BINDURA UNIVERSITY OF SCIENCE EDUCATION

DEPARTMENT OF STATISTICS AND MATHEMATICS MT109

MATHEMATICS FOR CHEMISTS

JUN 2025

Total marks: 100

Time: 3 Hours

Candidates should attempt **ALL** questions in section A and **TWO** questions in section B. Marks will be allocated as indicated.

SECTION A (40 marks)

A1. (a) The polynomial $ax^3 + x^2 + bx + 3$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $2x - 1$ and that when $p(x)$ is divided by $x + 2$, the remainder is 5.

Find the values of a and b. [5]

(b) Let $f(x) = -2x^2 + 1, g(x) = x^2$ and $h(x) = 2 - 3x$. Find $f \text{ o } g \text{ o } h(x)$. [3]

A2. (a) The equation of a curve is $y = \cos^3 x \sqrt{\sin x}$. It is given that the curve has one stationary point in the interval $0 < x < \frac{1}{2}\pi$. Find the x −coordinate of this stationary point, giving your answer correct to 3 significant figures. [5]

(b) The equation of a curve is $x^3 + 3x^2y - y^3 = 3$.

(i) Show that $\frac{dy}{dx} = \frac{x^2+2xy}{y^2-x^2}$. [5]

(ii) Find the coordinates of the points on the curve where the tangent is parallel to the x −axis. [5]

A3. The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = xe^{y-x}$$

and $y = 0$ when $x = 0$.

(a) Solve the differential equation, obtaining an expression for y in terms of x. [7]

(b) Find the value of y when $x = 1$, giving your answer in the form $a - \ln b$, where a and b are integers. [2]

A4. Express $\frac{x^3-18x-21}{(x+2)(x-5)(x^2-1)}$ into partial fractions. [5]

A5. Solve the equation $\ln e^{2x} + 3 = 2x + \ln 3$, giving your answer correct to 3 decimal places. [3]

SECTION B (60 marks)

Candidates may attempt **TWO** questions being careful to number them B6 to B8.

B6. (a) The parametric equations of a curve are $x = \frac{1}{\cos t}, y = \ln \tan t$, where $0 < t < \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = \frac{\cos t}{\sin^2 t}$. [4]

(ii) Find the equation of the tangent to the curve at the point where $y = 0$. [4]

(b) The constant p is such that $\int_1^p x^2 \ln x \, dx = 4$. Show that $p = \left(\frac{35}{3\ln p - 1}\right)^{\frac{1}{3}}$. [5]

(c) The coefficient of x^3 in the expansion of $(p + \frac{1}{p}x)^4$ is 144.

Find the possible values of the constant p. [5]

(d) (i) Solve the equation $6\sqrt{y} + \frac{2}{\sqrt{y}} - 7 = 0$. [4]

(ii) Hence, solve the equation $6\sqrt{\tan x} + \frac{2}{\sqrt{\tan x}} - 7 = 0$ for $0^0 \leq x \leq 360^0$. [4]

(e) Given that $x^2 - 4xy - 2y^2 = 12$, find $\frac{d^2y}{dx^2}$ at $(2, -2)$. [4]

B7. (a) A curve is such that the gradient at a general point with coordinates (x, y) is proportional to $\frac{y}{\sqrt{x+1}}$. The curve passes through the points with coordinates $(0,1)$ and $(3, e)$. By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x. [9]

(b) Let $f(x) = \frac{14-3x+2x^2}{(2+x)(3+x^2)}$

(i) Express f(x) into partial fractions. [5]

(ii) Hence, obtain the expansion of $f(x)$ in ascending of x, up to and including the term in x^2. [5]

(c) A line with equation $y = mx - 6$ is a tangent to the curve with equation $y = x^2 - 4x + 3$. Find the possible values of the constant m, and the corresponding coordinates of the points at which the line touches the curve. [6]

(d) Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$

$$g(x) = x^2 + 4x + 13.$$

By first expressing each of $f(x)$ and $g(x)$ in completed square form, express $g(x)$ in the form $f(x+p) + q$, where p and q are constants. [5]

A8. (a) (i) A geometric progression is such that the second term is equal to 24% of the sum to infinity. Find the possible values of the common ratio. [5]

(ii) The first, second and third terms of an arithmetic progression are $a, 2a$ and a^2 respectively, where a is a positive constant. Find the sum of the first 50 terms of the progression. [6]

(b) The solutions of the equation $5|x| = 5 - 2x$ are $x = a$ and $x = b$, where $a < b$. Find the value of $|3a - 1| + |7b - 1|$. [5]

(c) Evaluate $g^{-1}(y)$ given that $g(y) = \frac{-3y+5}{12-7y}$. Hence, determine $g^{-1}(3)$. [6]

(d) Determine the stationary points on the curve $y = 4x^3 + 9x^2 - 30x + 10$ and determine the nature of each stationary point. [8]