BINDURA UNIVERSITY OF SCIENCE EDUCATION

Faculty of Science and Engineering

Department of Engineering and Physics

BACHELOR OF SCIENCE EDUCATION HONOURS DEGREE

Physics

AUG 2024

PH104

Optics and Modern Physics

Duration: Three (3) Hours

Answer ALL parts of Section A and any THREE questions from Section B. Section A carries 40 marks and each question of Section B carries 20 marks.

Clearly show ALL working

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

SECTION A

Attempt all parts of question 1

1. (a) The first excited state of an iron atom ^{53}Fe collapses to the ground state by emitting a photon of energy 14.4 keV. The lifetime of the excited state is 141 ns. model. Estimate the linewidth ΔE (in eV) of the spectral line and find the recoil energy (in eV) of the atom.

[5]

(b) A Bohr hydrogen atom at rest in free space undergoes a transition from The energy level $n = 2$ to $n = 1$, emitting a photon in the positive x-direction. Find the recoil velocity of the atom. [Hint: You may need to use the first two terms of the expansion, $\sqrt{1+x} \approx 1 + x/2$.]

[5]

(c) To image a virus of linear size d, through what voltage must electrons (of mass m and charge e) in an electron microscope be accelerated. Assume that the electrons' motion is non-relativistic and express your answer in terms of d, e, m and $\hbar$.

[5]

(d) A photon strikes at electron at rest and scatters from it elastically. If the photon ends up with a wavelength equal to three times the Compton wavelength and loses half its energy in the collision, at what angle relative to its initial propagation direction is the photon scattered? (Express this angle in radians, between 0 and π.)

[5]

(e) An optical system consists of two thin lenses in air, separated by a distance d. The lenses have focal lengths f_A and f_B, which may be positive and or negative. Assume the input and output reference planes are located on the lenses. Find the system matrix and use it to deduce the system matrix for back-to-back lenses. What is the effective focal length of the composite lens?

[6]

(f) A small object is placed on axis at a distance of 16 cm from the left end of a long, plastic rod with a polished spherical end of radius 4 cm as indicated in Figure 1.

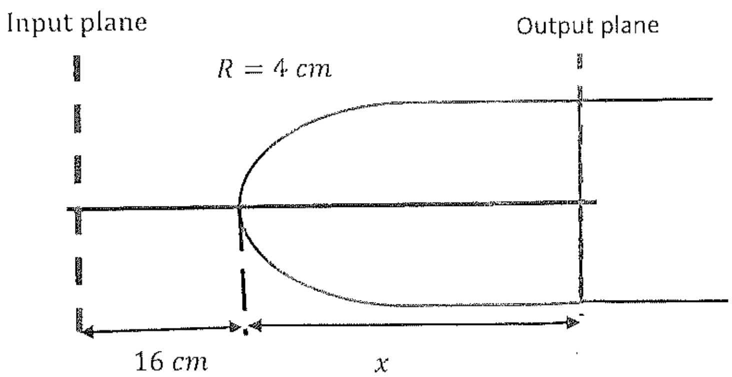


Figure 1: Schematic defining the problem

The index of refraction of the plastic is 1.50 and the object is in air. The image is formed at the output reference plane. Find the image distance x and the lateral magnification, m.

[8]

(g) A thick lens whose index of refraction is n_L and whose thickness is t, is immersed in an environment of index of refraction n The ray transfer matrix for the tick lens is:

$$\mathbb{M} = \begin{bmatrix} 1 & 0 \\ \dfrac{n_L - n}{nR_2} & \dfrac{n_L}{n} \end{bmatrix} \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{n - n_L}{n_L R_1} & \dfrac{n}{n_L} \end{bmatrix}$$

Deduce the thin lens ray-transfer matrix in terms of the lensmaker's formula.

[6]

SECTION B

Attempt any three (3) questions

2. (a) Assuming that all are small with respect to the horizontal, derive the transfer matrix which acts on a ray of light crossing a flat interface between two materials with indices of refraction n_1 and n_2.

[5]

(b) An object is located a distance a to the left of a single flat interface between two materials with indices of refraction n_1 and n_2. A distance b to the right of this interface is a concave mirror with radius R.

