BINDURA UNIVERSITY OF SCIENCE EDUCATION MT102

BACHELOR OF SCIENCE EDUCATION

LINEAR MATHEMATICS 1 AUG 2024

Time: 3 Hours

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B.

Each question should start on a fresh page.

**SECTION A** (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A6.

A1. Given that **u** = 3**i**−2**j**+6**k** and **v** = 2**i**+3**j** − 4**k**, find **u** × **v** and a unit vector perpendicular to the plane containing the vectors **u** and **v**. [5]

A2. Find the cartesian equation of a line in space which passes through the points P(2, 3, −2) and Q(5,0, −2). [4]

A3. Given $z_1 = 2 + i$ and $z_2 = 4 - 2i$. Find

(a) $z_1\overline{z_2}$ (b) $\frac{z_1}{z_2}$. [5]

A4. (i) Find $A^{-1}$ (the inverse of A) where $A = \begin{bmatrix} -1 & 3 & 6 \\ 2 & 5 & -2 \\ 4 & 1 & 3 \end{bmatrix}$. [7]

(ii) Let A, B be square matrices. Outline any six properties of determinants. [4]

A5. Find the solution of the following system of linear equations using Gauss elimination method

$$x_1 + 2x_2 + 3x_3 = 8$$
$$2x_1 + 3x_2 + 2x_3 = 10$$
$$3x_1 + x_2 + 2x_3 = 7.$$ [6]

A6. (a) Find the real numbers $x$ and $y$ for which

$$\frac{x}{1+2i} + \frac{y}{3+2i} = \frac{5+6i}{-1+8i}.$$

[5]

(b) Show that $-1 + \sqrt{3i}$ is a solution of the equation: $z^4 + 5z^2 + 2z + 20 = 0$. [4]

**SECTION B (60 marks)**

Candidates may attempt TWO questions being careful to number them B7 to B9.

B7. (a) (i) Define a matrix which is in row echelon form. [4]

(ii) Find the solution to the linear system of equations,

$$x_1 + 2x_2 + 3x_3 = 17$$
$$3x_1 + 2x_2 + x_3 = 1$$
$$3x_1 - 5x_2 + x_3 = -5$$

using Crammers rule. [8]

(b) Solve for $x$ given that $\begin{vmatrix} x & -1 \\ 3 & 1-x \end{vmatrix} = \begin{vmatrix} 1 & 0 & -3 \\ 2 & x & -6 \\ 1 & 3 & x-5 \end{vmatrix}$. [5]

(c) Determine whether the following matrix is singular or not where $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 4 & 1 \end{bmatrix}$. [3]

(d) Solve the following system of simultaneous equations using Gauss elimination.

$$x - 3y + 2z + w = -4$$
$$2x - 6y + z + 4w = 1$$
$$-x + 2y + 3z + 4w = 12$$
$$-y + z + w = 0.$$

[10]

B8. (a) Given the points $A\ (0,1,0)$; $B(1,2,1)$; and $C\ (2,2,-1)$, find

(i) the angle $B\,\widehat{A}\,C$ formed in the triangle ABC,

(ii) the area of the triangle ABC.

(iii) the equation of the plane that passes through the points A, B, C. [12]

(b) Find the volume of the parallelepiped formed by the vectors $\boldsymbol{u} = <1, 2, 4>, \boldsymbol{v} = <2, 4, 1>$ and $\boldsymbol{w} = <5, 1, 0>$. [4]

(c) Given that $\boldsymbol{a} = 3\boldsymbol{i} - 2\boldsymbol{j} + 6\boldsymbol{k}$ and $\boldsymbol{b} = 2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$, find $\boldsymbol{a}\,.\,\boldsymbol{b}$. [2]

(d) Find the parametric equation of the line $l_3$ that is parallel to the vector $3i - 2j + 7k$ and it passes through the point of intersection of the lines;

$$l_1: F = 7i + 6j + 8k + t(5i + 4j + 5k)$$
$$l_2: \frac{x-8}{6} = \frac{y-7}{4} = \frac{z-9}{6}.$$

[7]

(e) If $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{u} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, Find $|(2\mathbf{v} + \mathbf{u}) \times (\mathbf{v} - 2\mathbf{u})|$. [5]

B9. (a) Express in the form $a + bi$ the complex number $\dfrac{-4+6i}{5-i}$. [2]

(b) Solve the equation $z^4 + z^3 + 3z^2 + 7z + 20 = 0$ given that $1 + 2i$ is a root of the equation. [6]

(c) (i) Use Euler's formula to show that $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ and $\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$. [3]

(ii) Hence show that $\sin^4\theta = \frac{1}{8}\cos 4\theta - \frac{1}{2}\cos 2\theta + \frac{3}{8}$. [6]

(c) (i) State De Moivres's formula. [2]

(ii) Prove the following identity using Demoivre's theorem

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$

[6]

(d) Express the complex number $-1 + \sqrt{3}i$ in polar form, and find its modulus, argument and conjugate, illustrating these on the argand diagram. [5]

**END OF QUESTION PAPER**