BINDURA UNIVERSITY OF SCIENCE EDUCATION

MT507: ADVANCED NUMERICAL METHODS

Time : 3 hours

AUG 2024

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

Q1. (a) Give and explain the 'Intermediate Value Theorem' [2]

(b) Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in $[1, 2]$ and use the Bisection method to find the approximate root. [8]

(c) Suppose a continuous function f and its derivative f' have values given in the table:

x	2.0	2.5	3.0
f'(x)	0.4	0.6	0.8
f(x)	5		

Given that $f(2) = 5$, use Euler method with two steps of 0.5 to approximate the value of $f(3)$. [5]

(d) Show that the error from Newton's method can be given by

$$\varepsilon_{n+1} = \frac{-f''(\xi)e_n^2}{2f'(x_n)}$$

, where $\xi\varepsilon(x_n, x_{n+1})$ [5]

(e) Find the least squares solution to the system of linear equations:

$$-x + 2y = 4$$

$$2x - 3y = 1$$

$$-x + 3y = 2$$

[5]

Q2. (a) Apply Gauss Elimination with scaled partial pivoting to solve:

$$\begin{aligned} x_1 - x_2 + 2x_3 + x_4 &= 1 \\ 3x_1 + 2x_2 + x_3 + 4x_4 &= 1 \\ 5x_1 - 8x_2 + 6x_3 + 3x_4 &= 1 \\ 4x_1 + 2x_2 + 5x_3 + 3x_4 &= -1 \ . \end{aligned}$$

using 4-digit arithmetic with rounding. [15]

(b) Solve Equations

$$x + y + 4z = 6$$
$$5x - y + 2z = 12$$
$$3x + 8y - 2z = -25$$

using Gauss Seidel method up to the eighth iteration. [10]

Q3. (a) Use the Relaxation Method to solve the following system of linear equations:

$$9x - 2y + z = 50$$
$$x + 5y - 3z = 18$$
$$-2x + 2y + 7z = 19$$

[15]]

(b) If the Trapezoidal Rule is used to estimate $\int_0^2 e^{-x^2} dx$ with an error of at most 5.0×10^{-6}, how many points should be used? [10]

Q4. (a) Find $y(0.2)$ for $y = \frac{x-y}{2}$, $y(0) = 1$, with step length 0.1 using Runge-Kutta 4 method. [8]

(b) By doing seven iterations, use the Gauss Jacobi method to find the approximate solutions to the system of linear equations:

$$x + 7y + 2z = -17$$
$$2x + y - 4z = -3$$
$$4x + 2y + z = 3$$

Initial $(x, y, z) = (1, 1, 0)$ [17]

Q5. (a) Solve the second order differential equation using 4 equal nodes:

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 0$$

, for $y(2) = 0.008$ and $y(6.5) = 0.003$. [12]

(b) Solve Equations

$$x + y + z = 3$$
$$2x - y - z = 3$$
$$x - y + z = 9$$

using LU decomposition using Gauss Elimination method. [13]

END OF QUESTION PAPER