(i) Use the matrix method to find the location of the image (that is, the horizontal distance x from the mirror.)

[10]

(ii) Show that the condition that must be satisfied for a real image is

$$\frac{2}{R}\left(a + b\frac{n_1}{n_2}\right) > \frac{n_1}{n_2}$$

Check the limiting case $n_1 = n_2$.

[5]

3. (a) Explain the Heisenberg Uncertainty Principle (HUP) in position and momentum, $\Delta p \Delta x \geq \hbar/2$.

[4]

(b) Show that the uncertainty principle for a free particle in one-dimension can be written as $\Delta\lambda\Delta x \geq \lambda^2/4\pi$ where the symbols have their usual meanings.

[6]

(c) A particle of mass m moves in one-dimensional potential

$$V(x) = F_0|x|$$

where F_0 is a positive constant. Use the HUP to estimate the minimum total energy of the particle as a function of m, F_0 and $\hbar$.

[10]

4. (a) Write down the mathematical expression proposed by Erwin Schrodinger in 1926 that describes how matter waves propagate on the line $x \in [-\infty, +\infty]$. Define all symbols used.

[3]

(b) A particle of mass m propagates on the line $x \in [-\infty, +\infty]$ and has the following quantum wave function at some time

$$\psi(x) = \begin{cases} 0 & \text{for } x < -a \\ A(a - |x|) & \text{for } -a \leq x \leq +a \\ 0 & \text{for } x > a \end{cases}$$

(i) Find A using the normalisation condition.

[4]

(ii) Find the probability that a measurement of the particle's position will reveal it to be in the range $[0, a/2]$.

[5]

(iii) Calculate $\langle x \rangle$ and $\langle x^2 \rangle$ for this state.

[8]

5. A ray of light starts at a distance d_1 from a thin lens (of focal length f) and is observed a distance d_2 away from the lens on its right hand side.

(a) Derive the ray transfer matrix, $\mathbb{T}_{s-\ell-s}$.

[5]

(b) Show that the imaging condition leads to the lens formula

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}$$

[4]

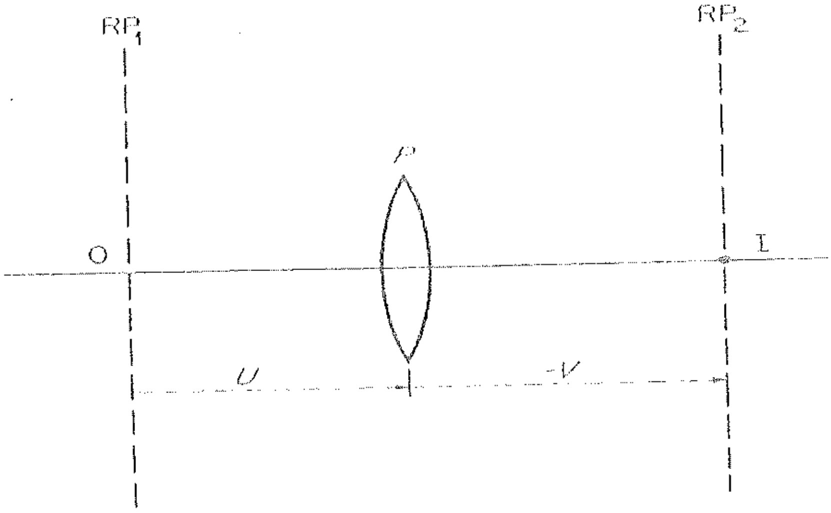
(c) Find the magnification. Is the image upright or inverted? Explain your answer.

[5]

(d) Describe the form of the image for the case of $d_1 < f$.

[4]

6. An object is located at a distance U to the left of a thin lens and its image is formed a distance V to the left of the same lens as shown in the figure below:



(a) If now the object is displaced axially a small distance dU to the left, find an expression for the longitudinal magnification dU/dV.

[6]

(b) Prove that the distance between the real object and real image formed by a thin positive lens cannot be less than four times the focal length.

[8]

(c) An optical tracking device has a lens of focal length 10 cm that forms the image of a moving object on a screen. If the object is moving away at a speed of 10 ms^{-1}, how fast must the lens be moving and in what direction in order to keep the object in focus at the instant when it is 80 cm away from the lens.

[6]