BINDURA UNIVERSITY OF SCIENCE EDUCATION

MT101: CALCULUS 1

AUG 2024.

Time : 3 hours

Answer ALL questions in Section A and at most TWO questions in section B.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

A1. Verify that $f(x) = x^3 - x^2 - 6x + 2$ satisfies the hypothesis of Rolle' theorem for the interval $[0, 3]$, then find all c that satisfy the conclusion. [7]

A2. Prove that if a sequence converges, then its limit is unique. [8]

A3. Solve the inequality $|x - 4| < 3$. [5]

A4. Given $f(x) = \frac{x+1}{x-1}$, prove that this function is bijective. [10]

A5. (a) Write down any six indeterminate forms. [3]

(b) Find the area bounded by $y = cos(x), y = 0, x = 0$ and $x = \frac{3\pi}{2}$. [7]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B6 to B8.

B6. (a) Evaluate $lim_{x \to \infty}(\frac{sin(x)^2}{xsin(x)})$. [5]

(b) Give an $\epsilon - \delta$ definition of the limit of a function. [3]

(c) Use the $\epsilon - N$ definition of a sequence to show that a sequence whose n^{th} term is given by $a_n = 3 - \frac{1}{7n^2}$ converges to 3. [5]

(d) Show that the sequence $U_n = \frac{2n-7}{3n+2}$ is monotonic increasing. [5]

(e) State the Mean Value Theorem of differential calculus. [2]

(f) Given that $f(x) = \sqrt{25-x^2}$ satisfies the Mean Value Theorem on the interval $[-3, 4]$, find the value $C \in [-3, 4]$. [3]

(g) Given that $y = \frac{x^4}{4x-3}$, sketch the graph showing clearly its asymptotes and stationary points. [7]

B7. (a) Verify that the function $f(x) = x^3 + x - 1$ satisfies the Mean Value Theorem in the interval $[0, 2]$. [10]

(b) The graph of the parabola $x = y^2 - 2$ is NOT the graph of a function of x. Explain why? [4]

(c) Find the relative extrema for the function $f(x) = 2x^3 - 3x^2 - 36x + 14$. [6]

(d) Consider the function $f(x) = \frac{x^2-4}{x^2-1}$. State the domain and range of the function. [5]

(e) Determine the derivative of $y = sin^{-1}(x)$. [5]

B8. (a) State the Second Fundamental theorem of calculus. [3]

(b) Show that $\int \frac{1}{a^2+b^2x^2}dx = \frac{1}{ab}arctan(\frac{bx}{a}) + k$. [6]

(c) hence evaluate $\int \frac{1}{2x^2+9}dx$. [3]

(d) Evaluate the following integrals:

(i) $\int_0^{\frac{1}{2}} arcsin(x)dx$. [5]

(ii) $\int \frac{2x^2-5x+2}{x^3+x}dx$. [5]

(e) Find the area of the region bounded by $f(x) = 4ax$ and $g(x) = \frac{x^2}{4a}$ in the first quadrant. [8]

END OF QUESTION PAPER