NOV 2024

BINDURA UNIVERSITY OF SCIENCE EDUCATION

DEPARTMENT OF STATISTICS AND MATHEMATICS

AEH202/MTE2101/EEE3101: ENGINEERING MATHEMATICS 3

Time: 3 hours

Candidates may attempt ALL questions in Section A and Section B. Each question should start on a fresh page.

SECTION A (40 marks)

1. Starting from the generating function of the Bessel function of the first kind

$$e^{\frac{1}{2}x(t-\frac{1}{t})} = \sum_{n=-\infty}^{\infty} [t^n J_N(X)], n \in \mathbf{Z}$$

Show that

$$\acute{J}_n(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$$

[10]

2. Use integration to find the Laplace Transform of

$$f(t) = t^n, t \geq 0$$

where

$$n \neq \ldots - 4, -3, -2, -1, 0.$$

[12]

3. The generating function for the Legendre's polynomials $P_n(x)$, satisfies

$$(1 - 2xt + t^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty}[t^n P_n(x)].$$

Use this relationship to prove that

$$P_n(-x) = (-1)^n P_n(x).$$

[10]

4. 20 grams of salt are dissolved into a beaker containing 1 litre of a certain chemical.
The mass of salt, M grams, which remains undissolved t seconds later, is modelled by the differential equation

$$\frac{d\mathrm{M}}{dt} + \frac{2\mathrm{M}}{20-t} + 1 = 0, t \geq 0.$$

Show clearly that

$$\mathrm{M} = \frac{1}{10}(10 - t(20-t).$$

[8]

SECTION B (60 marks)

5. The region R in the $x-y$ plane is the ellipse with equation

$$2x^2 + xy + 2y^2 = 15$$

The surface with equation $z = f(x,y)$ is given by

$$f(x,y) = xy, x \in \mathbf{R}, y \in \mathbf{R}.$$

Determine the maximum value of f and the minimum value of f whose projection onto the $x-y$ plane is the region R.
Give the corresponding x and y coordinates in each case. [15]

6. Solve the heat equation for $u = u(x,t)$

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}, 0 \leq x \leq 5, t \geq 0,$$

Subject to the conditions
$u(0,t) = 0$, $u(5,t) = 0$ and $u(x,0) = \sin \pi x - 37\sin(\frac{1}{5}\pi x) + 6\sin(\frac{9}{5}\pi x)$.
You must derive the standard solution of the heat equation in variable separate form. [15]

7.

$$f(x) = xe^{-2x}, x \geq 0.$$

Find, by direct integration, the Fourier transform of $f(x)$. [10]

8.
$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0$$

The above partial differential equation is Laplace's equation in two dimensional Cartesian system of coordinates.
Show clearly that Laplace's equation in the standard two dimension Polar system of coordinates is given by

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2} = 0.$$

[20]

End