

54 AUG 2023

Time : 3 hours

Answer ALL questions in Section A and at most TWO questions in section B.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1.

Define:

- (a) A Differential Equation. [2]
- (b) A linear Differential equation. [2]
- (c) A partial Differential equation. [2]
- (d) the order of a Differential equation. [2]
- (e) An inhomogeneous Differential equation. [2]

A2. Consider the differential equation $y'' - 5y' + 6y = 0$

- (a) Show that e^{2x} and e^{3x} are linearly independent solutions of this equation on the interval $-\infty < x < \infty$. [3]
- (b) Write down the general solution of the given differential equation. [3]
- (c) Find the solution that satisfies the conditions $Y(0) = 2$ and $Y'(0) = 3$. [3]
- (d) Explain why the solution in (c) is unique and determine the interval over which it is defined. [3]

A3. consider a point $x = x_0$ for a second order homogeneous differential equation $y'' + P(x)y' + Q(x)y = 0$

- (a) Ordinary point. [2]
- (b) Singular point. [2]
- (c) Regular singular point. [2]
- (d) Irregular (essential) singular point. [2]

END OF QUESTION PAPER

- B7. (a) Define the following terms: [2]
 (i) Gamma function. [2]
 (ii) Bessel function of the first kind of order p . [2]
 (b) Given that $\Gamma(1.5) = 0.8862$ and $\Gamma(p+1) = p\Gamma(p)$, find $\Gamma(3.5)$. [2]
 (c) Find $L^{-1}\left\{\frac{1}{s-1}\right\}$ by the method of convolutions. [9]
 (d) Use power series to solve the equation $y'' - 2xy' + y = 0$. [15]

- B6. (a) Use the method of variation of parameters to solve $4y'' + 36y = 3\sec(x)$. [15]
 (b) Find the Bessel function (solutions) of the differential equation $x^2 y'' + xy' + (x^2 - p^2)y = 0$ where p is a parameter. [15]
 B5. (a) Solve the differential equation $y'' + y = 0$ using the power series method. [15]
 (b) Use the method of undetermined coefficients to solve $y' - 5y = x^2 e^x - xe^{5x}$. [8]
 (c) Find the eigenvalues and eigenfunctions of $y'' - 4\lambda y' + 4\lambda^2 y = 0$; $y(0) = 0$; $y'(1) + y(1) = 0$. [7]

Candidates may attempt TWO questions being careful to number them B5 to B7.

SECTION B (60 marks)

- A4. (a) Show that the function defined by $f(x) = (2x^2 + 2e^{3x} + 3)e^{-2x}$ satisfies the differential equation $y' + 2y = 6e^x + 4xe^{-2x}$ and the condition $f(0) = 5$. [5]
 (b) Compute the Wronskian of the following functions from a fundamental set of solutions of the given homogeneous equations $\cos(3x)$, $e^{3x}\sin(3x)$, $st(3x)$. [5]