



BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE EDUCATION

SCIENCE & MATHEMATICS EDUCATION DEPARTMENT

DM007: VECTORS AND MATRICES

15 JUN 2023

TIME: 3 HOURS

### INSTRUCTIONS

Answer **TWO** questions from Section A and **TWO** questions from Section B

**SECTION A:** Answer any **two** questions from this section

1. (a) Find a Matrix **X** satisfying the matrix equation below:

$$3X + \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ -1 & -2 \end{bmatrix} \quad [10]$$

- (b) Use Cofactors to determine the inverse of the matrix below:

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -3 \\ 3 & 2 & 4 \end{bmatrix} \quad [15]$$

2. Use the row transformations of the matrix to solve the system of equations below:

$$2x + y + z = 8$$

$$x - 2y - 3z = 5$$

$$3x + 2y + 4z = 10$$

[25]

3. Use Cramer's rule to solve the system:

$$2x - y = 0$$

$$3x + z = 7$$

$$2x + 3z = 1$$

[25]

**SECTION B:**

Answer any **two** questions from this section. Each question carries 25 Marks

4.

Relative to a fixed origin  $O$ , the point  $A$  has position vector  $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$

and the point  $B$  has position vector  $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$

The line  $l_1$  passes through the points  $A$  and  $B$ .

(a) Find the vector  $\overrightarrow{AB}$ .

(b) Hence find a vector equation for the line  $l_1$

The point  $P$  has position vector  $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$

Given that angle  $PBA$  is  $\theta$ ,

(c) show that  $\cos \theta = \frac{1}{3}$

The line  $l_2$  passes through the point  $P$  and is parallel to the line  $l_1$

(d) Find a vector equation for the line  $l_2$

5.

Points  $A(2, 2, 5)$ ,  $B(1, -1, -4)$ ,  $C(3, 3, 10)$  and  $D(8, 6, 3)$  are the vertices of a pyramid with a triangular base.

- (i) Calculate the lengths  $AB$  and  $AC$ , and the angle  $BAC$ .
- (ii) Show that  $\overrightarrow{AD}$  is perpendicular to both  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$ .
- (iii) Calculate the volume of the pyramid  $ABCD$ .

[The volume of the pyramid is  $V = \frac{1}{3} \times \text{base area} \times \text{perpendicular height}$ .]

6.

Relative to a fixed origin  $O$ , the point  $A$  has position vector  $(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ , and the point  $B$  has position vector  $(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$ .

The line  $l$  passes through the points  $A$  and  $B$ .

- (a) Find the vector  $\overrightarrow{AB}$ .
- (b) Find a vector equation for the line  $l$ .

The point  $C$  has position vector  $(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$ .

The point  $P$  lies on  $l$ . Given that the vector  $\overrightarrow{CP}$  is perpendicular to  $l$ ,

- (c) find the position vector of the point  $P$ .

**END OF PAPER**