BINDURA UNIVERSITY OF SCIENCE EDUCATION

ENGINEERING AND PHYSICS DEPARTMENT

PH101: MECHANICS AND OSCILLATIONS AUG 2024

DURATION: THREE HOURS

Answer **ALL** parts of Section A and any **THREE** questions from Section B. Section A carries 40 marks and Section B carries 60 marks.

SECTION A

1 (a) (i) Use dimensional analysis to show that the expression $v = v_0 + at$ is dimensionally correct, where v and v_0 represent velocities, a is acceleration, and t is a time interval. [3]

(ii) Suppose we are told that the acceleration a of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r, say r^n, and some power of v, say v^m. Determine the values of n and m and write the simplest form of an equation for the acceleration. [4]

(b) A person walks 3 km due East and then 2 km due North. What is his displacement vector? [5]

(c) Find the sum of the following two vectors: [4]

$$\vec{A} = 8\vec{i} + 3\vec{j}$$
$$\vec{B} = -5\vec{i} - 7\vec{j}$$

(d) A block of mass m is pushed up a rough inclined plane of angle θ by a constant force $\vec{F}$ parallel to the incline, as shown in Fig. 1. The displacement of the block up the incline is $\vec{d}$. Calculate the work done for $m = 2\ kg, \mu_k = 0.5$, $\theta = 30°, F = 20N$, and $d = 5m$. [5]

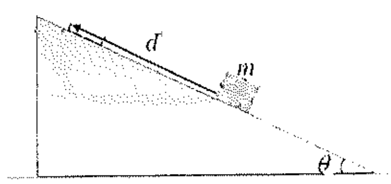


Figure 1.1

e) A disk of mass $M = 8\ kg$ and radius $R = 0.5$ m accelerates about its massless axle from rest to an angular speed $\omega = 8.5\ rad/s$ in a time $\Delta t = 2\ s$, see Figure 2.

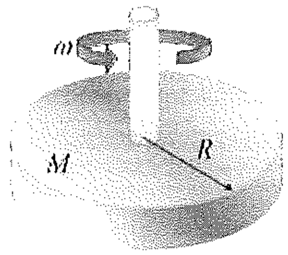


Figure 2

Find:

(i) the angular momentum of the disk **[5]**

(ii) the required constant torque used for this acceleration. **[5]**

f) A car accelerates uniformly from rest to a speed of 100 km/h in 18 s.

(i) Find the acceleration of the car. **[3]**

(ii) Find the distance that the car travels. **[3]**

(iii) If the car brakes to a full stop over a distance of 100 m, then find its uniform deceleration. **[3]**

Section B

2 Figure 2.1 shows the following three vectors.

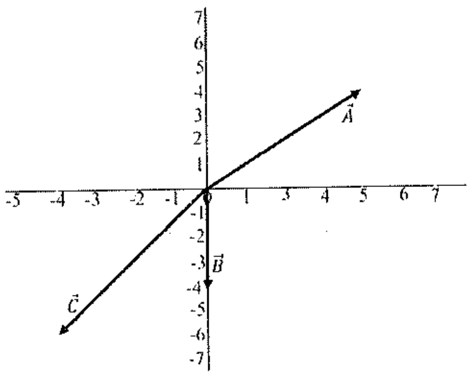


Figure 2.1

Find

(a) the vector sum $\vec{R}$of these three vectors? **[10]**

	(b)	the magnitude of $R^{\rightarrow}\vec{R}$?	[5]
	(c)	the angle measured from the $+x$ direction, (θ) ?	[5]

3 A ball thrown from the top of a building has an initial speed of ***20 m/s*** at an angle of $\mathbf{30}^{\circ}$ above the horizontal. The building is 40 m high and the ball takes time $\acute{t}$ before hitting the ground, see the Fig. 3.1. Take $g\ =\ \mathbf{10}\ m/s^2$.

