BINDURA UNIVERSITY OF SCIENCE EDUCATION SFM222

HBSc.IN STATISTICS & FINANCIAL MATHEMATICS

RISK THEORY

OCT 2024

Time : 3 hours

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

A1. (a) Briefly explain the following theories;

(i) Utility theory,

(ii) Ruin theory,

(iii) Credibility theory. [3,3,3]

A2. State any four classes of utility functions and their domains. [4]

A3. If $S_1, S_2, ..., S_m$ are mutually independent random variables such that S_i has a compound Poisson distribution with parameter λ_i and density function of claim amount $P_i(x)$, $i = 1, 2, ..., m$. Show that

$$S = S_1 + S_2 + ... + S_m$$

has a Poisson distribution with $\lambda = \sum_{i=1}^{m} \lambda_i$ and

$$P(x) = \sum_{i=1}^{m} \frac{\lambda_i}{\lambda} P_i(x)$$

[6]

A4. The operation of 'convolution' calculates the distribution of X and Y from those of two independent variables X and Y. Show that for the continuous case. [6]

A5. (a) Let

$$u(x) = -e^{-\alpha x}, \quad \alpha > 0.$$

Show that $u(x)$ may serve as a utility function. [5]

(b) Let G be the amount that the insured is prepared to pay for complete financial protection. Find G. [4]

(c) Suppose $\alpha = 10$ and the decision maker has two random economic prospects available. One outcome denoted by A is $N(5, 2)$ and the other B is $N(6, 2.5)$. Which prospect will be preferred? [6]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B6 to B8.

B6. (a) Prove that the exponential distribution is memoryless. [6]

(b) The number of claims for an individual insurance policy in a policy period is modeled by the geometric distribution. The individual claim, when it occurs, is modeled by the exponential distribution with parameter λ (i.e. the mean individual claim amount is $\frac{1}{\lambda}$. Find the

(i) compound distribution function, F_Y, [3]

(ii) mean and the variance of the random sum Y, [2, 4]

(iii) moment generating function, $M_Y(t)$ and the cumulant functions, $\Phi_Y t$, [3, 3]

(iv) measure of skewness $\Phi_Y^{(3)}(0)$. [4]

(c) Suppose that S has a compound Poisson distribution with $\mu = 0.8$ and individual claim amounts are 1, 2, or 3 with probabilities 0.25, 0.375 and 0.375 respectively. Find the probability function and distribution function of aggregate claims for $x = 0, 1, ..., 6$. [5]

B7. (a) Suppose the distribution of the number of claims, N, is the negative binomial, that is,

$$P(N = n) = \left(\begin{array}{c} r+n-1 \\ n \end{array} \right) p^r q^n, \quad n = 0, 1, 2, ...$$

where $r > 0$, $0 < p < 1$ and $q = 1 - p$.

(i) Find the moment generating function of N, $M_N(t)$. [6]

(ii) Find the moment generating function of S, $M_S(t)$ in terms of $M_X(t)$. [4]

(b) Prove that ruin probability for $u \geq 0$ satisfies

$$\psi(u) = \frac{e^{-Ru}}{E[e^{-}RU(T)|T < \infty]}$$

[10]

(c) A decision makers utility of wealth function is given by

$$u(w) = w - 0.001w^2, \quad w < 50$$

The decision maker will retain wealth of amount w with probability p and suffer a financial loss of amount c with probability $1 - p$. Consider the values of w, c and p given in the table below:
Find the maximum insurance premium that the decision maker will pay for complete insurance and comment on the two premiums for the two different cases. [10]

Wealth(w)	Loss (c)	Probability (p)
15	10	0.5
20	10	0.5

B8. (a) Suppose that S has a compound Poisson distribution with $\lambda = 2$ and $p(x) = 0.1x,\ x = 1, 2, 3, 4$. Find the probabilities that aggregate claims equal 0,1,2,3 and 4. [10]

(b) The number of claims N in an policy is such than $N \sim Geometric(p)$, $o < p < 1$ while the claim amounts X follows an exponential with parameter 1. Find the distribution of S, the aggregate claims. [10]

(c) Derive the adjustment coefficient, $\widetilde{R}$ for the discrete time model, given that G_n is normally distributed with mean, μ and variance σ^2. [10]

END OF QUESTION PAPER