

BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE EDUCATION

Diploma in Science Education Part 1.1

DM001: Algebra Duration 3 hours

Semester Examinations

OCT 2023

INSTRUCTIONS

Answer all questions in Section A and any **two** questions from Section B

Section A: (40 marks)

A1 Express (a) $\frac{2x-3}{x^3+10x}$, [6]

(b) $\frac{6x}{x^2-4}$ in partial fractions [4]

A2 (a). Given that a geometric sequence has third term 21.6 and the sum to infinity is 150, find the common ratio and the first term a . [6]

(b). Evaluate $\sum_{r=1}^{20} \ln 2^r$. [4]

A3. (a) Write down in logarithmic form:

(i) $10^{-3} = 0.001$ (ii). $3^p = r$ [4]

(b). Solve the equation $3^{4x} - 3^{2x+1} + 10 = 0$ [6]

A4. (a) Determine the nature of roots of the equation: $3x^2 = 5x - 2$. [4]

(b) Find a quadratic equation whose roots are: $-2 \pm \sqrt{5}$. [6]

Section B [60 marks]

Answer **two** questions from this section being careful to number them B5 to B7.

B5. (a) (i). Show that $a^3 - b^3 \equiv (a - b)(a^2 + ab + b^2)$. [5]

(ii). Hence, factorize completely the expression: $8 - 27x^3$ [5]

(b). Solve the inequality $\frac{4-x}{x+2} > 0$ [5]

(c). Solve the equation: $\sqrt{(2x-1)} = x - 2$. [7]

(d). (i). Express $3\log x - \frac{1}{2}\log y$ as a single logarithm. [3]

(ii). Show that $\log_a b \times \log_b c = \log_a c$. [5]

B6. (a) The roots of the equation $9x^2 + 6x + 1 = 4kx$ where k is a constant are denoted by α and β .

(i) Show that the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is $x^2 + 6x + 9 = 4kx$. [10]

(ii) Find the set of values of k for which α and β are real. [10]

(b). Find the range of values of x for which $(x - 4) < x(x - 4) \leq 5$. [10]

B7. (a). (i) Determine the greatest or least value of the function $g(x) = 2x^2 + 5x + 1$ [7]

(ii). Hence, draw a sketch of the graph of $g(x)$. [3]

(b). Show that $\log_a x + \log_a y = \log_a xy$. [5]

(c) (i). Prove the identity $\log_a b \equiv \frac{1}{\log_b a}$. [5]

(ii). Hence, solve the equation $\log_2 x + \log_x 2 = 2$. [6]

(d) Solve the inequality $|3x - 2| > 4$. [4]