BINDURA UNIVERSITY OF SCIENCE EDUCATION

DME001 : INTRODUCTORY MATHEMATICS

JUN 2024

Time : 3 hours

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A6.

A1. (a) Evaluate the following indicial expression, giving the final answers as an exact simplified fraction: $125^{\frac{2}{3}} \times 625^{\frac{3}{4}}$ [3]

(b) Solve the following logarithmic equation for x, $\log(x-1) + \log x = \log_2 1$. [4]

A2. Given that $z_1 = 3 + 2i$ and $z_2 = 5 - 7i$, find:

(a) $z_1 + z_2$ [1]

(b) $z_1 z_2$ [2]

(c) $\dfrac{z_1}{z_2}$ [2]

A3. Simplify the following:

(a) $(2 + \sqrt{5}) + (5 + 2\sqrt{3})$ [2]

(b) $(2 + \sqrt{5})(3 - \sqrt{5})$ [3]

(c) $\dfrac{8}{\sqrt{2} + 5}$ [3]

A4. (a) Find the remainder when $3x^3 - x^2 - 5x + 2$ is divided by $2x + 2$. [2]

(b) Given $x^2 + 2x - 3$ is a factor of $f(x)$, where $f(x) \equiv x^4 + 6x^3 + 2ax^2 + bx - 3a$. Find the value of a and of b. [4]

A5. (a) Find the set of values of x , that satisfy the following inequality.

$\frac{3}{x-4} \geq 1$ [4]

(b) Express $\frac{13}{(2x+3)(x^2+1)}$ into partial fractions. [5]

A6. Given a curve, $y = ax^2 + bx + c$, the curve passes through $(0, -4)$, its gradient at $x = -0.5$ is -2 and the second derivative is 10, find the constants a, b, and c. [5]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B7 to B9.

B7. (a) Find $\frac{dy}{dx}$ in each of the following:

(i) $y = 4x^4 - 5x^{-3} + 2x^3$. [2]

(ii) $y = \frac{2e^x}{2x-2e^x}$. [4]

(iii) $y = In(7x^2 - 3)^4$. [4]

(b) A curve is defined by the parametric equations:
$x = 120t - 4t^2$ and $y = 60t - 6t^2$.
Find the value of $\frac{dy}{dx}$ at each of the points where the curve crosses the $x - axis$. [9]

(c) Find the coordinates of the stationery points whose equation is $y = 2x^3 - 3x^2 + 2$ and determine their nature. [11]

B8. (a) Find the equation of the tangent to the curse $y = 8x - x^2$ at the point $(2; 12)$ [5]

(b) Show that $\tan(2A) = \frac{2\tan(A)}{1-\tan^2(A)}$ [9]

(c) A curve is given by $y^3 + y^2 + y = x^2 - 2x$.

(i) Show that at the origin, $\frac{dy}{dx} = -2$ and $\frac{d^2y}{dx^2} = -6$ [5]

(ii) Hence or otherwise give the McLaurin's series for y as far as x^2. [5]

(d) Calculate the area under the graph $y = 4x - x^3$ between $x = 0$ and $x = 2$. [6]

B9. (a) Express the equation $5sin3x = 4cos3x$ in the form $tan3x = k$ where k is a constant. [2]

(b) For the following, find:

(i) $\int (x^2 + \frac{4}{x^3} - 5)dx$ [2]

(ii) $\int \frac{x^2+4}{x^2}dx$ [3]

(iii) $\int sinx In(cosx)dx.$ [4]

(iv) $\int_0^1 xe^{x^2}dx.$ [7]

(c) A curve has an equation $y = (4-x^2)^{-\frac{1}{2}}$ for $-1 \leq x \leq 1$. The region R is enclosed by $y = (4-x^2)^{-\frac{1}{2}}$, the x-axis and the line $x = -1$ and $x = 1$. Find the exact value of the area R. [5]

(d) The sum to infinity of a geometric series is three times the first term of the series. The first term of the series is a.

(i) Show that the common ratio of the geometric series is $\frac{3}{2}$ [2]

(ii) The third term of the geometric series is 81.

(a) Find the sixth term of the series. [2]

(b) Find the value of a as a fraction. [3]

END OF QUESTION PAPER