BINDURA UNIVERSITY OF SCIENCE EDUCATION

BACHELOR OF SCIENCE IN OPTOMETRY

OPTC102: FOUNDATION OF MATHEMATICS

Time: 3 hours

Candidates may attempt **ALL** questions in Section A and at most **TWO** questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them 1 to 6.

1. Solve for x if

$$\begin{vmatrix} x & -1 \\ 3 & 1-x \end{vmatrix} = \begin{vmatrix} 1 & 0 & -3 \\ 2 & x & -6 \\ 1 & 3 & x-5 \end{vmatrix}$$

[4]

2. If $(x+iy)^2 = 3+4i$ find x and y, where $x, y \in \mathbb{R}$ [5]

3. (a) Define a scalar and vector product of two vectors $\mathbf{u}$ and $\mathbf{v}$ and illustrate these geometrically. [4]

   (b) Given $\mathbf{u} = 3i + 2j + 5k$ and $\mathbf{v} = 6i + 7j + 9k$, find $\mathbf{u} \times \mathbf{v}$ and a unit vector perpendicular to the plane containing the vectors $\mathbf{u}$ and $\mathbf{v}$. [4]

4. (a) Let $\mathbf{A} = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$. Find $\mathbf{A}^{-1}$. [4]

   (b) What do you understand by the order. type, homogeneity and linearity of differential equations. [3]

5. Reduce the following system of linear equations to its row-achelon-form so that the solution to the system is self evident.

$$\begin{aligned} x_1 + 3x_2 + 5x_3 &= 14 \\ 2x_1 - x_2 - 3x_3 &= 3 \\ 4x_1 + 5x_2 - x_3 &= 7 \end{aligned}$$

[8]

6. Prove using Euler's formula i.e $e^{i\theta} = \cos\theta + i\sin\theta$ that

   (a) $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$. [4]

   (b) $\sin\theta = \frac{1}{2}(e^{i\theta} - e^{-i\theta})$. [4]

SECTION B (60 marks)

Candidates may attempt **TWO** questions being careful to number 7 to 9.

7. (a) Use Gauss elimination to find the rank of the matrix **A** and a solution to the following system of inhomogeneous simultaneous linear equations, $\mathbf{A}\mathbf{x} = b$.

$$\begin{aligned} 2x_1 + x_2 + 2x_3 + x_4 &= 5 \\ 4x_1 + 3x_2 + 7x_3 + 3x_4 &= 8 \\ -8x_1 - x_2 - x_3 + 3x_4 &= 4 \\ 6x_1 + x_2 + 2x_3 + x_4 &= 1 \end{aligned}$$

[10]

   (b) Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}$, calculate A^TA. [2]

   (c) Find the solution of the linear system:

$$\begin{aligned} 2x - 3y + z &= 0 \\ 5x + 4y + z &= 10 \\ 2x - 2y - z &= -1 \end{aligned}$$

   using Cramer's rule. [8]

   (d) Consider the system

$$\begin{aligned} x + 2y + 3z &= 1 \\ 2x + ky + 6z &= 6 \\ -x + 3y + (k-3)z &= 0 \end{aligned}$$

   For what values of k will it have no solution and unique solution. [10]

8. (a) Find the solution of $\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ of

$$\begin{pmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 13 \\ 4 \end{pmatrix}$$ using LU decomposition. [12]

(b) If $z = \cos\theta + i\sin\theta$ show that $z^n + \frac{1}{z^n} = 2\cos n\theta$. [8]

(c) Solve the following differential equations

i. $\frac{dy}{dx} + y + 4 = 0$ [5]

ii. $e^x dx + 6dy = 0$ [5]

(d) Find the area of the parallelogram with vectors $u = 2i - j + 2k$ and $v = 3i + 4j + k$ as sides. [4]

9. (a) If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, prove that $(z_1)(z_2) = r_1r_2\{\cos((\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\}$. [4]

(b) Prove the identities

i. $\sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta$. [5]

ii. $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$. [5]

(c) Find a complex number z which satisfy $z^3 = 4i$. [12]

(d) What is the motivation behind the LU decomposition method of solving systems of linear equations? What is the limitation of the method? [4]