BINDURA UNIVERSITY OF SCIENCE EDUCATION

HONOURS DEGREE IN STATISTISTICS AND FINANCIAL MATHEMATICS (HBScSFM)

SFM113/ MT303: Probability Theory and Statistics 1

Time: 3 Hours

SECTION A (40 Marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

A1. Define the following expressions.

a. Sample space [2]
b. Independent events [2]
c. Random variable [2]
d. Monotone property of probability. [2]

A2. Show that the Poisson distribution with index λ has mean λ and variance λ. [10]

A3. Suppose X and Y are independent events, prove that

i. X' and Y' are independent. [4]
ii. $P(X'|Y) = P(X')$. [5]

A4. a. Let X and Y be events and let $X \subset Y$. Show that $P(X) < P(Y)$. [5]
b. State the two properties of the legitimacy of a probability mass function, $p(x)$. [2]

A5. A continuous random variable X, with mean unit, has probability density function $f_X(x)$ is given by:

$$f_X(x) = \begin{cases} a(b-x)^2 & 0 \leq x \leq b \\ 0 & otherwise \end{cases}$$

Find the value of a and b. [6]

SECTION B (60 Marks)

Candidates may attempt TWO questions being careful to number them B6 to B8.

B6. a. State and prove the Bayes theorem. [10]

b. In my home town, its rainy one third of the days. Given that it is rainy, there will be heavy traffic with probability 0.5, and given it is not rainy, there will be heavy traffic with probability 0.25. If its rainy and there is heavy traffic, I arrive late for work with probability 0.5. On the other hand, the probability of being late is reduced to 0.125 if it is not rainy and there is no heavy traffic. In the other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25. You pick a random day:

i. What is the probability that it's not raining and there is heavy traffic and am not late. [4]

ii. What is the probability that I am late? [4]

iii. Given that I arrive late at work, what is the probability that it rained that day? [5]

c. State and prove the memoryless property of the exponential distribution. [7]

B7. a. The joint probability mass function of two random variable X and Y is given by:

Y|X	2	3
1	$\frac{3k}{2}$	$\frac{4k}{3}$
2	k	$\frac{5k}{6}$
3	$\frac{5k}{6}$	$\frac{2k}{3}$

i. Find the value of k. [1]

ii. Hence evaluate $P(X > Y)$. [3]

iii. Find the marginal distribution function of X and Y. [3]

iv. Find the correlation of X and Y and comment. [8]

v. Are X and Y independent? [1]

b. Let X and Y have the joint probability density function,

$$f_{X,Y}(x, y) = kxy \qquad 0 < x < 4, \quad 1 < y < 5.$$

i. Find the value of the constant k. [3]

ii. Find $P(1 < x < 2, \ 2 < y < 3)$ [3]

iii. Find the marginal distribution of X and Y. [2, 2]

iv. Find the conditional distribution of Y given $X = x$. [4]

B8. a. Let X be a geometric distribution

$$p(x) = pq^{x-1} \quad x = 1, 2, \ldots \quad 0 < p < 1$$

i. Find the probability generating function of X. [7]

ii. Hence find the mean and the variance. [8]

iii. State and prove the Markov inequality. [7]

iv. Suppose the test score of a student is a random variable with mean 50 and variance 25. Give the lower bound to ta probability that a student score will be between 40 and 60. [4]

v. For events A and B, prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ [4]