BINDURA UNIVERSITY OF SCIENCE EDUCATION

PHYSICS AND ENGINEERING DEPARTMENT

MPH503: ELECTRICITY AND MAGNETISM [3]

TIME: 3 HOURS

AUG 2024

INSTRUCTIONS

Answer **question one** in Section A and **any three** questions from Section B. Section A carries 40 marks and each question in Section B carries 20 marks.

Physical constants

Electronic charge, $e = 1.6 \times 10^{-19}\ C$
Boltzmann's constant, $k = 1.38 \times 10^{-23}\ J\,K^{-1}$
Mass of an electron, $m_e = 9.11 \times 10^{-31}\ kg$
Permittivity of free space, $\varepsilon_o = 8.85 \times 10^{-12}\ F\,m^{-1}$
Permeability of free space, $\mu_o = 4\pi \times 10^{-7}\ H\,m^{-1}$
Electric potential at infinity, $V_\infty = 0$
Coulomb's constant, $k_e = 9 \times 10^{9}\ N\,m\,C^{-2}$

SECTION A

1 (a) What is the equation in cylindrical coordinates of a cone $x^2 + y^2 = z^2$? [4]

(b) Convert the point (6, 8, 4.5) in Cartesian coordinate system to cylindrical coordinate system. [6]

(c) Compute the divergence of the vector field $\vec{F}(x, y, z) = x^2y\vec{\imath} + xyz\vec{\jmath} - x^2y^2\vec{k}$. [6]

(d) Determine the curl of the vector field $\vec{E} = yz^2\vec{\imath} + xy\vec{\jmath} + yz\vec{k}$. [6]

(e) Given that $\boldsymbol{F}(x, y, z)$ and that $\boldsymbol{G}(x, y, z)$ are vector fields, what, if anything, is wrong with the following expression: $\nabla \times \boldsymbol{G} = \nabla \cdot \boldsymbol{F}$. [4]

(f) Define a conservative vector field?. Determine whether the following vector field is conservative or not:

$\vec{B} = x^2y\vec{\imath} + xyz\vec{\jmath} - x^2y^2\vec{k}$. [7]

(g) What are the limitations of Coulomb's law of electrostatics? Give any three. [3]

(h) A long straight wire has a circular loop with a radius of 5 *cm*. The current flowing through a closed loop of this wire is 2 *A*. Show that the magnetic field due to this loop has magnitude 8×10^{-6} T. [4]

SECTION B

2 (a) The electric field at 2 *cm* from the centre of long copper rod of radius 1 *cm* has a magnitude 3 *N/C* and directed outward from the axis of the rod.

(i) How much charge per unit length exists on the copper rod? [3]

(ii) What would be the electric flux through a cube of side 5 *cm* situated such that the rod passes through opposite sides of the cube perpendicularly? [3]

(b) The charge density $\rho(r)$ of a solid ball of radius R depends on the radial coordinate and is given by $\rho(r) = Cr^{\alpha}$ where C and α are constants. Show that

(i) the net charge Q_{net} of such a ball is $Q_{net} = \frac{4\pi}{\alpha+3} CR^{\alpha+3}$. [6]

(ii) outside the ball ($r > R$), the electric field is $E(r) = \frac{Q_{net}}{4\pi\varepsilon_o r^2}$. [2]

(iii) inside the ball ($r < R$), the electric field is $E(r) = \frac{Q_{net}}{4\pi\varepsilon_o R^2}\left(\frac{r}{R}\right)^{\alpha+1}$. [6]

3 (a) For a point charge Q, the potential V is given by

$$V = \frac{Q}{4\pi\varepsilon_o r}$$

Verify, using spherical polar coordinates, that

$$\vec{E} = -\vec{\nabla}V = \frac{Q}{4\pi\varepsilon_o r^2}\hat{r}$$ [6]

(b) A certain magnetic field (in cylindrical polar coordinates) is given by

$$\boldsymbol{B} = B_o \frac{\rho}{1+\rho^2}\hat{\phi} + \alpha\hat{z}$$

where B_o and are α constants.