(a) Find the time t_1 for the ball to reach its highest point. **[3]**

(b) How high will it rise? **[3]**

(c) How long will it take to return to the level of the thrower? **[3]**

(d) Find the time of flight $\acute{t}$. **[4]**

(e) What is the horizontal distance covered by the ball during this time? **[3]**

(f) What is the velocity of the ball before striking the ground? **[7]**

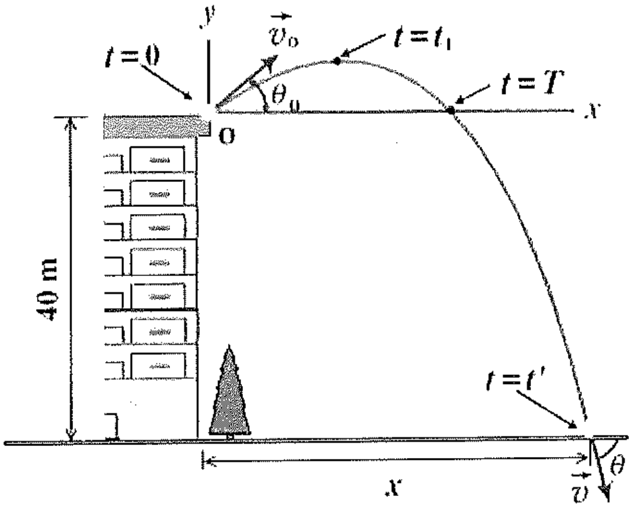
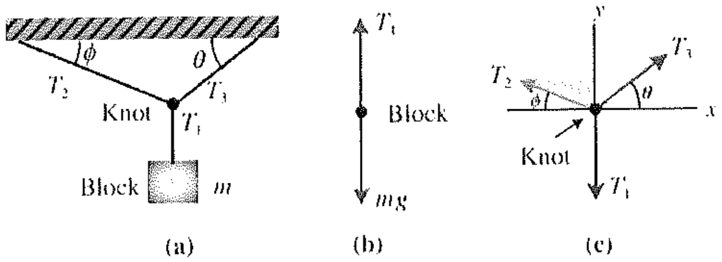


Figure 3.1

4 a) A block of mass $m\ =\ 21\ kg$ hangs from three cords as shown in part (a) of Figure. 4.1. Taking $sin\ \theta\ =\ 4/5$, $cos\ \theta\ =\ 3/5$, $sin\emptyset\ =\ 5/13$, and $cos\ \emptyset\ =\ 12/13$, find the tensions in the three cords. **[15]**



Fugure 4.1

b) Discuss frictional force. **[5]**

5 A particle oscillates with a simple harmonic motion along the x axis. Its displacement
from the origin varies with time according to the equation:

$$x = (2m)\cos\left(0.5\pi t + {}^{\pi}/_{3}\right)$$

where t is in seconds and the argument of the cosine is in radians.
Find:

a) the amplitude, frequency, and period of the motion. **[4]**

b) the velocity and acceleration of the particle at any time. **[6]**

c) both the maximum speed and acceleration of the particle. **[4]**

d) the displacement of the particle between $t = 0$ and $t = 2s$. **[6]**

6 A block of mass $m = 400\ g$ is attached to a light spring of force constant $k_H = 10\ Nm^{-1}$, see Figure 6.1 (a). The block is pushed against the spring from $x = 0$ to $x_i = -10cm$, see Figure 6.1 (b), and then released to oscillate on a horizontal frictionless surface.

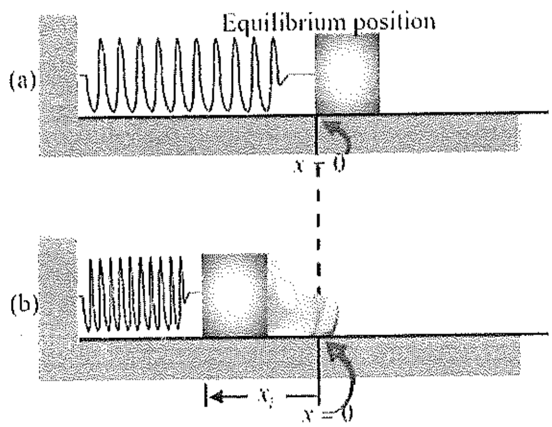


Figure 6.1

a)	Find:	
b)	the angular frequency and the period of the block-spring system.	[6]
c)	the maximum speed and maximum acceleration of the block.	[6]
	the position, speed, and acceleration of the block at any time.	[8]

