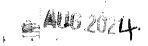
BINDURA UNIVERSITY OF SCIENCE EDUCATION

MT101: CALCULUS 1



Time: 3 hours

Answer ALL questions in Section A and at most TWO questions in section B.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

- A1. Verify that $f(x) = x^3 x^2 6x + 2$ satisfies the hypothesis of Rolle' theorem for the interval [0, 3], then find all c that satisfy the conclusion. [7]
- A2. Prove that if a sequence converges, then its limit is unique. [8]
- A3. Solve the inequality |x-4| < 3. [5]
- **A4.** Given $f(x) = \frac{x+1}{x-1}$, prove that this function is bijective. [10]
- A5. (a) Write down any six indeterminate forms. [3]
 - (b) Find the area bounded by y = cos(x), y = 0, x = 0 and $x = \frac{3\pi}{2}$. [7]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B6 to B8.

- **B6.** (a) Evaluate $\lim_{x\to\infty} \left(\frac{\sin(x)^2}{x\sin(x)}\right)$. [5]
 - (b) Give an $\epsilon \delta$ definition of the limit of a function. [3]
 - (c) Use the ϵN definition of a sequence to show that a sequence whose n^{th} term is given by $a_n = 3 \frac{1}{7n^2}$ converges to 3. [5]

- (d) Show that the sequence $U_n = \frac{2n-7}{3n+2}$ is monotonic increasing. [5]
- (e) State the Mean Value Theorem of differential calculus. [2]
- (f) Given that $f(x) = \sqrt{25 x^2}$ satisfies the Mean Value Theorem on the interval [-3, 4], find the value $C \in [-3, 4]$.
- (g) Given that $y = \frac{x^4}{4x-3}$, sketch the graph showing clearly its asymptotes and stationary points. [7]
- B7. (a) Verify that the function $f(x) = x^3 + x 1$ satisfies the Mean Value Theorem in the interval [0,2].
 - (b) The graph of the parabola $x = y^2 2$ is NOT the graph of a function of x. Explain why? [4]
 - (c) Find the relative extrema for the function $f(x) = 2x^3 3x^2 36x + 14$. [6]
 - (d) Consider the function $f(x) = \frac{x^2 4}{x^2 1}$. State the domain and range of the function. [5]
 - (e) Determine the derivative of $y = \sin^{-1}(x)$. [5]
- B8. (a) State the Second Fundamental theorem of calculus. [3]
 - (b) Show that $\int \frac{1}{a^2 + b^2 x^2} dx = \frac{1}{ab} \arctan(\frac{bx}{a}) + k.$ [6]
 - (c) hence evaluate $\int \frac{1}{2x^2+9} dx$. [3]
 - (d) Evaluate the following integrals:
 - (i) $\int_0^{\frac{1}{2}} \arcsin(x) dx.$ [5]

(ii)
$$\int \frac{2x^2 - 5x + 2}{x^3 + x} dx.$$
 [5]

(e) Find the area of the region bounded by f(x) = 4ax and $g(x) = \frac{x^2}{4a}$ in the first quadrant. [8]

END OF QUESTION PAPER