BINDURA UNIVERSITY OF SCIENCE EDUCATION MT108

BACHELOR OF SCIENCE EDUCATION HONOURS DEGREE

LINEAR ALGEBRA

Time : 3 hours AUG 2023

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A7.

A1. Find the parametric equation of a line in space which passes through the points $P(1, 0, -1)$ and $Q(3, -2, -3)$. [4]

A2. Let $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$ be vectors in R^2. Show that the Weighted Euclidean inner product $< \mathbf{u}, \mathbf{v} >= 3u_1v_1 + 2u_2v_2$ satisfies all the inner product. [6]

A3. (a) Define the term linear combination of vectors. [2]

(b) Given the vectors $\mathbf{u} = (1, 2, -1)$ and $\mathbf{v} = (6, 4, 2)$, show that $\mathbf{w} = (9, 2, 7)$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$. [4]

A4. Let A= $\begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}$

(a) Find the adjoint of the matrix A, [5]

(b) Hence or otherwise find A^{-1}, the inverse of matrix A. [2]

A5. Find the solution of the following system of linear equations using Gauss elimination method.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 5, \\ 2x_1 + 5x_2 + 3x_3 &= 3, \\ x_1 + 8x_3 &= 17. \end{aligned}$$

[6]

A6. (a) Verify the Cayley Hamilton theorem using $A = \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}$. [4]

(b) Hence find A^4 using the Cayley Hamilton theorem. [2]

A7. Prove that $||v_1 + v_2 + ... + v_n||^2 = ||v_1||^2 + ||v_2||^2 + ... + ||v_n||^2$ given that the set $S = \{v_1, v_2 ... v_n\}$ form an orthogonal basis. [5]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B8 to B10.

B8. (a) Let A be a non singular matrix. Give a formular connecting A^{-1} , det (A) and adj (A). [2]

(b) Find the solution of the linear system:

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 4, \\ x_1 - 2x_2 + x_3 &= 6, \\ x_1 - 12x_2 + 5x_3 &= 10, \end{aligned}$$

using Cramer's rule. [8]

(c) Find x such that det $(A) = 0$ if

$$A = \begin{bmatrix} 1 & x & x \\ -x & -2 & x \\ x & x & 3 \end{bmatrix}.$$

[8]

(d) Use Gauss elimination method to find a solution to the following system of inhomogeneous simultaneous linear equations.

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 1, \\ 2x_1 - x_2 + x_4 &= -2, \\ x_1 - x_2 - x_3 - 2x_4 &= 4, \\ 2x_1 - x_2 + 2x_3 - x_4 &= 0. \end{aligned}$$

[12]

B9. (a) Find area of a triangle with the vertices $M(2,1,0)$, $N(3,-1,1)$ and $R(1,0,2)$. [8]

(b) Given that $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, find $\mathbf{u} \times \mathbf{v}$ and a unit vector perpendicular to the plane containing the vectors $\mathbf{u}$ and $\mathbf{v}$. [6]

(c) Prove that if $\mathbf{u}$ and $\mathbf{v}$ are vectors in a real inner product space then, $\| \mathbf{u} + \mathbf{v} \| \leq \| \mathbf{u} \| + \| \mathbf{v} \|$. [6]

(d) Let A $= \begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \in M_{2\times 2}$. Show that $\| \frac{1}{\|A\|} A \| = 1$. [5]

(e) Find the area of the parallelogram with vectors $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ as sides. [5]

B10. (a) (i) Define the term linear independence of vectors. [2]

(ii) Determine whether the vectors $\mathbf{v}_1 = (1, 2, -3), \mathbf{v}_2 = (1, -3, 2)$ and $\mathbf{v}_3 = (2, -1, 5)$ in R^3 are linearly dependent or not. Justify your answer. [6]

(b) Show that W is not a subspace of $V = \mathbf{R}^3$ where W consists of those vectors whose first component is non negative, that is, $W = \{(a, b, c) : a \geq 0\}$. [5]

(c) Determine whether the vectors $(1, 1, 1), (1, 2, 3)$ and $(2, -1, 1)$ spans $\mathbf{R}^3$. [5]

(d) Find the dimension and a basis of the solution space W of the following homogenous system:

$$x + 2y + 2z - s + 3t = 0$$

$$x + 2y + 3z + s + t = 0$$

$$3x + 6y + 8z + s + 5t = 0.$$

[7]

(e) Let $B = \begin{pmatrix} 0 & -1 & 4 & 1 \\ 1 & 1 & 2 & -1 \\ -3 & 0 & -1 & 0 \\ 2 & -1 & 0 & 1 \end{pmatrix}$

Find the rank and nullity of matrix B. [5]

END OF QUESTION PAPER