BINDURA UNIVERSITY OF SCIENCE EDUCATION

MT210: ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

Time : 3 hours

JAN 2025

Answer ALL questions in Section A and at most TWO questions in section B.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1.

Define:

(a) A Differential Equation. [2]

(b) A linear Differential equation. [2]

(c) A partial Differential equation. [2]

(d) The order of a Differential equation. [2]

(e) An inhomogeneous Differential equation. [2]

A2. Consider the differential equation $y'' - 5y' + 6y = 0$

(a) Show that e^{2x} and e^{3x} are linearly independent solutions of this equation on the interval $-\infty < x < \infty$. [3]

(b) Write down the general solution of the given differential equation. [3]

(c) Find the solution that satisfies the conditions $Y(0) = 2$ and $Y'(0) = 3$. [3]

(d) Explain why the solution in (*c*) is unique and determine the interval over which it is defined. [3]

A3. Consider a point $x = x_0$ for a second order homogeneous differential equation $y'' + P(x)y' + Q(x)y = 0$

(a) Ordinary point. [2]

(b) Singular point. [2]

(c) Regular singular point. [2]

(d) Irregular (essential) singular point. [2]

A4. (a) Show that the function defined by $f(x) = (2x^2 + 2e^{3x} + 3)e^{-2x}$ satisfies the differential equation $y' + 2y = 6e^x + 4xe^{-2x}$ and the condition $f(0) = 5$. [5]

(b) Compute the Wronskin of the following functions from a fundamental set of solutions of the given homogeneous equations $cos(3x)$, $e^{3x}sin(3x)$, $sin(3x)$. [5]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B5 to B7.

B5. (a) Solve the differential equation $y'' + y = 0$ using the power series method. [15]

(b) Use the method of undetermined coefficients to solve $y' - 5y = x^2e^x - xe^{5x}$. [8]

(c) Find the eigenvalues and eigenfunctions of $y'' - 4\lambda y' + 4\lambda^2 y = 0$; $y(0) = 0$; $y'(1) + y(1) = 0$. [7]

B6. (a) Use the method of variation of parameters to solve $4y'' + 36y = 3cosec(x)$. [15]

(b) Find the Bessel function (solutions) of the differential equation $x^2y'' + xy' + (x^2 - p^2)y = 0$ where p is a parameter . [15]

B7. (a) Define the following terms;

(i) Gamma function. [2]

(ii) Bessel function of the first kind of order p. [2]

(b) Given that $\Gamma(1.5) = 0.8862$ and $\Gamma(p + 1) = p)$, find $\Gamma(3.5)$. [2]

(c) Find $L^{-1}\{\frac{1}{(s-1)^2}\}$ by the method of convolutions. [9]

(d) Use power series to solve the equation $y'' - 2xy' + y = 0$. [15]

END OF QUESTION PAPER