BINDURA UNIVERSITY OF SCIENCE EDUCATION

MT109 : MATHEMATICS for Chemists

AUG 2023

Time : 3 hours

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A6.

A1. (a) Evaluate the following indicial expression, giving the final answers as an exact simplified fraction: $(36^{\frac{1}{2}} + 16^{\frac{1}{4}})^{\frac{2}{3}}$ [3]

(b) Solve the following logarithmic equation for x, $\log_a(x^2 - 18) - \log_a x = \log_a 3$. [4]

A2. Given that $z_1 = 7 + 6i$ and $z_2 = 8 - 3i$, find:

(a) $z_1 - z_2$ [1]

(b) $z_1 z_2$ [2]

(c) $\frac{z_1}{z_2}$ [2]

A3. Simplify the following:

(a) $(-4 + \sqrt{3}) + (5 + 2\sqrt{3})$ [2]

(b) $(2\sqrt{7} + \sqrt{5})(\sqrt{7} - 3\sqrt{5})$ [3]

(c) $\frac{3 - \sqrt{5}}{\sqrt{5} - 5}$ [3]

A4. (a) Find the remainder when $3x^3 - x^2 - 5x + 2$ is divided by $2x + 2$. [2]

(b) Given $x^2 + 2x - 3$ is a factor of $f(x)$, where $f(x) \equiv x^4 + 6x^3 + 2ax^2 + bx - 3a$. Find the value of a and of b. [4]

A5. (a) Find the set of values of x , that satisfy the following inequality.

$$\frac{5x}{x^2+4} < x$$ [4]

(b) Express $\dfrac{2x + x^2 - 1}{x(x^2-1)}$ into partial fractions. [5]

A6. Given a curve, $y = ax^2 + bx + c$, the curve passes through $(0, -4)$, its gradient at $x = -0.5$ is -2 and the second derivative is 10, find the constants a, b, and c. [5]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B7 to B9.

B7. (a) Find $\dfrac{dy}{dx}$ in each of the following:

(i) $y = 3x^3 + 36x - 504$. [2]

(ii) $y = 4Sin^2 2x$. [4]

(iii) $y = In(2x^2 + 1)$. [4]

(b) A curve is defined by the parametric equations:
$x = 120t - 4t^2$ and $y = 60t - 6t^2$.
Find the value of $\dfrac{dy}{dx}$ at each of the points where the curve crosses the $x - axis$. [9]

(c) Find the coordinates of the stationery points whose equation is $y = x^4 - 4x^3 + 27$ and determine their nature. [11]

B8. (a) Show that $\sin(75^0) = \frac{1+\sqrt{3}}{2\sqrt{2}}$ [5]

(b) Show that $\tan(A + B) = \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}$ [9]

(c) Use Taylor series to expand $sin(x + \dfrac{\Pi}{6})$ in ascending powers of x as far as the power of the term in x^4. [10]

(d) Using the binomial expansion, or otherwise, express $(1+2x)^4$ in the form $1+ax+bx^2 + 32x^3 + 16x^4$ where a and b are integers. [6]

B9. (a) Express the equation $5sin2x = 4cos2x$ in the form $tan2x = k$ where k is a constant. [2]

(b) Integrate the following expressions with respect to x:

(i) $3x^4 - 4x^{\frac{3}{4}} + 24$ [2]

(ii) $\frac{x^2+4}{x^2}$ [3]

(iii) $\frac{4}{2x+3}$ from 0 up to 3. [3]

(iv) x^2e^{4x} from 0 up to 1. [8]

(c) A curve has an equation $y = (4-x^2)^{-\frac{1}{2}}$ for $-1 \leq x \leq 1$. The region R is enclosed by $y = (4-x^2)^{-\frac{1}{2}}$, the x-axis and the line $x = -1$ and $x = 1$. Find the exact value of the area R. [5]

(d) The sum to infinity of a geometric series is three times the first term of the series. The first term of the series is a.

(i) Show that the common ratio of the geometric series is $\frac{3}{2}$ [2]

(ii) The third term of the geometric series is 81.

(a) Find the sixth term of the series. [2]

(b) Find the value of a as a fraction. [3]

END OF QUESTION PAPER