(i) show that $\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$ [6]

(ii) find the current $\boldsymbol{I}$. [8]

4 A circular loop of radius R in the xy plane carries a steady current I, as shown in Fig. 4.1.

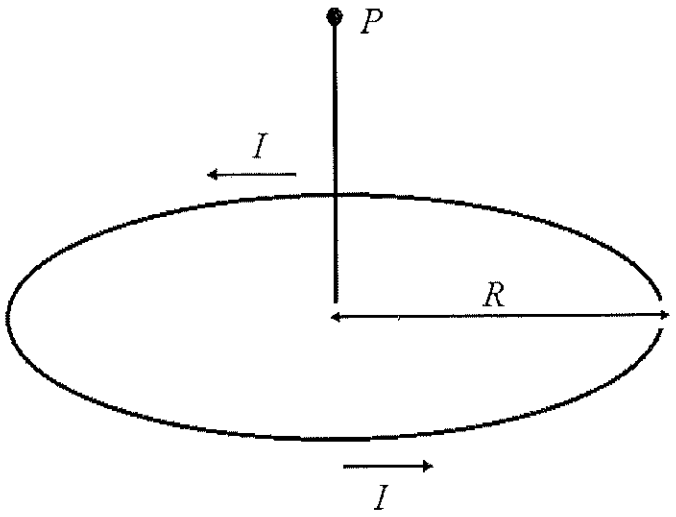

Fig. 4.1

Show that the magnetic field at a point P on the axis of the loop, at a distance z from the centre is $\frac{\mu_o IR^2}{2(R^2+z^2)^{3/2}}$ [20]

5 You are required to use Ampere's law in this question. Fig. 5.1(a) shows a long, straight wire of radius a. The wire carries a current I_o which is distributed uniformly over its cross-section. A cross-section of the same wire showing the radius a and the Ampere's loop of radius r is shown in Fig. 5.1(b).

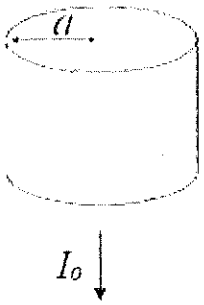

Fig. 5.1(a)

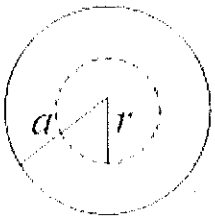

Fig. 5.1(b)

(a) Show that the magnetic field inside the wire is $B = \frac{\mu_o I_o}{2\pi}\frac{r}{a^2}(r \leq a)$ and magnetic field outside the wire is $B = \frac{\mu_o I_o}{2\pi r}(r \geq a)$. [15]

(b) Sketch a graph to show the variation of B with r. [5]

6 The rectangular conducting loop shown in Fig. 6.1 rotates at 6000 revolutions per minute in a uniform magnetic field of flux density given by

$$\boldsymbol{B} = \hat{y}50\ mT.$$

The loop has an internal resistance is 0.5 Ω.

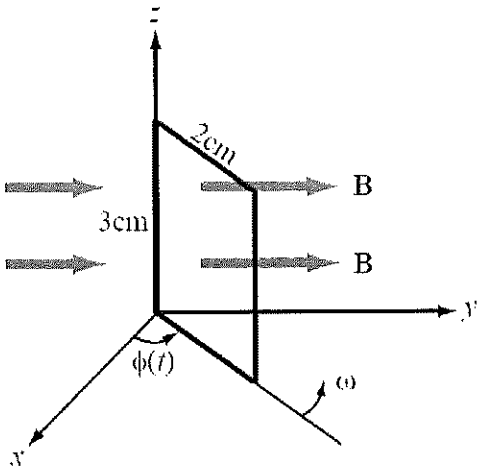

Fig. 6.1

(a) Show that the current induced in the loop is 37.7 sin (200πt) *mA*. [15]

(b) Determine the direction of the current. [5]

END OF EXAMINATION