BINDURA UNIVERSITY OF SCIENCE EDUCATION SFM214

BACHELOR OF STATISTICS AND FINANCIAL MATHEMATICS

MULTIVARIATE METHODS

APR 2025

Time : 3 hours

Candidates should attempt ALL questions in section A and at most TWO questions in section B.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A6

A1. (a) Define multivariate analysis. [2]

(b) State any three applications of multivariate techniques. [3]

A2. The following are five measurements on the variables x_1, x_2, and x_3.

x_1	9	2	6	5	8
x_2	12	8	6	4	10
x_3	3	4	0	2	1

Find the

(a) mean vector matrix, $\bar{\mathbb{X}}$ [2]

(b) sample variance-covariance matrix, $\mathbb{S}_n$ [4]

(c) correlation matrix, $\mathbb{R}$. [3]

A3. Consider a random vector $\mathbb{X}' = [X_1, X_2, X_3, X_4]$ with mean vector $\mu'_X = [3, 2, -2, 0]$ and variance-covariance matrix

$$\sum_X = \begin{bmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{bmatrix}.$$

Let

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & 1 & 1 & -3 \end{bmatrix}$$

(a) Find E(AX), the mean of AX. [2]

(b) Find Cov(AX), the variances and covariances of AX. [3]

A4. (a) Let X be a normally distributed random vector with

$$\mu = \begin{pmatrix} -3 \\ 1 \\ -2 \end{pmatrix} \text{ and } \sum = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

Determine whether the following are independent or not and justify your answers.

(i) X_1 and X_3 [2]

(ii) $X_1 + X_3$ and $X_1 - 2X_2$ [3]

(iii) X_1 and $X_1 + 3X_2 - 2X_3$. [3]

(b) Let X be $\mathbb{N}_3(\mu, \sum)$ with $\mu = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\sum = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 3 & 3 \end{pmatrix}$

Find the distribution of $3X_1 - 2X_2 + X_3$. [3]

A5. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$. Find the eigenvalues λ_1 and λ_2 of A, and the associated normalized eigenvalues $\hat{e}_1$ and $\hat{e}_2$. [6]

A6. Suppose

$$\sum = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{bmatrix}$$

is the variance-covarince matrix for random variables X_1, X_2 and X_3. Find the correlation matrix $\mathbb{R}$ for $\sum$. [4]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B7 to B9.

B7. (a) Consider the seven pairs of measurements of two variables x_1 and x_2.

Variable 1 (x_1)	3	4	2	6	8	2	5
Variable 2 (x_2)	5	5.5	4	7	10	5	7.5

(i) Construct a scatter plot of the data and the marginal dot diagrams. [4]

(ii) Infer the sign of the sample covariance S_{12} from the scatter plot. [2]

(b) Consider the following five pairs of measurements of two variables x_1 and x_2.

Variable 1 (x_1)	9	2	6	5	8
Variable 2 (x_2)	3	4	0	2	1

Find the sample mean vector matrix $\bar{\mathbb{X}}$, the sample variance-covariance matrix $\mathbb{S}_n$ and the sample correlation matrix $\mathbb{R}$. [13]

(c) Consider a bivariate normal population with mean $\mu_1 = 0$, $\mu_2 = 2$, $\sigma_{11} = 2$, $\sigma_{22} = 1$, and $\rho_{12} = 5$.

(i) Find the bivariate normal density function for the above normal population. [6]

(ii) Write out the squared generalised distance expression $(x-\mu)'\sum^{-1}(x-\mu)$ as a function of x_1 and x_2. [2]

(iii) Determine and sketch the constant density-contour that contains 50% of the probability for (ii) above. [3]

B8. (a) You are given the random vector $\mathbb{X}' = [X_1, X_2, X_3, ...X_5]$ with mean vector $\mu'_X = [2, 4, -1, 3, 0]$ and variance-covariance matrix

$$\sum = \begin{bmatrix} 4 & -1 & \frac{1}{2} & -\frac{1}{2} & 0 \\ -1 & 3 & 1 & -1 & 0 \\ \frac{1}{2} & 1 & 6 & 1 & -1 \\ -\frac{1}{2} & -1 & 1 & 4 & 0 \\ 0 & 0 & -1 & 0 & 2 \end{bmatrix}$$

Partition X as

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \dots \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} X^{(1)} \\ \dots \\ X^{(2)} \end{bmatrix}$$

Let $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$ and consider the linear combinations $AX^{(1)}$ and $BX^{(2)}$.

Find

(i) $\mathrm{E}(\mathrm{AX}^{(1)})$. [3]

(ii) $\mathrm{E}(\mathrm{BX}^{(2)})$. [3]

(iii) $\mathrm{Cov}(BX^{(2)})$. [3]

(iv) $\mathrm{Cov}(AX^{(1)})$. [3]

(v) $\text{Cov}(AX^{(1)}, BX^{(2)})$ [6]

(b) Three observations were done on two random samples X_1 and X_2 and the results are given below:

$$X = \begin{pmatrix} 6 & 9 \\ 10 & 6 \\ 8 & 3 \end{pmatrix}$$

(i) Evaluate the observed Hotelling's two sample T^2-statistic for $\mu_0' = [9, 5]$ [8]

(ii) Find the sampling distribution of T^2 in (i) above. [4]

B9. (a) Consider a sample of 10 observations recorded on two variables X_1 and X_2. Suppose that the mean vector matrix for the sample is

$$\bar{\mathbb{X}} = \begin{pmatrix} 192 \\ 278.4 \end{pmatrix}$$

and the sample variance-covariance matrix is

$$\mathbb{S} = \begin{pmatrix} 121.78 & 76.3 \\ 76.3 & 92.9 \end{pmatrix}.$$

Calculate the 95% simultaneous confidence interval for the mean μ. [12]

(b) Consider the following independent samples of populations subjected to three different treatments.

Population 1	6	5	8	4	7
Population 2	3	1	2		
Population 3	2	5	3	2	

(i) Breakdown the observations into $x_{ij} = \bar{x} + (\bar{x}_i - \bar{x}) + (x_{ij} - \bar{x}_i)$. [7]

(ii) Construct an ANOVA table for the data. [8]

(iii) Determine whether there is treatment effect on the sample data at 1% level of significance. [3]

END OF QUESTION PAPER