Bindura University of Science Education

Faculty of Science Education

JAN 2025

Science and Mathematics Education Department

Programme: HBSc Ed (Mathematics)

Course: MT320: Algebra | Duration: Three hours

Semester Examinations

Instructions to candidates

(i) Answer all questions in Section A and two questions from Section B.

(ii) Begin each question on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

A1. Define the following
- **(a)** An equivalence relation [2]
- (b) Homomorphism of a group [2]
- **(c)** A ring [7]

A2. Distinguish between
- (a) Group and a field [8]
- **(b)** Bijection and injection [4]

A3. Let A be a set of non-zero integers and let ~be a relation on $A \times A$ defined by $(a_1, a_2) \sim (b_1, b_2)$ whenever $a_1 b_2 = a_2 b_1$. Show that $\sim$ is an equivalence relation. **[8]**

A4. Suppose $(G,*), where\ G = \{1, -1, i, -i\}\ and\ H = \{-1, -i\}$. List the left cosets of H in G. [4]

A5. Let H and K be subgroups of G. Show that $H \cap K$ is also a subgroup of G. [5]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them **B6 to B8**

B6. (a) (i) Draw a Cayley table of $(Z_5, +)$ [5]

(ii) Is $(Z_5, +)$ a group? Give reasons. [4]

(iii) State the neutral element of $(Z_5, +)$ [2]

(b) Show that the inverse element of a group is unique. [5]

(c) Define a mapping and give two types of mappings. [4]

(d) If S is the set of all 2×2 matrices of the form $\begin{pmatrix} a & b \\ 2b & a \end{pmatrix}$ where $a, b \in \mathbb{R}$. Show that S forms a group under addition of matrices. [10]

B7. (a) Define the terms

(i) Homomorphism of a group [2]

(ii) Monomorphism of a group [2]

(iii) Isomorphism of a group [2]

(b) Let $(G, *)$ and $(H, +)$ be groups

(i) Define $(G \times H, \Delta)$ the direct product of G and H. [4]

(ii) Show that $(G \times H, \Delta)$ is a group [10]

(iii)show that G and H are abelian if and only if $G \times H$ is abelian [6]

(c) Prove that $\theta: R \to S$ is a ring homomorphism then $ker\theta$ is an ideal of R [4]

B8. (a) Let G be any group and $H \leq G$. We say that x is congruent to y modulo H (written $x \equiv y \ (mod\ H)$ if $x^{-1}y \in H, where\ x, y \in G$. Prove that the congruency modulo H is an equivalence relation in G. [6]

(b) Suppose $\mathcal{J}\sqrt{2} = \{m + n\sqrt{2}: m, n \in \mathbb{Z}\}$, then show that $\mathcal{J}\sqrt{2}$ is a homomorphism of a ring [10]

(c) let K be a ring of all 2×2 matrices of the form $\begin{pmatrix} y & x \\ -x & y \end{pmatrix}$ $where\ x, y \in \mathbb{R}$ and we have a field of complex numbers. Define a mapping

$$\wp: \mathbb{C} \Rightarrow K$$
$$x + iy \Rightarrow \begin{pmatrix} y & x \\ -x & y \end{pmatrix}$$

Show that $\wp$ is an isomorphism. [10]

(a) Let $f: G \to H$ be a homomorphism then show that ker (f) is a subgroup of G [4]

END OF PAPER