BINDURA UNIVERSITY OF SCIENCE EDUCATION SFM222

HBSc. IN STATISTICS & FINANCIAL MATHEMATICS

RISK THEORY

Time : 3 hours

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

A1. Define the following terms:

(a) Risk reserve, [2]

(b) safety loading, [2]

(c) ruin time, [2]

(d) reinsurance. [2]

A2. State and prove Jensens inequality. [6]

A3. Suppose that airplanes arrive at an airport following a Poisson process with parameter λ, and the number of passengers in each airplane follows a certain distribution. Then, the number of passengers arriving at the airport follows a compound Poisson process $\{Y(s), s \geq 0\}$, following

$$Y(s) = \Sigma_{i=1}^{N(s)} D_i,$$

where $\{N(s), S \geq 0\}$ is the Poisson process that the airplanes follow, and D_i is the distribution that the passengers follow. Find

(a) $E(Y)$, [3]

(b) $Var(Y)$, [4]

(c) $M_Y(t)$. [4]

A4. Distinguish the following:

(a) Compound and Mixed models. [4]

(b) individual and Collective models [3]

A5. Let N be a discrete random variable that gives the probability of n events occurring in s seconds. The occurrence of these events follows the exponential distribution. Prove that

$$P(N(s) = n) = \frac{e^{-\lambda s}(\lambda s)^n}{n!}$$

[8]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B6 to B8.

B6. (a) The total claims paid in a year can be 0, 5, 10, or 20, with respective probabilities 0.5, 0.25, 0.2, and 0.05. An annual premium of 6 is collected at the beginning of each year. Interest of 10% is earned on any funds available at the beginning of the year (which includes the premium collected at that time). Claims are paid at the end of the year. Calculate the ruin probability $\psi(3, 3)$ exactly. [20]

(b) Assume that the distribution of claim amount X_i, $i = 1, 2, ...$, has the density function

$$f_{X_i}(x) = \frac{1}{3}e^{-3x} + \frac{16}{3}e^{-6x},$$

$x > 0$. Given that $c = 1$ and $\lambda = 3$, evaluate μ, θ and k. [3,4,3]

B7. (a) Prove that the probability of ruin is expressed by means of the adjustment coefficient as

$$\psi(u) = \frac{e^{-Ru}}{E[e^{-RU(T)|T<\infty}]},$$

for $u \geq 0$. [10]

(b) Let τ_i be independently and identically distributed random variables with the exponential distribution. Take

$$T_n = \tau_1 + \tau_2 + ... + \tau_n.$$

Prove that T_n has the gamma distribution. [10]

(c) For a fixed fair net premium P prove that the stop loss reinsurance yields the smallest variance $Var(R(X))$ of all reinsurance policies. [10]

B8. (a) A die randomly selected from a pair of fair, six-sided dice, A and B. Die A has its faces marked with 1, 2, 3, 4, 5, and 6. Die B has its faces marked with 6, 7, 8, 9, 10, 11. The selected die is rolled four times. The results of the first three rolls are 1,2, and 3. Determine the Buhlmann credibility estimate of the expected value of the result of the fourth roll. [10]

(b) Loss size for an insurance coverage follows a two parameter Pareto distribution with parameters $\alpha = 3$ and θ. θ Varies by insured according to an exponential distribution. An insured has 4 losses in the first year with average size 1,000 and 8 losses in the second year with average size 1,300. The resulting Buhlmann-Straub estimate of average loss size for this insured is 1,100. Determine the mean of the exponential distribution. [10]

(c) Given a utility function $u(x)$, how can we approximate the maximum premium P^+ for a risk X? [10]

END OF QUESTION PAPER