of solid states

$$\psi(x) = u(x)e^{jkx}$$
$$\frac{\partial^2 u_1(x)}{\partial x^2} + 2jk\frac{\partial u_1(x)}{\partial x} - \left(k^2 - \alpha^2\right) u_1(x) = 0$$
$$\alpha^2 = \frac{2mE}{\hbar^2}$$
$$\frac{2m}{\hbar^2}(E - V_0) = \alpha^2 - \frac{2mV_0}{\hbar^2} = \beta^2$$
$$\frac{\partial^2 u_2(x)}{\partial x^2} + 2jk\frac{\partial u_2(x)}{\partial x} - \left(k^2 - \alpha^2 + \frac{2mV_0}{\hbar^2}\right) u_2(x) = 0$$
$$P'\frac{\sin\alpha a}{\alpha a} + \cos\alpha a = \cos ka$$
$$P' = \frac{mV_0 ba}{\hbar^2}$$
$$\frac{1}{\hbar}\frac{\partial E}{\partial k} = \frac{p}{m} = v$$
$$\frac{1}{m} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k^2}$$
$$g(E) = \frac{4\pi(2m)^{\frac{3}{2}}}{h^3}\sqrt{E}$$
$$\frac{N(E)}{g(E)} = f_{\mathrm{F}}(E) = \frac{1}{1+\exp\left\{\frac{(E-E_F)}{k_B T}\right\}}$$

Semiconductor

$$N_c = 2\left(\frac{2\pi m_n^* k_B T}{h^2}\right)^{\frac{3}{2}}$$
$$N_v = 2\left(\frac{2\pi m_p^* k_B T}{h^2}\right)^{\frac{3}{2}}$$
$$n(E) = g_c(E) f_{\mathrm{F}}(E)$$
$$E_{\mathrm{Fi}} - E_{\mathrm{midgap}} = \frac{3}{4}k_B T \ln\left(\frac{m_p^*}{m_n^*}\right)$$
$$n_i = N_c \exp\left\{\frac{-(E_c - E_{\mathrm{Fi}})}{k_B T}\right\}$$
$$n_0 = N_c \exp\left\{\frac{-(E_c - E_{\mathrm{F}})}{k_B T}\right\}$$
$$p_0 = N_v \exp\left\{\frac{-(E_{\mathrm{F}} - E_v)}{k_B T}\right\}$$
$$n_i^2 = N_c N_v \exp\left\{\frac{-(E_c - E_v)}{k_B T}\right\}$$
$$n_0 = n_i \exp\left\{\frac{(E_{\mathrm{F}} - E_{\mathrm{Fi}})}{k_B T}\right\}$$
$$p_0 = n_i \exp\left\{\frac{-(E_{\mathrm{F}} - E_{\mathrm{Fi}})}{k_B T}\right\}$$
$$n_d = \frac{N_d}{1+\frac{1}{2}\exp\frac{E_d - E_F}{k_B T}} = N_d - N_d^+$$
$$p_a = \frac{N_a}{1+\frac{1}{4}\exp\frac{E_F - E_a}{k_B T}} = N_a - N_a^-$$
$$n_0 = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$
$$p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$
$$E_c - E_F = k_B T \ln\left(\frac{N_c}{n_0}\right)$$
$$E_F - E_{Fi} = k_B T \ln\left(\frac{n_0}{n_i}\right)$$
$$E_F - E_v = k_B T \ln\left(\frac{N_v}{p_0}\right)$$
$$E_{Fi} - E_F = k_B T \ln\left(\frac{p_0}{n_i}\right)$$
$$n_0 p_0 = n_i^2$$

Properties of Si and GaAs

Property		Si	GaAs
Atom density (cm^{-3})		5.02×10^{22}	4.43×10^{22}
Atomic weight		28.09	144.64
Crystal structure		Diamond	Zincblende
Density (g/cm^3)		2.329	5.317
Lattice constant (Å)		5.43102	5.6533
Dielectric constant		11.9	12.9
Electron affinity χ (V)		4.05	4.07
Energy gap (eV)		1.12 (indirect)	1.42 (direct)
Effective density of states in conduction band, N_C (cm^{-3})		2.8×10^{19}	4.7×10^{17}
Effective density of states in valence band, N_V (cm^{-3})		2.65×10^{19}	7.0×10^{18}
Intrinsic carrier concentration n_i (cm^{-3})		9.65×10^{9}	2.1×10^{6}
Effective mass (m^*/m_0)	Electrons	$m_l^* = 0.98$	0.063
		$m_t^* = 0.19$	
	Holes	$m_{lh}^* = 0.16$	$m_{lh}^* = 0.076$
		$m_{hh}^* = 0.49$	$m_{hh}^* = 0.50$
Drift mobilities (cm^2/V-s)	Electrons μ_n	1,450	8,000
	Holes μ_p	500	400
Saturation velocity (cm/s)		1×10^{7}	7×10^{6}
Breakdown field (V/cm)		$2.5–8\times10^{5}$	$3–9\times10^{5}$
Minority-carrier lifetime (s)		$\approx10^{-3}$	$\approx10^{-8}$
Index of refraction		3.42	3.3
Optical-phonon energy (eV)		0.063	0.035
Melting point (°C)		1414	1240
Linear coefficient of thermal expansion $\Delta L/L\Delta T$ (°C^{-1})		2.59×10^{-6}	5.75×10^{-6}
Thermal conductivity (W/cm-K)		1.56	0.46
Thermal diffusivity (cm^2/s)		0.9	0.31
Specific heat (J/g-°C)		0.713	0.327
Heat capacity (J/mol-°C)		20.07	47.02
Young's modulus (GPa)		130	85.5