MECHANICS FORMULA SHEET

0.1: Physical Constants

Speed of light	c	3×10^{8} m/s
Planck constant	h	6.63×10^{-34} J s
	hc	1242 eV-nm
Gravitation constant	G	6.67×10^{-11} m^3 kg^{-1} s^{-2}
Boltzmann constant	k	1.38×10^{-23} J/K
Molar gas constant	R	8.314 J/(mol K)
Avogadro's number	N_A	6.023×10^{23} mol^{-1}
Charge of electron	e	1.602×10^{-19} C
Permeability of vacuum	μ_0	$4\pi \times 10^{-7}$ N/A^2
Permitivity of vacuum	ϵ_0	8.85×10^{-12} F/m
Coulomb constant	$\frac{1}{4\pi\epsilon_0}$	9×10^{9} N m^2/C^2
Faraday constant	F	96485 C/mol
Mass of electron	m_e	9.1×10^{-31} kg
Mass of proton	m_p	1.6726×10^{-27} kg
Mass of neutron	m_n	1.6749×10^{-27} kg
Atomic mass unit	u	1.66×10^{-27} kg
Atomic mass unit	u	931.49 MeV/c^2
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/(m^2 K^4)
Rydberg constant	R_∞	1.097×10^{7} m^{-1}
Bohr magneton	μ_B	9.27×10^{-24} J/T
Bohr radius	a_0	0.529×10^{-10} m
Standard atmosphere	atm	1.01325×10^{5} Pa
Wien displacement constant	b	2.9×10^{-3} m K

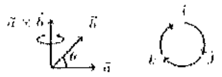
1.1: Vectors

Notation: $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

Magnitude: $a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Dot product: $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z = ab\cos\theta$

Cross product:



$$\vec{a}\times\vec{b} = (a_y b_z - a_z b_y)\hat{i} + (a_z b_x - a_x b_z)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$$

$$|\vec{a}\times\vec{b}| = ab\sin\theta$$

1.2: Kinematics

Average and Instantaneous Vel. and Accel.:

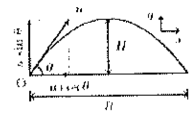
$$\vec{v}_{av} = \Delta\vec{r}/\Delta t, \qquad \vec{v}_{inst} = d\vec{r}/dt$$
$$\vec{a}_{av} = \Delta\vec{v}/\Delta t, \qquad \vec{a}_{inst} = d\vec{v}/dt$$

Motion in a straight line with constant a:

$$v = u + at, \quad s = ut + \tfrac{1}{2}at^2, \quad v^2 = u^2 + 2as$$

Relative Velocity: $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$

Projectile Motion:



$$x = ut\cos\theta, \quad y = ut\sin\theta - \tfrac{1}{2}gt^2$$
$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$$
$$T = \frac{2u\sin\theta}{g}, \quad R = \frac{u^2\sin 2\theta}{g}, \quad H = \frac{u^2\sin^2\theta}{2g}$$

1.3: Newton's Laws and Friction

Linear momentum: $\vec{p} = m\vec{v}$

Newton's first law: inertial frame.

Newton's second law: $\vec{F} = \frac{d\vec{p}}{dt}$, $\vec{F} = m\vec{a}$

Newton's third law: $\vec{F}_{AB} = -\vec{F}_{BA}$

Frictional force: $f_{static,\,max} = \mu_s N$, $f_{kinetic} = \mu_k N$

Banking angle: $\frac{v^2}{rg} = \tan\theta$, $\frac{v^2}{rg} = \frac{\mu + \tan\theta}{1 - \mu\tan\theta}$

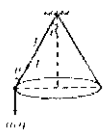
Centripetal force: $F_c = \frac{mv^2}{r}$, $a_c = \frac{v^2}{r}$

Pseudo force: $\vec{F}_{pseudo} = -m\vec{a}_0$, $F_{centrifugal} = -\frac{mv^2}{r}$

Minimum speed to complete vertical circle:

$$v_{min,\,bottom} = \sqrt{5gl}, \quad v_{min,\,top} = \sqrt{gl}$$

Conical pendulum: $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$



1.4: Work, Power and Energy

Work: $W = \vec{F}\cdot\vec{S} = FS\cos\theta$, $W = \int \vec{F}\cdot d\vec{S}$

Kinetic energy: $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

Potential energy: $F = -\partial U/\partial x$ for conservative forces.

$$U_{\text{gravitational}} = mgh, \quad U_{\text{spring}} = \frac{1}{2}kx^2$$

Work done by conservative forces is path independent and depends only on initial and final points: $\oint \vec{F}_{\text{conservative}}\cdot d\vec{r} = 0$.

