BINDURA UNIVERSITY OF SCIENCE EDUCATION

DEPARTMENT OF STATISTICS AND MATHEMATICS

MTE2101: ENGINEERING MATHEMATICS 3

Time: 3 hours

Candidates may attempt ALL questions in Section A and Section B. Each question should start on a fresh page.

SECTION A (40 marks)

1. Use variation of parameters to determine the specific solution for the following differential equation
$$x^2\frac{d^2y}{dx^2} - 7x\frac{dy}{dx} + 16y = 16\ln x,$$
given further that $y = \frac{1}{2}$, $\frac{dy}{dx} = 2$ at $x = 1$. [10]

2. Find the general solution of the following equation.
$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 15\sqrt{x}e^{2x}.$$
[12]

3. Find, as a series, a solution of the following differential equation
$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = e^{2x}$$
Give the final answer in simplified form up and including the term in x^8. [10]

4. Use the Leibniz rule to find a general solution, as an infinite series, for the following differential equation
$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2xy = 3,$$
subject to the boundary conditions $y = 0, \frac{dy}{dx} = 1$ at $x = 0$. [8]

SECTION B (60 marks)

5. The steady state temperature distribution $\Phi = \Phi(r, \theta)$ in a circular thin metal disc of radius 1, satisfies Laplace's Equation in plane polar coordinates

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} = 0.$$

Given further that

$$2\Phi(1, \theta) + \frac{\partial \Phi}{\partial r}(1, \theta) = 100 - 2\cos 2\theta$$

determine a simplified expression for $\Phi(r, \theta)$.
[You are expected to derive the general solution of the partial equation in variable separate form] [20]

6. It is given that $z = z(x, t)$ satisfies the waves equation

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{4}\frac{\partial^2 z}{\partial t^2},$$

subject to the initial conditions

$$z(x, 0) = e^{-x^2}, -\infty < x < \infty$$

and

$$\frac{\partial z}{\partial t}(x, 0)$$

   (a) Determine the solution of this wave equation.
   (b) Sketch the wave profiles for $t = i, i = 0, 1, 2, 3$.

You may use without proof the standard D'Alembert's solution for the wave equation. [20]

7. The temperature distribution $\theta(x, t)$ along a thin bar of length 2 m satisfies the partial differentiation

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{9}\frac{\partial \theta}{\partial t}, 0 \leq x \leq 2, t \geq 0$$

Initially the bar has a linear temperature distribution, with temperature 0 ° at one end of the bar where $x = 0$ m, and temperature 50° at the other end where $x = 2$m.

At time $t = 0$ the temperature is suddenly dropped to $0°$ at both ends of the rod and maintained at $0°$ for $t \geq 0$.
Assuming the rod is insulated along its length, determined an expression for $\theta(x, t)$ and hence show that
[You must derive the standard solution of the heat equation in variable separate form]

$$\theta(x,t) = \frac{100}{\pi}\sum_{n=1}^{\infty}\{\frac{(-1)^{n+1}}{n}\exp[-\frac{9n^2\pi^2 t}{4}]\sin[\frac{n\pi x}{2}]\}$$

[20]

End