BINDURA UNIVERSITY OF SCIENCE EDUCATION

MT011: PURE MATHEMATICS 4

Time : 2 hours

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1. (a) If $a = 3 + 4i$ and $b = 12 - 5i$, find the modulus and argument of $\frac{b}{a}$. [4]

(b) Express the following complex number in trigonometric form:$-1 + \sqrt{3}i$. [4]

(c) Simplify $\frac{Cos5\theta + isin5\theta}{Cos2\theta - isin2\theta}$ [3]

(d) Show that $(Z, *)$ is not a group where $*$ is the operation defined by $n * m = nm - n - m + 1$. [5]

A2. (a) Using a method based on De Moivre's Theorem, prove the identity that

$$Cos3\theta \equiv 4Cos^3\theta - 3Cos\theta.$$

[6]

(b) Prove by induction that $\sum_{r=1}^{n} r = \frac{n}{2}(n+1)$ for all positive integral values of n. [5]

(c) State the three conditions that an equivalence relation satisfies. [3]

A3. Use Cramers rule to solve:

$x + y = 3$

$x - y = 3i.$ [5]

A4. Solve the differential equation $\frac{dy}{dx} = xy$ given that $x = 0$ and $y = 1$. [5]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B5 to B7.

B5. (a) Write out a group table for $Z_2 \times Z_2$. To which familiar group is it isomorphic? [10]

(b) Given that $Z_1 = 2 - 3i$ and $z_2 = -2 - i$, find:

(i) $|z_1 - z_2|$ [3]

(ii) $arg(z_1 + z_2)$ [4]

(c) If $z = \cos\theta + i\sin\theta$, show that

$$\sin^7\theta = \frac{1}{64}(35\sin\theta - 21\sin 3\theta + 7\sin 5\theta - \sin 7\theta).$$ [7]

(d) Show that $(P \to Q) \wedge (Q \to P)$ is NOT a tautology. [6]

B6. (a) Prove by induction that $9^n - 1$ is divisible by 8 for all positive integral values of n. [9]

(b) Find the particular solution of the following differential equation:

$$(1 - x^2)\frac{dy}{dx} - xy + 1 = 0$$

given that $y = \frac{\pi}{2}$ when $x = 0$. [10]

(c) Solve the following second order differential equation,

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = Cos2x,$$

given that $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$ [11]

B7. (a) Find the value of x for which $\begin{pmatrix} 2 & x \\ 5 & 3 \end{pmatrix}$ has no inverse. [3]

(b) For the matrices $A = \begin{pmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$, verify that$(AB)^t = B^tA^t$. [6]

(c) Let $A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 1 & 2 & 5 \end{pmatrix}$ find the adjoint of matrix A. [15]

(d) Hence or otherwise find A^{-1}, the inverse of matrix A. [6]

END OF QUESTION PAPER