Work-energy theorem: $W = \Delta K$

Mechanical energy: $E = U + K$. Conserved if forces are conservative in nature.

Power $P_{\text{av}} = \frac{\Delta W}{\Delta t}$, $P_{\text{inst}} = \vec{F}\cdot\vec{v}$

1.6: Rigid Body Dynamics

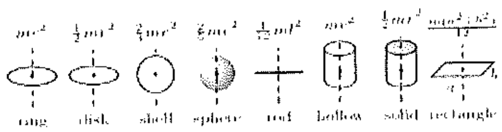
Angular velocity: $\omega_{\text{av}} = \frac{\Delta\theta}{\Delta t}$, $\omega = \frac{d\theta}{dt}$, $\vec{v} = \vec{\omega}\times\vec{r}$

Angular Accel.: $\alpha_{\text{av}} = \frac{\Delta\omega}{\Delta t}$, $\alpha = \frac{d\omega}{dt}$, $\vec{a} = \vec{\alpha}\times\vec{r}$

Rotation about an axis with constant α:

$$\omega = \omega_0 + \alpha t, \quad \theta = \omega t + \frac{1}{2}\alpha t^2, \quad \omega^2 - \omega_0^2 = 2\alpha\theta$$

Moment of Inertia: $I = \sum_i m_i r_i^2$, $I = \int r^2 dm$



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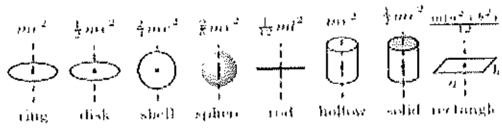


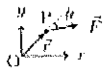
Theorem of Parallel Axes: $I_\parallel = I_{\text{cm}} + md^2$

Theorem of Perp. Axes: $I_z = I_x + I_y$

Radius of Gyration: $k = \sqrt{I/m}$

Angular Momentum: $\vec{L} = \vec{r}\times\vec{p}$, $\vec{L} = I\vec{\omega}$

Torque: $\vec{\tau} = \vec{r}\times\vec{F}$, $\vec{\tau} = \frac{d\vec{L}}{dt}$, $\tau = I\alpha$

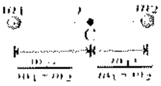


1.5: Centre of Mass and Collision

Centre of mass: $\vec{r}_{\rm cm} = \frac{\sum \vec{r}_i m_i}{\sum m_i}$, $\vec{r}_{\rm cm} = \frac{\int \vec{r}\,dm}{\int dm}$

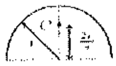
CM of few useful configurations:

1. m_1, m_2 separated by r:



2. Triangle (CM ≡ Centroid) $y_c = \frac{h}{3}$

3. Semicircular ring: $y_c = \frac{2r}{\pi}$



4. Semicircular disc: $y_c = \frac{4r}{3\pi}$

5. Hemispherical shell: $y_c = \frac{r}{2}$

6. Solid Hemisphere: $y_c = \frac{3r}{8}$

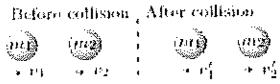
7. Cone: the height of CM from the base is $h/4$ for the solid cone and $h/3$ for the hollow cone.

Motion of the CM: $M = \sum m_i$

$$\vec{v}_{\rm cm} = \frac{\sum m_i \vec{v}_i}{M}, \quad \vec{p}_{\rm cm} = M\vec{v}_{\rm cm}, \quad \vec{a}_{\rm cm} = \frac{\vec{F}_{\rm ext}}{M}$$

Impulse: $\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$

Collision:



Momentum conservation: $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$
Elastic Collision: $\frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 = \frac{1}{2}m_1{v_1'}^2 + \frac{1}{2}m_2{v_2'}^2$
Coefficient of restitution:

$$e = \frac{-(v_1' - v_2')}{v_1 - v_2} = \begin{cases} 1, & \text{completely elastic} \\ 0, & \text{completely in-elastic} \end{cases}$$

If $v_2 = 0$ and $m_1 \ll m_2$ then $v_1' = -v_1$.
If $v_2 = 0$ and $m_1 \gg m_2$ then $v_2' = 2v_1$.
Elastic collision with $m_1 = m_2$: $v_1' = v_2$ and $v_2' = v_1$.

