Bindura University of Science Education

Faculty of Science Education

Department of Science and Mathematics Education

Programme: Diploma in Science Education (Mathematics & Geography)

Course: DM008: Numerical Methods Duration: Three hours

Semester Examinations

JUN 2024

Instructions to candidates

(i) Answer all questions in Section A and two questions from Section B.

(ii) Begin each question on a fresh page.

Section A [40 marks].

Answer all questions from this section being careful to number them **A1** to **A4.**

A1. (a) Expand $ln(2-x)$ each of the following functions as a series of ascending powers of x up to and including the term in x^4. **[5]**

(b). (i) Show graphically that the equation $18lnx + x^2 = 0$ has only one real root. **[3]**

(ii) Verify that this root lies between 0.5 and 1. **[2]**

A2. (a) Show that the equation $x^3 - 5x = 3$ has root between 2 and 3. **[3]**

(b). Use linear interpolation once to find an approximation to this root to 3 decimal places.**[7]**

A3. (a). Establish that the equation $e^x = 20sinx$ has a root, $\alpha \in (2, 3)$. **[3]**

(b). Given that the root, α, to the equation $e^x = 20sinx$ is close to 2.3, use Newton-Raphson method once to find an approximation to α correct to 3 decimal places. **[7]**

A4. By substituting 0.08 for x in $(1+x)^{\frac{1}{2}}$ and its expansion find $\sqrt{3}$ to 4 significant figures. **[10]**

Section B:[60 marks]

Answer **two** questions from this section being careful to number them **B5** to **B7.**

B5. (a) Express $3cosxsinx$ as a series of ascending powers of x up to and including the term in x^4. **[6]**

(b) The variables x and y are related by the law: $ay = b^x$. The following table gives the set of values for x and y.

x	5	6	7	8
y	1.07	2.13	4.27	8.53

(i). Express this relationship in linear form. **[4]**

(ii). Hence, by drawing a straight line graph find approximate values of a and b. **[6]**

(c). Use the expansion of $ln\left(\frac{1+x}{1-x}\right)$, with $x = \frac{1}{3}$ to find $ln2$ correct to 3 decimal places. **[8]**

(d). Given that $\theta = \frac{3\pi}{8}$ radians is an approximation for the root of the equation $\theta = tan\theta - \theta$, use the Newton-Raphson Method to obtain a second approximation giving your answer correct to 3 decimal places. **[6]**

B6 (a) (i) Use Simpson's rule with 7 ordinates to find an estimate for the value of $\int_0^{0.6} xe^x\,dx$. **[7]**

(ii) Find by analytic means the exact value of $\int_0^{0.6} xe^x\,dx$. **[5]**

(iii) Hence, determine the absolute error that resulted from use of Simpson's rule. **[4]**

(b) If x is small enough that terms involving x^5 and higher powers of x can be ignored, use the Binomial Theorem to show that $\frac{1}{\sqrt{1+x^2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^4$. **[6]**

Hence, show that the approximate value of the integral $\int_0^{0.1} \frac{1}{\sqrt{1+x^2}}dx = 0.0998$. **[8]**

B7 (a). If x_1 is the first approximation of the root of the equation $f(x) = 0$ and x_2 is the second approximation of the root:

(i) State the algebraic connection between x_1 and x_2. **[2]**

(ii) Hence, show that $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$. [4]

(iii) Apply the Newton-Raphson method to find the root of the equation $x + \frac{4}{x^2} - 1 = 0$, with $x_0 = -1$, giving your answer to 3 significant figures. [7]

(b) (i) Show that if x is a fixed point of the iteration, $x_{n+1} = \sqrt{3x_n + 2}$, then x satisfies the equation: $x^2 - 3x - 2 = 0$. [2]

(ii) Perform 4 iterations of for $x_{n+1} = \sqrt{3x_n + 2}$, using $x_0 = 1$ to obtain x_4. [4]

(c). (i) Use linear interpolation to find the root of the equation $e^x = 3x + 1$ to 3 decimal places. [5]

(ii) It is known that x and y are relaed by the law $ae^y = x^2 - bx$. Explain how you would reduce the relation to the form $Y = mX + c$. [6]

END OF PAPER