Bindura University of Science Education

Programmes: HBSc Ed (Mathematics)

Course: MT314: Mechanics Duration: Three hours

Semester Examinations

JAN 2025

Instructions to candidates

(i) Answer all questions in Section A and two questions from Section B.

(ii) Begin each question on a fresh page.

Section A [40 marks].

Answer all questions from this section being careful to number them **A1** to **A5.**

A1. The y-coordinate of a particle is given by $y = 4t^2 - 3t$ where y is in metres and t is in seconds. Also, the particle has an acceleration in the in the $x-$ direction is given by $a_x = 12tms^{-2}$. If the velocity of the particle in the $x-$direction is $4ms^{-1}$ when $t = 0$, calculate, the magnitude of the velocity, $\overline{v}$, and the acceleration, $\overline{a}$, of the particle when $t = 2s$. [10]

A2. Prove that $|\vec{v}| = \sqrt{(\dot{r}^2 + (r\dot{\theta})^2)}$, where $\overline{v}$, denotes the velocity in (r, θ) form. [8]

A3. Prove the change in linear momentum of a body in any given interval is equal to the geometric sum of the impulses of all the forces acting on the particle during that time interval. **[8]**

A4.(a) Define the terms:

(i). virtual displacement **[3]**

(ii). virtual work **[3]**

(b). State the necessary and sufficient condition for the equilibrium of a system of bodies subjected to ideal constraints. **[3]**

A5. Prove that the change in linear momentum of a particle in any given interval is equal to the geometric sum of all the forces acting on the particle during that time interval. **[5]**

Section B. [60 marks]

Answer **two** questions from this section being careful to number them **B5** to **B7.**

B5. (a). The y-coordinate of a particle is given by $y = 4t^2 - 3t$ where y is in metres and t is in seconds. Also the particle has an acceleration in the $x -$ direction given by $a_x = 12tms^{-2}$. If the velocity of the particle in the x –direction is $4ms^{-1}$ when $t = 0$, calculate, the magnitude of the velocity, $\vec{v}$, and the acceleration, $\vec{a}$, of the particle when $t = 2s$. **[10]**

(b). Distinguish between rotational motion and translational motion of a rigid body. **[4]**

(c). Prove that in translational motion of a rigid body all particles of the body move along similar paths and have at any instant the same velocity and acceleration. **[8]**

(d). The equation of motion of an accelerated wheel is $\theta = \frac{9}{32}t^3$. Determine the linear velocity and acceleration of a point lying at a distance $r = 0.8m$ from the axis of rotation at the instant when its tangential and normal accelerations are equal. **[8]**

B6. (a). Define the term angular momentum. **[3]**

(b). (i). Prove that the moment about a fixed point O of all the forces acting on a rigid body of mass M is equal to the time rate of change of angular momentum of M about O. **[8]**

(ii). A particle with a mass of $4kg$ has position vector given by $\vec{r} = 3t^2\,\boldsymbol{i} - 2t\boldsymbol{j} - 3t\boldsymbol{k}$, where $\vec{r}$ is in metres and t is in seconds. For $t = 3s$, determine the magnitude of the angular momentum of the particle and the moment of all the forces acting on the particle both about the origin of coordinates. **[10]**

(c). Prove that the path followed by a particle launched at angle α to the horizontal is parabolic. **[9]**

B7 (a). State and prove the parallel axis theorem. **[13]**

(b). Prove that the loss in kinetic energy of a system of bodies colliding in a perfectly inelastic collision is equal to the kinetic energy the system would have had if its bodies had moved with lost velocities. **[10]**

(c). The motion of a body in polar coordinates is given by: $\theta = 0.2t + 0.02t^3$ and $r = 0.2 + 0.04t^2$ where θ is in radians and r is in meters. Calculate the magnitude of the velocity at the instant when $t = 3s$. **[7]**

END OF PAPER