BINDURA UNIVERSITY OF SCIENCE EDUCATION

MT312: PARTIAL DIFFERENTIAL EQUATIONS AND FOURIER SERIES

Time : 3 hours

Answer ALL questions in Section A and at most TWO questions in section B.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1. (a) Define a partial differential equation. [2]

(b) Define the order of a partial differential equation. [2]

(c) Find $u(x, y)$ when $\frac{\partial u}{\partial y} = xy$. [3]

(d) Classify the partial differential equation $u_{xx} + 2yu_{xy} + xu_{yy} - ux = 0$ [4]

A2. (a) Find the general solution of the partial differential equation $\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x} = y$. [5]

(b) Find the equation of the characteristic curve for the partial differential equation $u_x + yu_y = 0$. [4]

A3. Given the partial differential equation $4\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$

(a) Classify the partial differential equation. [2]

(b) Apply the method of characteristic curves to reduce the partial differential equation to $\frac{\partial^2 u}{\partial w^2} = 0$. [8]

(c) Find the general solution of $\frac{\partial^2 u}{\partial w^2} = 0$. [5]

A4. (a) Define the term Fourier Series. [3]

(b) State without proof the Bessel inequality. [2]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B5 to B7.

B5. (a) Find the Fourier transform of

$$f(x) = \begin{cases} \frac{1}{2k}; |x| \leq k \\ 0; |x| > k, k > 0 \end{cases}$$

[8]

(b) Find the fourier half-range sine series. [8]

(c) The fourier series of $f(x) = x^2, -\pi \leq x \leq \pi, f(x + 2\pi) = f(x)$ is given by $S(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^2}{n^2} cos(nx)$. Use Parseval's identity to show that $\sum_{n=1}^{\infty} (\frac{1}{n^4}) = \frac{\pi^4}{90}$. [8]

(d) Consider the function $f(t) = cos(2t)$, find the period $f(t)$. [6]

B6. (a) Verify $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin(mwt)cos(nwt)dt = 0$ for all m and n. [15]

(b) Solve $y^2 u_{xx} - u_{yy} - 4y^2 u_x + (4y + \frac{1}{y})u_y = 0, (y \neq 0)$. [15]

B7. (a) Consider the set of function, $\phi(x) = \sqrt{\frac{2}{\pi}} sin(nx), n = 1, 2, ...$ Is ϕ_n an orthonormal set in the interval $0 \leq x \leq \pi$. [15]

(b) Solve the heat flow equation $\frac{\partial u}{\partial t} = \frac{a^2 \partial^2 u}{\partial x^2}, x > 0, t > 0$ given that $u(x, 0) = 0$.

$$u(0, t) = \begin{cases} 100; 0 < t < t_0 \\ 0; t > t_0 \end{cases}$$

[15]

END OF QUESTION PAPER