Conservation of $\vec{L}$: $\vec{\tau}_{\rm ext} = 0 \implies \vec{L} = \text{const.}$

Equilibrium condition: $\sum \vec{F} = \vec{0}$, $\sum \vec{\tau} = \vec{0}$

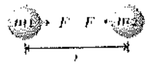
Kinetic Energy: $K_{\rm rot} = \frac{1}{2}I\omega^2$

Dynamics:

$$\vec{\tau}_{\rm cm} = I_{\rm cm}\vec{\alpha}, \quad \vec{F}_{\rm ext} = m\vec{a}_{\rm cm}, \quad \vec{p}_{\rm cm} = m\vec{v}_{\rm cm}$$
$$K = \frac{1}{2}m{v_{\rm cm}}^2 + \frac{1}{2}I_{\rm cm}\omega^2, \quad \vec{L} = I_{\rm cm}\vec{\omega} + \vec{r}_{\rm cm} \times m\vec{v}_{\rm cm}$$

1.7: Gravitation

Gravitational force: $F = G\frac{m_1m_2}{r^2}$



Potential energy: $U = -\frac{GMm}{r}$

Gravitational acceleration: $g = \frac{GM}{R^2}$

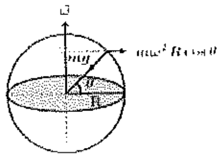
Variation of g with depth: $g_{\rm inside} \approx g\left(1 - \frac{h}{R}\right)$

Variation of g with height: $g_{\rm outside} \approx g\left(1 - \frac{2h}{R}\right)$

Effect of non-spherical earth shape on g:
$g_{\rm at\ pole} > g_{\rm at\ equator}$ ($\because R_e - R_p \approx 21$ km)

Effect of earth rotation on apparent weight:

$$mg_\theta' = mg - m\omega^2R\cos^2\theta$$



Orbital velocity of satellite: $v_o = \sqrt{\frac{GM}{R}}$

Escape velocity: $v_e = \sqrt{\frac{2GM}{R}}$

Kepler's laws:

First: Elliptical orbit with sun at one of the focus.
Second: Areal velocity is constant. ($\because d\vec{L}/dt = 0$).
Third: $T^2 \propto a^3$. In circular orbit $T^2 = \frac{4\pi^2}{GM}a^3$.

1.8: Simple Harmonic Motion

Hooke's law: $F = -kx$ (for small elongation x)

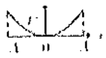
Acceleration: $a = \frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$

Time period: $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

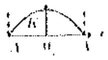
Displacement: $x = A\sin(\omega t + \phi)$

Velocity: $v = A\omega\cos(\omega t + \phi) = \pm\omega\sqrt{A^2 - x^2}$

Potential energy: $U = \frac{1}{2}kx^2$

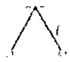


Kinetic energy $K = \frac{1}{2}mv^2$



Total energy: $E = U + K = \frac{1}{2}m\omega^2A^2$

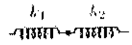
Simple pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$



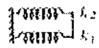
Physical Pendulum: $T = 2\pi\sqrt{\frac{I}{mgd}}$

Torsional Pendulum $T = 2\pi\sqrt{\frac{I}{\kappa}}$

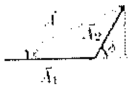
Springs in series: $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$



Springs in parallel: $k_{eq} = k_1 + k_2$



Superposition of two SHM's:



$x_1 = A_1\sin\omega t, \quad x_2 = A_2\sin(\omega t + \delta)$

$x = x_1 + x_2 = A\sin(\omega t + \epsilon)$

$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta}$

$\tan\epsilon = \frac{A_2\sin\delta}{A_1 + A_2\cos\delta}$

END OF EXAM