Note: All properties at room temperature.

Appendix F
Properties of Important Semiconductors

	Semiconductor		Crystal Struct.	Lattice Const. at 300 K (Å)	Bandgap (eV)		Band	Mobility at 300 K (cm^2/V-s)		Effective Mass		$\varepsilon_s/\varepsilon_0$
					300 K	0 K		μ_n	μ_p	m_n^*/m_0	m_p^*/m_0	
	C	Carbon (diamond)	D	3.56683	5.47	5.48	I	1,800	1,200	0.2	0.25	5.7
	Ge	Germanium	D	5.64613	0.66	0.74	I	3,900	1,900	1.64^l, 0.082^t	0.04^{lh}, 0.28^{hh}	16.0
	Si	Silicon	D	5.43102	1.12	1.17	I	1,450	500	0.98^l, 0.19^t	0.16^{lh}, 0.49^{hh}	11.9
IV-IV	SiC	Silicon carbide	W	a=3.086, c=15.117	2.996	3.03	I	400	50	0.60	1.00	9.66
III-V	AlAs	Aluminum arsenide	Z	5.6605	2.36	2.23	I	180		0.11	0.22	10.1
	AlP	Aluminum phosphide	Z	5.4635	2.42	2.51	I	60	450	0.212	0.145	9.8
	AlSb	Aluminum antimonide	Z	6.1355	1.58	1.68	I	200	420	0.12	0.98	14.4
	BN	Boron nitride	Z	3.6157	6.4		I	200	500	0.26	0.36	7.1
	"	"	W	a=2.55, c=4.17	5.8		D			0.24	0.88	6.85
	BP	Boron phosphide	Z	4.5383	2.0		I	40	500	0.67	0.042	11
	GaAs	Gallium arsenide	Z	5.6533	1.42	1.52	D	8,000	400	0.063	0.076^{lh}, 0.5^{hh}	12.9
	GaN	Gallium nitride	W	a=3.189, c=5.182	3.44	3.50	D	400	10	0.27	0.8	10.4
	GaP	Gallium phosphide	Z	5.4512	2.26	2.34	I	110	75	0.82	0.60	11.1
	GaSb	Gallium antimonide	Z	6.0959	0.72	0.81	D	5,000	850	0.042	0.40	15.7
	InAs	Indium arsenide	Z	6.0584	0.36	0.42	D	33,000	460	0.023	0.40	15.1
	InP	Indium phosphide	Z	5.8686	1.35	1.42	D	4,600	150	0.077	0.64	12.6
	InSb	Indium antimonide	Z	6.4794	0.17	0.23	D	80,000	1,250	0.0145	0.40	16.8
II-VI	CdS	Cadmium sulfide	Z	5.825	2.5		D			0.14	0.51	5.4
	"	"	W	a=4.136, c=6.714	2.49		D	350	40	0.20	0.7	9.1
	CdSe	Cadmium selenide	Z	6.050	1.70	1.85	D	800		0.13	0.45	10.0
	CdTe	Cadmium telluride	Z	6.482	1.56		D	1,050	100			10.2
	ZnO	Zinc oxide	R	4.580	3.35	3.42	D	200	180	0.27		9.0
	ZnS	Zinc sulfide	Z	5.410	3.66	3.84	D	600		0.39	0.23	8.4
			W	a=3.822, c=6.26	3.78		D	280	800	0.287	0.49	9.6
IV-VI	PbS	Lead sulfide	R	5.9362	0.41	0.286	I	600	700	0.25	0.25	17.0
	PbTe	Lead telluride	R	6.4620	0.31	0.19	I	6,000	4,000	0.17	0.20	30.0

D = Diamond, W = Wurtzite, Z = Zincblende, R = Rock salt. I, D = Indirect, direct bandgap. l, t, lh, hh = longitudinal, transverse, light-hole, heavy-hole effective mass.

Group	1	2		3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Period																			
1	1 H																		2 He
2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca		21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr		39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	*	71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	**	103 Lr	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Uub	113 Uut	114 Uuq	115 Uup	116 Uuh	Uus	Uuo
*Lanthanoids			*	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb		
**Actinoids			**	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No		

End of exam