Bindura University of Science Education

Faculty of Science Education

Department of Science & Mathematics Education

AUG 2023

Programme: HBSc Ed (Mathematics)

Course: MT314: Mechanics Duration: **Three** hours

Semester Examinations

Instructions to candidates

(i) Answer all questions in Section A and two questions from Section B.

(ii) Begin each question on a fresh page.

Section A [40 marks].

Answer all questions from this section being careful to number them **A1** to **A5.**

A1. The y-coordinate of a particle is given by $y = 4t^2 - 3t$ where y is in metres and t is in seconds. Also the particle has an acceleration in the $x-$direction given by $a_x = 12tms^{-2}$. If the velocity of the particle in the x −direction is $4ms^{-1}$ when $t = 0$, calculate, the magnitude of the velocity, $\vec{v}$, and the acceleration, $\vec{a}$, of the particle when $t = 2s$. **[10]**

A2. Prove that $|\vec{v}| = \sqrt{(\dot{r}^2 + (r\dot{\theta})^2)}$, where $\vec{v}$, denotes the velocity in (r, θ) form. **[8]**

A3. (a). If the scalar field $\Omega = xy^2z^3 - x^3y^2z$, find $grad\Omega$ at the point $(1, -1, 1)$. **[4]**

(b). A car P increases its speed at a constant rate of $1.2ms^{-2}$ as it rounds a curve whose radius of curvature is $200m$. If the magnitude of the total acceleration of the car is $1.3ms^{-2}$ determine the velocity of the car. **[6]**

A4. Prove that the projection of the acceleration on the normal to the path of a particle is equal to the second power of the numerical value of the velocity divided by the radius of curvature **[7]**

A5. Prove that the change in linear momentum of a particle in any given interval is equal to the geometric sum of all the forces acting on the particle during that time interval. **[5]**

Section B: [60 marks]

Answer **two** questions from this section being careful to number them **B6** to **B8.**

B6. (a). Define the following terms

(i) Inertia

(ii) Angular momentum

(iii) Rigid body **[6]**

(b). (i) Find the *curl* for the vector field $\vec{F} = (z, x, y)$. **[5]**

(ii) Find the divergence of the vector field $\vec{H} = (x^2, 3y, x^3)$. **[5]**

(iii) Find the directional derivative of $\psi = x^2 + y^2 - z$ at $P(1,1,-2)$ in the direction of $\vec{n} = (1,1,2)$. **[6]**

(c). The position vector of a particle of mass 2kg is given by

$r = 8ti + t^2\ j\ - \frac{1}{2}\ (t^3 - 1)k$ where t is time in seconds from the start of motion and where r is expressed in metres. For the condition when t = 2s, determine the power *P* developed by a force F = 20i – 16k which acts on the particle and the angular momentum of the particle. **[3,5]**

B7. (a). Distinguish between direct central impact and oblique central impact. **[5]**

(b). State and prove Carnot Theorem that characterize the loss of kinetic energy during collision of a system of bodies. **[15]**

(c). Define the terms;

(i). translational motion of a rigid body. **[2]**

(ii). rotational motion of a rigid body. **[2]**

(d). Prove that in translational motion of a rigid body all particles of the body move along similar paths and have at any instant the same velocity and the same acceleration. **[6]**

B8. (a). A $10 - kg$ block is moving to right with a velocity of $0.6m/s$ on a horizontal surface when a force Q is applied to it at time $t = 0s$. The variation of the force Q with time was that during the interval $0s$ to $2s$, $Q = 36N$ and for the interval $2s$ to $4s$, $Q = 72N$. The coefficient of kinetic friction, $u_k = 0.3$. Determine the velocity of the block when $t = 4s$. **[8]**

(b). (i). Define the term constrained motion. **[3]**

(ii). State the Fundamental Law of Dynamics of rigid bodies. **[3]**

(iii). A load of weight W starts moving from rest along a smooth horizontal plane under the action of a force of R. The magnitude of R increases proportionally with time, that is, $R = kt$. Develop the equation of motion for the load. **[10]**

(c). Prove that the moment about a fixed point O of all the forces acting on a mass M equals the rate of change of angular momentum. **[6]**

END OF PAPER