Bindura University of Science Education

Faculty of Science Education

Science and Mathematics Education Department

Programme: HBSc Ed (Mathematics)

Course: MT320: Algebra   Duration: Three hours

Semester Examinations

Instructions to candidates

(i) Answer all questions in Section A and two questions from Section B.

(ii) Begin each question on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

A1. Define the following terms
- **(a)** Set [2]
- **(b)** Neutral element [2]
- (c) Equivalence relation [2]

A2. (a) Distinguish between a ring and a group [8]

(b) Give two examples of rings [2]

A3. Prove the following De Morgan's theorem

a) $(A \cup B)' = (A' \cap B')$ [5]

b) $(A \cap B)' = (A' \cup B')$ [5]

A4. Show that $x \equiv y\ (mod\ m)$ written $x - y\ (mod\ m)$ if $x - y$ is divisible by 9 is an equivalence relation. [8]

A5. Let $\mathbb{R}$ be a ring of all 2×2 matrices and then S be a subset of R such that $S = \left\{\begin{pmatrix} a & 0 \\ b & c \end{pmatrix}\right\}$ *where* $a, b, c, d \in \mathbb{R}$. Find the left and right ideal. [6]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B6 to B8

B6. (a) (i) Draw a Cayley table of $(Z_5, +)$ [5]

(ii) Is $(Z_5, +)$ a group? Give reasons. [4]

(iii) State the neutral element of $(Z_5, +)$ [2]

(b) Show that the inverse element of a group is unique. [5]

(c) Define a mapping and give two types of mappings. [4]

(d) If S is the set of all 2×2 matrices of the form $\begin{pmatrix} a & b \\ 2b & a \end{pmatrix}$ where $a, b \in \mathbb{R}$. Show that S forms a group under addition of matrices. [10]

B7. (a) Define the terms

(i) Homomorphism of a group [2]

(ii) Monomorphism of a group [2]

(iii) Isomorphism of a group [2]

(b) Let $(G, *)$ and $(H, +)$ be groups

(i) Define $(G \times H, \Delta)$ the direct product of G and H. [4]

(ii) Show that $(G \times H, \Delta)$ is a group [10]

(iii) Show that G and H are abelian if and only if $G \times H$ is abelian. [6]

(c) Prove that $\theta: R \to S$ is a ring homomorphism then $ker\theta$ is an ideal of R. [4]

B8. (a) Let G be any group and $H \leq G$. We say that x is congruent to y modulo H (written $x \equiv y \ (mod\ H)$) if $x^{-1}y \in H, where\ x, y \in G$. Prove that the congruency modulo H is an equivalence relation in G. [6]

(b) Suppose $\mathcal{J}\sqrt{2} = \{m + n\sqrt{2}: m, n \in \mathbb{Z}\}$, then show that $\mathcal{J}\sqrt{2}$ is a homomorphism of a ring [10]

(c) let K be a ring of all 2×2 matrices of the form $\begin{pmatrix} y & x \\ -x & y \end{pmatrix}$ $where\ x, y \in \mathbb{R}$ and we have a field of complex numbers. Define a mapping

$$\wp: \mathbb{C} \Rightarrow K$$
$$x + iy \Rightarrow \begin{pmatrix} y & x \\ -x & y \end{pmatrix}$$

Show that $\wp$ is an isomorphism. [10]

(d) Let $f: G \to H$ be a homomorphism then show that ker (f) is a subgroup of G [4]

END OF PAPER