Bindura University of Science Education

Faculty of Science Education 5 - AUG 2023

Department of Science & Mathematics Education

Programmes: HBSc Ed (Mathematics)

Course: MT207: Analysis Duration: **3** hours

Semester Examinations

Instructions to candidates

(i) Answer all questions in Section A and two questions from Section B.

(ii) Begin each question on a fresh page.

Section A [40 marks].

Answer all questions from this section being careful to number them **A1** to **A4.**

A1. (a) Prove that a convergent sequence of real numbers has a unique limit. **[7]**

(b). Find the least upper bound and greatest lower bound for the set.

$$A = \left\{ x \in \mathbb{R}: \frac{\sqrt{x+2,}}{x},\ x \geq 2 \right\}.$$

[4]

(c) Prove that the multiplicative inverse of a non-zero element of a field $\mathbb{R}$ is unique **[5]**

A2. Let . Prove that $\big||x| - |y|\big| \leq |x \pm y| \leq |x| + |y|$ for any $x, y \in \mathbb{R}$. **[9]**

A3. Prove that $\lim_{x \to -2} \frac{x^2-4}{x+2} = -4$. **[7]**

A4.(i) State the axiom of completeness . **[3]**

(ii). Show that the function $f(x) = \sqrt{x+2}$ is differentiable. **[5]**

Section B:[60 marks]

Answer **two** questions from this section being careful to number them **B5** to **B7.**

B5.(a). Define the terms.

(i). monotone sequence. **[2]**

(ii). Cauchy sequence. **[2]**

(b) A sequence (a_n) of real numbers is defined by $a_1 = 1$ and $a_{n+1} = \frac{2a_n+7}{5}$.

(i). Prove that (a_n) is a bounded monotone increasing sequence. **[7]**

(ii). Hence, determine its limit. [3]

(c) . Prove that $f(t) = t^2$ is uniformly continuous on $[2, 4]$. **[8]**

(d). Prove that there exists a real number x such that x^2=2. **[8]**

B6. (a). Define a cut in $\mathbb{R}$. **[3]**

(b). Prove that if an ordered pair (A, B) of non-empty subsets of $\mathbb{R}$ form a cut in $\mathbb{R}$, then there is a unique element ε that satisfies: $a \leq \varepsilon, \forall a \in A$ and $\varepsilon \leq \forall b \in B$. **[15]**

(c). State and prove the nested cells property in $\mathbb{R}$ **[12]**

B7. (a). State and prove the Mean Value Theorem **[10]**

(b). Define the following terms:

(i). a partition P of an interval $[a, b]$, **[3]**

(ii). a lower and upper Riemann sum of a function f with respect to the partition P **[4]**

(c). Prove that $\int_0^1 t^2 dt$ is a Riemann integrable function **[6]**

(d). Show that $\sqrt{2} - \sqrt{7}$ cannot be rational. **[7]**

END OF PAPER